# Structure Change of Cooper Pairs in Color Superconductivity

## – Crossover from BCS to BEC ? –

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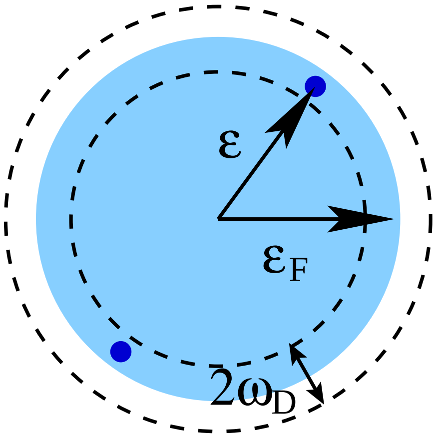
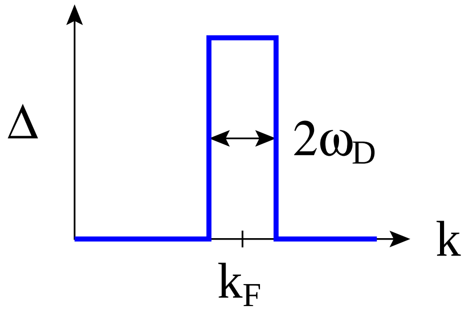
1. Structural change of Cooper pairs and momentum-dependent gap in color superconductivity
   H.Abuki, T.Hatsuda and K.I. PRD65 ('02) 074014
2. Crossover from BCS to BEC in 2 color QCD
   G. Baym, T.Hatsuda and K.I., in preparation

# Difference from the usual BCS in metals

- BCS superconductivity in metals

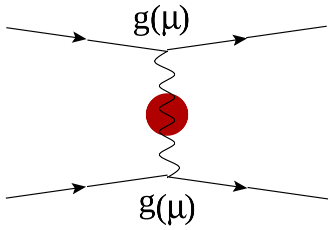
  Attractive force between electrons is effective for $|\epsilon - \epsilon_{\rm F}| < \omega_{\rm D}$ ("Debye cutoff"), and is very weak.




- Color superconductivity in Quark Matter

  Attractive force between quarks is effective for all quarks in the Fermi sea and becomes large as the density decreases.




$$g(\mu) \to 0 \quad \text{as} \quad \mu \to \infty$$

Non-BCS like coupling dependence:

$$\Delta(\mu) \sim \mu \exp\{-c/g\} \quad \Leftarrow \text{magnetic gluons}$$

# We study

- How color superconductivity changes as the density decreases ?

  High density: weak-coupling picture based on QCD
  Low density: effective description based on 4Fermi models

  $\Rightarrow$ want to connect high and low densities by using a single model

- "Structure" of the Cooper pairs and its change

  wave function, size of the Cooper pairs

  $\Leftarrow$ momentum dependence of the gap function

- Departure from the weak-coupling BCS picture ?

  Large coupling allows to form a bound state of quark pair which induces transition from BCS to "strong coupling superconductivity" or "BEC"?

1. Detailed study of the Cooper pairs:
   $\Rightarrow$ Two flavor, 3 color QCD
2. Transition from BCS to BEC:
   $\Rightarrow$ Two flavor, 2 color QCD

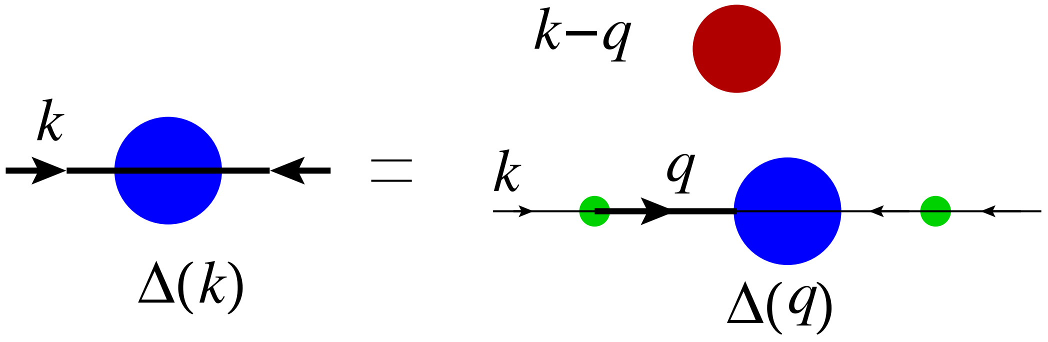
# Momentum dependent gap equation

Nambu Gor′kov rep. $\Psi = \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}, \quad \psi^c = C\bar{\psi}^T$

$$\begin{aligned} iS_F(x-y) &\equiv \langle T\Psi(x)\bar{\Psi}(y)\rangle \\ &= \begin{pmatrix} \langle T\psi(x)\bar{\psi}(y)\rangle & \langle T\psi(x)\bar{\psi}^c(y)\rangle \\ \langle T\psi^c(x)\bar{\psi}(y)\rangle & \langle T\psi^c(x)\bar{\psi}^c(y)\rangle \end{pmatrix} \end{aligned}$$

Schwinger-Dyson equation for self energy $\Sigma = S_0^{-1} - S^{-1}$

$$\Sigma(k) = \begin{pmatrix} M(k) & \Delta(k) \\ \tilde{\Delta}(k) & M(-k) \end{pmatrix} = \int \frac{d^4q}{(2\pi)^4}\, g^2 \Gamma^a_\mu S_F(q) \Gamma^a_\nu D^{\mu\nu}(k-q)$$




$$\Gamma^a_\mu = \begin{pmatrix} \gamma_\mu T^a & 0 \\ 0 & -\gamma_\mu (T^a)^T \end{pmatrix}$$

Perform the angular and frequency integrals

$\Rightarrow$ Momentum dependent gap equation

# Gap equation: $N_f = 2,\ N_c = 3$

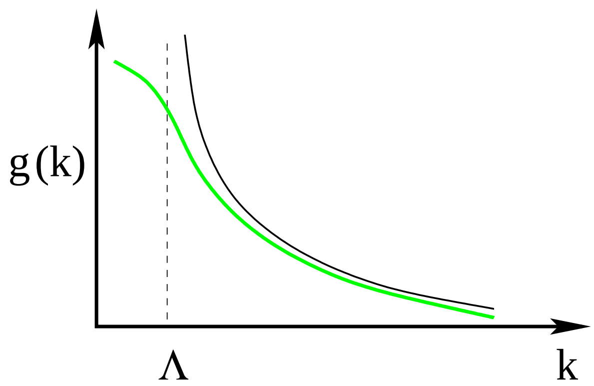
with H. Abuki and T. Hatsuda

- Gap structure $\quad \Delta \sim \epsilon_{ij}\epsilon_{ab3}\langle \psi_i^a C\gamma_5 \psi_j^b \rangle$
- Ignore the mass function $M(k) = 0$
- Include both magnetic and electric gluons

$$D_{\mu\nu}(p) = -\frac{P^T_{\mu\nu}}{\mathbf{p}^2 + i(\pi/4)m_D^2|p^0/\mathbf{p}|} - \frac{P^L_{\mu\nu}}{\mathbf{p}^2 + m_D^2}$$

- Include antiquark pairing
- Momentum dependent coupling (Higashijima, Miranski)
  $\Leftarrow$ taming the IR behavior $\ p_c^2 = 1.5\Lambda_{QCD}^2$

$$g^2(q,k) \propto \frac{1}{\ln[(p_{max}^2 + p_c^2)/\Lambda_{QCD}^2]}, \quad p_{max} = \max(q,k)$$



# Gap equation: $N_f = 2$, $N_c = 2$

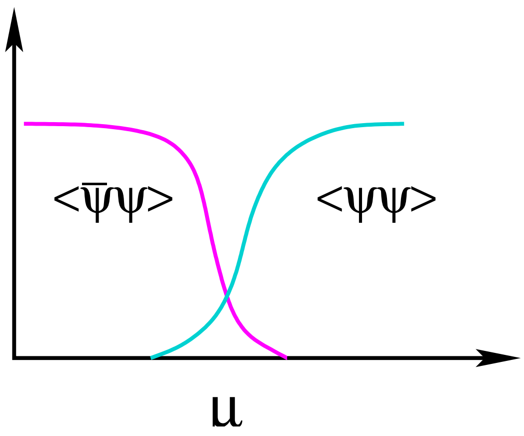
with T. Hatsuda and G. Baym

- Diquark condensate is gauge invariant ("baryon")

$$\Delta \sim \epsilon_{ij}\epsilon_{ab}\langle \psi_i^a C\gamma_5 \psi_j^b \rangle$$

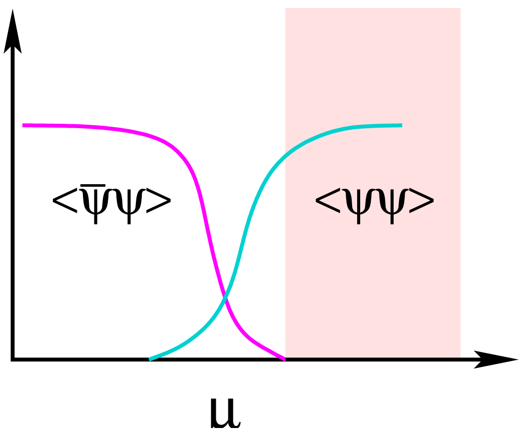
- Retain $M(k) \neq 0$ $\Rightarrow$ Chiral condensate
- $\Rightarrow$ Need to solve coupled equations for $M(k)$ and $\Delta(k)$ to study moderate density.

Naively, we expect




$\Rightarrow$ Treat high and low densities separately

# 2 color QCD at high density




$M(k) = 0 \quad \Rightarrow \quad$ similar to SU$_c$(3) case

Gap equation for $\Delta(p = |\mathbf{p}|)$

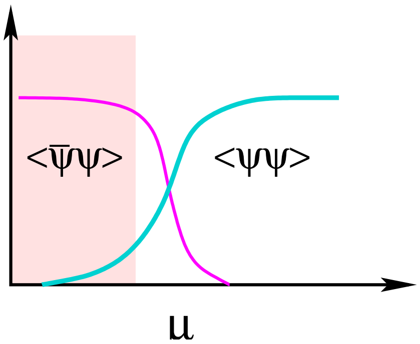
$$\Delta(p) \approx \frac{3g^2}{16} \int \frac{d^3\mathbf{q}}{(2\pi)^3} D_{\mu\mu}(\mathbf{p} - \mathbf{q}) \frac{\Delta(q)}{\sqrt{(q-\mu)^2 + \Delta^2(q)}}$$

Characteristic $g$ dependence of $\Delta(k = \mu)$

$$\Delta(k = \mu) \sim \mu \, e^{-c/g(\mu)}$$

increases as $\mu$ becomes small.

# 2 color QCD at low density




Assume $M(k) \gg \Delta(k) \quad (k = |\mathbf{k}|)$

$$\Delta(p) \approx \frac{3g^2}{16}\int \frac{d^3\mathbf{q}}{(2\pi)^3} D_{\nu\nu}(\mathbf{p}-\mathbf{q}) \frac{2\sqrt{q^2 + M^2(q)}}{q^2 + M^2(q) - \mu^2}\,\Delta(q)$$

$$M(p) \approx \frac{3g^2}{8}\int \frac{d^3\mathbf{q}}{(2\pi)^3} D_{\nu\nu}(\mathbf{p}-\mathbf{q}) \frac{M(q)}{\sqrt{q^2 + M^2(q)}}\,\Theta\left(\sqrt{q^2 + M^2(q)} - \mu\right)$$

Define $\phi(q) \equiv \dfrac{\Delta(q)}{q^2 + M^2 - \mu^2}$ then diquark gap eq. becomes

"bound state equation" [Leggett '80, Noziéres & Schmitt-Rink '85]

$$(\mathbf{k}^2 + M^2)\phi(k) - \int [d\mathbf{q}] V_{\mathbf{k},\mathbf{q}}\phi(q) = \mu^2\phi(k)$$

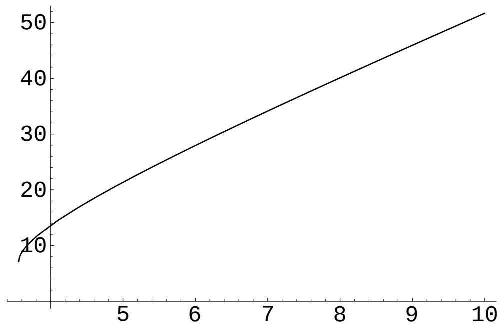
and this has a solution $\mu = 0$ because two gap equations becomes equivalent at $\mu = 0$. (cf: Pauli-Gürsey sym.)

Condensation of tightly bound pairs
$\Rightarrow$ Bose-Einstein Condensation

# Density dependence of $\Delta$ in 2 color QCD

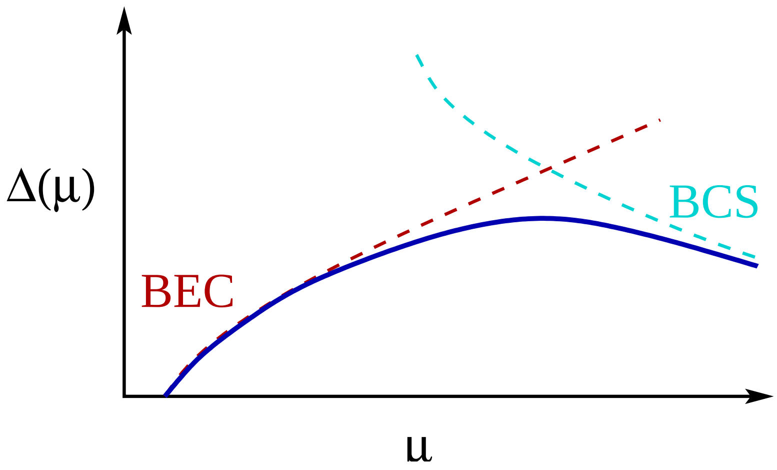
Approximate solutions: as $\mu$ becomes small

High density: $\Delta(\mu) \sim \mu e^{-c/g}$ becomes large
Low density: $\Delta(\mu)$ becomes small




$\Delta$ vs $\mu$

Therefore




Similar "turnover" of the diquark condensate is seen in the lattice results. [Aloisio, et al. '00, Kogut, et al. '01]

# Summary

- Momentum dependence of the gap $\Delta$ changes considerably as the density is changed. This directly affects the structure of the Cooper pairs.
- At high density, a sharp peak in $\Delta(k)$ at Fermi surface: Cooper pairs are formed in a small region around Fermi surface. Large Cooper pair, consistent with weak-coupling BCS picture.
- At low density, no sharp peak: Cooper pairs are formed even away from Fermi surface. Small Cooper pair size shows a clear departure from the weak-coupling BCS picture.
- In 2 color QCD, smooth transition from BCS to BEC indeed occurs. Density dependence of $\Delta$ is different in two pictures. Diquark gap equation becomes a bound-state equation for a Cooper pair and has a solution at small $\mu$.

# Lattice results

R. Aloisio et al. Phys.Lett.B493:189-196,2000

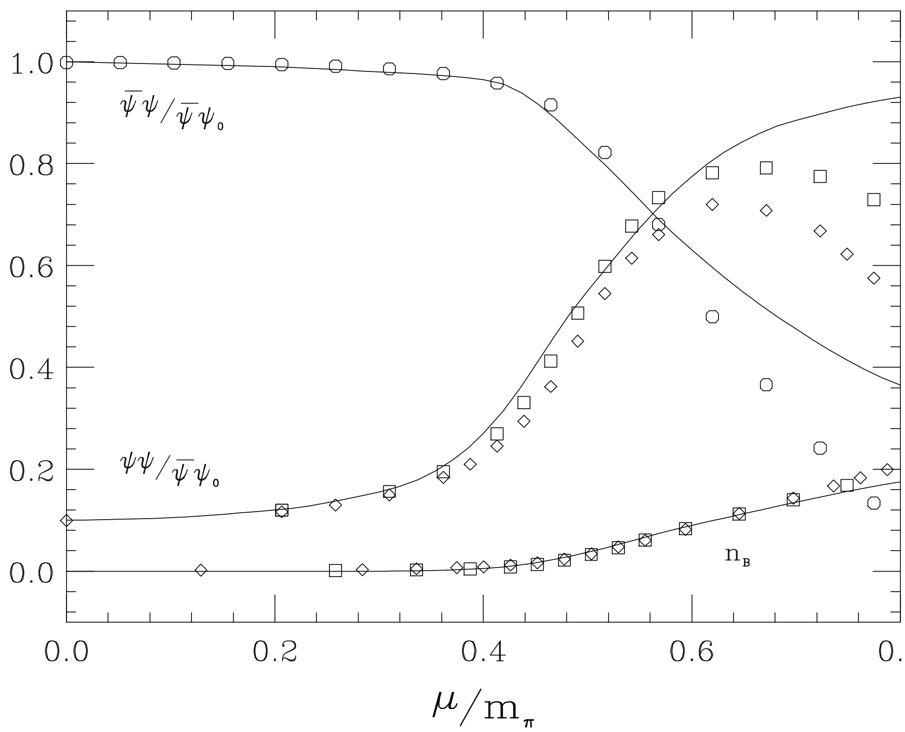



Fig. 4