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Recycler Closed Orbit Distortion Expected from Random Magnetic Element Strength and Alignment Errors

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The Recycler Ring is constructed from 344 combined function and 88 discrete quadrupole magnets. Magnet-to-magnet variations in strength and alignment will result in a Recycler closed orbit that differs from the design central orbit. The purpose of this note is to calculate the relationship between the expected closed orbit distortion and expected randomly distributed strength and alignment errors.

Because the distribution of errors is random, and therefore not known a priori, one can only calculate a range of closed orbit errors that might be expected at a particular point around the circumference of the Recycler. We calculate for an ensemble of error distributions the rms of the collection of correspondingly generated orbit distortions at any point within the ring. The rms of this collection of orbit distortions is denoted σ_X. The rms distribution of closed orbits can be calculated by applying a simple statistical argument to the standard closed orbit formula. The result is:

$$\frac{\sigma_x^2(s)}{\beta_x(s)} = \frac{1}{8\sin^2(\pi\nu_x)}\sum_i \beta_{x_i}(\sigma_{\theta_i})^2 \qquad (1)$$

where σ_{θ_i} is the rms bending angle error through the i^{th} element. For a dipole magnet σ_{θ_i} is $\sigma_{BL}/(B\rho)$ (horizontally) or $\sigma_\phi \times BL/(B\rho)$ where ϕ is the roll angle (vertically). For any magnet containing a field gradient, σ_{θ_i} is just $\sigma_d \times B'L/(B\rho)$ where d is the transverse displacement.

The sensitivity of lattice RRV6 to random strength and alignment variations has been evaluated using expression (1). Results for $\sigma_x/\sqrt{\beta}$ are given in Table 1 below for the corresponding errors as noted. The interpretation is that at any given point in the ring, s, there would be a ~2/3 chance of observing an orbit distortion of $<(\sigma_x/\sqrt{\beta})\times\sqrt{\beta(s)}$ if magnetic elements were fabricated and aligned to the noted tolerances.

A total rms orbit distortion at any point due to a different set of errors can be calculated by noting that distortions are linear in the corresponding errors and that all distortions add in quadruature. Note also that the rms orbit distortion around the entire ring can be calculated by multiplying the coefficients in the table by $\sqrt{<\beta>}$. For example, a magnet-to-magnet (rms) strength variation of 2×10^{-4}, accompanied by an rms displacement error of 0.25 mm, and an rms roll angle of 0.5 mrad would produce an rms orbit distortion of 6.5 mm as measured at the horizontal BPMs (β=54 m), and 7.8 mm as measured at the vertical BPMs (β=54 m). In both planes the orbit error is dominated by the displacement error contribution.

Conclusions

Table 1 shows that transverse alignment errors dominate for the range of strength and alignment tolerances that could probably be achieved. This has consequences on the required magnet-to-magnet uniformity required from the gradient magnet production. For example, if we believe that magnets will be aligned transversely with an rms error of 0.25 mm, then an rms magnet-to-magnet strength variation of 3.7×10^{-4} contributes only 10% to the expected closed orbit error. In this case there is no real reason to shuffle magnets as alignment errors will dominate. Likewise, this analysis shows that for an rms alignment error of 0.25 mm achieving a distribution of roll angles with rms < 0.38 mrad will contribute only an additional 10% to the closed orbit.

Table 1: Recycler lattice RRV6 rms orbit errors generated by various magnet strength and alignment variations.

	Orbit Error ($\sigma_x/\sqrt{\beta_x}$)	Orbit Error ($\sigma_y/\sqrt{\beta_y}$)
Gradient Magnet (4.1m)		
integrated dipole strength σ_{BL}/BL=.0001	9.6×10^{-5} $m^{1/2}$	
transverse displacement σ_d=.00025 m	5.9×10^{-4} $m^{1/2}$	6.2×10^{-4} $m^{1/2}$
roll σ_ϕ=.0001		9.9×10^{-5} $m^{1/2}$
Gradient Magnet (2.7m)		
integrated dipole strength σ_{BL}/BL=.0001	4.4×10^{-5} $m^{1/2}$	
transverse displacement σ_d=.00025 m	5.4×10^{-4} $m^{1/2}$	5.6×10^{-4} $m^{1/2}$
roll σ_ϕ=.0001		4.6×10^{-5} $m^{1/2}$
Quadrupole magnets		
transverse displacement σ_d=.00025 m	3.2×10^{-4} $m^{1/2}$	3.4×10^{-4} $m^{1/2}$