Generation of mixed-element meshes using a flexible refinement approach: Advantages and limitations

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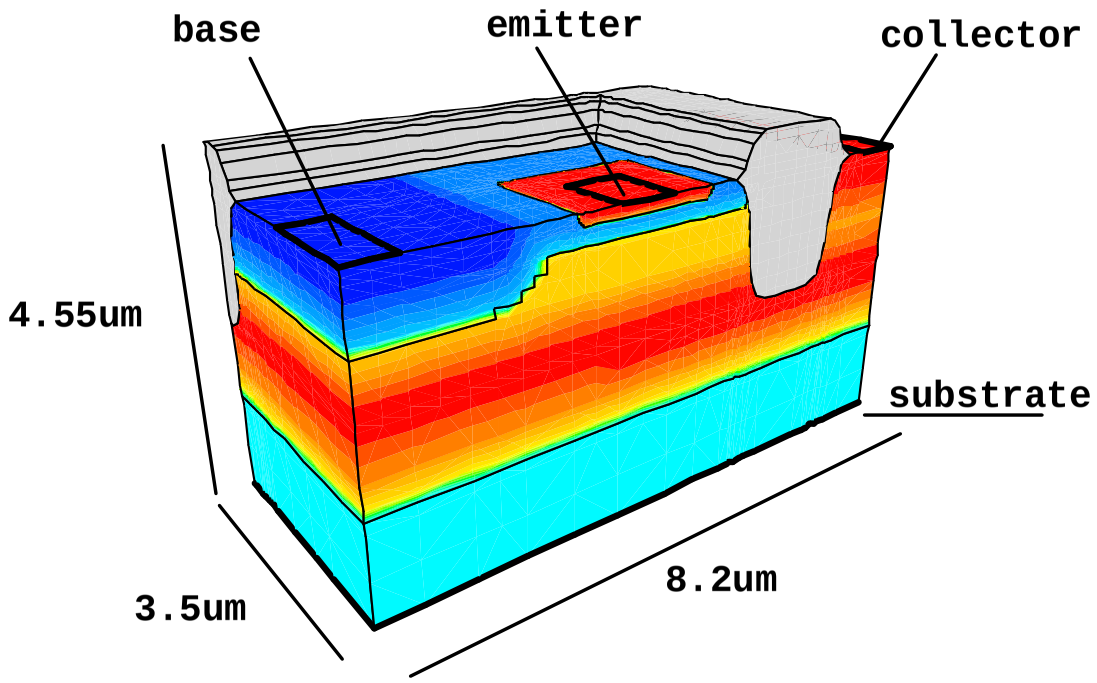
Overview

- Introduction
- Basic strategy: Element set and refinement strategy
- Mixed-element mesh generation algorithm
- Modeling Complex examples
- Advantages and Limitations

Introduction: motivation

3-D Mesh generation for polyhedral geometries



Introduction: original motivation (Cont.)

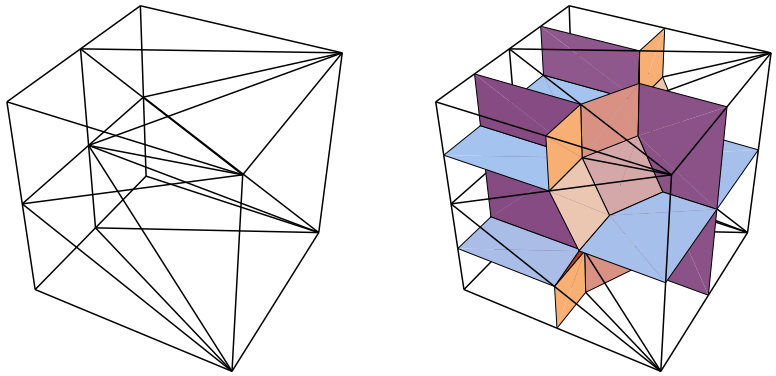
Condition: appropriate for the control volume method

Desired Mesh Properties:

- Delaunay mesh
- no Voronoi point outside a region boundary
- fit geometries as exact as possible
- have as few points as possible

Introduction: original motivation (Cont.)

Definition 1 *A tessellation T of a set of points is a Delaunay tessellation if there exists a point-free circumsphere for each tessellation element*



Small Delaunay mesh and its Voronoi diagram

Introduction: Modified octrees approach

- Geometry enclosed by a cuboid
- Use of cubes as internal nodes
- Refinement into eight similar cubes
- Tetrahedra as final elements

Basic strategy

- Domain enclosed by a cuboid
- Different types of internal nodes: cuboids, prisms, pyramids and tetrahedra
- Refinement only in the required direction
- Refinement at any position
- Different element types as final elements

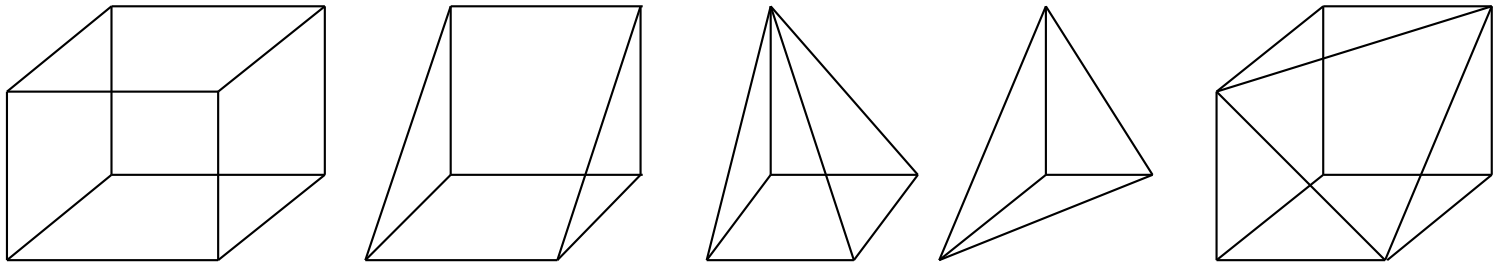
Basic strategy (cont.)

Context

- Domain geometry represented using a set of general polyhedra
- Tessellation of the general polyhedra into basic elements that fit the geometry
- Density requirements satisfied using iterative refinement
- Final mesh composed of several elements such as: tetrahedra, pyramids, prisms and bricks

Set of macro elements

Theorem 1 *Let P be a set of convex polyhedra. P leads to Delaunay meshes if each polyhedron $p \in P$*
(i) has co-circular vertices
(ii) can be refined in such a way that all newly generated polyhedra also belong to P (P is closed).



Brick

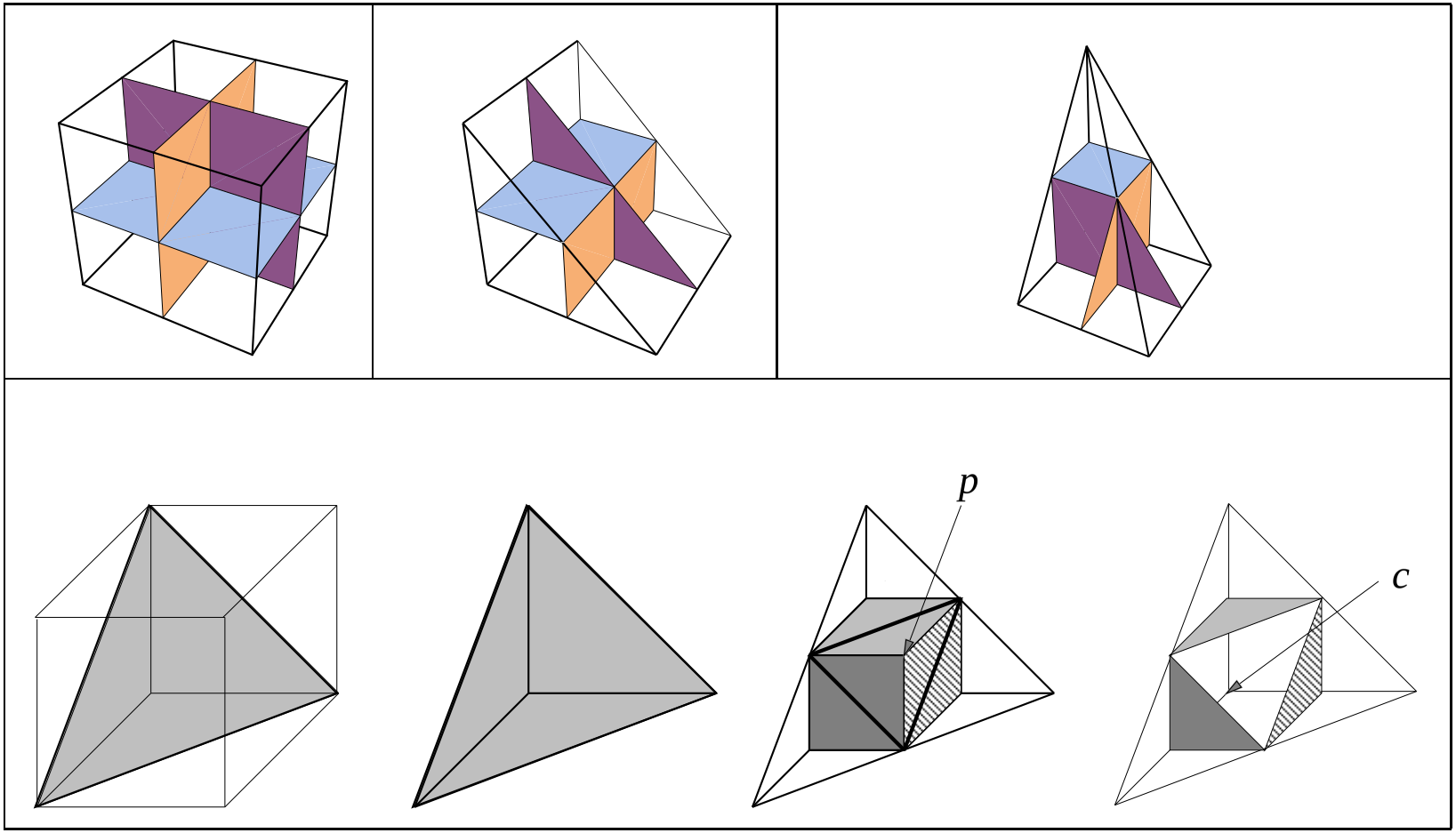
Rectangular Prism

Rectangular Pyramid

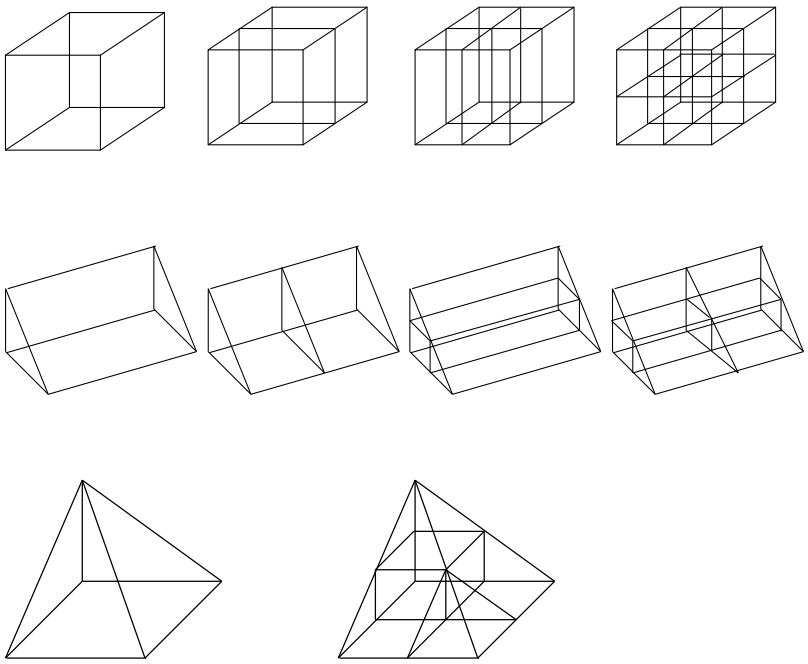
Rectangular Tetrahedron

Rectangular Tetrahedron complement

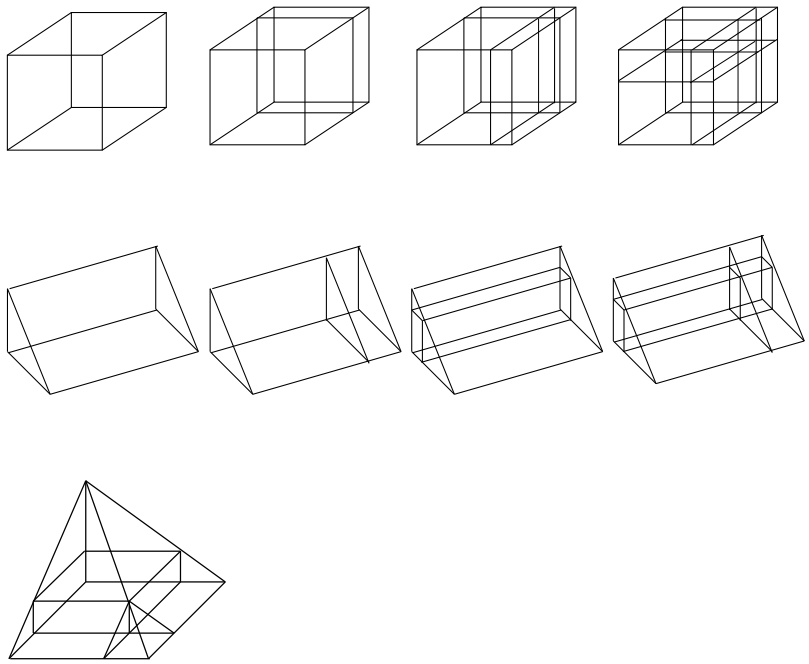
Element set and their Voronoi region



Element Refinement



Bisection approach



Flexible-refinement approach

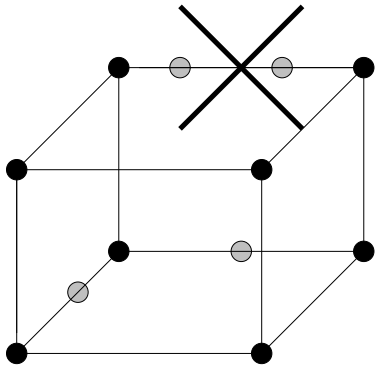
Characterization of 1-irregular elements

Definition 2 *Let l be a 1-irregular macro-element. l is a well-shaped if no Voronoi point of l lies outside its convex hull (in this case, the 1-irregular macro-element itself).*

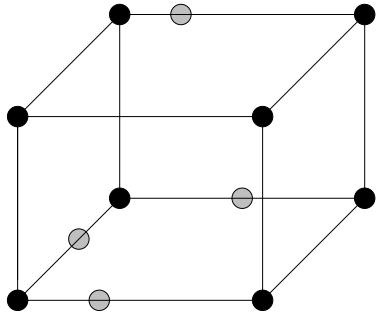
Theorem 2 *Let $S \subset R^n$, n $\leq$ 3 be a set of points, C the convex hull of S, and T a Delaunay tessellation of S. Then no Voronoi point of S lies outside C if and only if for each face f_{ijk} in 3-D of T on the surface of C, the circumsphere of f_{ijk} with the center in the middle of f_{ijk} is point-free.*

1-irregular element partition

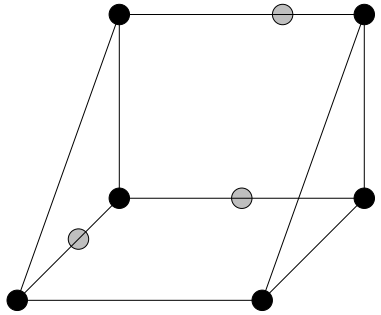
1-irregular condition



2-irregular element

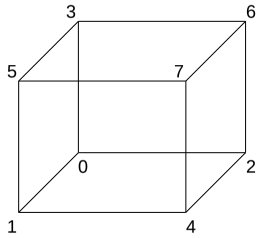


1-irregular element

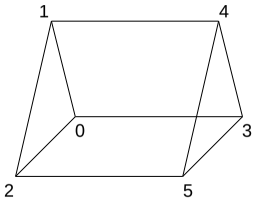


1-irregular element

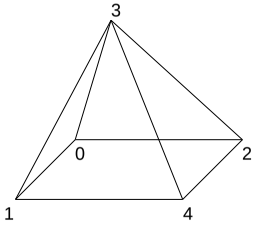
Current set of final elements



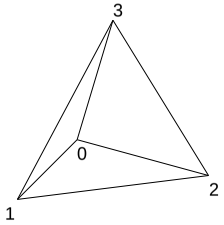
(a)



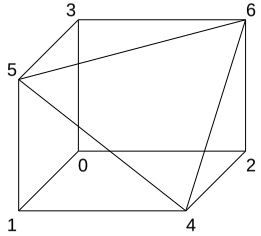
(b)



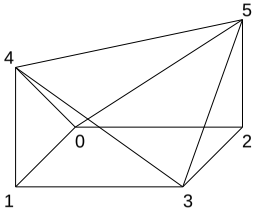
(c)



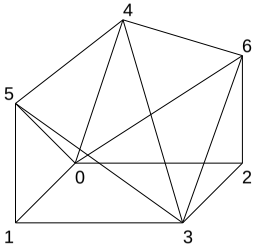
(d)



(e)



(f)

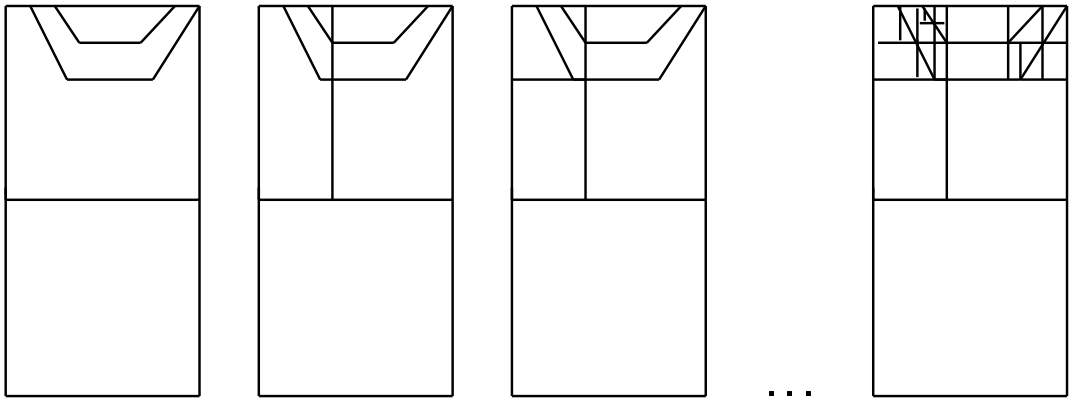


(g)

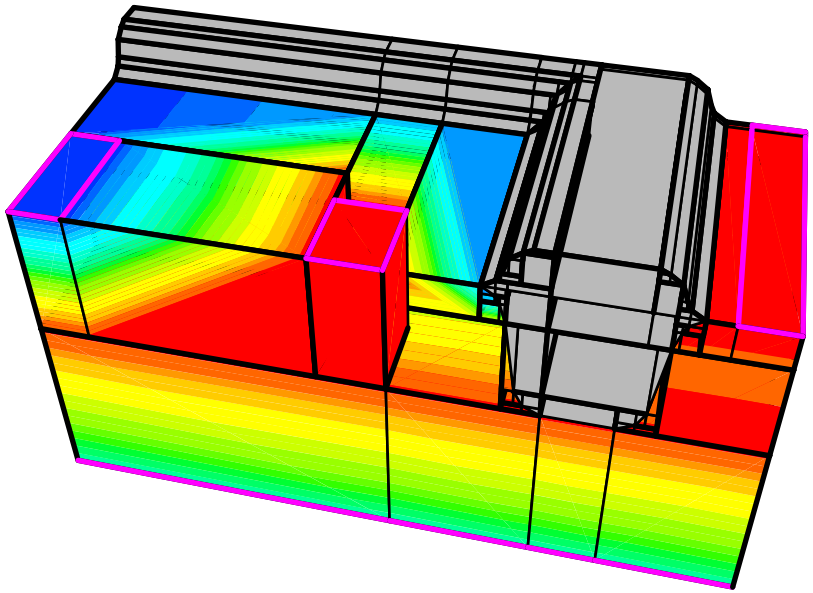
Mixed-element mesh generation algorithm

- Generation of an initial macro-mesh using
- Mesh density using iterative refinement
- Generation of a proper 1-irregular mesh
- Final mesh

Mixed element algorithm: generation of the macro mesh

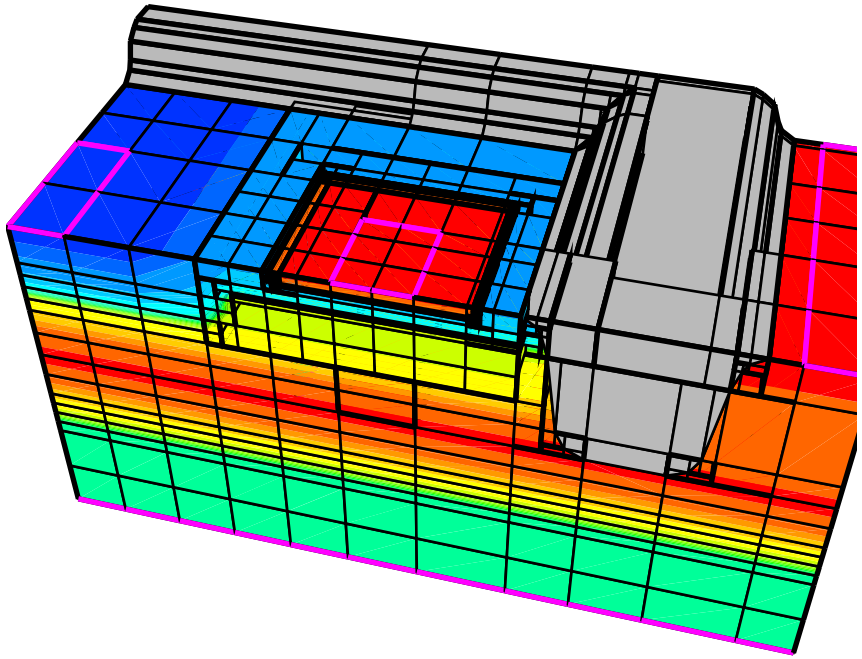


Mixed element algorithm: generation of the initial macro



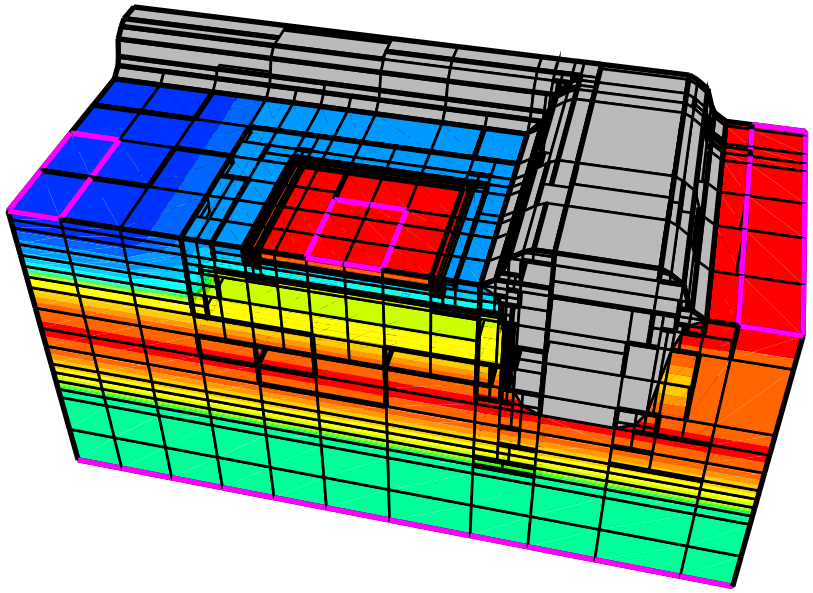
Fitting the device geometry: 554 points

Mixed element algorithm: Fulfilling the mesh density



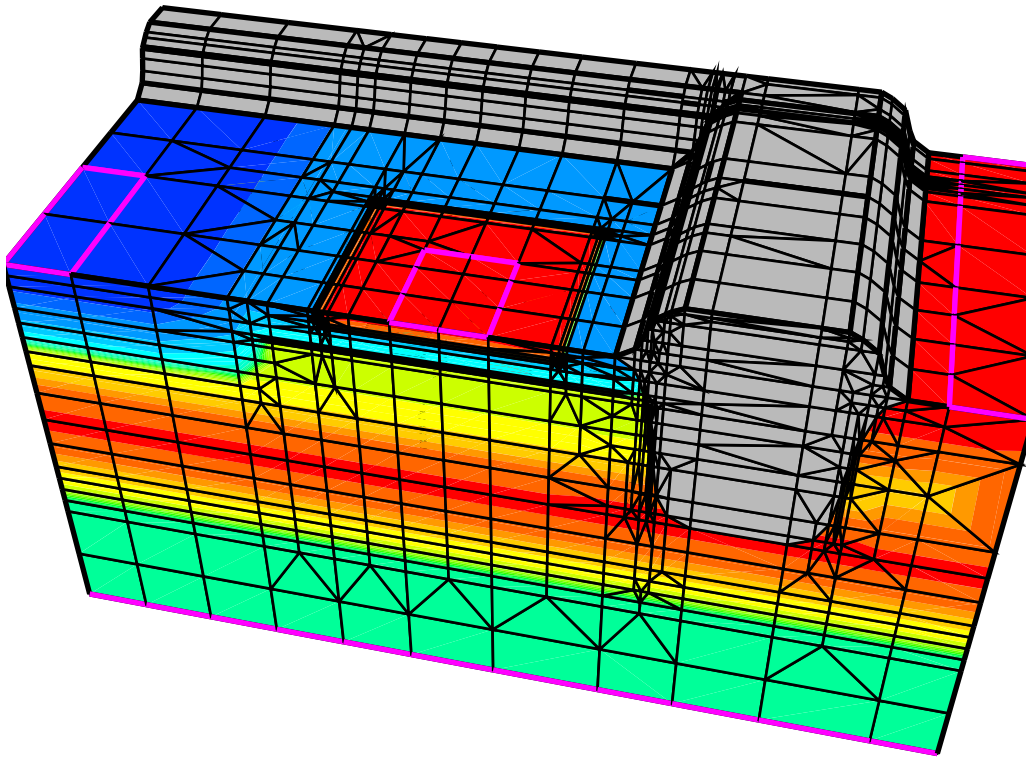
Getting the desired mesh density: 3030 points

Mixed element algorithm: generating a 1-irregular mesh



Making the mesh density 1-irregular: 6230 points

Mixed element algorithm: generating a final mesh (Cont.)



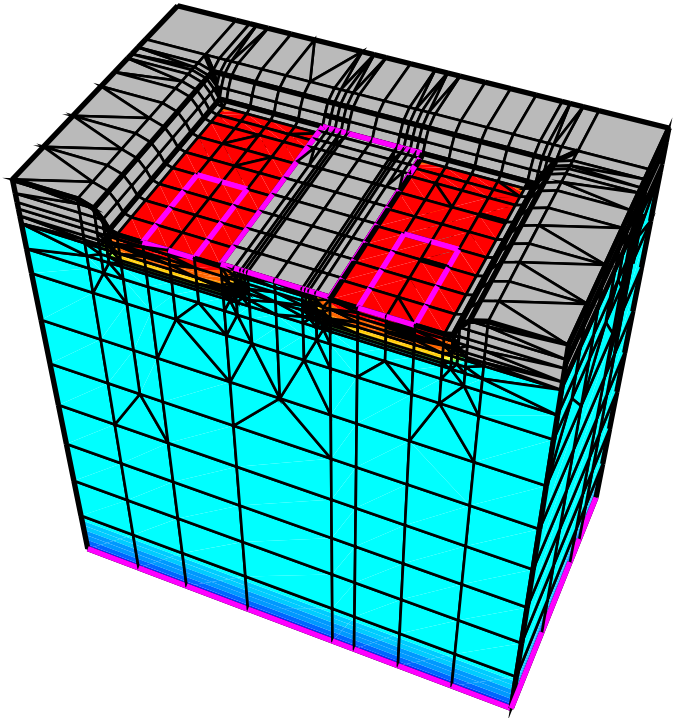
Final mesh: 11,403 points and 16,509 elements

Mixed element algorithm: generating a final mesh (Cont.)

	Cuboid	Prism	Pyramid	Tetr.	Tetr. Compl.	Deformed Prism	DBC	Total
ecl	1418	2029	7880	4612	570	0	0	16509
locos	701	1089	3471	1912	187	0	0	7630

Number of terminal elements

Another example



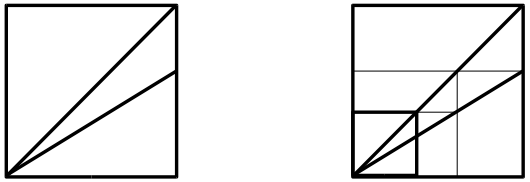
Final mesh: 4939 points

Critical parts

- Detection of fractal configurations
- Generation of proper 1-irregular elements
- Recognition of final elements from co-spherical point configurations

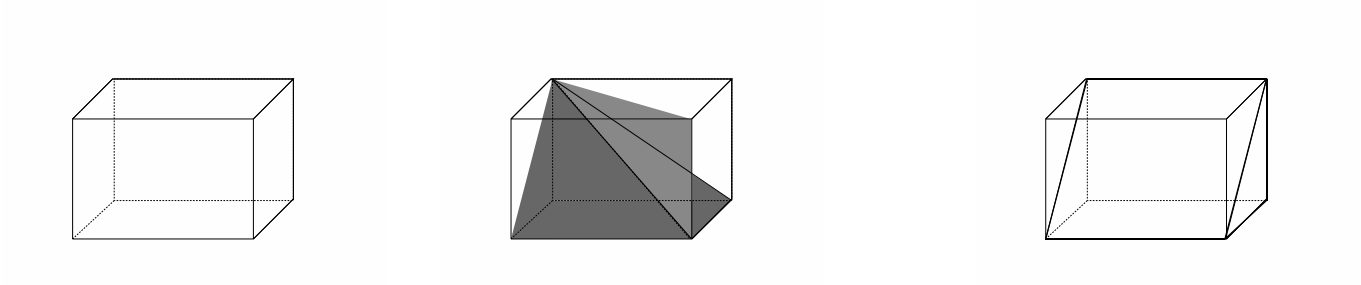
Critical parts: Detection of fractal configuration en 2D

Terminal configurations

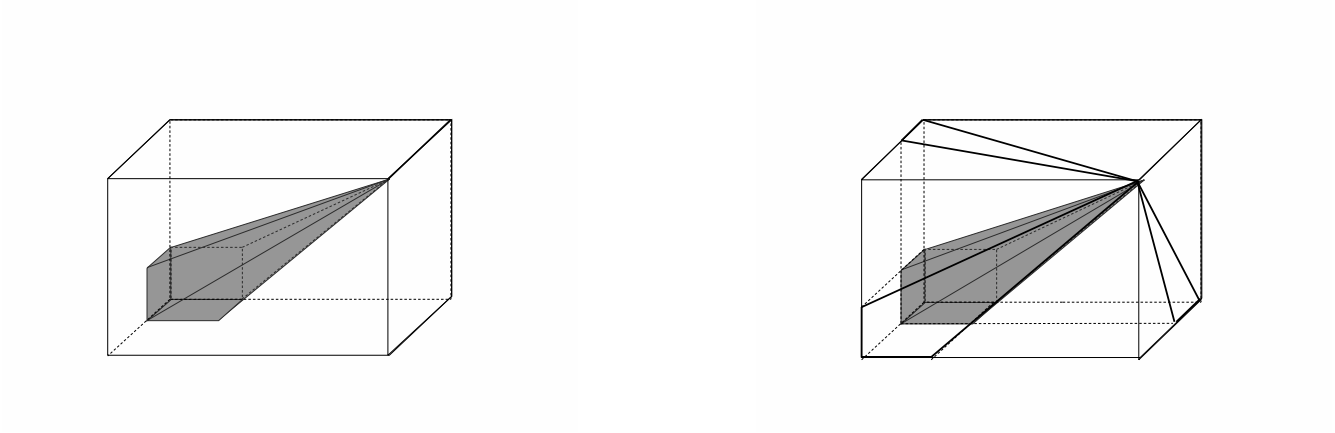


Fractal configuration

Critical parts: Detection of fractal configuration en 3D



Terminal configurations



Fractal configuration

Critical parts: (cont.)

- Generation of proper 1-irregular elements
  - Strategies are locally decided
  - element refinement
  - specific edge point insertions
- Recognition of final elements from co-spherical point configurations
  - configurations and tessellations stored in a table (pattern-wise)
  - algorithm to tessellate the rest of the configurations

Good characteristics

- A flexible refinement allows:
  - a quick and efficient fitting of the device geometry
  - to increase the point density in the required direction
- The use of several of final elements reduces:
  - the number of final elements and edges
  - too small and large dihedral angles
  - vertex-edge connectivities

Limitations

Limitations of the method:

- Fit better well-oriented geometry boundaries
- Model better well-oriented flows

The current implementation needs to be improved in :

- the quality of the boundary elements inside fractal configurations
- the strategy used to generate proper 1-irregular elements
- the robustness of the algorithm that recognizes final elements