GR Boson stars

Dae-Il Choi
Laboratory for High Energy Astrophysics, NASA Goddard Space Flight Center, Green belt, MD, 20771
Universities Space Research Association, 7501 Forbes Boulevard, #206, Seabrook, MD 20706

Kevin Lai
University of British Columbia
(Dated: July 17, 2003)

PACS numbers:

Introduction – Would like to study several problems with boson stars in axisymmetry.

Formulation – The Lagrangian for Einstein-Klein-Gordon system is

$$L = \frac{R}{16\pi} - \frac{1}{2} g^{\mu\nu} \phi^*_{;\mu} \phi_{;\nu} - \frac{1}{2} m^2 |\phi|^2 - \frac{1}{4} \lambda |\phi|^4 \tag{1}$$

where m is the mass of the bosonic particle and λ is the strength of the self-interaction.

The energy-mometum tensor is given by

$$T^\mu_\nu = \frac{1}{2} g^{\mu\sigma} (\phi^*_{;\sigma} \phi_{;\nu} + \phi_{;\sigma} \phi^*_{;\nu}) - \frac{1}{2} \delta^\mu_\nu (g^{\lambda\sigma} \phi^*_{;\lambda} \phi_{;\sigma} + m^2 |\phi|^2 + \frac{1}{2} \lambda |\phi|^4) \tag{2}$$

Initial data – Here we are interested in setting up ground state (stable) boson star configurations with a given angular momentum (zero or non-zero). I will describe non-rotating stars only here. Kevin will have a detailed description for rotating boson stars.

For non-rotating stars, I am interested in stationary solutions in spherically symmetry.

The spacetime line element in its most general format in spherical symmetry is given by

$$ds^2 = (-\alpha^2 + a^2\beta^2) dt^2 + 2a^2 \beta dt dr + a^2 dr^2 + r^2 b^2 d\Omega^2 \tag{3}$$

Now, I will make the following choices:

$$a = b (= \psi^2) \tag{4}$$

$$\beta = 0 \tag{5}$$

Then, we get

$$ds^2 = -\alpha^2 dt^2 + a^2 (dr^2 + r^2 d\Omega^2) ds^2 = -\alpha^2 dt^2 + \psi^4 (dr^2 + r^2 d\Omega^2) \tag{6}$$

Since we are looking for stationary solutions, complex scalar fields have the form:

$$\phi = \phi_0(r) e^{-i\omega t} \tag{7}$$

I will also use Maximal slicing condition and assume time-symmetry, i.e., $K_{ij} = 0$.

Thus, I also a coupled system of equations which consist of the Hamiltonian constraint equation ($G_{tt} = 8\pi T_{tt}$), Maximal slicing condition, and Klein-Gordon equation.

These equations are given as follows.

Hamiltonian constraint equation (HCE):

$$G_{00} = -4 \frac{\alpha^2}{\psi^5 r} (r \psi_{,rr} + 2\psi_{,r}) \tag{8}$$

$$T_{00} = \frac{1}{2} [\omega^2 \phi^2 + \alpha^2 (\psi^{-4} \phi^2_{,r} + m^2 \phi^2 + \frac{\lambda}{2} \phi^4)] \tag{9}$$

Therefore, HCE is given by

$$\psi_{,rr} + \frac{2}{r} \psi_{,r} = -\pi \psi^5 (\frac{\omega^2 \phi^2}{\alpha^2} + \psi^{-4} \phi^2_{,r} + m^2 \phi^2 + \frac{\lambda}{2} \phi^4) \tag{10}$$

Maximal slicing condition is from the evolution equation, $K_{,t} = 0$, and it gives rise to an elliptic equation for lapse function, α.

$$\alpha\Big|_i^i = \alpha(K_{ij}K^{ij} + 4\pi(S + \rho)) \tag{11}$$

Each term in the maximal slicing condition is:

$$\alpha\Big|_i^i = \frac{1}{\psi^6 r^2}(r^2\psi^2\alpha^{'})^{'} \tag{12}$$

$$\rho = \frac{1}{2}(\frac{\omega^2}{\alpha^2}\phi^2 + \frac{1}{\psi^4}(\phi_{,r})^2 + m^2\phi^2 + \frac{\lambda}{2}\phi^4) \tag{13}$$

$$S = tr(S_{ij}) = \frac{3}{2}(\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 - \frac{1}{2}\frac{1}{\psi^4}(\phi_{,r})^2 \tag{14}$$

$$S + \rho = (2\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 \tag{15}$$

Therefore, Maximal slicing condition is

$$\frac{1}{\psi^6 r^2}(r^2\psi^2\alpha^{'})^{'} = 4\pi\alpha(2\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 \tag{16}$$

Finally Klein-Gordon equation with a given metric of the form, eqn (6), is given by

$$(\alpha r\psi^2)\partial_{rr}\phi + (2\alpha\psi^2 + \alpha_{,r}r\psi^2 + 2\alpha r\psi\psi_{,r})\phi_{,r} = \alpha\psi^6 r(m^2 - \frac{\omega^2}{\alpha^2} + \lambda|\phi|^2)\phi \tag{17}$$

Finally combining equations, (10,16,17), we get,

$$\psi^{'} = \Psi \tag{18}$$

$$\Psi^{'} = -\frac{2}{r}\Psi - C\pi(\psi\Phi^2 + \psi^5(\frac{\omega^2}{\alpha^2} + m^2 + \frac{\lambda}{2}\phi^2)\phi^2) \tag{19}$$

$$\phi^{'} = \Phi \tag{20}$$

$$\Phi^{'} = -(\frac{2}{r} + \frac{A}{\alpha} + \frac{2\Psi}{\psi})\Phi + \psi^4(m^2 - \frac{\omega^2}{\alpha^2} + \lambda\phi^2)\phi \tag{21}$$

$$\alpha^{'} = A \tag{22}$$

$$A^{'} = -(\frac{2}{r} + 2\frac{\Psi}{\psi})A + 4C\pi\psi^4\alpha(2\frac{\omega^2}{\alpha^2} - m^2 - \frac{\lambda}{2}\phi^2)\phi^2 \tag{23}$$

Note here a constant C. This was introduced to accomodate the difference of convention in defining stress energy tensor $T_{\mu\nu}$. $C = 1$ is the conventional unit that I have been using and $C = 2$ is the unit used by `graxi`.

This system of equations constitute an eigenvalue problem for ω and the corresponding eigen-solutions for α, ψ, ϕ are the configurations for a stable ground state boson star.

[1] A. Brandt, Math. Comput. **31**, 333 (1977).