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MINIMIZING LONG-TERM WIND SET-UP ERRORS IN ESTIMATED MEAN ERIE AND SUPERIOR LAKE LEVELS

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MINIMIZING LONG-TERM WIND SET-UP ERRORS IN ESTIMATED MEAN ERIE AND SUPERIOR LAKE LEVELS[1]

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ABSTRACT. Errors in computed mean lake levels, caused by wind set-up, are estimated from linearized hydrodynamic shallow-water equations applied to Lakes Erie and Superior forhistorical and current gage networks. Observations of maximum unit error (that results from a unit wind stress) with each of the historical networks and with the current network are consistent with lake orientation and network placement considerations. Optimum network gage selections are made from the 16 available Lake Erie gages to minimize mean square set-up error estimated over one season's wind data, mean square total error estimated from daily data for 12 years, and mean square total error with constraints on the network size. Optimum network gage selections for Lake Superior are made from the 10 available gages to minimize mean square total error estimated from 12 years of daily data with and without constraints on the network size. It is not possible to eliminate wind set-up errors in mean lake levels if Thiessen weights must be used (although errors can be kept quite small); without this constraint, wind set-up errors can be eliminated from mean lake level computations. This allows selection of the weights that minimize other types of errors. The differences in net basin supply and lake volume computations that result by using optimum weights instead of Thiessen weights appear significant on both Lakes Erie and Superior.

1. INTRODUCTION

Hydrologists record lake surface elevations to estimate temporal changes in water storage resulting from the water balance between all inputs and outputs and from thermal changes. However, spatial variations in surface elevation (Forrester, 1980) must be accounted for in determining the proper mean (spatial average) lake level to use as the index of lake storage. Atmospheric pressure differences cause lake level differences between locations. Surface wind stress induces a tilt of the lake surface, referred to as set-up. The passage of pressure systems or the build-up and decay of wind stress may initiate oscillations, referred to as seiches, of bodies of water at their natural frequencies. Uneven evaporation and precipitation over a lake surface and tidal effects can cause small differences between different points on the lake. Systematic changes also exist at gage locations such as gravitational effects (Feldscher and Berry, 1968), land subsidence or emergence, lake surface slopes induced by local inflows and outflows, and wind or wave shelters or accentuators.

When selecting a network of gages to monitor lake surface elevations, hydrologists avoid or account for locations with systematic changes. Like-

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wise, hydrostatic analyses allow consideration of fluctuations due to atmospheric pressure changes (Quinn, 1976). Selecting an appropriate averaging time interval filters errors from short-term set-ups, seiches, tides, and temporal variations in evaporation and precipitation out of many computations for individual gages. The spatial averaging of surface elevations from several gages about a lake also reduces the error in the mean since all these errors are compensating across a lake. Selecting an appropriate monitoring gage network further minimizes (even eliminates in some cases) long-term wind set-up errors that cannot be filtered by choice of a time interval.

This report considers the minimization of long-term wind set-up errors by selecting monitoring networks of gages for Lakes Erie and Superior. Normally, selection of a network consists of choosing the number and locations of the monitoring gages. Here, consideration is limited to the locations where gages already are placed about Lakes Erie and Superior. Previous network selections on Lake Erie (Quinn and Derecki, 1976) and on Lake Superior (Quinn and Todd, 1974) considered Thiessen networks that use available gages by adding one gage at a time, in their historical order, until changes in computed beginning-of-month levels are inappreciable. Minimizations of set-up and total errors with and without restriction to Thiessen weights are considered here. Minimization, constrained to Thiessen weights, allows testing of elimination of wind set-up errors with a conventional weighting technique and determination of how much better we can do with Thiessen networks than is currently being done. Minimization without this constraint reveals the minimum number of gages required to eliminate wind set-up errors, shows how we can reduce other errors, and determines how much better we can do than is possible with the best Thiessen network.

2. LAKE ERIE

There are currently 16 locations on the shore of Lake Erie where water-level gages are maintained by either the National Ocean Survey of the U.S. Department of Commerce or the Canadian Hydrographic Service; see Fig. 1 (two others at Point Pelee are not considered here). All gages are in stilling wells designed to filter out high-frequency water-level fluctuations. The data are reduced to hourly scaled values relative to the International Great Lakes Datum of 1955 (Dohler, 1961). The locations shown in Fig. 1 are listed in Table 1 clockwise around the lake from Buffalo. Data from these gages are routinely used to estimate mean Lake Erie water levels by Thiessen weighting; gages with missing data are removed from the weighting by adjusting the remaining weights to compensate. Thus all gages with data are used when available; published values of water surface elevations from these 16 gages are represented as being accurate to within 0.02 ft (about 6 mm).

The free-surface circulation model of Schwab et al. (1981) was run on a 2-km grid of Lake Erie for a uniform eastward wind stress of 1 dyne cm^{-2} and for a uniform northward wind stress of 1 dyne cm^{-2} to model wind set-ups. Each run was for 5 days to simulate long-term residual set-ups as found typically over a week or month; water level displacements for the two runs are shown in Fig. 2 and summarized in Table 1 for the 16 gage locations. The model is a hydrodynamic model based on a finite difference solution of the linearized, depth-integrated, shallow water momentum and continuity equations;

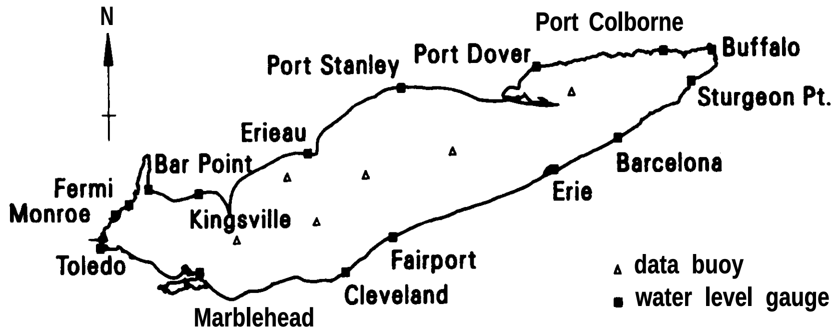

Figure 1. --Locations of Lake Erie gages and data buoys.

TABLE 1. --Lake Erie water-level gages, locations, and unit stress responses

Gage number[a]	First date and order[b]	Location	Latitude (degrees north)	Longitude (degrees east)	Eastward stress response[c] (mm)	Northward stress response[c] (mm)
1	1887 (1)	Buffalo	42.87750	-78.89083	95	50
2	1969 (14)	Sturgeon Pt.	42.69083	-79.04778	86	34
3	1961 (10)	Barcelona	42.34306	-79.59722	67	17
4	1959 (7)	Erie	42.15417	-80.07556	57	3
5	1981 (16)	Fairport	41.75000	-81.28333	8	-21
6	1900 (2)	Cleveland	41.54083	-81.63556	-6	-38
7	1960 (9)	Marblehead	41.54444	-82.73139	-71	-50
8	1906 (3)	Toledo	41.69333	-83.47222	-184	-66
9	1975 (15)	Monroe	41.89833	-83.36167	-145	-17
10	1964 (12)	Fermi	41.95972	-83.25833	-131	-8
11	1967 (13)	Bar Point	42.05000	-83.11100	-115	8
12	1963 (11)	Kingsville	42.02167	-82.73483	-76	-1
13	1959 (6)	Erieau	42.26033	-81.91450	-24	4
14	1927 (5)	Port Stanley	42.65900	-81.21333	-1	32
15	1960 (8)	Port Dover	42.78083	-80.20167	32	36
16	1912 (4)	Port Colborne	42.87400	-79.25333	73	43

[a]Numbers are assigned clockwise around the lake from Buffalo.
[b]The order is chronologic, starting with Buffalo in 1887.
[c]Response to 1 dyne cm^{-2} steady uniform 5 days of wind stress.

it has been used for forecasting storm surges (Schwab, 1978) and for inversely determining wind stress from water-level fluctuations (Schwab, 1982). Because the equations are linear, water-level displacements for an arbitrary steady stress can be computed by superposition of the results in Table 1 for the eastward unit stress and the northward unit stress after they are scaled by the actual components of stress in those directions.

The hydrodynamic model used to construct Table 1 neglects forces due to atmospheric pressure gradients, as well as density variations in the lake, and takes bottom stress as inversely proportional to the square of the depth. Boundary conditions are that there is no water transport normal to the shore-

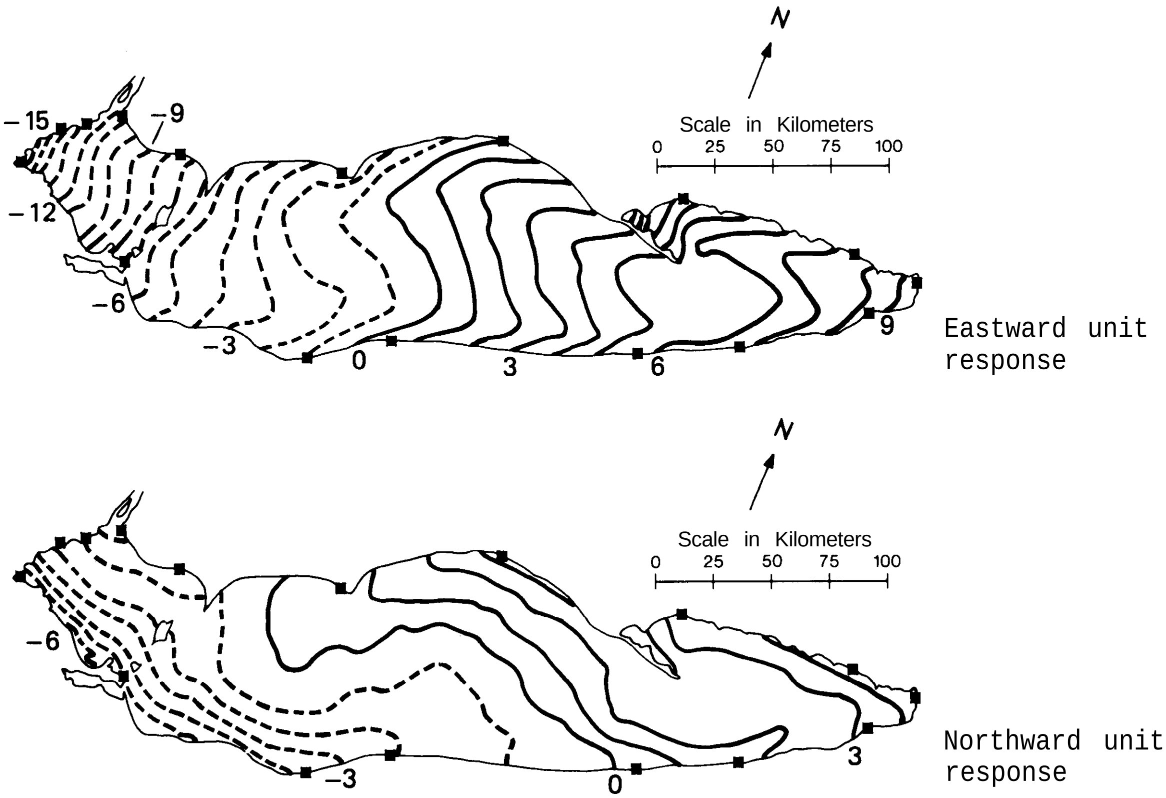

Figure 2. --Linearized hydrodynamic Lake Erie water surface response.

line (including no outflows or inflows to the lake) and that the wind stress is described spatially and temporally over the lake. This numerical model has the advantage that set-ups from the model, resulting from spatially variable wind stresses, in turn yield the vector average wind stress when inverted to solve for an equivalent spatially uniform wind stress (Schwab, 1982). The results of this model and superposition of model results are very good for Lake Erie storm surges when spatially averaged winds are used and compare well with observations (Schwab, 1978). Wind set-ups are often considered to result from spatially averaged wind stresses; Simon (1975) derived spatially averaged wind stress over Lakes Erie and Ontario by comparing modeled and observed water levels. In using this model and its results in Table 1 to estimate long-term wind set-ups, we assume that the temporally averaged wind set-up is given by the shallow water equations applied to temporally averaged wind stresses. Simon (1975) found that the relationships between observed and modeled wind set-ups on Lakes Erie and Ontario were remarkably similar in comparisons of water levels averaged over periods ranging from 5 h to 1 day.

3. LONG-TERM LAKE ERIE SET-UP ERROR

The long-term wind set-up error at gage i is a function of wind stress:

$$s_i(T, B) = e_i\ T \sin(B) + n_i\ T \cos(B) \qquad (1)$$

where $s_i(T, B)$ = long-term wind set-up error at gage i for a wind stress of magnitude T with bearing B; B = the angle in radians clockwise from north to the stress vector; and e_i and n_i = water level displacements at gage i resulting from 1 dyne cm^{-2} eastward and northward wind stresses, respectively. The weighted mean of the gage readings is usually taken to determine the (spatial) average lake level as an estimate of the "equivalent" level pool elevation (the elevation of a level pool with the same storage volume). The square set-up error in the weighted mean is

$$z^2(T, B, w_1, \ldots, w_n) = \left[\sum_{i=1}^{N} w_i s_i(T, B) \right]^2$$

$$= T^2 \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j C_{ij} \tag{2}$$

where $z^2(T, B, w_1, \ldots, w_N)$ = long-term wind square set-up error in the weighted mean for stress of magnitude T from direction B; w_i and w_j = averaging weights associated with gages i and j, respectively, for each of N gages, and

$$C_{ij} = e_i e_j \sin^2(B) + (e_i n_j + n_i e_j)\sin(B)\cos(B) + n_i n_j \cos^2(B) \tag{3}$$

Normalizing, let

$$Z^2(B, w_1, \ldots, w_N) = z^2(T, B, w_1, \ldots, w_N) / T^2 \tag{4}$$

where upper case and lower case letters denote different symbols throughout this report.

The historical sequence of networks on Lake Erie may be determined by starting with Buffalo in 1887 and identifying each of the networks used in the past by adding one gage at a time, in the chronological order indicated in Table 1; all gages were used in the past in each network as data were available. Analysis of long-term wind set-up error in mean Lake Erie water surface elevations as a function of stress direction gives a fast assessment of these networks. The unit root square set-up error Z (obtained by using Thiessen weights for w_i, i = 1, ... N) is plotted against direction B in polar coordinates in Fig. 3 for each of the last 15 historical networks to give an assessment of those networks. The networks are denoted in Fig. 3 by using binary notation where each place i in the network identifier (numbered from right to left) corresponds to gage i in Table 1. Thus, for example, network 000000000000001 consists of gage 1 (Buffalo), and network 000000000100001

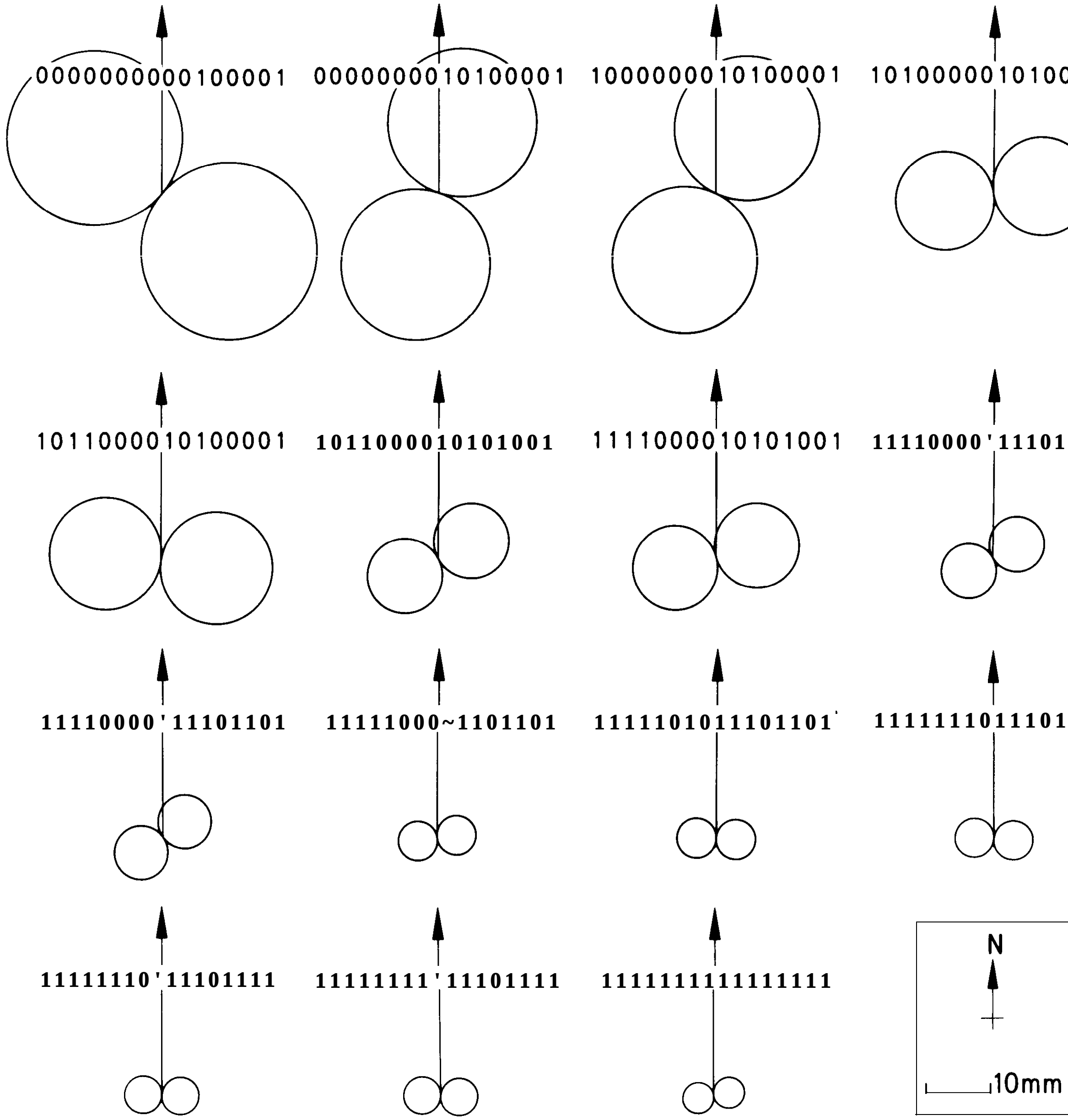

Figure 3. --Historical Thiessen network directional unit set-up error in mean Lake Erie level for 2- through 16-gage networks.

consists of gages 1 and 6 (Buffalo and Cleveland, respectively).

With only Buffalo in the network (0000000000000001), the unit root square set-up error is large and the maximum (107 mm) occurs for wind stresses with bearing 62 degrees (62 degrees clockwise from north) or bearing 242 degrees. This is easily seen from Fig. 1, since Buffalo is located at the end of Lake Erie's long axis, which has these bearings. The second network (0000000000100001) adds Cleveland to reduce the maximum error to 25 mm and shifts it to bearings 130 and 310 degrees (see Fig. 3). By having the second gage also along the long axis of the lake but toward the other end of the lake, set-up errors along this axis are greatly reduced because of partial compensation (drop at one gage is offset by rise at the other); however, set-up errors almost perpendicular to this axis do not compensate. By adding a third gage (Toledo) at the opposite end of the lake from Buffalo and somewhat removed from Cleveland in the direction of the minor axis of the lake, the third network (0000000010100001) reduces maximum error a little further (to 21 mm) and shifts it to bearings 18 and 198 degrees, still fairly close to perpendicular to the long axis. The fourth gage added (Port Colborne), being fairly close to the first (Buffalo) in the network (1000000010100001), reduces the maximum error only a little (less than 1 mm) and shifts it inappreciably. Further additions allow compensation of set-up errors along the minor axis, reducing the maximum error greatly and shifting it generally to an east-west orientation.

It is interesting to note that adding a gage to the five-gage network (1010000010100001) to get the six-gage network (1011000010100001) actually increases the maximum unit set-up error (from 14 to 16 mm); this also happens between the 7-gage network (1011000010101001) and the 8-gage network (1111000010101001) and between the 11- and 12-gage networks (1111100011101101 to 1111101011101101). In these cases, the Thiessen weightings are redistributed such that set-up errors are less compensated. Another interesting aspect of these analyses is that after about nine gages are present in the network (1111000011101001), little can be gained by adding later gages in a Thiessen-weighted network as the maximum unit set-up error is very small (7.8 mm). By adding another 7 gages to get the 16-gage network, we reduce the error further to only 4.4 mm. Looking at historical records at each of the gages instead of modeled wind set-up errors, Quinn and Derecki (1976) also settled on this nine-gage network by comparing changes in computed beginning-of-month mean lake levels that resulted with the historical networks considered in sequence as done here; they found the changes to be relatively very small after the first nine gages were present.

The results of Fig. 3 represent a unit root square set-up error corresponding to unit wind stress from each direction. Actual stress magnitude varies with direction and time in general, and historical data may be used to provide an idea of this variation and its effect on the error in the estimated mean lake level. For example, mean square set-up error (mse) is the expected value E of the square set-up error z^2 with the expectation taken over all values of stress magnitude and direction:

$$\begin{aligned} mse(w_1, \ldots w_N) &= E[z^2(T, B, wl, \ldots w_N)] \\ &= \int [z^2(T, B, w_1, \ldots, w_N)]\, dF(T, B) \end{aligned} \tag{5}$$

where E is the expectation operator and F(T, B) is the joint cumulative distribution function on (T, B); it is equal to the joint probability that the stress direction is less than B and the stress magnitude is less than T.

Equation (5) may be rewritten by substituting (2):

$$mse(w_1, \ldots . w_N) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j K_{ij} \tag{6}$$

where

$$K_{ij} = e_i e_j \int T^2 \sin^2(B) dF(T, B) + (einj + niej) \int T^2 \sin(B)\cos(B) dF(T, B)$$
$$+ n_i n_j \int T^2 \cos^2(B) dF(T, B) \tag{7}$$

The mean square set-up error may be estimated from time series data; a small set of data is available from 1979 when the Canada Centre for Inland Waters operated six meteorological buoys in Lake Erie from May to October (Schwab, 1982). The locations are shown in Fig. 1. They measured wind speed, wind direction, air temperature (all at 4 m above the water surface), and water temperature at lo-minute intervals. Hourly averages of these parameters were used to compute hourly values of vector wind stress (Schwab et al., 1981). Monthly values of wind speed, wind direction (determined from vector-averaged wind), air-water temperature difference, wind stress magnitude, and wind stress direction are given in Table 2. Equation (7) may be approximated by using the relative frequencies for each period in Table 2:

$$K_{ij} = e_i e_j \sum_{k-l}^{M} T_k^2 F_k \sin^2(B_k) + (einj + niej) \sum_{k=1}^{M} T_k^2 F_k \sin(B_k)\cos(B_k)$$

$$+ \quad n_i n_j \sum_{k-l}^{M} T_k^2 F_k \cos^2(B_k) \tag{8}$$

where Fk, Tk, and Bk are given by columns 2, 6, and 7, respectively, in Table 2 and M = the number of periods represented in Table 2. Equations (6) and (8) are used with the data in Table 2 to determine the set-up error associated with each of the historical networks; these errors are reported in Table 3 and range from 45 mm for only 1 gage at Buffalo to 2 mm for all 16 gages.

4. MINIMUM SET-UP ERROR FOR LAKE ERIE

The estimated Thiessen-weighted mean square set-up error of (5) may be minimized in an optimization to select some subset of the gages, from those in Table 1, whose set-up error is smallest of all:

$$\underset{q}{\text{MIN}}\ mse(w_1, \ldots . w_N), \quad \text{subject to (s.t.) } wl, \ldots . w_N = \text{Thiessen weights} \tag{9}$$

where q denotes a network number in which the <u>i</u>th binary bit from the right corresponds to gage i in Table 1 (as used to denote the historical networks in

TABLE 2. --Monthly mean values of meteorological parameters over Lake Erie from May to October 1979

Period	Relative frequency[a]	Wind Speed (m/s)	Wind Bearing[b]	$(T_a-T_w)^c$ (deg. C)	Stress Magnitude (dyne/cm^2)	Stress Bearing[b]
11-31 May	0.124	4.24	51	0.87	0.119	20
1-30 Jun	0.185	4.30	207	1.14	0.100	264
1-31 Jul	0.191	4.29	252	0.03	0.144	267
1-31 Aug	0.191	5.27	235	-0.56	0.204	254
1-30 Sep	0.185	5.71	222	-1.59	0.067	325
1-20 Oct	0.124	7.20	257	-3.48	0.847	262

[a]Relative frequency is number of days in period divided by total.
[b]Units are degrees clockwise from north.
[c]Air temperature minus water surface temperature.

TABLE 3. --Historical Thiessen networks on Lake Erie and root-mean-square set-up errors

Network number	Set-up error (mm)
0000000000000001	45.2
0000000000100001	7.76
0000000010100001	5.14
1000000010100001	5.73
1010000010100001	6.18
1011000010100001	6.79
1011000010101001	4.49
1111000010101001	5.29
1111000011101001	3.25
1111000011101101	3.06
1111100011101101	2.45
1111101011101101	2.44
1111111011101101	2.39
1111111011101111	2.29
1111111111101111	2.27
1111111111111111	1.95

Fig. 3 and Table 3); a "1" in position i means that gage i is included in the network and a "0" means it is not (and its weight w_i is zero). With 16 gages, there are 65,535 possible networks q, from 0000000000000001 to 1111111111111111. All these networks were analyzed to find the 65,535 sets of Thiessen weights corresponding to them; the Thiessen weighting algorithm described by Croley and Hartmann (1985) and associated database management techniques (Croley and Hartmann, 1986) made these calculations feasible at a resolution of 1 km^2. The solution to (9) and the nine next best networks are

TABLE 4. --Minimum mean-square-set-up-error[a] Thiessen networks on Lake Erie

Network number	Set-up error (mm)
1001010000111011	0.0327
1001110000111000	0.0332
1000100000010001	0.0359
1001010000111111	0.0399
1001010000111110	0.0546
0100100000100110	0.0631
0001110000101110	0.0639
1001010000111010	0.0653
0101100000111100	0.0775
1001110000101110	0.0777

[a]Subject to the constraint that all gage weights be Thiessen weights.

identified in Table 4 along with the root of the value of the objective function in (9). The Thiessen network that minimizes the estimated mean-square set-up error is seen from Table 4 (and Table 1) to consist of the eight gages at Buffalo, Sturgeon Point, Erie, Fairport, Cleveland, Bar Point, Erieau, and Port Colborne. It is interesting that none of the top ten networks identified in Table 4 contains gage 7, 8, 9, 10, or 14 (Marblehead, Toledo, Monroe, Fermi, or Port Stanley, respectively). It is difficult to assess the utility of these gages, however, since the estimated mean is based only on the few observations in Table 2. However, for the data in Table 2 at least, the historical networks have much larger set-up error (see Table 3) than is necessary when existing gages in a Thiessen network (see Table 4) are used.

It well may be that there are weights (other than Thiessen weights) that would be better for averaging gage readings in a network to minimize set-up errors. If our optimization is not restricted to Thiessen weights, then locations are accommodated where wind set-up is very much uncharacteristic (as in shallow bays or protected areas in the northeast section of the lake). Thiessen weights are based on the value at a gage applying uniformly halfway to the next gage; this is not true for arbitrary gage locations or for all areas of the lake. The optimization of (9) is reformulated by removing the constraint that gage weights be Thiessen weights.

$$\min_{(w_1,\ldots,w_N)} \operatorname{mse}(w_1, \ldots, w_N) \tag{10}$$

Equation (5) can be rewritten, for those cases in which the weights sum to unity, as

$$\operatorname{mse}(w_1, \ldots, w_N) = \int \Big[\sum_{i=1}^{N} w_i g_i - L\Big]^2 \, dF(g_1, \ldots, g_N) \tag{11}$$

where L = level pool elevation and gi = level pool elevation plus long-term wind set-up at gage i (which would be the gage reading corrected for all errors except set-up):

$$g_i = T^e e_i + T^n n_i + L, \quad T^e = T\cos(B),\, T^n = T\sin(B) \tag{12}$$

where T^e and T^n are eastward and northward components of T respectively. The optimization of (10) now can be seen as minimizing the mean square set-up error between the estimator, $\Sigma w_i g_i$ (summation over i from 1 to N is to be understood henceforth) and the equivalent level-pool elevation L. Without the constraint of Thiessen weights, it is possible to make this error zero; for any three-gage network, we can choose weights such that $\Sigma w_i g_i = L$ (zero error). This is so because any three gages (i, j, and m) give three simultaneous linear equations in three unknowns (T^e, T^n, and L), allowing the exact determination of the equivalent level-pool elevation L, if the equations are not degenerate [the vector (ei, ni) is not a scalar multiple of the vector (e_j, nj) where i and j are any two of the three gage numbers]:

$$\begin{aligned} g_i &= T^e e_i + T^n n_i + L \\ g_j &= T^e e_j + T^n n_j + L \\ g_m &= T^e e_m + T^n n_m + L \end{aligned} \tag{13}$$

Solving for L,

$$\begin{aligned} L &= w_i g_i + w_j g_j + w_m g_m \\ w_i &= (e_m n_j - e_j n_m)\,/\,D_{ijm} \\ w_j &= (e_i n_m - e_m n_i)\,/\,D_{ijm} \\ w_m &= (e_j n_i - e_i n_j)\,/\,D_{ijm} \\ D_{ijm} &= e_m n_j - e_j n_m + e_i n_m - e_m n_i + e_j n_i - e_i n_j \end{aligned} \tag{14}$$

Thus, without the constraint of Thiessen weights, it is possible to completely eliminate the long-term wind set-up error from the computation of the (spatial) mean lake level.

5. TOTAL ERROR FOR LAKE ERIE

Any three-gage network (gages i, j, and m), with weights given by (14), gives a long-term wind set-up error of zero (regardless of the magnitude or direction of the wind stress) and hence is better in this sense than any Thiessen network; for N = 16 gages, there are 560 possible three-gage networks. However, inspection of actual data reveals that the value of L computed from (14) for each three-gage network is different than the others. This is due to the existence of real-world errors, which have not been accounted for in (12) or (13); rewriting (12) to account for these errors gives

$$h_{ik} + dik + bi = T^e_k e_i + T^n_k n_i + Lk \tag{15}$$

where T^e_k and T^n_k are eastward and northward components of the wind stress at time k, respectively; Lk is the level-pool elevation at time k; hik = measured or reported lake level at gage i at time k; dik = the random error at gage i at time k due to measurement, processing, and reporting; and bi = systematic error at gage i resulting from the earlier use of an unlevel pool to transfer elevations across the lake (leveling error) and modeling error resulting from the inexactness of: the linear response assumption of (15), the assumptions of application of the hydrodynamic response model to Lake Erie, and the assumptions that only long-term set-up errors are present. The random errors are expected to be stationary, to be independent of each other in both space and time, to be identically distributed from one gage to another, and to have zero means. The systematic errors, although different at different gages, are taken as constant with time. We can find estimators of Lk, T^e_k, and T^n_k that minimize the sum of square errors for all gages in an effort to best fit (15) to available data:

$$\Sigma d^2_{ik} = (Lk - hik - bi + T^e_k e_i + T^n_k n_i)^2 \tag{16}$$

This is, of course, not the same as the optimizations of Sec. 3 since here we are minimizing errors associated with each gage in addition to the long-term wind set-up errors considered solely there. By differentiating the sum in (16) with respect to Lk, T^e_k, and T^n_k, setting the derivatives equal to zero (to get the "critical equations"), and solving these simultaneous linear critical equations, the estimators (referred to as the spatial-optimum estimators) are found to be

$$\hat{L}_k = \Sigma w_i(h_{ik} + b_i) \tag{17}$$

$$\hat{T}^e_k = \Sigma r_i(h_{ik} + bi) \tag{18}$$

$$\hat{T}^n_k = \Sigma u_i(h_{ik} + b_i) \tag{19}$$

where the "hat" notation (^) denotes the least squares estimator, where there are at least three gages in the network [sufficient information to estimate the three parameters in (17), (18), and (19)], and where, for N > 2,

$$w_j = x_j \;/\; \Sigma x_i,$$

$$x_j = e_j(\Sigma n_i \;\; \Sigma e_i n_i - \Sigma e_i \; \Sigma n_i^2) + n_j(\Sigma e_i \; \text{Ceini} - \Sigma n_i \; \Sigma e_i^2)$$

$$+ \Sigma e_i^2 \; \Sigma n_i^2 - (\Sigma e_i n_i)^2,$$

$$r_j = \{e_j[N \; \Sigma n_i^2 - (\Sigma n_i)^2] + n_j(\Sigma e_i \; \Sigma n_i - N \; \Sigma e_i n_i) \tag{20}$$

$$+ \Sigma n_i \; \Sigma e_i n_i - \Sigma e_i \; \Sigma n_i^2\} \;/\; \Sigma x_i,$$

$$u_j = \{n_j [N \; \Sigma e_i^2 - (\Sigma e_i)^2] + e_j(\Sigma e_i \; \Sigma n_i - N \; \Sigma e_i n_i)$$

$$+ \Sigma e_i \; \Sigma e_i n_i - \Sigma n_i \; \Sigma e_i^2\} \;/\; \Sigma x_i,$$

It is useful to note $\Sigma r_i n_i$ = Cuiei = $\Sigma w_i e_i = \Sigma w_i n_i = 0$ and $\Sigma r_i e_i$ = Cuini = 1. From (15) summed over a large number M of periods (k = 1, ... M),

$$b_j \simeq \Sigma r_i(H_i + bi) + \Sigma u_i(H_i + bi) + \Sigma w_i H_i - H_j, \quad j = 1, \ldots, N \tag{21}$$

since $(\sum_{k=1}^{M} d_{ik})/M$ is very small for M large, where

$$Hi = (\sum_{k=1}^{M} h_{ik})/M, \qquad i = 1, \ldots . N \tag{22}$$

We can also get (17)-(22) by least squares regression, as above for (17)-(20) but including the bi variables. This requires minimizing the sum of squared errors from all gages and all time periods,

$$\sum_{k=1}^{M} \sum_{i=1}^{N} d_{ik}^2 = \sum_{k=1}^{M} \sum_{i=1}^{N} (Lk - hik - bi + T_k^e e_i + T_k^n n_i)^2$$

with respect to L_k, T_k^e, T_k^n, and b_i.

However, (17)-(22) are not independent (nor are the critical equations that produce them); i.e., they are not sufficient to uniquely identify all parameters (Lk, $\hat{T}_k^e$, T_k^n, b_i, k = 1, ... M, i = 1, ... N). While this may not be apparent at first inspection, it can be demonstrated by noting that the following are all different sets of solutions to (17)-(22) (there are multiple solutions in each set for multiple values of the arbitrary constants B and C):

$\{\hat{L}_k, \hat{T}_k^e, \hat{T}_k^n, b_j\}$

$= \{\Sigma w_i h_{ik} + C, \Sigma r_i h_{ik}, \Sigma u_i h_{ik}, B[(\Sigma r_i H_i)e_j + (\Sigma u_i H_i)n_j + \Sigma w_i H_i - H_j] + C\}$

$= \{\Sigma w_i h_{ik} + C, \Sigma r_i h_{ik}, \Sigma u_i h_{ik} - \Sigma u_i H_i, (\Sigma r_i H_i)e_j + \Sigma w_i H_i - H_j + C\}$

$= \{\Sigma w_i h_{ik} + C, \Sigma r_i h_{ik} -$ CriHi, $\Sigma u_i h_{ik}, (\Sigma u_i H_i)n_j + \Sigma w_i H_i - H_j + C\}$

$= \{\Sigma w_i h_{ik} + C, \Sigma r_i h_{ik} -$ CriHi, $\Sigma u_i h_{ik} - \Sigma u_i H_i, \Sigma w_i H_i - H_j + C\}$

$= \{\Sigma w_i h_{ik} - \Sigma w_i H_i + C$, Crihik - CriHi, $\Sigma u_i h_{ik} - \Sigma u_i H_i, - H_j + C\}$

This means that (15) has more parameters than can be estimated from the available data (Draper and Smith, 1966); other information is necessary to uniquely identify all parameters. Here, (17)-(20) are used for Lk, T_k^e, and T_k^n, and other information is required to identify b_j.

The mgan square error of the spatial-optimum level-pool lake level estimator L_k is $E[(L_k - L_k)^2]$; using (15) and (17) gives

$$E[(\hat{L}_k - L_k)^2] = E[\{\Sigma wi(T_k^e e_i + T_k^n n_i + Lk - dik) - L_k\}^2] = E[(\Sigma w_i d_{ik})^2]$$

$$= \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j E[d_{ik} d_{jk}] = \Sigma w_i^2 E[d^2], \; N > 2 \tag{23}$$

since $\Sigma w_i e_i = \Sigma w_i n_i = 0$ [which means that the set-up error in the lake level estimate is zero, $\Sigma w_i(T_k^e e_i + T_k^n n_i) = 0$] and since the measurement errors d at each gage are independent and identically distributed with zero mean. Equation (23) gives us a measure of the estimation error associated with (17). Likewise, for any estimator defined as a weighted sum of gage heights,

$$L_k' = \Sigma w_i' h_{ik} \qquad (\text{where } \Sigma w_i' = 1) \tag{24}$$

the error of estimate, $E[(L_k' - L_k)^2]$, is

$$E[(L_k' - L_k)^2] = E[\{\Sigma w_i'(T_k^e e_i + T_k^n n_i + Lk - bi - dik) - L_k\}^2]$$

$$= E[\{\Sigma w_i'(T_k^e e_i + T_k^n n_i - b_i)\}^2] + \Sigma(w_i')^2 E[d^2] \tag{25}$$

6. MINIMUM TOTAL ERROR FOR LAKE ERIE

The mean square error of estimate mse (expected value of the square error of the estimator) can be computed from (20) and (23) for any spatial-optimum network if the mean square gage error $E[d^2]$ is known; it is denoted here for network q as

$$\mathrm{mse}(q) = E[(\hat{L}_k - L_k)^2] \tag{26}$$

The minimization of the mean square error in (26) is equivalent to the minimization of the sum of the squares of the weights (Σw_i^2) since $E[d^2]$ is constant (but unknown):

$$\underset{q}{\mathrm{MIN}}\ \mathrm{mse}(q) = \underset{(w_1,\ldots,w_N)}{\mathrm{MIN}} \Sigma w_i^2 E[d^2] = E[d^2] \underset{(w_1,\ldots,w_N)}{\mathrm{MIN}} \Sigma w_i^2 \tag{27}$$

The 65,535 possible networks, which can result from 16 gages, were analyzed by computing the spatial-optimum estimator weights from (20) and the sum of square weights as in (27) for all networks with three or more gages. For networks with fewer than three gages, it is not possible to compute the spatial-optimum estimators [since three independent equations like (17)-(19) are not then available). The results were searched to identify the minimum in (27); the best ten networks are identified in Table 5.

By using the estimators associated with the best network (all 16 gages) on a suitable data set, the mean square error of estimate of (23) is estimated, for the case where all gage biases are zero (bi = 0, i = 1, ... N), as

$$\hat{E}[(\hat{L}_k - L_k)^2]$$

$$= \Sigma w_i^2 \{ \sum_{k=1}^{M} \sum_{j-l}^{N_k} [(\Sigma r_{ik} h_{ik}) e_j + (\Sigma u_{ik} h_{ik}) n_j + (\mathrm{Cwikhik}) - h_{jk}]^2 / N_k \} / M \tag{28}$$

where the k subscript denotes the <u>k</u>th period in a data set; note that the sums on i and j from 1 to N_k are for the Nk reporting gages in the network at period k (those with no missing data). The parameters w_{ik}, rik, and u_{ik} are the spatial optimum for those gages reporting each period k from the best network that minimized the total error of (23) (all 16 gages). Daily data for the period 1973-1984 were assembled for each of the 16 gages identified in Table 1 for Lake Erie. Data were missing for part of this period, particularly for the gages at Fairport and Monroe. Available daily data were averaged over the month for each gage to estimate the average monthly lake level at each gage for each month of the data period. Thus, the short-term errors discussed in the Introduction were filtered from the monthly estimates. Some months contained few or no data at some gages, and so some average monthly lake levels are poorly estimated or missing. In making the estimate of (28), those periods in the data set for which Nk < 3 were excluded since there are no spatial-optimum estimators defined for fewer than three gages in a network. The root-mean-square error estimate is also tabulated in Table 5.

If we can tolerate sub-optimal solutions we can further reduce the number of gages necessary. Note from Table 5 that eliminating one or two certain gages from the 16-gage solution entails only a small penalty in terms of additional error. For example, eliminating gage 11 (Bar Point) increases the root-mean-square error by only about 0.4%; eliminating gages 10 and 11 (Fermi

TABLE 5. --Best spatial-optimum networks on Lake Erie

Network number	Root-mean-square error (mm)
1111111111111111	4.623
1111101111111111	4.640
1111110111111111	4.652
1111111011111111	4.657
1111100111111111	4.687
1111011111111111	4.688
1111101011111111	4.692
1101111111111111	4.698
1111110011111111	4.708
1111001111111111	4.720

and Bar Point) increases the root mean square error by only about 1.4%. As we allow more sub-optimal solutions [say with mean square error arbitrarily small, $mse(q) \leq a$], we can reduce the number of gages required further. This is expressed formally as

$$\min_{q} \Sigma I(q, i) \text{ s.t. } mse(q) \leq a \tag{29}$$

where I(q, i) is the indicator function such that it equals "1" if gage i is in network q (the ith bit from the right is "1") and "0" if it is not. The problem statement of (29) is to find a network that minimizes the number of gages such that the error is not greater than a. The optimization of (29) for all values of a is difficult to make; optimizations that are computationally more expedient are

$$\min_{q} mse(q) \text{ s.t. } \Sigma I(q, i) \leq p \tag{30}$$

Equation (30) yields the same information as (29) if $a1 > a2 > . . . > a_N$ where

$$a_p = \min_{q} mse(q) \text{ s.t. } \Sigma I(q, i) = p$$

The successive values of a_p, which decrease as p increases, serve as step points in the solution of (29); others are omitted. Again, the 65,535 possible networks were searched for p = 1, ... 16, in (30) to identify the best for each network size; the solutions are identified in Table 6, and the minimum gage count of (29) is plotted against the maximum tolerable error a in Fig. 4. Generally, as the maximum tolerable error allowed, a, decreases in Fig. 4, the optimization of (29) is more constrained and the minimum number of gages required in the optimum network increases. Figure 4 and Table 6 provide

TABLE 6. --Best spatial-optimum networks of each size and associated errors for Lake Erie

Number of gages	Network number	Root-mean square-error (mm)	Normalized error Observed[a]	Normalized error Theoretical[b]	Relative difference (%)
3	0101000000100000	9.941	0.577596	0.577350	0.043
4	0000100001001001	8.607	0.500068	0.500000	0.014
5	0011000001010010	7.697	0.447227	0.447214	0.003
6	0001100001010101	7.027	0.408255	0.408248	0.002
7	0001000101100111	6.505	0.377970	0.377964	0.002
8	1000101001101110	6.085	0.353554	0.353553	0.000
9	1000010011101111	5.737	0.333335	0.333333	0.001
10	1001110001111110	5.443	0.316230	0.316228	0.001
11	1010001011111111	5.190	0.301525	0.301511	0.005
12	1101000111111111	4.969	0.288696	0.288675	0.007
13	1111100011111111	4.796	0.278657	0.277350	0.471
14	1111100111111111	4.687	0.272308	0.267261	1.888
15	1111101111111111	4.640	0.269583	0.258199	4.409
16	1111111111111111	4.623	0.268598	0.250000	7.439

[a]Observed normalized error = $\hat{E}[(\hat{\ell}_k - L)^2]^{1/2} / \hat{E}[d^2]^{1/2} = (\Sigma w_i^2)^{1/2}$.
[b]Theoretical normalized error = $(1/N)^{1/2}$.

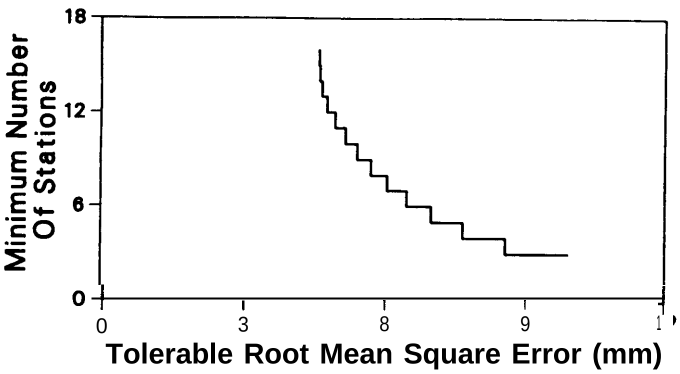

Figure 4. --Number of gages required to limit error in spatial optimum network for Lake Erie.

the basis for the trade-off between errors in the mean lake level and network size. For example, consider the solution for a maximum tolerable error of 0.02 ft (6.096 mm) which is equivalent to the supposed accuracy of the individual gages. The optimum set of gages is identified from Table 6 as q = 1000101001101110, a minimum of eight gages. Adding additional gages to the optimum network can reduce the root-mean-square error of the estimate of level-pool elevation by about 31% (to about 4.6 mm), with the 16 gages currently used.

The minimums in (27) or (30) yield weights that are close to uniform; indeed, if the weights were not constrained to spatial-optimum networks, then

$$\underset{(w_1,\ldots,w_n)}{\mathrm{MIN}} \quad \Sigma w_i^2 \text{ s.t. } \Sigma w_i = 1 \tag{31}$$

yields

$$w_i = 1/N, \quad i = 1, \ldots, N$$

$$[\min_{(w_1,\ldots,w_N)} \Sigma w_i^2]^{1/2} = [1/N]^{1/2}, \; N > 2 \tag{32}$$

The spatial-optimum gage weights for the 10- through 16-gage networks are presented in Table 7. The weights are closer to uniform for the 10- through 12-gage networks than they are for the 13- through 16-gage networks. Inspection of observed normalized error in Table 6 reveals that the root sum of square weights for each constrained optimization is indeed very close to the value given by (32) (found in Table 6 as theoretical normalized error). If some of the existing gages were to be relocated and the rest removed, the best locations for those gages would be where the model-derived wind set-ups yield weights [eqs. (20)] that are uniform. The best spatial-optimum networks identified for different network sizes in Table 6 have their root sum of square errors very close to (32) (the relative difference in Table 6 is close to zero, particularly for the best spatial-optimum networks with 3 to 12 gages). Therefore, little improvement can be expected by relocating the gages in those networks. Significant improvements are possible by relocation of gages for spatial-optimum networks with greater than 12 gages, since the normalized error is significantly different than the theoretical; i.e., better 16-gage networks than the one considered here for Lake Erie are possible if some of the gages are relocated. The only other way to decrease the network error would be to reduce the random error at each gage; this is true in general, regardless of network size.

TABLE 7.--Selected Lake Erie spatial-optimum network gage weights

Gage number	Location	Weights for optimum networks						
		10-gage	11-gage	12-gage	13-gage	14-gage	15-gage	16-gage
1	Buffalo		0.0921	0.0846	0.0682	0.0694	0.0691	0.0682
2	Sturgeon Pt.	0.1004	0.0917	0.0843	0.0724	0.0765	0.0778	0.0779
3	Barcelona	0.1002	0.0913	0.0840	0.0763	0.0815	0.0836	0.0846
4	Erie	0.1000	0.0909	0.0837	0.0798	0.0871	0.0904	0.0922
5	Fairport	0.0997	0.0904	0.0831	0.0841	0.0885	0.0909	0.0928
6	Cleveland	0.0995	0.0900	0.0828	0.0883	0.0948	0.0985	0.1014
7	Marblehead	0.0994	0.0897	0.0822	0.0883	0.0851	0.0846	0.0856
8	Toledo		0.0895	0.0813	0.0870	0.0653	0.0569	0.0541
9	Monroe			0.0822		0.0467	0.0345	0.0288
10	Fermi		0.0909				0.0328	0.0269
11	Bar Point	0.1002						0.0199
12	Kingsville	0.1001			0.0739	0.0552	0.0468	0.0429
13	Erieau	0.1001		0.0832	0.0752	0.0656	0.0613	0.0593
14	Port Stanley		0.0917		0.0684	0.0552	0.0488	0.0451
15	Port Dover			0.0840	0.0690	0.0614	0.0574	0.0549
16	Port Colborne	0.1005	0.0919	0.0844	0.0691	0.0678	0.0666	0.0653

Once the best spatial-optimum network is determined, its weights can be used to estimate the error associated with other kinds of networks (such as a Thiessen network); this error is given by (25) and is estimated relative to spatial-optimum estimators, for the case where all gage biases are zero ($b_i = 0$, $i = 1, \ldots N$), by

$$\hat{E}[(L'_k - L_k)^2] = \sum_{k=1}^{M} \{ \sum_{j=1}^{N_k} w'_{jk}[(\Sigma r_{ik}h_{ik})e_j + (\Sigma u_{ik}h_{ik})n_j] \}^2 / M$$

$$+ \Sigma(w'_i)^2 \{ \sum_{k=1}^{M} \sum_{j=1}^{N_k} [(\Sigma r_{ik}h_{ik})e_j + (\Sigma u_{ik}h_{ik})n_j + (\Sigma w_{ik}h_{ik}) - h_{jk}]^2/N_k \} / M \qquad N > 2 \tag{33}$$

Those periods in the data set for which $N_k < 3$ are excluded since there are no spatial-optimum estimators defined for fewer than three gages in a network. This exclusion is useful in assessing the errors associated with the current uses of the existing gages. The mean square error of estimate for a network q now becomes, from eq. 33

$$mse(q) \simeq \hat{E}[(L'_k - L_k)^2] \tag{34}$$

This error can be minimized over the selected data set by choosing a network of gages that gives the smallest value of (34).

Ideally, an optimization of (34) should use the best Thiessen network for each month of the data set that is possible from a given set of gages:

$$\underset{Q}{MIN} \sum_{k=1}^{M} [\underset{q \varepsilon Q \cdot v_k}{MIN} \; mse(q)] \tag{35}$$

where v_k denotes which gages have data at time k (it is a number like q (in which a "1" in position i, numbered from the right, means that gage i has data at time k and a "0" means data are missing at gage i at time k); Q = a number representing a set of gages (same binary convention that we use for networks). The notation $Q \cdot v_k$ signifies the intersection; the notation $q \varepsilon Q \cdot v_k$ (read as "q within the set $Q \cdot v_k$") identifies that set of networks q with gages from the set Q that have data at time k. The problem statement of (35) is to find a set of gages that minimizes the sum of errors over all periods of the data set where the best Thiessen network is used for each period. This optimization is very difficult to make since the computational requirements are very large to find the best Thiessen network for each period for each different set of gages, Q. Instead, a slightly different optimization is considered that uses all gages possible for each period. This is usually done in practice in near-real-time and is the only approach feasible when historical data are being reduced:

$$\min_{Q} \sum_{k=1}^{M} mse(Q \cdot v_k) \qquad (36)$$

The 65,535 possible networks were again analyzed by using spatial-optimum network weights for all gages with data (vk) for each period of the data set and appropriate Thiessen weights for all gages in each network with data ($Q \cdot v_k$) from each period in (33) to compute the root mean square error of the Thiessen estimator. The results were searched to identify the minimum in (36); the best was found to be a 14-gage network (1111110101111111) with an estimated root mean square error of 5.371 mm. Likewise, the trade-off between number of gages and tolerable error, expressed similarly to (29) and (30), but for Thiessen networks [with mse(q) from (34) and all gages possible used in each period of the data set], is given as

$$\min_{Q} \sum_{k=1}^{M} mse(Q \cdot v_k) \qquad \text{s.t.} \quad \Sigma I(Q,i) \leq p \qquad (37)$$

The 65,535 possible networks were again searched to determine the solutions to (37) for p = 1, ... 16; the results are identified in Table 8 and are plotted in Fig. 5.

TABLE 8.--Best Thiessen networks and associated errors for Lake Erie

Number of gages	Network number	Root-mean square-error (mm)
1	0000000000010000	18.274
2	0010000000100000	12.249
3	0000100000011000	10.304
4	0100100000101000	8.741
5	0010100000101100	7.802
6	0010010000111100	7.067
7	0011010000111100	6.615
8	0111100001101100	6.248
9	0111100001111010	5.863
10	0111100001111110	5.620
11	0111100101111110	5.470
12	1111100101111110	5.403
13	1111110101111110	5.383
14	1111110101111111	5.371
15	1111111101111111	5.377
16	1111111111111111	5.441

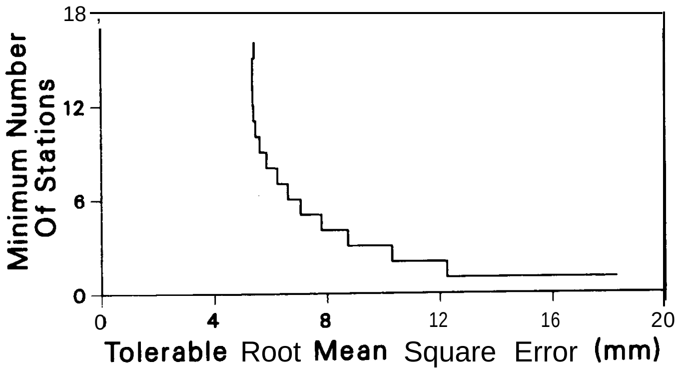

Figure 5. --Number of gages required to limit error in Thiessen network for Lake Erie.

7. SPATIAL-OPTIMUM AND THIESSEN WEIGHTING DIFFERENCES FOR LAKE ERIE

It is not possible to decide which method (spatial-optimum or Thiessen) is better on the basis of analysis of lake levels (comparisons show only that they are different), although the spatial-optimum method should be better since the long-term set-up error is explicitly considered and eliminated. We can look, however, at the differences between the two methods to understand their magnitude and distribution. Computations of beginning-of-month spatial-mean lake levels, monthly lake storage changes, and net basin supplies with both of the methods illustrate these differences.

Beginning-of-month (BOM) lake levels for all gages on Lake Erie were computed by temporally averaging hourly levels (reported by the National Ocean Survey of the National Oceanic and Atmospheric Administration in Rockville, Maryland, and by the Water Survey of Canada of Environment Canada in Guelph, Ontario, Canada) for the last day of the preceding month and the first day of the month for which the BOM level is desired. This eliminates some short-term fluctuations associated with storms, winds, and atmospheric pressure fluctuations. The BOM gage levels were combined by means of both spatial-optimum and Thiessen weightings to determine spatial mean BOM lake levels for Lake Erie. The 14-gage Thiessen network identified in Table 8 with the least root-mean-square error and the 16-gage spatial optimum network identified in Table 6 were used to represent the best of both methods. The differences between BOM lake levels computed with these two methods (Table 9) are found by subtracting the Thiessen weighted BOM levels from the spatial-optimum-weighted BOM levels. These differences can be further subdivided as follows (for the case where all gage biases are zero) from (15), (17), and (24):

$$\hat{L}_k - \hat{L}'_k = \Sigma w'_{ik} d_{ik} - \Sigma w'_{ik} (\hat{T}^e_k e_i + \hat{T}^n_k n_i \qquad (38)$$

where dik denotes the estimate of random error at gage i at time k; it is determined by replacing variables with their estimates in (15). Equation (38) separates the difference in BOM levels between spatial-optimum and Thiessen weightings (Table 9) into a "random" gage error component (called the residual) and a long-term wind set-up error component [the two sums on the right side of the equality in (38) from left to right respectively]. The

TABLE 9.--Absolute difference in BOM Lake Erie levels[a]
(based on spatial-optimum and Thiessen lake level estimates)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	44.9	-17.0	-2.9	-3.9	-8.8	0.4	-6.5	-7.9	0.5	-7.4	24.9	13.6
1974	6.1	10.5	-2.0	1.6	3.3	-1.2	-0.6	6.1	6.4	0.9	-1.8	-22.2
1975	4.5	-8.9	5.2	5.2	-7.6	-1.9	-5.4	-1.5	-19.1	1.8	0.7	25.4
1976	-9.7	-1.5	-12.4	-9.3	-4.3	-5.2	-1.0	-3.2	4.0	-3.4	-3.1	19.8
1977	38.6	-4.9	-5.0	2.4	-3.6	-7.7	6.5	0.7	-4.2	-6.1	-6.3	11.1
1978	-9.1	-5.6	-5.8	-0.7	3.7	-2.6	-0.8	-3.5	-4.8	2.6	1.2	15.4
1979	0.6	6.1	-2.8	-3.6	-2.3	-3.9	-6.6	-1.2	-1.3	-5.1	-4.7	16.1
1980	-1.0	0.0	1.0	-8.4	-1.8	-2.2	0.9	-1.6	-0.1	-3.0	26.1	8.6
1981	-9.4	-3.3	0.3	-5.4	3.4	3.0	-3.2	-1.9	-6.2	0.5	-3.4	-8.0
1982	5.3	-3.5	-3.4	3.0	-2.6	-6.6	2.3	5.6	-4.9	-1.1	-4.7	0.7
1983	10.0	6.7	-3.6	-4.7	-1.8	-2.3	-7.6	0.8	-1.8	-9.3	-7.0	30.9
1984	6.1	-3.8	19.7	0.1	0.1	0.7	-5.0	-3.3	0.9	-9.4	-3.8	11.7

[a]In millimeters over the lake.

long-term wind set-up error is tabulated in Table 10, and the residual error is tabulated in Table 11; an entry in Table 9 is equal to the corresponding entry in Table 11 minus the corresponding entry in Table 10, by (38).

Table 9 shows that larger differences between the two methods occur in the winter and the fall of the year, corresponding approximately to times of larger differences in individual gage readings around the lake due to increased wind activity on the lake (both short- and long-term set-ups). The data in Table 10 reveal that although there are isolated months with (relatively) large long-term set-up error, the difference too is larger generally in the winter and fall of the year, although it is not large enough to account for

TABLE 10.--Long-term set-up error in BOM Lake Erie levels for best Thiessen network[a] (based on spatial-optimum lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	-3.6	5.5	0.7	1.7	1.5	-0.5	0.4	0.5	0.1	2.9	-5.8	-2.5
1974	-1.1	-2.4	0.1	0.5	0.8	0.8	-0.3	-1.4	-1.0	-5.7	0.8	7.1
1975	-3.1	1.3	-2.4	-1.6	0.9	-1.5	0.4	0.2	2.5	-1.3	-0.2	-11.8
1976	3.0	-0.6	3.5	-1.1	0.9	1.2	-3.0	-0.1	-1.3	0.8	-1.2	-5.4
1977	-8.7	-1.0	-0.9	-0.3	0.6	-0.6	-3.6	-0.4	0.0	0.8	3.0	-3.0
1978	0.1	1.0	0.9	-0.5	0.0	0.0	2.2	0.6	0.5	-0.3	0.1	-4.2
1979	-0.2	-0.5	0.8	0.8	-1.9	1.1	-2.9	0.3	0.9	1.1	1.0	-5.9
1980	-0.2	-0.6	-0.5	1.1	0.0	0.0	-0.6	0.3	-0.2	0.2	-5.8	-1.2
1981	1.8	0.5	-2.5	-0.9	0.0	-0.2	1.4	1.1	0.5	-0.6	1.3	3.3
1982	-0.5	0.7	0.8	-2.6	0.8	-0.2	-0.3	-0.8	0.9	0.5	0.6	0.6
1983	-2.2	-0.2	0.5	2.3	2.4	-0.6	0.2	-1.5	1.1	1.7	1.7	-10.5
1984	0.6	-0.3	-15.4	-0.4	-11.1	0.1	1.4	0.5	-0.6	1.8	0.5	-3.0

[a]In millimeters over the lake.

TABLE 11.--Gages "random" error in BOM Lake Erie levels for best Thiessen network[a] (based on spatial-optimum lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	41.3	-11.6	-2.1	-2.2	-7.3	-0.2	-6.0	-7.4	0.6	-4.5	19.1	11.1
1974	5.0	8.0	-1.9	2.1	4.1	-0.4	-0.9	4.8	5.3	-4.8	-1.0	-15.1
1975	1.4	-7.6	2.8	3.6	-6.7	-3.4	-5.0	-1.3	-16.7	0.6	0.5	13.6
1976	-6.7	-2.1	-8.8	-10.4	-3.4	-4.0	-4.0	-3.3	2.7	-2.6	-4.4	14.4
1977	29.9	-5.9	-5.9	2.1	-3.0	-8.3	2.9	0.3	-4.2	-5.3	-3.3	8.1
1978	-9.0	-4.6	-4.9	-1.1	3.7	-2.6	1.4	-2.9	-4.4	2.3	1.3	11.2
1979	0.4	5.6	-2.1	-2.8	-4.2	-2.8	-9.5	-0.9	-0.4	-4.0	-3.7	10.2
1980	-1.2	-0.6	0.5	-7.3	-1.8	-2.1	0.3	-1.4	-0.4	-2.8	20.3	7.5
1981	-7.5	-2.8	-2.1	-6.4	3.4	2.7	-1.8	-0.9	-5.7	-0.1	-2.1	-4.7
1982	4.9	-2.8	-2.6	0.4	-1.8	-6.7	2.0	4.8	-4.0	-0.6	-4.2	1.3
1983	7.8	6.5	-3.1	-2.4	0.7	-3.0	-7.3	-0.7	-0.6	-7.7	-5.2	20.4
1984	6.7	-4.1	4.3	-0.3	-11.1	0.8	-3.6	-2.8	0.4	-7.6	-3.3	8.7

[a]In millimeters over the lake.

the total difference between the methods in Table 9. Most of the difference is found in the residual of Table 11. Many of the errors associated with short- to intermediate-term set-ups, seiches, and other sources, that were not eliminated by the two-day average to determine the BOM level, are reflected in the "random" gage error component of Table 11 (meaning that this residual reflects more than just random gage errors). The pattern of winter and fall error distribution fbr this residual (in Table 11) is the same as the pattern for the total difference and long-term set-up error (in Tables 9 and 10 respectively).

In the majority of months (106 out of 144), adding the absolute magnitudes of long-term set-up error (Table 10) and the residual (Table 11) produces the total absolute error (sign of terms in Tables 10 and 11 are opposite); in only 38 out of 144 months, the residual and the long-term set-up errors are partially compensating (same sign of terms in Tables 10 and 11). In the 106 noncompensating months, which include all months with large total differences (greater than 10 mm), the average absolute total error is 7.3 mm consisting of 1.8-mm average absolute long-term set-up error and 5.5-mm average absolute residual. In the 38 compensating months, the average absolute long-term set-up error is 1.2 mm and the average absolute residual is 3.6 mm but, because of partial compensation, the total is only 2.4 mm. The differences in these averages for the noncompensating and the compensating cases point up the effect of BOM gage reading differences on the mean BOM levels under the two weighting methods. When the gages around the lake differ greatly because of set-up (both long- and short-term), seiches, and other factors, the mean BOM levels produced by spatial-optimum and Thiessen weightings differ the most; the residual and the long-term set-up error are noncompensating and combine to give large total differences, since similar set-up conditions are present in both components. Taken as a whole, without regard to compensating or noncompensating differences, the average absolute total difference is 6.0 mm, the average absolute long-term set-up error is 1.7 mm, and the average absolute residual is 5.0 mm.

Errors in computed spatial-mean BOM lake levels are translated into other quantities. In particular, the monthly change in storage in a lake is computed from the spatial-mean BOM lake levels; the monthly Net Basin Supply (NBS), which consists of lake precipitation plus basin runoff minus lake evaporation, is estimated from these computed changes in storage and river inflows and outflows. It is useful to assess the differences that exist in these quantities when Thiessen weights are replaced with spatial-optimum weights. The monthly change in storage in Lake Erie for month i in Table 12 is BOM level for month i + 1 minus BOM level for month i; the spatial-optimum results minus the Thiessen results are given in Table 13. (Note that the differences in Table 13 between storage changes computed with spatial-optimum and storage

TABLE 12. --Lake Erie monthly change in storage[a]
(based on spatial-optimum and Thiessen lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	-77	-27	273	11	19	53	-99	-104	-174	-45	-71	34
1974	112	24	174	41	51	-30	-98	-103	-139	-146	3	69
1975	47	111	60	-11	18	43	-107	55	-90	-119	-68	27
1976	-51	201	192	-1	-3	-12	-18	-113	-130	-155	-132	-55
1977	-171	26	283	187	-56	-17	-31	-28	52	-180	-4	108
1978	-31	-76	257	137	2	-59	-119	-104	-87	-85	-60	-25
1979	-30	-4	233	250	54	-16	-17	-29	-58	-121	37	85
1980	-63	-42	181	122	-23	19	-6	-12	-130	-150	-109	-54
1981	-151	215	19	157	43	125	-44	-94	-10	-47	-82	-17
1982	-60	17	288	69	-18	-4	-86	-157	-47	-138	97	135
1983	-70	-12	76	156	104	29	14	-91	-180	-101	94	-3
1984	-123	197	139	-119	141	18	-71	-54	-79	-70	-47	54

[a]In millimeters over the lake.

TABLE 13. --Absolute difference between Lake Erie monthly changes
in storage[a] (based on spatial-optimum and Thiessen lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	-62	14	-1	-5	9	-7	-1	8	-8	32	-11	-7
1974	4	-12	4	2	-4	1	7	0	-5	-3	-20	27
1975	-13	14	0	-13	6	-3	4	-18	21	-1	25	-35
1976	8	-11	3	5	-1	4	-2	7	-7	0	23	19
1977	-43	0	7	-6	-4	14	-6	-5	-2	0	17	-20
1978	3	0	5	4	-6	2	-3	-1	7	-1	14	-15
1979	5	-9	-1	1	-2	-3	5	0	-4	0	21	-17
1980	1	1	-9	7	0	3	-3	2	-3	29	-17	-18
1981	6	4	-6	9	0	-6	1	-4	7	-4	-5	13
1982	-9	0	6	-6	-4	9	3	-10	4	-4	5	9
1983	-3	-10	-1	3	-1	-5	8	-3	-8	2	38	-25
1984	-10	23	-20	0	1	-6	2	4	-10	6	15	-10

[a]In millimeters over the lake.

changes computed with Thiessen BOM lake levels are found by subtracting successive values in Table 9. For example, the difference of -62 mm in Table 13 between storage changes for January 1978 is equal to the -17.0 mm BOM level difference for February 1978 in Table 9 minus the 44.9 mm BOM level difference for January 1978, rounded after subtraction). By subtracting the Detroit (input) river flow and adding the Niagara and Welland (output) flows to the change in storage in Table 12, the monthly net basin supply is determined; it is presented in cubic meters per second (by dividing by the area of the lake and by the number of seconds in a month) in Table 14. 'The difference in NBS computed from lake levels determined with spatial-optimum weights and NBS computed from lake levels determined with Thiessen weights is the same as the difference in monthly storage in Table 13; it is presented in cubic meters per second in Table 15 for convenience. Differences in NBS are patterned similarly to those observed for spatial-mean BOM lake levels; the large differences occur mainly in the fall and winter. The average absolute difference in NBS for the two weighting methods is 81.4 cms.

TABLE 14. --Lake Erie monthly net basin supply[a]
(based on spatial-optimum and Thiessen lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	69	754	3620	1551	1525	1753	131	-228	-1190	-40	-231	861
1974	1778	1098	2687	1852	1828	793	-253	-505	-1010	-1112	469	1340
1975	1486	2180	1629	917	1172	1122	-353	1063	-200	-455	172	1006
1976	1146	3386	3032	1307	1085	776	550	-499	-537	-748	-510	341
1977	-118	1084	2917	2589	731	924	632	773	1552	-908	683	2227
1978	835	470	3346	2509	1484	783	-281	-268	-413	-398	-326	428
1979	934	1018	2520	2951	1310	628	358	294	-46	-456	902	1477
1980	208	267	2478	2314	990	1317	840	864	-384	-682	-582	-96
1981	-199	2601	541	2005	1127	2246	298	-101	590	183	-28	608
1982	1099	1533	3625	1760	1050	1276	35	-801	4	-925	1560	2007
1983	196	502	1229	2268	1955	861	804	-114	-1166	-497	1546	1179
1984	311	2490	2157	1662	2363	839	-65	-94	-387	-409	-122	856

[a]Average over the month in cubic meters per second.

8. LAKE SUPERIOR

Sections 8-11 consider the minimization of long-term wind set-up errors by selecting a monitoring network of gages for Lake Superior; consideration is limited to the locations about Lake Superior where gages already are placed. There are currently 10 locations on the shore of Lake Superior where permanent water-level gages are maintained by either the National Ocean Survey of the U.S. Department of Commerce or the Canadian Hydrographic Service (see Fig. 6). The locations shown in Fig. 6 are listed in Table 16 clockwise around the lake from Point Iroquois. Data from these gages are routinely used to estimate mean Lake Superior water levels by Thiessen weighting; gages with missing data are removed from the weighting by adjusting the remaining weights to compensate. Thus all gages with data are used when available.

TABLE 15.--Absolute difference between Lake Erie monthly net basin supplies[a] (based on spatial-optimum and Thiessen lake level)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	-595	151	-10	-48	88	-68	-14	81	-78	310	-112	-72
1974	42	-132	34	18	-43	6	64	2	-54	-26	-202	256
1975	-129	150	0	-127	55	-35	38	-169	208	-11	246	-337
1976	78	-111	29	50	-8	42	-21	69	-74	2	227	180
1977	-417	0	71	-60	-39	141	-56	-46	-19	-2	173	-194
1978	33	-2	50	43	-60	17	-25	-13	74	-13	141	-142
1979	53	-95	-7	13	-15	-27	52	-1	-37	4	206	-164
1980	9	11	-91	66	-4	30	-24	15	-29	279	-173	-173
1981	59	38	-56	87	-3	-61	12	-41	67	-38	-46	128
1982	-84	1	61	-56	-38	88	31	-101	38	-35	54	89
1983	-32	-109	-11	29	-5	-52	80	-25	-75	23	376	-238
1984	-94	241	-188	0	6	-57	16	40	-102	54	153	-96

[a]Average over the month in cubic meters per second.

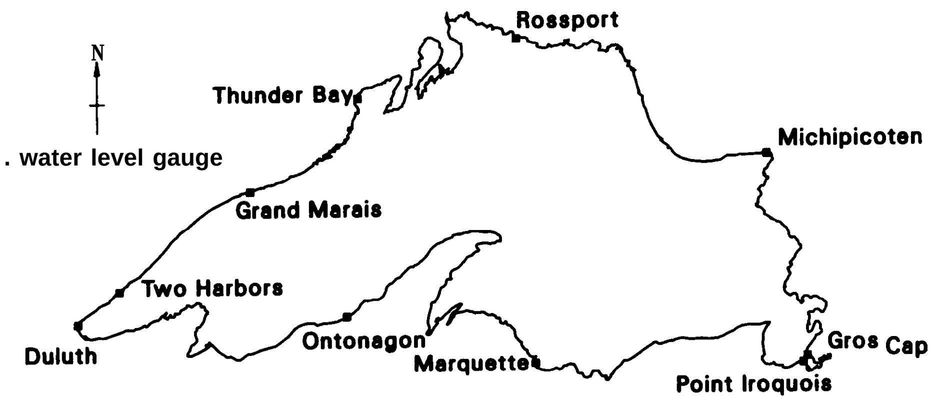

Figure 6.--Locations of Lake Superior gages.

The free-surface circulation model of Schwab et al. (1981) was run on a 15-km grid of Lake Superior for a uniform eastward wind stress of 1 dyne cm^{-2} and for a uniform northward wind stress of 1 dyne cm^{-2} to model wind set-ups. Each run was for 5 days to simulate long-term residual set- ups as found typically over a week or month; water level displacements for the two runs are summarized in Table 16 for the ten gage locations.

9. LONG-TERM LAKE SUPERIOR SET-UP ERROR

The historical sequence of networks on Lake Superior may be determined by starting with Duluth (first in 1885) and identifying each of the networks used in the past by adding one gage at a time, in the chronological order indicated in Table 16; all gages were used in the past in each network as data were available. The unit set-up error (obtained by using Thiessen weights) is plotted against direction in polar coordinates in Fig. 7 for each of the ten historical networks, to assess those networks.

TABLE 16. --Lake Superior water-level gages, locations, and unit stress responses

Gage number[a]	First date and order[b]	Location	Latitude (degrees north)	Longitude (degrees east)	Eastward stress response[c] (mm)	Northward stress response[c] (mm)
1	1930 (6)	Point Iroquois	46.48500	-84.63111	30	-10
2	1891 (2)	Marquette	46.54500	-87.37833	4	-11
3	1959 (8)	Ontonagon	46.87778	-89.32083	-5	-5
4	1885 (1)	Duluth	46.77556	-92.09278	-25	-16
5	1941 (7)	Two Harbors	47.01750	-91.67500	-20	-2
6	1966 (9)	Grand Marais	47.74806	-90.34167	-14	-1
	1912 (3)	Thunder Bay	48.40950	-89.21700	-7	3
8	1967 (10)	Rossport	48.83383	-87.51950	1	
9	1915 (4)	Michipicoten	47.96217	-84.90050	14	5
10	1926 (5)	Gros Cap	46.52933	-84.58550	30	-10

[a]Numbers are assigned clockwise around the lake from Point Iroquois.
[b]The order is chronologic, starting with Duluth in 1885.
'Response to 1 dyne cm^{-2} steady uniform 5 days of wind stress.

With only Duluth in the network (0000001000), the maximum unit set-up error (30. mm) is larger than with any combination of gages; it has bearing 57 degrees (57 degrees clockwise from north) or bearing 237 degrees. This is easily seen from Fig. 6 since Duluth is located at the end of a long extension of Lake Superior that lies along these bearings. The maximum (30. mm) is much less than the maximum with the one historical gage on Lake Erie (Buffalo at 107 mm); this is understandable since there is neither the same fetch length for Duluth as for Buffalo, nor the same depth for Superior as for Erie (shallow Lake Erie responds quickly to wind stresses). The second network (0000001010) adds Marquette to reduce the maximum error to 12 mm and shift it to bearings 6 and 186 degrees (see Fig. 7). By having the second gage also along the long axis of the lake but toward the other end of the lake, set-up errors along this axis are greatly reduced because of partial compensation (drop at one gage is offset by rise at the other); however, set-up errors almost perpendicular to this axis do not compensate and the axis for the maximum unit set-up error associated with the two-gage network in nearly due north. By adding a third gage (Thunder Bay) at the opposite end of the lake in the direction of the minor axis of the lake, the third network (0001001010) reduces maximum error a little further (to 7.1 mm) and shifts it to bearings 32 and 212 degrees. The fourth gage added (Michipicoten), being nearer to Marquette, balances the distribution of gages about the lake (0101001010), reduces the maximum error to 2.4 mm, and shifts it almost back to north. Further additions do little more to allow compensation of set-up errors, but shift the directions of the maximum error generally to an east-west orientation.

It is interesting to note that adding a gage to the four-gage network (0101001010) to get the five-gage network (1101001010) actually increases the maximum unit set-up error (from 2.4 to 3.4 mm); this also happens between the

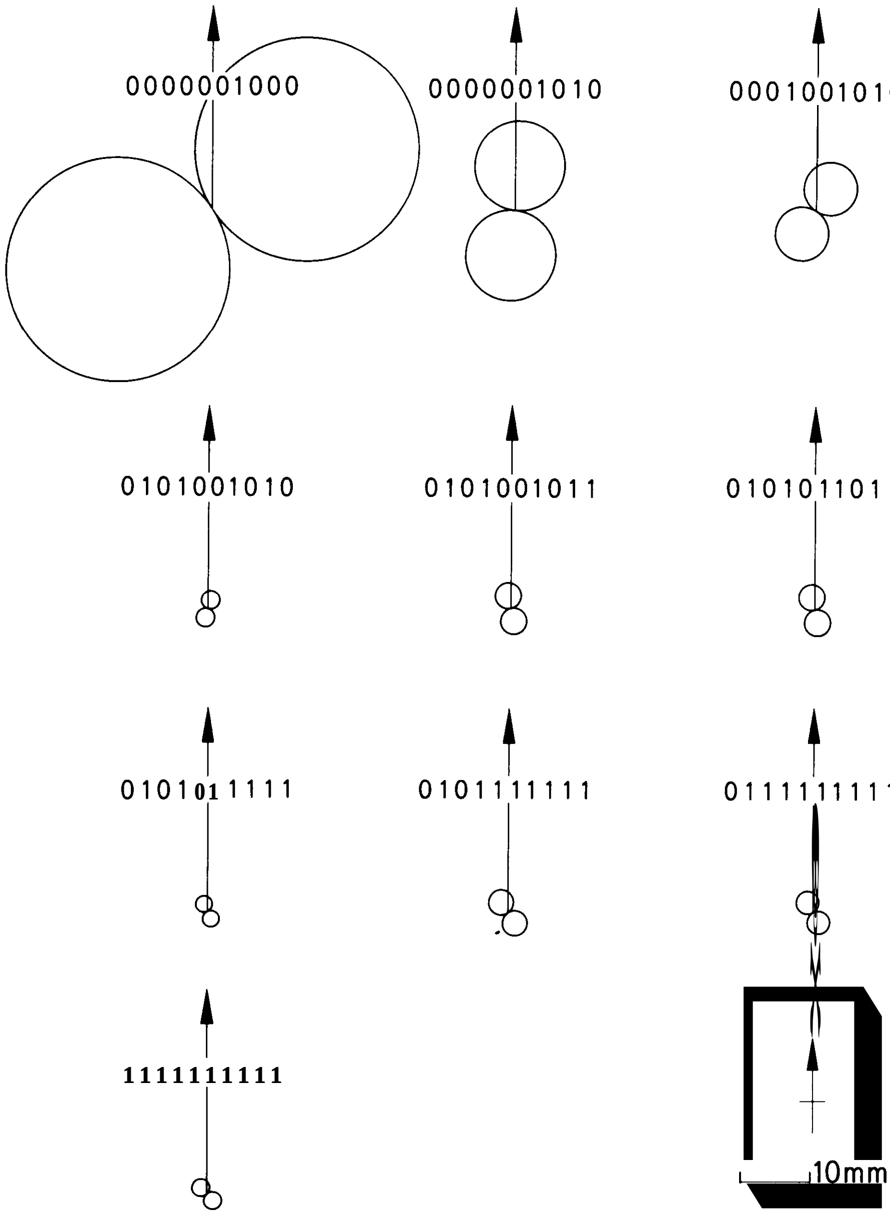

Figure 7. --Historical Thiessen network directional unit set-up error in mean Lake Superior level for two- through ten-gage networks.

seven- and eight-gage networks (1101011011 and 1101011111) and between the nine- and ten-gage networks (1101111111 and 1111111111). In these cases, the Thiessen weightings are redistributed such that set-up errors are less compensated. Another interesting aspect of these analyses is that after about four gages are present in the network (0101001010), little can be gained by adding later gages in a Thiessen-weighted network as the maximum unit set-up error is very small (2.4 mm). By adding another three gages to get the seven-gage network, we reduce the error further to only 2.1 mm; by adding yet another three gages to get the ten-gage network, we actually increase the error (2.3 mm). Looking at historical records at each of the gages instead of modeled wind set-up errors, Quinn and Todd (1974) settled on a nine-gage network (0111111111) by comparing changes in computed beginning-of-month mean lake levels that resulted with the historic networks considered in sequence as done here.

10. MINIMUM TOTAL ERROR FOR LAKE SUPERIOR

With ten gages, there are 1,023 possible networks, from 0000000001 to 1111111111. These networks were analyzed by computing the spatial-optimum estimator weights from (20) and the sum of square weights as in (27) for all networks with three or more gages. The results were searched to identify the minimum in (27); the ten best networks are identified in Table 17. The 'mean square error of estimate is estimated for the case where all gage biases are zero by (28) with daily data averaged to estimate the monthly lake level for each month of the period 1973-1984 for each of the ten gages identified in Table 16 for Lake Superior. Data in 1973 were missing at Two Harbors and Grand Marais; otherwise data availability is quite good for this period. The root-mean-square error estimate is also tabulated in Table 17.

The two gages located at Point Iroquois and Gros Cap are very close together (one in Canada and one in the United States) and both fall within the same 15-km cell in the numerical model; they both then have the same modeled set-up response to winds (see Table 16). Therefore, any three-gage network that has these two gages will not provide three independent equations [as in (17)-(19)] for the estimation of the lake level and wind stresses. This eliminates eight networks in addition to all those that have fewer than three gages. Likewise, note that networks 1111111110 and 0111111111 give the same root-mean-square error in Table 17 as do networks 0111110111 and 1111110110; in both cases, the networks differ only by the removal of either Point Iroquois or Gros Cap and the addition of the other. The root-mean-square error is estimated at values in excess of the supposed accuracy of the individual gages (6 mm); even though the long-term wind set-up error is zero for the spatial-optimum network on Lake Superior, the random gage errors mount for

TABLE 17.--Best spatial-optimum networks on Lake Superior.

Network Number	Root Mean Square Error (mm)
1111111111	7.520
111111~111	7.530
1111111101	7.599
1111110101	7.655
1111111110	7.656
0111111111	7.656
0111110111	7.665
1111110110	7.665
1111111011	7.796
1111111100	7.816

such a small network (compare the errors of the optimums for Lakes Erie and Superior as 4.6 mm for 16 gages and 7.5 mm for 10 gages, respectively).

Note again, as for Lake Erie, that eliminating one or two certain gages from the lo-gage solution entails only a small penalty in terms of the additional error that is consequent. For example, eliminating gage 4 (Duluth) increases the root mean square error by only about 0.1%; eliminating gages 2 and 4 (Marquette and Duluth) increases the root-mean-square error by only about 1.8%. As we allow more sub-optimal solutions, we can reduce the required number of gages further, as in Table 6 and Fig. 4 for Lake Erie. Again, the 1,023 possible networks were searched for p = 1, ... 10, in (30) to identify the best for each network size; the solutions are identified in Table 18 and the minimum gage count of (29) is plotted against the maximum tolerable error in Fig. 8.

The spatial-optimum network gage weights for the four- through ten-gage networks are presented in Table 19. Inspection of these weights reveals that the weights are closer to uniform for the four- through seven-gage networks than they are for the eight- through ten-gage networks. The relative differences between observed and theoretical normalized error in Table 18 reveal that little improvement can be expected by relocating the gages in the four-

TABLE 18. --Best spatial-optimum networks of each size and associated errors for Lake Superior

Number of gages	Network Number	Root-mean-square-error (mm)	Normalized error: Observed[a]	Normalized error: Theoretical[b]	Relative difference (%)
3	0100100100	12.16	0.583623	0.577350	1.087
4	0110010010	10.43	0.500357	0.500000	0.071
5	0111100010	9.374	0.449756	0.447214	0.568
6	0111110001	8.540	0.409776	0.408248	0.374
7	0111110101	7.905	0.379294	0.377964	0.352
8	1111110101	7.655	0.367295	0.353553	3.887
9	1111110111	7.530	0.361279	0.333333	8.384
10	1111111111	7.520	0.360837	0.316228	14.107

[a]Observed normalized error = $E[(\hat{L} - L)^2]^{1/2} / E[d^2]^{1/2} = (\Sigma w_i^2)^{1/2}$.
[b]Theoretical normalized error = $(1/n)^{1/2}$.

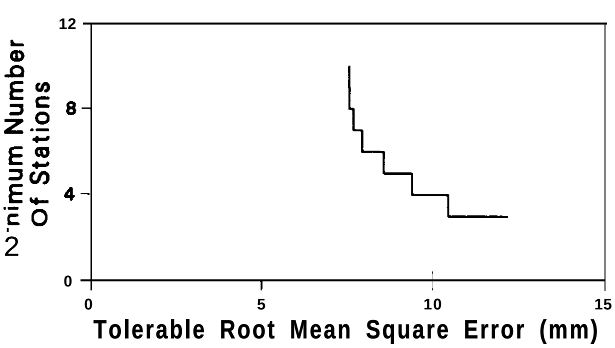

Figure 8. --Number of gages required to limit error in spatial-optimum network for Lake Superior.

TABLE 19. --Selected Lake Superior spatial-optimum network gage weights

Gage number	Location	Optimum networks						
		4-gage	5-gage	6-gage	7-gage	8-gage	9-gage	10-gage
1	Point Iroquois			0.1728	0.1301	0.0689	0.0528	0.0528
2	Marquette	0.2387	0.2406				0.0538	0.0470
3	Ontonagon				0.1311	0.1151	0.0977	0.0928
4	Duluth							0.0113
5	Two Harbors	0.2426		0.1859	0.1335	0.1389	0.1229	0.1165
6	Grand Marais		0.1905	0.1795	0.1375	0.1395	0.1279	0.1236
7	Thunder Bay		0.1856	0.1651	0.1486	0.1534	0.1532	0.1533
8	Rossport	0.2587	0.1817	0.1501	0.1599	0.1666	0.1783	0.1829
9	Michipicoten	0.2599	0.2016	0.1466	0.1592	0.1488	0.1606	0.1670
10	Gros Cap					0.0689	0.0528	0.0528

through seven-gage networks. They are already weighted close to uniformly [which (32) reveals as the best possible arrangement]. Significant improvements are possible by relocation of gages for spatial-optimum networks with the number of gages greater than 8, since the normalized error is significantly different than the theoretical; i.e., better ten-gage networks than the one considered here for Lake Superior are possible if some of the gages are relocated.

The best spatial-optimum network weights can be used to estimate the error associated with Thiessen networks; this error is given by (33), for the case where all gage biases are zero. All of these networks were analyzed to find the 1,023 sets of Thiessen weights corresponding to them; again, the Thiessen-weighting algorithm described by Croley and Hartmann (1985) and associated database management techniques (Croley and Hartmann, 1986a,b) made these calculations feasible at a resolution of 1 km^2. The 1,023 possible networks were then analyzed by using spatial-optimum network weights for all gages with data each period of the data set and appropriate Thiessen weights for all gages in each network with data from each period in (33) to compute the root-mean-square error of the Thiessen estimator. The results were searched to identify the minimum in (36); the best was found to be an eight-gage network (1111101101) with an estimated root-mean-square error of 8.680 mm. Likewise, the trade-off between number of gages and tolerable error of (37) was found by searching the 1,023 networks to determine the solutions to (37) for p = 1, ... 10; the results are identified in Table 20 and Fig. 9.

11. SPATIAL-OPTIMUM AND THIESSEN WEIGHTING DIFFERENCES FOR LAKE SUPERIOR

Again, although comparisons of spatial-optimum and Thiessen methods show only that they are different, we can look at the differences between the two methods to understand the difference magnitude and distribution. Computations of BOM spatial-mean lake levels, monthly lake storage changes, and NBS with both of the methods illustrate these differences.

BOM lake levels for all gages on Lake Superior were computed and combined by means of both spatial-optimum and Thiessen weightings to determine spatial-

TABLE 20. --Best Thiessen networks and associated errors for Lake Superior

Number of gages	Network number	Root-mean-square error (mm)
1	0001000000	21.790
2	0001000001	15.823
3	0101000010	12.943
4	0110100010	11.006
5	0110100110	9.809
6	0111100110	9.094
7	0111101101	8.902
8	1111101101	8.680
9	1111111101	9.167
10	1111111111	9.788

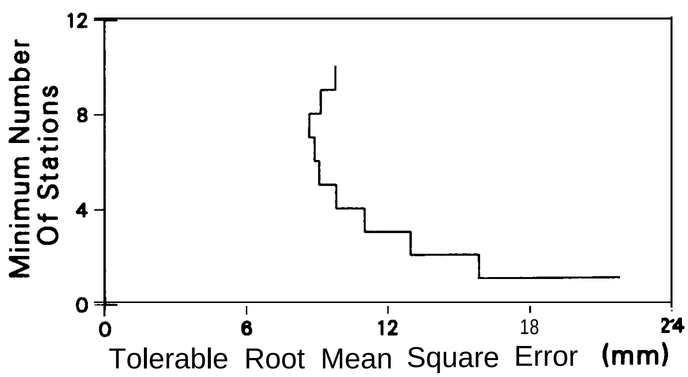

Figure 9. --Number of gages required to limit error in Thiessen network for Lake Superior.

mean BOM lake levels for the best eight-gage Thiessen network identified in Table 20 and the ten-gage spatial-optimum network identified in Table 18. The difference in BOM lake levels computed with these two methods (given in Table 21) is found by subtracting the Thiessen weighted BOM levels from the spatial-optimum weighted BOM levels. These are further subdivided with (38) into the residual and long-term wind set-up error component as was done with Lake Erie; the long-term wind set-up error is tabulated in Table 22, and the residual error is tabulated in Table 23; entries in Table 21 are equal to the corresponding entry in Table 23 minus the corresponding entry in Table 22.

Table 21 shows that the absolute sizes of differences between the two methods are fairly evenly distributed over the year but that the Thiessen weighted areal-mean BOM level most often exceeds the spatial-optimum areal-mean BOM level (118 out of 144 months have a negative value). The data in Tables 22 and 23 show that this bias corresponds to similar bias in both the long-term wind set-up error and the residual (which reflects unfiltered error components such as short-term set-up), indicating that gages are affected by wind set-up throughout the year from consistent directions. The average absolute difference between the areal-mean BOM lake levels produced by the two methods in Table 21 is 5.3 mm; the average absolute long-term set-up error is 2.9 mm (Table 22), and the average absolute residual is 3.6 mm (Table 23). These are quite a bit less than similar values for Lake Erie (Tables 9-11).

TABLE 21. --Absolute difference in BOM Lake Superior levels[a] (based on spatial-optimum and Thiessen Lake level estimates)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	11.5	-4.2	4.6	14.4	0.0	4.4	4.9	1.5	-0.6	0.1	6.7	6.9
1974	2.7	3.5	-8.3	-12.2	-8.7	-8.1	-3.2	0.0	2.9	-1.6	-11.1	-7.3
1975	-1.8	-10.8	-8.4	-8.8	-8.6	-6.2	-5.1	-2.1	-7.3	-0.5	-5.2	2.7
1976	-15.6	-7.9	-16.1	-10.0	-2.3	-4.4	-3.7	-2.8	-1.4	-2.5	-9.4	-4.2
1977	-6.5	1.4	-0.2	-5.7	-7.2	-3.4	0.0	-0.9	-1.1	-5.1	-6.4	-5.4
1978	-9.2	-7.6	-7.0	-11.4	-3.4	-5.6	-8.6	-5.4	-8.3	-6.4	1.3	-2.8
1979	-9.6	-9.7	-13.6	-10.9	-4.1	-5.6	-0.3	-2.0	-3.5	-1.9	-6.6	1.4
1980	-9.8	-11.8	-9.6	-10.0	-2.4	-5.3	-3.6	-2.7	-4.7	-4.9	4.5	-0.9
1981	-0.8	-10.1	-5.5	2.8	-5.5	-4.7	-3.1	-5.5	-9.8	-17.6	-9.5	-6.2
1982	-2.3	-11.0	-1.9	2.1	-4.9	-8.9	-2.5	-4.8	-5.7	-3.8	-3.0	-7.2
1983	-0.8	-4.1	-6.6	-10.2	-6.5	-3.4	-5.1	-1.2	-1.3	-4.6	-5.5	0.9
1984	-5.7	-1.6	-0.9	-0.5	3.6	-5.5	-2.8	-3.9	1.5	-3.1	1.5	-0.1

[a]In millimeters over the lake.

TABLE 22. --Long-term set-up error in BOM Lake Superior levels for best Thiessen network[a] (based on Spatial-Optimum Lake Levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	-7.4	8.9	1.5	-1.4	-1.4	-1.5	-2.1	-4.6	3.6	1.1	-3.8	-3.0
1974	-3.0	-6.5	1.1	2.4	2.0	-0.5	1.1	-0.6	-3.6	-3.3	5.1	2.1
1975	-2.6	4.1	0.9	2.5	3.7	0.3	3.5	0.2	2.8	-3.4	3.4	-4.0
1976	9.4	1.0	9.1	3.5	1.0	-1.9	-2.6	-2.5	-3.2	1.1	1.4	-0.8
1977	-1.2	-6.0	-2.0	2.3	2.2	-0.1	0.2	-1.0	1.0	0.4	6.7	3.9
1978	3.3	0.8	2.4	5.7	-2.7	2.3	4.4	2.9	4.5	1.9	-0.3	-0.1
1979	2.9	4.2	8.4	6.1	2.1	2.9	-1.2	0.1	5.9	0.8	8.6	-1.5
1980	5.0	5.9	3.1	6.2	0.3	2.5	2.0	1.4	2.3	2.1	-6.8	-1.2
1981	-0.0	5.6	1.4	4.9	0.8	4.5	1.1	3.7	4.1	6.2	6.2	3.7
1982	-1.5	5.1	0.7	-4.0	2.7	4.8	0.5	2.2	2.9	1.2	2.4	4.8
1983	-1.1	-0.0	2.0	5.9	1.7	0.7	3.6	-0.0	-1.9	3.5	4.4	-3.6
1984	2.3	2.1	-4.6	1.3	-6.6	4.9	1.9	2.3	0.1	1.5	-1.6	-2.0

[a]In millimeters over the lake.

Successive spatial-mean BOM lake levels were subtracted to estimate the monthly change in storage in Lake Superior; the spatial-optimum results are given in Table 24, and the differences from the Thiessen results are given in Table 25. By subtracting the Ogoki and Long Lac (input) diversions and adding the St. Marys (output) river flow to the change in storage in Table 24, the monthly net basin supply is determined (Table 26). Differences in NBS computed from lake levels based on spatial-optimum and on Thiessen weights are presented in Table 27. Differences in NBS between the two methods for Lake Superior are smallest during the summer and fairly evenly distributed in sign. The average absolute difference in NBS between the two weighting methods is 130.9 cms. Interestingly enough, the BOM lake level differences between the two methods are not as large for Lake Superior as they are for Lake Erie

TABLE 23.--Gages "random" error in BOM Lake Superior levels for best Thiessen Network[a] (based on spatial-optimum lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	4.1	4.6	6.1	13.0	-1.5	2.9	2.7	-3.1	2.9	1.2	3.0	3.9
1974	-0.2	-3.0	-7.2	-9.8	-6.7	-8.6	-2.2	-0.6	-0.7	-5.0	-6.0	-5.1
1975	-4.4	-6.7	-7.5	-6.3	-4.9	-5.9	-1.5	-1.9	-4.5	-3.9	-1.8	-1.3
1976	-6.1	-6.9	-7.1	-6.5	-1.3	-6.2	-6.4	-5.3	-4.6	-1.4	-8.0	-5.0
1977	-7.7	-4.5	-2.2	-3.4	-5.0	-3.5	0.2	-1.9	-0.1	-4.7	0.3	-1.6
1978	-5.9	-6.8	-4.6	-5.7	-6.1	-3.3	-4.2	-2.5	-3.8	-4.4	1.0	-2.9
1979	-6.7	-5.5	-5.2	-4.8	-2.0	-2.7	-1.5	-1.9	2.5	-1.1	1.9	-0.2
1980	-4.8	-5.9	-6.5	-3.8	-2.1	-2.8	-1.6	-1.2.	-2.4	-2.8	-2.3	-2.1
1981	-0.8	-4.4	-4.0	7.7	-4.7	-0.3	-2.0	-1.8	-5.8	-11.3	-3.3	-2.6
1982	-3.8	-5.9	-1.2	-2.0	-2.2	-4.2	-2.1	-2.6	-2.8	-2.6	-0.6	-2.4
1983	-1.9	-4.0	-4.6	-4.3	-4.8	-2.7	-1.5	-1.2	-3.2	-1.0	-1.1	-2.7
1984	-3.3	0.5	-5.6	0.8	-3.0	-0.5	-1.0	-1.6	1.6	-1.5	-0.1	-2.1

[a]In millimeters over the lake.

TABLE 24. --Lake Superior monthly change in storage[a] (based on spatial-optimum lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	-68	-52	63	47	163	95	47	67	-69	-46	-86	-85
1974	-62	-57	-60	108	97	109	62	63	-23	-14	-27	-93
1975	-47	-57	-64	27	76	99	16	-37	-24	-49	20	-73
1976	-88	-31	16	150	-3	52	9	-50	-88	-94	-112	-109
1977	-63	-23	84	69	33	63	81	76	156	-20	-56	-74
1978	-103	-68	-53	15	99	62	87	58	-15	108	-102	-94
1979	-74	-12	76	119	183	109	12	-28	-41	10	-65	-103
1980	-46	-69	-55	68	28	38	36	45	40	-61	-84	-102
1981	-83	5	13	112	48	114	-23	-39	-97	12	-55	-70
1982	-67	-46	-8	117	147	18	134	19	23	81	-25	-40
1983	-78	-61	-13	51	71	38	16	-4	-23	45	-18	-99
1984	-69	-50	-20	71	46	131	19	1	-31	-7	-49	-46

[a]In millimeters over the lake.

(average of 5.3 mm from Table 21 vs. average of 6.0 mm from Table 9), but the NBS differences between the two methods are larger for Lake Superior than they are for Lake Erie (average of 130.9 cms from Table 27 vs. 81.4 cms from Table 15). This is due to the consistent nature of the BOM lake level difference between the two methods for Lake Superior (uniform sign of the difference) and to the larger surface area of Lake Superior (1 mm depth on Lake Superior is equivalent to 3.2 mm on Lake Erie).

TABLE 25.--Absolute difference between Lake Superior monthly changes in storage[a] (based on spatial-optimum and Thiessen lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	-16	9	10	-14	4	0	-3	-2	1	7	0	-4
1974	1	-12	-4	4	1	5	3	3	-5	-9	4	6
1975	-9	2	0	0	2	1	3	-5	7	-5	8	-18
1976	8	-8	6	8	-2	1	1	1	-1	-7	5	-2
1977	8	-2	-5	-1	4	3	-1	0	-4	-1	1	-4
1978	2	1	-4	8	-2	-3	3	-3	2	8	-4	-7
1979	0	-4	3	7	-1	5	-2	-1	2	-5	8	-11
1980	-2	2	0	8	-3	2	1	-2	0	9	-5	0
1981	-9	5	8	-8	1	2	-2	-4	-8	8	3	4
1982	-9	9	4	-7	-4	6	-2	-1	2	1	-4	6
1983	-3	-3	-4	4	3	-2	4	0	-3	-1	6	-7
1984	4	1	0	4	-9	3	-1	5	-5	5	-2	-4

[a]In millimeters over the lake.

TABLE 26.--Lake Superior monthly net basin supply[a] (based on spatial-optimum lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	11	-177	3392	3030	6535	4572	3667	4764	1133	1195	-371	-246
1974	214	142	255	5523	5026	5814	4021	4022	1342	1610	1478	-155
1975	977	458	442	3448	5082	5611	2819	1122	1312	380	2551	-292
1976	-764	873	2417	6766	2254	3925	2499	456	-902	-1091	-1947	-1681
1977	-213	929	4289	3903	2666	3686	3921	3976	6758	2095	1323	230
1978	-1012	-156	497	2626	5336	4267	5068	4386	2273	-526	-490	-807
1979	-392	1458	4214	5652	7842	6138	3493	2333	1880	3057	538	-893
1980	661	-168	399	4278	3198	3436	3239	3464	3284	448	-354	-1277
1981	-683	2054	2243	5655	3922	6302	2140	1529	-783	1727	-505	-848
1982	-814	-295	1023	4990	6162	2625	6256	2951	3101	4949	1987	1516
1983	-153	179	2007	4019	5069	4070	3375	2832	1630	3077	1779	-713
1984	135	440	1620	4512	4119	6908	3528	3096	1885	1840	51	487

[a]Average over the month in cubic meters per second.

12. NET BASIN SUPPLY COMPARISONS

Although it has been possible only to look at the differences between the two weighting methods and the differences they induce into other computations, and although it has not been possible to determine, on the basis of these other computations, which method is "better," we can judge the relative worth of the two methods if we have independent data for comparing the computations. Net basin supply is given also as the algebraic sum of basin runoff to the lake, overlake precipitation, and overlake evaporation. If these quantities are available independently from lake level computations (i.e., overlake evaporation was not derived from NBS in a water balance), then NBS may be computed

TABLE 27.--Absolute difference between Lake Superior monthly net basins supplies[a] (based on spatial-optimum and Thiessen lake levels)

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	483	-300	-301	458	-137	-15	103	66	-23	-204	-4	127
1974	-25	400	120	-112	-17	-154	-99	-88	143	290	-121	-169
1975	278	-82	13	-7	-74	-36	-92	159	-213	144	-251	560
1976	-234	269	-188	-243	63	-21	-27	-45	38	211	-167	73
1977	-244	57	168	47	-117	-107	29	6	125	41	-31	116
1978	-51	-19	136	-256	68	96	-99	89	-62	-235	130	210
1979	2	131	-81	-215	45	-168	52	46	-51	146	-254	344
1980	61	-74	12	-239	87	-52	-30	64	5	-288	170	-0
1981	282	-155	-254	263	-24	-52	73	133	246	-249	-104	-120
1982	267	-308	-123	222	122	-202	69	27	-59	-27	135	-198
1983	101	87	110	-118	-94	54	-120	3	104	27	-203	202
1984	-123	-23	-14	-130	279	-84	33	-167	145	-140	51	119

[a]Average over the month in cubic meters per second.

also from these quantities for comparison with NBS computed from spatial-optimum and Thiessen BOM lake levels (referred to hereafter for convenience as NBSo and NBSt, respectively). Estimates of lake evaporation are available for both Lakes Erie and Superior, but there is more confidence in the Lake Superior estimates (Derecki, 1981) and they are used here for an evaluation of the two weighting methods in estimating spatial-mean lake levels. Table 28 contains NBS estimated from basin runoff and overlake precipitation and evaporation. The runoff was determined from extrapolation of all known (36) daily runoff gage records over the 34% ungaged land portion of the Lake Superior basin (Croley and Hartmann, 1986); the overlake precipitation was determined by areally averaging (with appropriate Thiessen weights each day) all known (177) daily precipitation gage records on the Lake Superior basin over the lake area (Croley and Hartmann, 1986). The overlake evaporation was determined by Derecki (1981) for the period: 1942-1975 and has since been extended to 1979 by using an improved mass transfer method based on wind, humidity, and air and water temperature records for Lake Superior, corrected for overwater

TABLE 28.--Net basin supply to Lake Superior computed from basin runoff, overlake precipitation, and overlake evaporation[a]

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	-144	-356	2839	3547	6433	5110	4558	4733	2108	1941	-37	-1267
1974	35	256	507	4596	5599	6070	4399	4609	1885	1522	1896	-430
1975	724	697	556	3178	5266	6128	3460	2289	1691	774	1561	-1386
1976	322	994	2246	5803	3122	4335	2830	1506	-104	-1219	-2012	-992
1977	-700	-13	3541	4260	3038	3900	4290	4591	6264	1545	9	-670
1978	-508	287	564	2765	5083	4453	5290	4453	2808	377	-777	-742
1979	-1036	1851	3898	4506	8067	6225	3897	3304	2420	2094	-1124	-1129

[a]Average over the month in cubic meters per second.

conditions. Although Derecki compared this evaporation with that obtained from a water balance for the lake, he did not use the water balance in determining this evaporation or the coefficients used in determining this evaporation.

The absolute difference between NBS of Table 28 and NBSo (contained in Table 26) is given in Table 29; the absolute difference between NBS and NBSt is given in Table 30. Both differences are variable, but the spatial-optimum weightings for BOM lake levels yield NBS values closer to those based on the sum of runoff and overlake precipitation and evaporation in Table 28. The average NBS in Table 28 is 2253 cms; the average NBS based on spatial-optimum weightings for BOM lake level determinations (NBSo) is 2189 cms; the average NBS based on Thiessen weightings (NBSt) is 2181 cms. The average absolute difference between NBS and NBSo (contained in Table 29) is 502 cms and the average absolute difference between NBS and NBSt is 521 cms. On the basis of these rather limited data, the spatial-optimum estimator appears to give slightly better agreement with independent determinations of NBS. Of course, no corrections were made to Lake Superior lake levels to account for thermal expansion or contraction, and this rather close comparison of the two methods may depend on that and other corrections.

TABLE 29.--Absolute difference between net basin supply computed from basin runoff and overlake precipitation and evaporation, and that computed from spatial-optimum BOM lake levels[a]

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	155	179	553	517	102	538	891	31	975	746	334	1021
1974	179	114	252	927	573	256	378	587	543	88	418	275
1975	253	239	114	270	184	517	641	1167	379	394	990	1094
1976	1086	121	171	963	868	410	331	1050	798	128	65	689
1977	487	942	748	357	372	214	369	615	494	550	1314	900
1978	504	443	67	139	253	186	222	67	535	903	287	65
1979	644	393	316	1146	225	87	404	971	540	963	1662	236

[a]Average over the month in cubic meters per second.

TABLE 30.--Absolute difference between net basin supply computed from basin runoff and overlake precipitation and evaporation and that computed from Thiessen BOM lake levels[a]

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1973	328	479	854	975	239	524	994	35	952	543	329	894
1974	204	514	372	1039	556	102	278	499	687	201	298	444
1975	24	156	127	277	110	481	548	1326	166	538	1241	534
1976	852	390	359	1206	931	389	304	1005	835	83	232	761
1977	731	886	580	404	254	107	397	621	368	510	1346	783
1978	453	424	203	117	185	283	123	156	473	668	157	275
1979	642	524	397	1361	270	81	456	1017	489	817	1916	108

[a]Average over the month in cubic meters per second.

13. SUMMARY

The error of estimation of the spatial mean lake level can be made small by selecting an appropriate network of monitoring lake-level gages and an appropriate lake-level estimator. Comparisons of different networks and spatial averaging techniques for existing Lake Erie and Lake Superior gages reveal that long-term wind set-up errors exist in past estimates of lake levels that were made with the historical networks and Thiessen weights. The unit set-up errors in lake level estimates (that result from unit wind stresses) are a function of stress direction for any Thiessen network and range historically on Lake Erie up to 107 mm for 1887-1900, up to 21-25 mm for 1900-1927, up to 11-16 mm for 1927-1960, and up to 4.4-7.8 mm for 1960-1986 (see Table 1 and Fig. 2). For Lake Superior, they range historically up to 30.7 mm for 1885-1891, up to 12 mm for 189-L-1912, up to 7.1 mm for 1912-1915, up to 2.4 mm for 1915-1926, and generally below 3.5 mm for 1926-1986 (see Table 16 and Fig. 7). A minimization of wind set-up error for the summer wind stress data of 1979 on Lake Erie revealed that actual set-up errors can be kept small with an eight-gage Thiessen network. However, wind set-up errors cannot be eliminated from the estimation of lake levels with Thiessen networks for even the mild conditions of this data set.

By removing the constraint that Thiessen weights be used, it is possible to eliminate long-term set-up errors. This allows the minimization of total errors to proceed by selection of spatial-optimum estimators; these estimators have the property that long-term set-up error in the estimate is zero and other errors are minimized. The best spatial-optimum estimator on Lake Erie uses all 16 of the present gages to give zero set-up error in the lake level estimate and a total error (by assuming no gage biases) of about 4.6 mm (see Table 5). The best Thiessen network on Lake Erie consists of 14 of the 16 gages and gives a total error (again by assuming no gage biases) of about 5.4 mm (see Table 7). Both of these are within the publicly acknowledged error range, reported by collection agencies, of 0.02 ft (6.096 mm); but the spatial-optimum estimator reduces the error of the best Thiessen network by another 17%. The best spatial-optimum estimator on Lake Superior uses all 10 of the present gages; the total error, assuming no gage biases, is about 7.5 mm (see Table 17). The best Thiessen network on Lake Superior consists of 8 of the 10 gages and gives a total error, by again assuming no gage biases, of about 8.7 mm (see Table 20).

Trade-offs between lake-level estimation error and network size reveal that the best spatial-optimum estimator, as a function of allowable network size, is uniformly superior to Thiessen estimators (see also Figs. 3 and 8). The trade-offs allow consideration of the expense (in terms of network size) associated with estimation error reduction; e.g., a network of 11 gages on Lake Erie can be used with the spatial-optimum estimator to achieve lower estimation errors than those associated with the current Thiessen estimator and current network (or with any Thiessen network). Likewise, use of the spatial-optimum estimator on Lake Erie with only eight gages at Sturgeon Point, Barcelona, Erie, Cleveland, Marblehead, Fermi, Kingsville, and Port Colborne, results in an estimation error of less than 0.02 ft. On Lake Superior, a network of seven gages can be used with the spatial-optimum estimator to achieve lower estimation errors than those associated with the best Thiessen network; this is a savings of only one gage as the best Thiessen network consists of eight gages.

Two optimizations are made in the selection of the best network of gages to use on Lake Erie or Superior for estimation of spatially averaged lake levels. The first is a minimization of the spatial sum of squared errors in a least-squares regression to determine the spatial-optimum estimators of lake level and wind stress [see (17)-(19)]. The optimum weights to be used with gage readings in a spatial average are observed to be about as good as can be for network sizes of 3 to 12 gages on Lake Erie and 4 to 7 gages on Lake Superior (as now located) since the network gage weights are very close to uniform [see (31) and (32)]. Improvements in larger networks (more than 12 gages on Lake Erie and more than 7 gages on Lake Superior) are possible by relocating the gages such that the modeled wind stress responses at these gages yield spatial-optimum weights close to uniform also.

The second optimization is a minimization of the error of estimate by selection of an appropriate spatial-optimum network; on both lakes, all present gages were found to comprise this network. The derivation of the spatial-optimum estimators (in the first optimization) and the error of estimate is given for any set of gage biases. These biases need not be known to select the network that minimizes the error of estimate (in the second optimization); however, they must be known to estimate the equivalent level-pool lake level, wind stress, and error of estimate. Gage biases cannot be estimated from gage level data alone; their estimation requires additional information such as the correct levels at one or more of the gages. Gage biases were taken as zero here in the estimation of errors associated with lake level estimates for the optimizations and trade-off analyses of both spatial-optimum and Thiessen networks. When information is available to determine gage biases, the derivations of lake level estimates [(17) and (24)] and errors of estimates [(23) and (25)] can be used as the basis for constructing estimators that include these biases.

There are other errors in computing spatially-averaged lake levels besides the ones considered here [see (15)], particularly for shorter-time-period temporal averages or instantaneous values that are largely filtered out in monthly averages. The errors calculated here do not reflect these other errors, and consideration of these other errors would lead to different techniques in estimation of spatially averaged lake levels for short time periods. However, use of the spatial-optimum lake level estimators, derived herein, would still reduce error associated with existing Thiessen networks to the extent that the random error in lake level estimates is reduced and the long-term wind set-up error in the estimate is eliminated.

Although comparisons of spatial-optimum and Thiessen weightings applied to lake level data sets reveal only differences between the methods, these differences correspond to what is expected from using spatial-optimum estimators instead of current practice. Average absolute errors in computed BOM lake levels for the period 1973-1984 are 6.0 mm for Lake Erie and 5.3 mm for Lake Superior (see Tables 9 and 21). Errors in computed spatial-mean BOM lake levels are translated into other quantities, such as monthly storage changes in a lake and computed NBS. The average absolute differences in NBS for the two weighting methods are 81.4 cms for Lake Erie and 130.9 cms for Lake Superior.

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