Title: With Poly-gon, Who is Left to March?

(Distinct activities linked by their properties)

SIMT SUMMER MATH INSTITUTE

WITH POLY-GON, WHO IS LEFT TO MARCH?

(ACTIVITIES LINKED TOGETHER BY THEIR PROPERTIES)

Brief Overview:

Students will use Geometer's Sketchpad to discover and represent the linear relationship between the number of sides and the sum of the angle measures of a polygon through graphing. In addition, students will discover and graph the non-linear relationship between the number of sides and the measure of each angle in regular polygons. Students will demonstrate their skills with translations, reflections, rotations and dilations of straight lines (segments) and quadrilaterals to represent band formations by using football field grids in the coordinate system. Specific quadrilateral shapes are formed by using coordinates of the band members.

NCTM Content Standard/National Science Education Standard:

Geometry:

- Analyze characteristics and properties of two and three dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Apply transformations and use symmetry to analyze mathematical situations.

Measurement:

- Understand measurable attributes of objects and the units, systems and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements.

Grade/Level:

Grades 10-12/ Geometry

Duration/Length:

Approximately4-5 fifty-minute class periods

Student Outcomes:

Students will:

- Use transformations with straight lines.
- Distinguish between different types of quadrilaterals
- Calculate the sum of the measures of the angles of any polygon
- Use the tool options found in Geometer's Sketchpad to draw regular polygons
- Calculate the measure of an individual angle of a regular polygon
- Use rotations feature in Geometer's Sketchpad to draw regular heptagons, nonagons, and decagons.

- Find the area of a triangle, a trapezoid and a parallelogram.
- Use the areas of trapezoids and triangles to replicate President James Garfield's Proof of the Pythagorean Theorem
- Find the distance between band members.

Materials and Resources:

- Overhead transparency of *President Garfield's Proof*
- Word Wall
- Geometer's Sketchpad (for teacher demonstration and student use)
- Worksheets (adapted from the suggested reference sources cited below)
  1) *Parrots*
  2) *Band Line Formations*
  3) *President Garfield's Proof*
  4) *Quads Quickstep Part 1*
  5) *Quads Quickstep Part 2*
  6) *Quads Quickstep Part 3*

Variety of pictures of quadrilaterals

Suggested Reference Sources:

- Springboard: Mathematics with Meaning, Geometry version, published by College Board, 2004 "Plenty of Polygons" Pages 37-44 and "Halftime Salute, pages 109-126
- Garfield's Proof, Geometry: Tools for a Changing World, Prentice-Hall Publishers, 2001, page 273

Development/Procedures:

Lesson 1

Preassessment – Have students plot and find the slope of any two given points. Pretend that your students are on a field strip visiting Washington, DC. Ask: How many degrees do the angles of the Pentagon have? How many degrees do the angles of the Octagon House have?

Launch – Demonstrate on Geometer's Sketchpad how to draw a regular polygon from the tool options located on the left side of the menu (Select polygons, then pentagon). Measure the interior angles, by highlighting in order any three consecutive vertices and clicking on Measure (angle). Calculate the total measure of the angles using the calculate feature (highlight one of the angles and multiply by 5). Construct all the diagonals from any one vertex. Count the number of triangles formed. Find the total number of degrees in a pentagon by multiplying (3)(180) = 540.

Teacher Facilitation – Assist students as they create sketches of a triangle, quadrilateral, hexagon, and octagon from the Geometer's Sketchpad tool options and complete the table from worksheet *Parrots*. (Note:Inside joke:When the worksheet is done, then Polly's gone.) Instruct students to go to Graph 1 on the worksheet, where they plot points and determine the slope and the equation of the linear data. Emphasize the meaning of discrete graphs. Use Table 1 to complete Table 2 and graph the data on Graph 2. Have students predict whether or not the points are in a linear pattern. Make Calculus reference to the concept of limits. (The limit in this case is 180 degrees).

Student Application--Students complete the tables from the worksheet *Parrots.* The diagonals of heptagon, nonagon, and decagon should be drawn directly onto the worksheet.

Using the answers from Table 1, students complete Table 2 and graph the points.

Embedded Assessment –Students will provide written explanations of how to find the sum of the measures of interior angles of any polygon and give the formula.

Reteaching/Extension –

- Work individually with students to reinforce concepts.
- Homework: Have students find the number of diagonals for a 17-sided figure. Research Karl Friedrich Gauss as he relates to this figure.

Computer-work: Explore the sum of the exterior angles of any polygon on the *Geometer's Sketchpad* . Then do the same for at least three other polygons.

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PARROTS

NAME_____

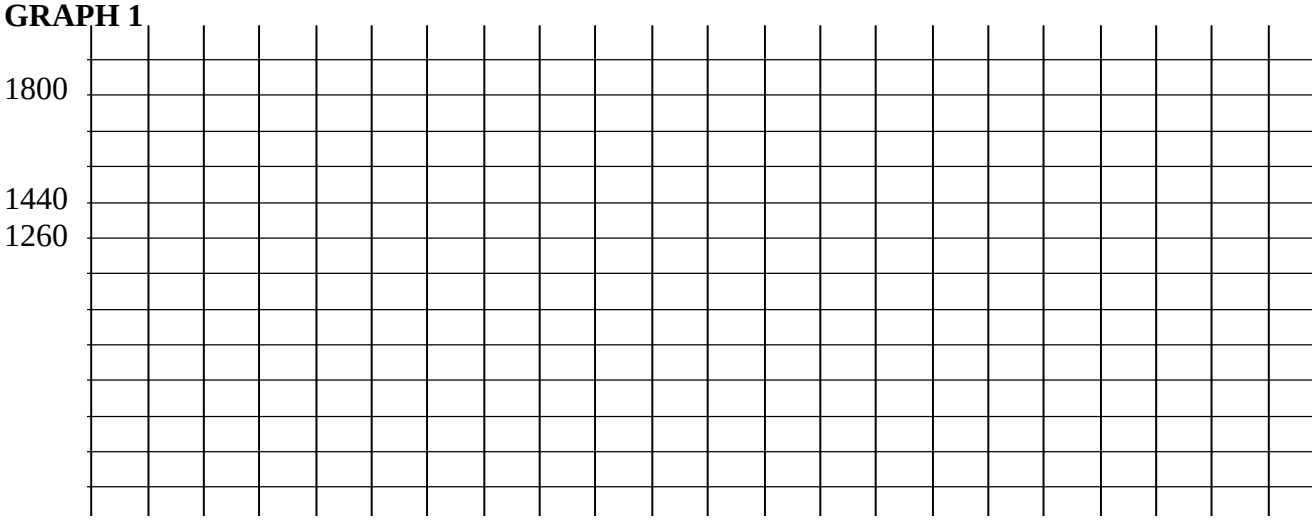
Draw the diagonals from one vertex for each polygon from Geometer's Sketchpad. Record the data in Table 1.

TABLE 1

Polygon	Number of Sides	Number of non-overlapping triangles	Sum of the Measures of the Interior Angles
Triangle	3	1	180 degrees (180 * 1)
Quadrilateral	4		
Pentagon	5	3	540 degrees (180 * 3)
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			

Using the data from Table 1, graph the ordered pairs (*number of sides*, *sum of the measures of the interior angles*)
Plot the points. The plotted points should appear collinear. The equation is y =_____. What is the slope?_____ Explain how you would use this equation to find the sum of the interior angle measures of any polygon. Write a formula for finding the sum of the measures of the interior angles of a polygon with N sides..

GRAPH 1



720

In a regular polygon, each interior angle has the same number of degrees. Complete Table 2.

TABLE 2

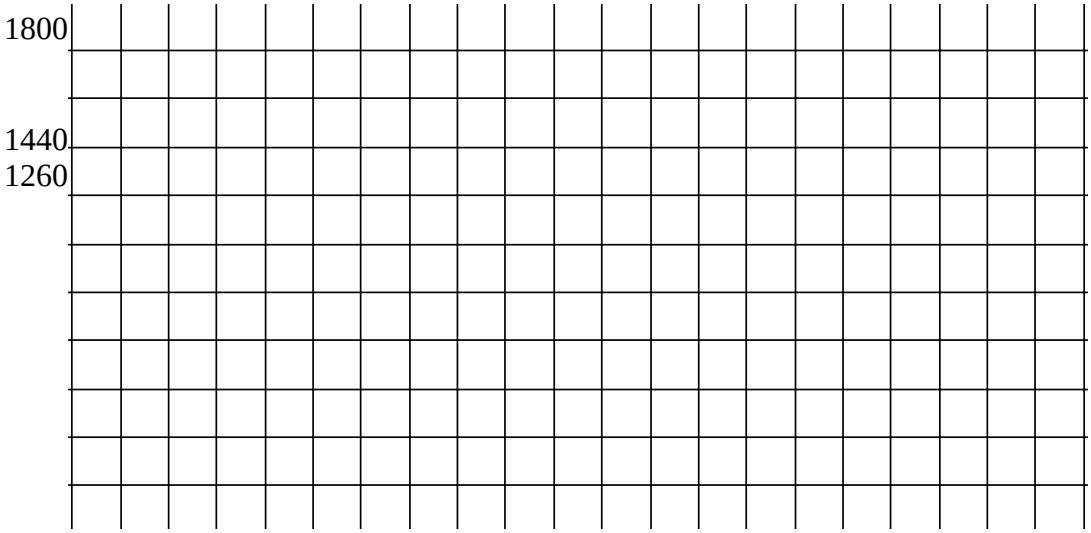
Regular Polygon	Number of Sides	Sum of Angles	Measure of Each Interior Angle
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Heptagon	7	900	128 4/7 degrees (900/7)
Octagon			
Nonagon			
Decagon			

GRAPH 2

Using the data from Table 1, graph the ordered pairs

(*number of sides*, *measure of each interior angle*)

Plot the points. Do the points appear collinear? As the number of sides increase, the measure of each interior angle gets closer and closer to _____degrees, but never actually reaches _____degrees. In Calculus this is called a **limit.**



NAME_____ANSWER KEY _____

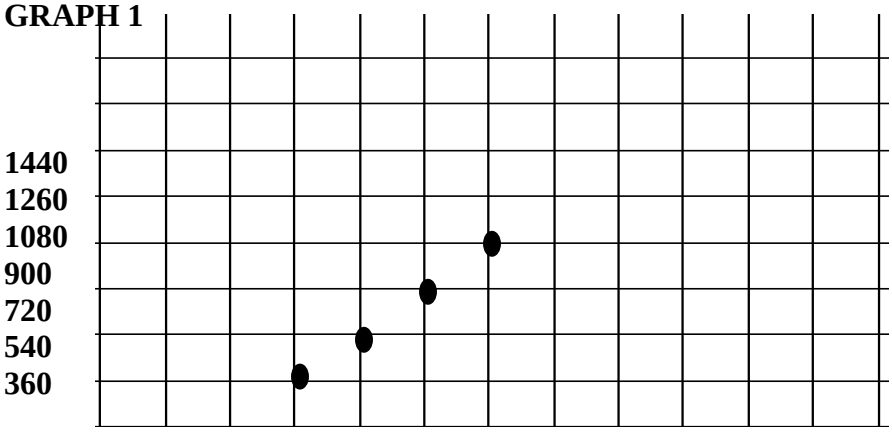
Draw the diagonals from one vertex for each polygon from Geometer's Sketchpad in Table 1.

TABLE 1

Polygon	Number of Sides	Number of non-overlapping triangles	Sum of the Measures of the Interior Angles
Triangle	3	1	180 degrees (180 * 1)
Quadrilateral	4	2	360
Pentagon	5	3	540 degrees (180 * 3)
Hexagon	6	4	720
Heptagon	7	5	900
Octagon	8	6	1080
Nonagon	9	7	1260
Decagon	10	8	1440

Using the data from Table 1, graph the ordered pairs (*number of sides, sum of the measures of the interior angles*)
Plot the points. The plotted points should appear collinear. The equation is y =___180 (x-2)_____. What is the slope?___180___ Explain how you would use this equation to find the sum of the interior angle measures of any polygon. Write a formula for finding the sum of the measures of the interior angles of a polygon with N sides.

GRAPH 1



180

3 4 5 6 7 8 9

TABLE 2 **ANSWER KEY**

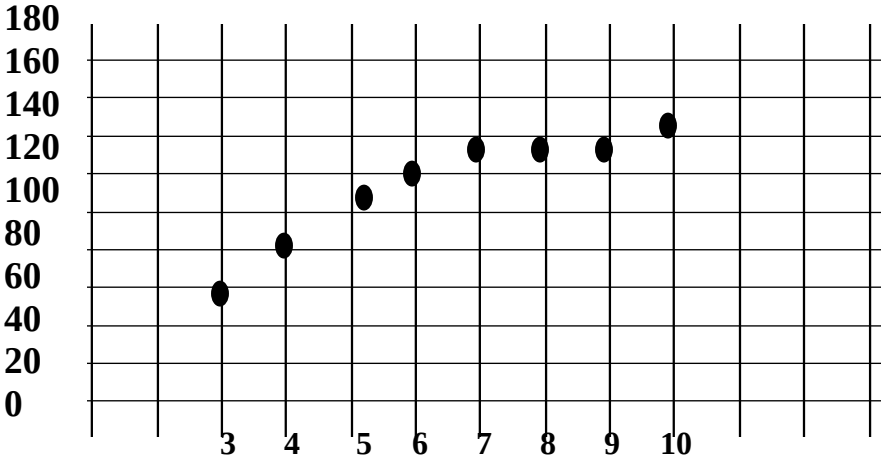
Regular Polygon	Number of Sides	Sum of Angles	Measure of Each Interior Angle
Triangle	3	180	60
Quadrilateral	4	360	90
Pentagon	5	540	108
Hexagon	6	720	120
Heptagon	7	900	128 4/7 degrees (900/7)
Octagon	8	1080	135
Nonagon	9	1260	140
Decagon	10	1440	144

GRAPH 2

Using the data from Table 1, graph the ordered pairs

(*number of sides, measure of each interior angle*)

Plot the points. Do the points appear collinear? As the number of sides increase, the measure of each interior angle gets closer and closer to ___180_____degrees, but never actually reaches _____180_____degrees. In Calculus this is called a **limit.**



Lesson 2

Preassessment— Have students review finding the distance between two points by either the distance formula or the Pythagorean Theorem.

Launch – Lead a discussion of football halftime game shows. Distribute the worksheet Band Line Formations.

Teacher Facilitation-- Assist students in using Geometer's Sketchpad to simulate the worksheet grid and in completing the first task on the worksheet. **Note: Each unit on the grid represents 10 yards on a football field.** Demonstrate how to transform (slide and flip) a line segment.

Student Application—Students have to plot the points representing the first and last members of the Band Line; measure the length of the band line. Hint: Divide to find the distance between between band members. Check answers using the distance formula when appropriate, translate and reflect the Band Members as directed by the Band Director. Complete worksheet *Band Line Formation*s.

Embedded Assessment—Indicate, using coordinates, where a specific band member will be after each translation and reflection.

Reteaching/Extension—Animate a particular band member's sojourn from the beginning formation to the final formation using Geometer's Sketchpad.

BAND LINE FORMATIONS

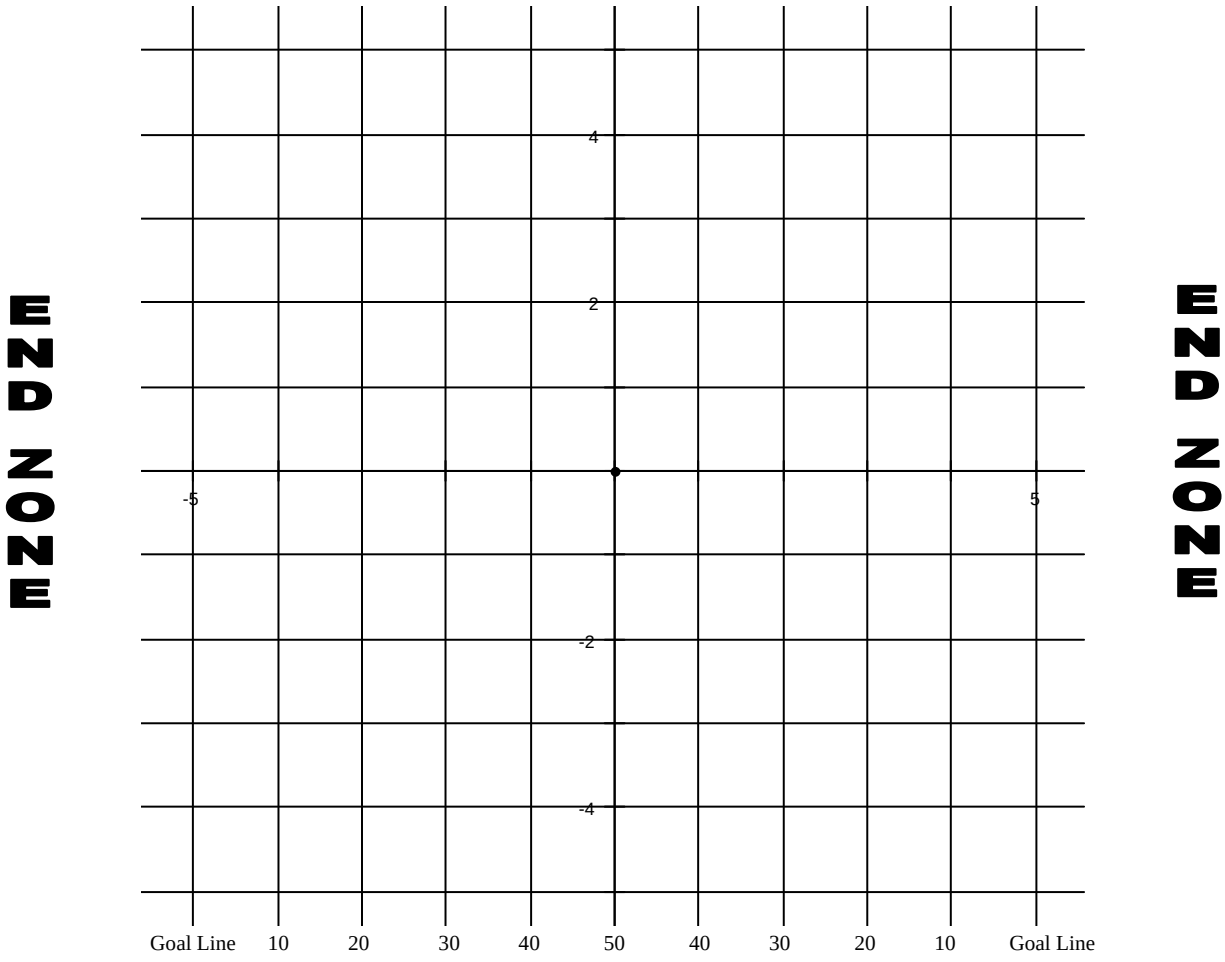
NAME_____

Eastern's award winning Marching Band is ready for the Homecoming Half-time Show. Different formations using quadrilaterals and translations will be featured in their show.

1) Your task is to use Geometer's Sketchpad to create a football-field grid, with the y-axis as the 50 yard line. Complete the chart below, then create a Sketchpad model of the football field.

Left Side of Field			Right Side of Field	
Abscissa	Yard Line		Abscissa	Yard Line
-5	Goal Line		1	40
-4				30
	20			20
	30			10
-1	40			Goal Line

NOTE: The abscissa is the x-coordinate.



2) David starts at the 30-yard line (left of 50-yard line), which is (-2,0). The band director asked David's sister Jennifer to start the halftime show on the 50- yard line, 15 yards above the x-axis. State Jennifer's starting coordinates. (_____)

3) How far apart are David and Jennifer? _____yards

4) David and Jennifer are at opposite ends of a line of 11 band members. If the band members between David and Jennifer are evenly spaced, determine, using Geometer's Sketchpad, the approximate distance between each band player in the line. (The band's line formation is a line segment).

5) When the song begins, David and Jennifer's line will march to a new location on the field. David marches to coordinates (-3, -1.5). Jennifer marches to coordinates (-1, 0).

A) Draw David and Jennifer's line in their new location on the grid on page 1 and on Geometer's Sketchpad. Label it line # 2. What is Jennifer's new yard-line?

B) How far did David and Jennifer each travel to get to their new locations? _____

C) Did the length of David and Jennifer's line change? Explain

6) Some band members think that the movement described in question 5 is called a translation (slide) Do you agree? _____

7) Translate David and Jennifer's line from question 5 so that Jennifer ends up at the center (50-yard-line)of the football field. Label it line # 3.

A) What are David and Jennifer's coordinates now? David _(_____) Jennifer (_____)

B) How many yards did Jennifer move?_____yards

C) How many yards did David move?_____yards

8) For the next formation, Jennifer marches in place and David moves forward, crossing the 50 yard line and stopping on the 30-yard line. The rest of the band line also shifts to the right side of the 50 yard line.

A) What are David's new coordinates? (_____)

B) How far did David travel to get to his new location? _____yards.

C) Draw David and Jennifer's line in its new location on the grid on

page 1 and on Geometer's Sketchpad. Label it line # 4.

9) Some band members think that the type of movement described in question 8 is called a reflection (also called a flip). Do you agree?_____

10) To get to the band's final line formation, reflect David and Jennifer's line across the right-side 30-yard line. Label it line # 5.

A) Write instructions below to tell the band line members how they should move to get to their new locations.

B) What are David and Jennifer's coordinates now? David (_____) Jennifer (_____)

C) How far did Jennifer move? _____yards

D) How far did David move?_____yards

E) Did all of the other band members move the same distance as David and Jennifer? Justify with Geometer's Sketchpad.

Lesson 3

Preassessment—Identify the type of quadrilateral as seen from a variety of pictures on paper or Geometer's Sketchpad, using angles, sides and diagonals. What properties distinguish a trapezoid from a parallelogram, a parallelogram from a rectangle, and a rhombus from a square?

Launch – NOTE: This lesson is in three worksheets: *Quad Quickstep, part 1*(Trapezoid), *part 2* (Parallelogram) and *part 3* (Rhombus, Square). Using Sketchpad, draw any trapezoid and measure the angles to determine which adjacent angle combinations will yield 180 degrees. Then draw any parallelogram and measure the sides and angles to display some of the properties of a parallelogram.

Teacher Facilitation—Distribute worksheets. On each worksheet, monitor students to insure that they are creating the correct quadrilateral as defined by the given points. Students may work collaboratively.

NOTE: Isosceles trapezoid is referenced Quad Quickstep part 1. Display overhead transparency of President Garfield's Trapezoidal Proof of the Pythagorean Theorem.

Student Application—Worksheet *Quad Quickstep, part 1*: Using Geometer's Sketchpad, students mark the midpoints of each of the two legs of the trapezoid, draw the median, and measure the lengths of the bases and the median. Students should verify the relationship between the sum of the lengths of the bases and the median.

Worksheet *Quad Quickstep, part 2:* Using Geometer's Sketchpad, students draw the diagonals of the parallelogram to find the location of the drum major, marking that intersection, estimate the coordinates of the intersection and investigate its accuracy by finding the midpoints of both diagonals, and compare the lengths of the diagonals to determine if the parallelogram is a rectangle.

Worksheet *Quad Quickstep, part 3:* Students form a figure with four congruent sides as defined by the worksheet. Students should make the connection that the diagonals of a square are congruent, unlike the diagonals of a rhombus.

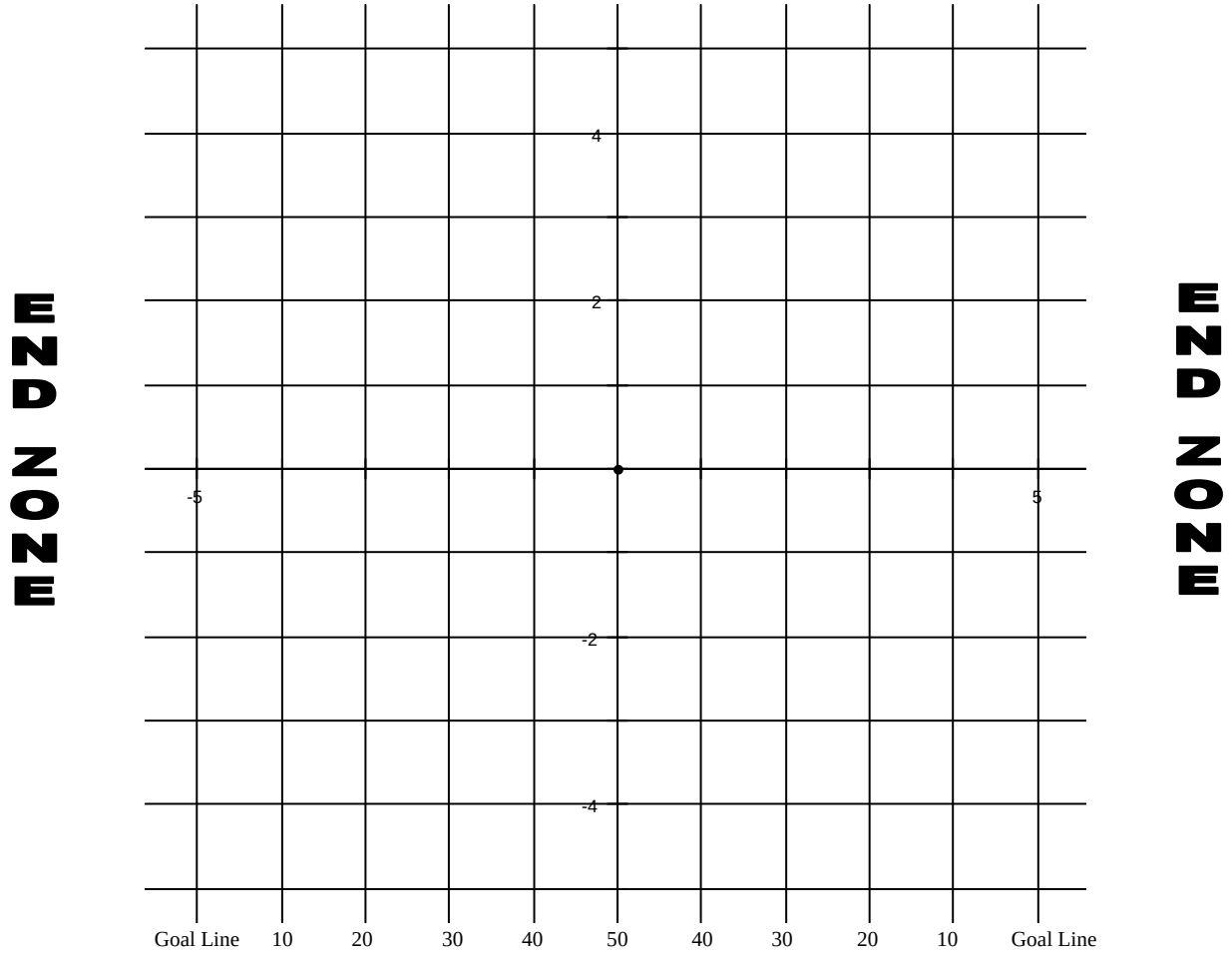
Embedded Assessment—See question 7 on *Quad Quickstep*, part 3, where the students construct the square from the given coordinates and find the lengths of each side and diagonal.

From the square in question #6, identify and justify the types of transformations used to get the square in question #7.

Reteaching/Extension— Use *Geometer's Sketchpad* to construct an altitude of a parallelogram (Worksheet *Quad Quickstep, part 2)* and find the area of that figure. Find the area of the squares in the third Worksheet (*part 3).* Students form a rhombus using the following coordinates: (-2, .5), (0, -.5), (0, 2), (-2, -2). Then they rotate the figure 90 degrees, and identify the coordinates of each vertex. They then translate 40 yards and identify the new coordinates.

Quadrilateral Quickstep Part 1 **NAME**_____

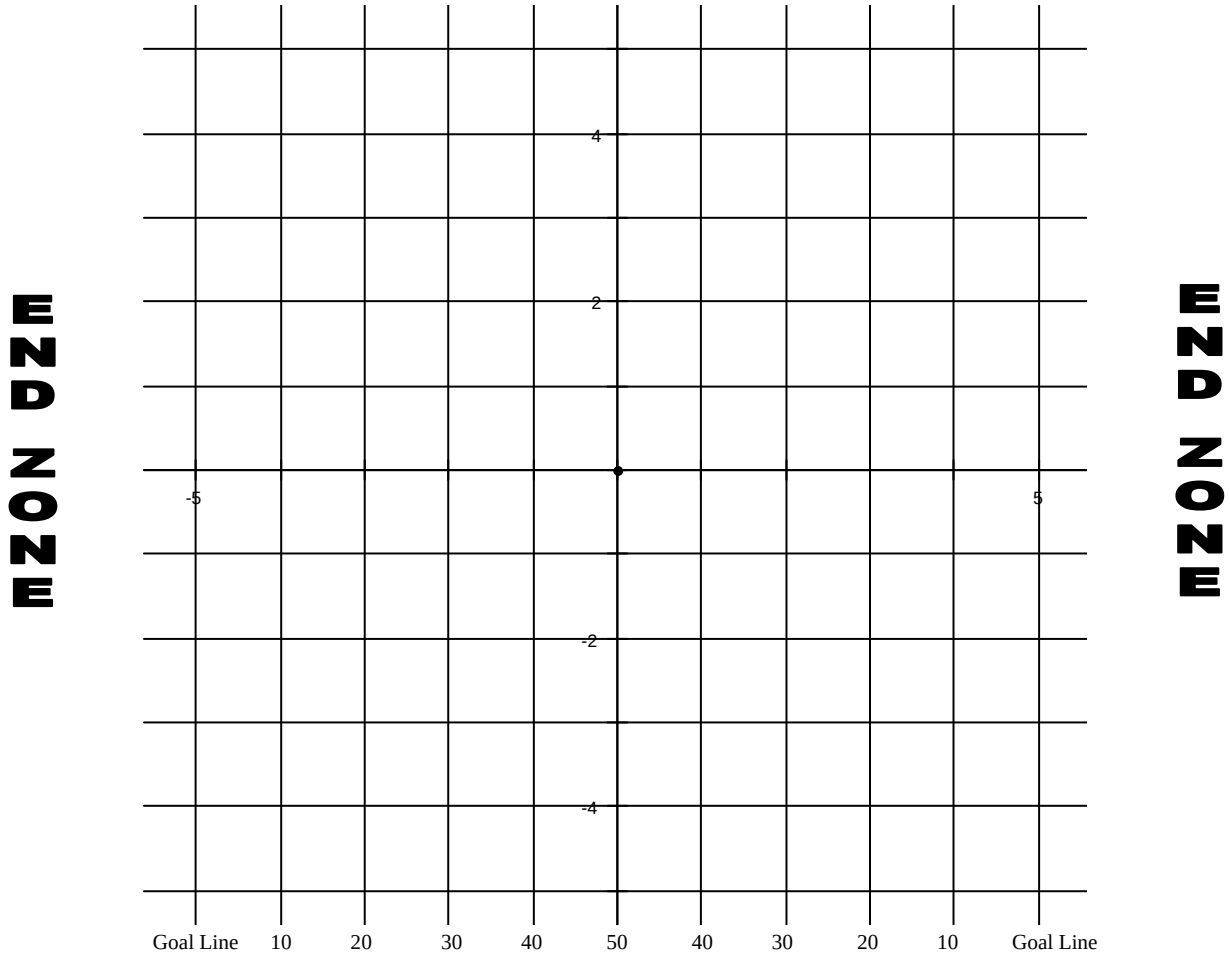
At the beginning of the bands' next song, David and Jennifer's line is part of a quadrilateral formation with the other band members. Recall that at the end of their last song, David's final position was the ordered pair (2, -.5) and Jennifer's coordinates were (4, 0). The other band members, Tapscott and Pamela, are at (4,1.5) and (0, .5), respectively. These four vertices, in order, form a quadrilateral.



1) Plot and label the vertices with the first letter of each band member's name. Draw the quadrilateral. Use Geometer's Sketchpad to create a model of the diagram.
2) What type of quadrilateral is DJTP?
3) Verify that quadrilateral DJTP is a trapezoid by the showing that the bases are parallel and that the legs are not parallel. (HINT: When the sides are parallel, the same side interior angles must be supplementary.

4) A trapezoid whose legs are congruent is called an isosceles trapezoid. Is DJTP an isosceles trapezoid? Justify.

For the next formation, Jennifer and David will each march half-way and directly towards Tapscott and Pamela. Use Geometer's Sketchpad to identify their new locations and connect them with a line segment. This segment, which joins the midpoints of the non-parallel sides of the trapezoid, is called the **median** of the trapezoid.



5) What are the coordinates of the midpoints of the legs of the trapezoid ?

6) Plot and connect these points on the grid to form the median (also known as the midsegment.

7) How long is the median?

Median = $\sqrt{\qquad\qquad\qquad\qquad}$

8) Show that the length of the median is half the sum of the lengths of the bases of the trapezoid. Use Geometer's Sketchpad to verify this relationship.

Base 1 = $\sqrt{\qquad\qquad\qquad\qquad}$

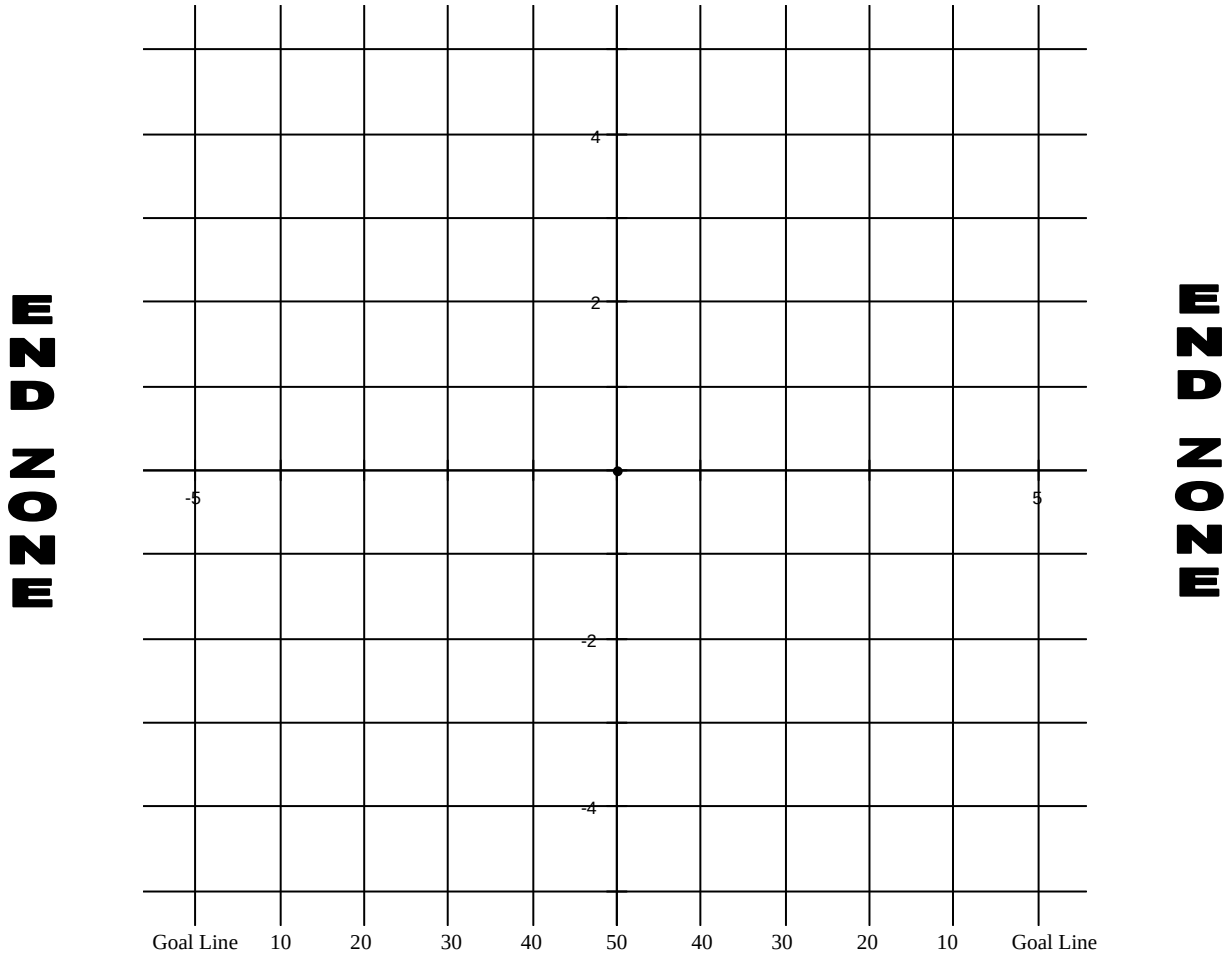
Base 2 = $\sqrt{\qquad\qquad\qquad\qquad}$

Quad Quickstep Part 2

NAME_____

Halftime Salute PARALLELOGRAM

The band is now going to form a parallelogram. David, Jennifer, Tapscott, Pamela march to new positions D (2, -2.5), J (5, 0) , T (3, 2) and P (0, -.5) to form the parallelogram.
(Recall: In a parallelogram, opposite sides are parallel and congruent)

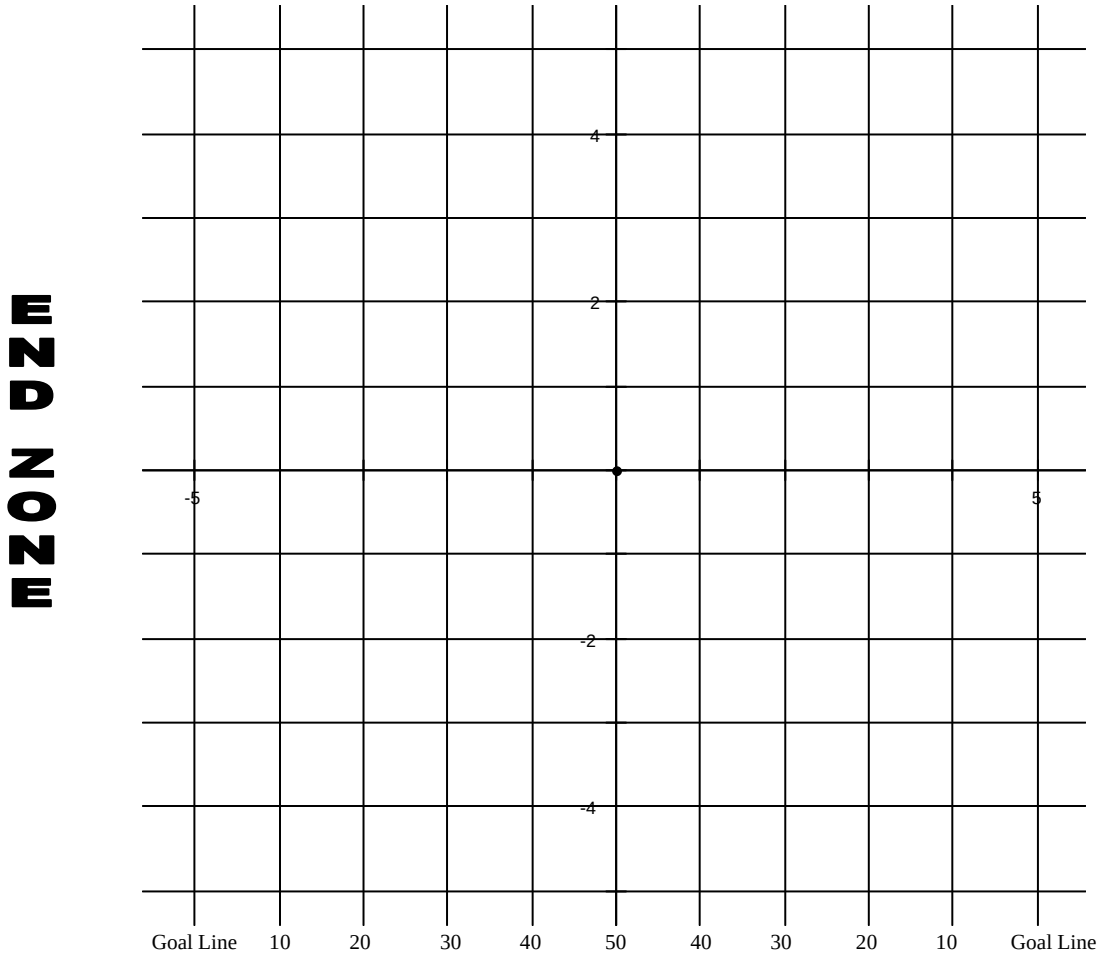


1. Plot and label the vertices of the parallelogram on the grid.

2. Use the Geometer's Sketchpad to verify that the quadrilateral DJTP is a parallelogram by showing that both pairs of opposite sides are parallel. Show that all of the adjacent angles must add up to 180 degrees. Verify by your angle measurements that this is NOT a rectangle.

3) Verify that DJTP is a parallelogram by showing that both pairs of opposite sides are congruent.

During this song, the drum major moved to the center of the parallelogram The center is the intersection of the diagonals of the parallelogram.



4) Draw the diagonals of the parallelogram on the grid and estimate the
4) Draw the diagonals of the parallelogram on the grid and estimate the coordinates of the intersection point. Intersection = ( )

5) Use the midpoint formula to determine the midpoint of each diagonal.

A) Midpoint of TD = ((2 + 3)/2, (2-2.5)/2) = **( )**

B) Midpoint of PJ = ((5+0)/2, (-.5+0)/2)) = ( )

C) The drum major is located at ( ).

6) Some members of the band think that the final formation looks like a rectangle, but are not sure. How can you help them decide one way or other? Recall: The diagonals of a rectangle are congruent.

Length TD = $\sqrt{\qquad\qquad\qquad\qquad}$

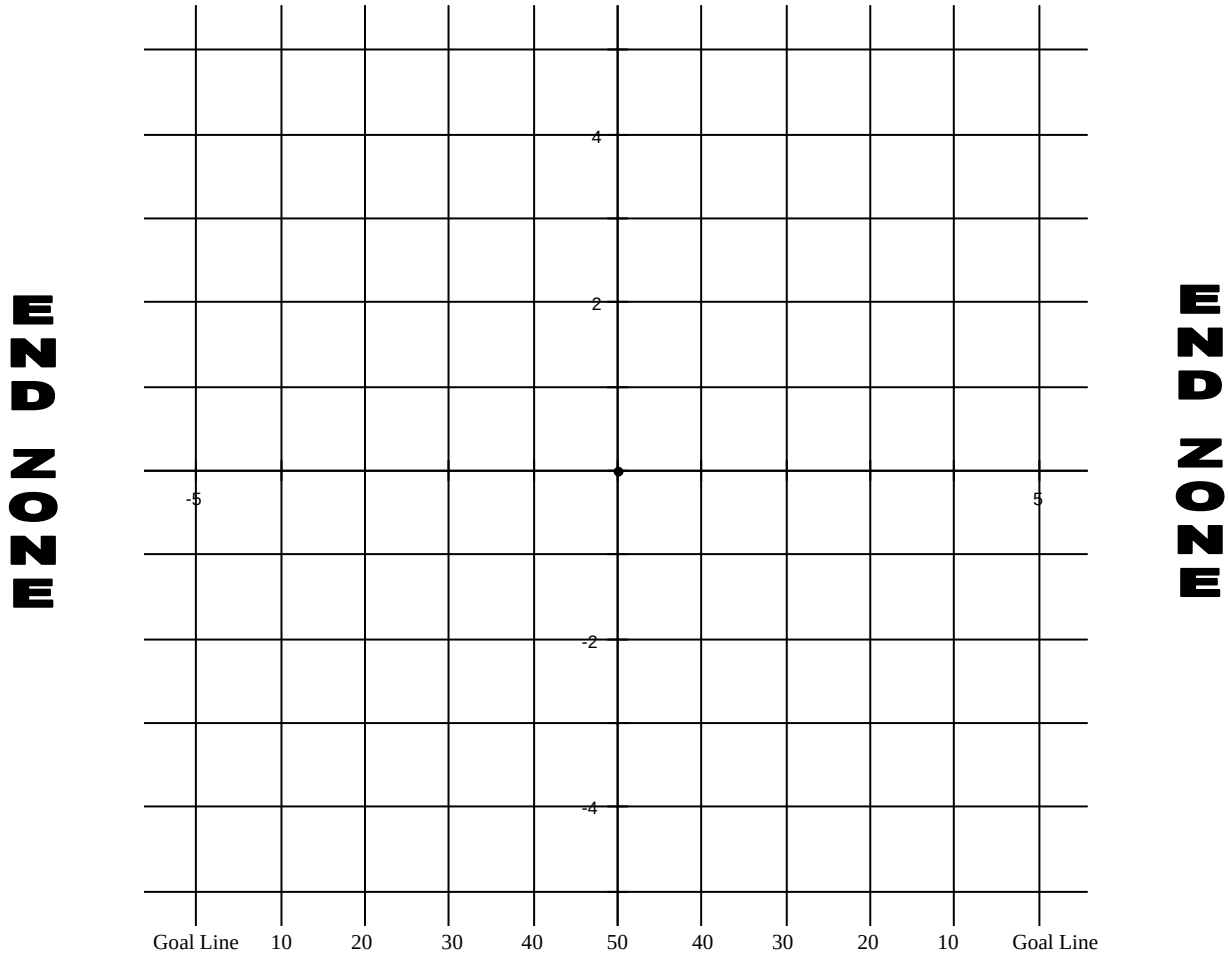
Length PJ = $\sqrt{\qquad\qquad\qquad\qquad}$

Justify why you say it is or is not a rectangle.

Quadrilateral Quickstep Part 3

NAME_____

As the band members begin to play the next song, they form a horizontal line from the 50 yard line to the goal line on the right.



1) Pamela is on the left end and Jennifer is on the right end of the horizontal line. What are their coordinates? Pamela ( ) Jennifer ( )

2) David is standing halfway between Pamela and Jennifer and Tapscott is standing next to him. Identify David's starting coordinates? David ( ) (NOTE: Tapscott has become the 12^{th} band member).

3) The band now moves into a vertical line formation as the song begins. David marches 5 yards toward the Home side along the 25-yard line and Tapscott marches 5 yards toward the Away side along the 25-yard line. Pamela and Jennifer march in place. The rest of the band line splits with every other band member either marching toward the Home side with David or toward the Away side with Tapscott stopping at 5 yard intervals. The band forms a new quadrilateral. Connect Pamela and Jennifer to the band members closest to the Home and Away sides of the field.
NOTE: Those 10 band members are all lined up along the 25 yard line.

4) What is the length of each side of the parallelogram? (They are all the same).

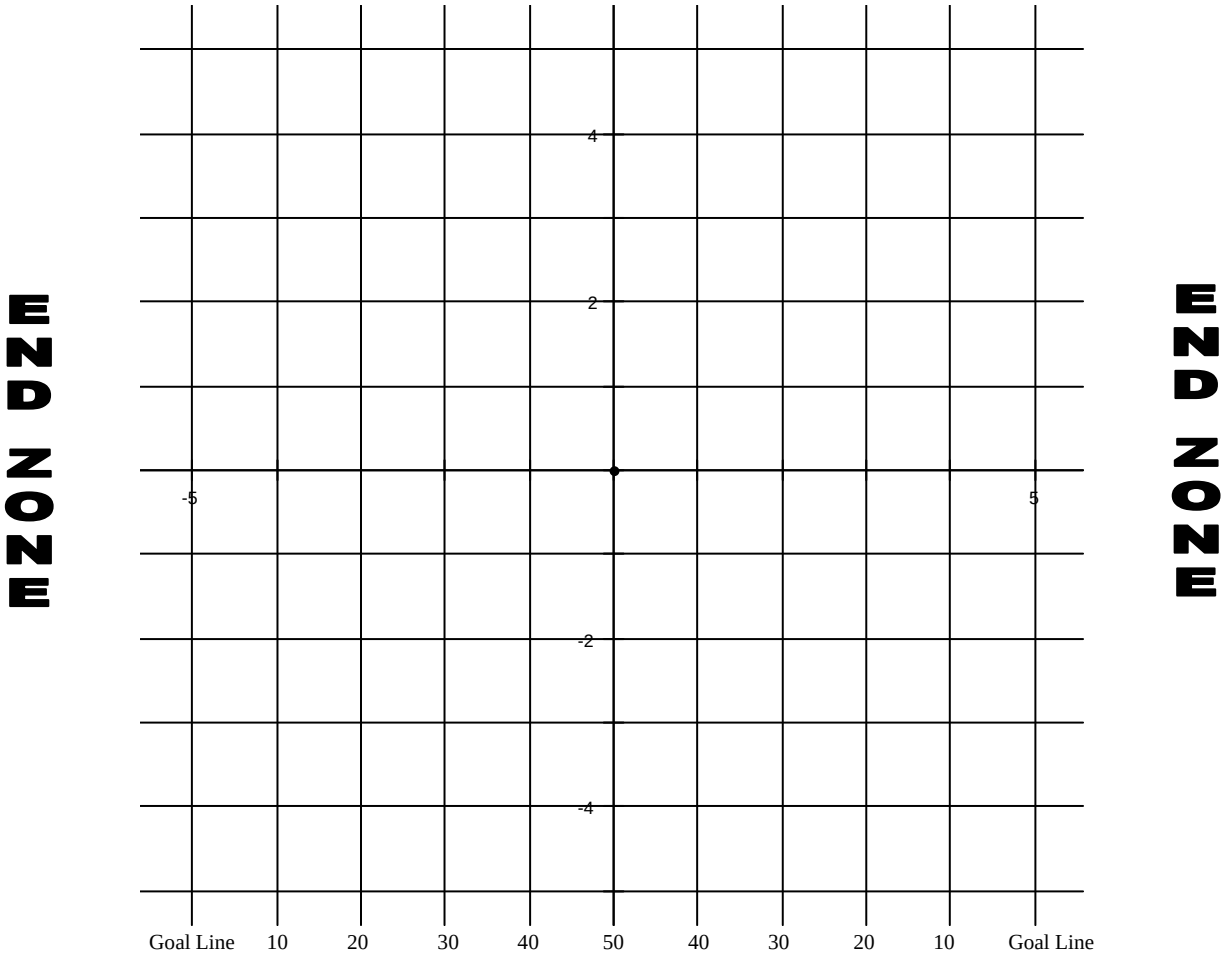
5) What do we call a parallelogram with all 4 sides congruent?_____

6) A) What is the name for the special kind of rhombus formed above?

B) Draw the diagonals. What is the length of the diagonals of the square?_____

C) What is the measure of each angle where these diagonals intersect? _____

For the band's final song, David, Jennifer, Tapscott, Pamela re the vertices of one final rhombus with the rest of the band along the figure's sides. Their coordinates are D (3.5, -2), J (4.5, 1), T (1.5, 2) and P (.5, -1)



7) A) Plot the coordinates of the vertices and draw the lines that form the rhombus on the diagram above.

B) Use Geometer's Sketchpad to determine the measure of each angle.

C) Determine the length of each side.

D) Draw the diagonals and find their lengths.

E) Verify that rhombus DJTP is a square.

F) Which transformations were used to get to this final square formation from question 6? (translation, reflection, rotation, dilation)

COMMENT:

1) Journal Writing Assignment: Identify two objects each from the real world that are 3-sided, 4-sided, 5-sided, 6-sided and 8-sided using their mathematical names. State the locations and uses thereof. State the sum of the measures of the interior angles.
   Document your answers using Geometer's Sketchpad.

2) Use Geometer's Sketchpad, construct these four non-regular quadrilaterals. Determine which coordinates are needed to construct a trapezoid, parallelogram, a rectangle and a rhombus. Justify your conclusions.

3) Use Geometer's Sketchpad to form the band into a Pentagon, Hexagon or an Octagon.

Write the coordinates of each vertex and verify the angle measurements.
If you are looking for a challenge, transform the polygon and note the image coordinates.

WORD WALL

For Quad Quickstep

Teachers should include these mathematical words in the class discussions:

Geometer's Sketchpad
Transformations
Translation
Reflection
Rotation
Line
Perpendicular Line
Segment
Ray
Angle
Coordinate Plane/ Cartesian Plane/Coordinate System/Coordinate Grid
Abscissa
Ordinate
Vertical
Horizontal
Axis/Axes
Quadrilateral
Parallelogram
Trapezoid
Rhombus/Rhombi
Midpoint
Median
Diagonal
Altitude
Construct
Measure

FORMULAS:

Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Midpoint Formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

VOCABULARY for "Parrots"
Geometer's Sketchpad
Polygons
Concave
Convex
Regular Polygons
Equilateral Polygons
Equiangular Polygons
Triangle
Quadrilateral
Pentagon
Hexagon
Octagon
Heptagon
Octagon
Nonagon
Decagon
Dodecagon
Interior Angles
Remote Interior Angles
Exterior Angles
Adjacent Angles
Diagonals
Discrete
Limits
Construct
Measure

THEOREMS:
Polygon Interior Angle-Sum Theorem
Exterior Angle Theorem
Polygon Exterior Angle-Sum Theorem

FORMULAS:

Sum of the interior angles of a polygon of n sides: $(n-2)\,180$

Measure of each interior angle of a regular polygon: $\frac{(n-2)180}{n}$

Authors:

Name: David Sidewater
School: Coolidge High School
County/Jurisdiction: District of Columbia Public Schools Washington, DC

Name: E. Marilyn Williams
School: Eastern High School
County/Jurisdiction: District of Columbia Public Schools Washington, DC