Estimating Forward-Looking Euler Equations with GMM and Maximum Likelihood Estimators: An Optimal Instruments Approach

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Abstract

We compare different methods for estimating forward-looking output and inflation equations and show that weak identification can be an issue in conventional GMM estimation. GMM and maximum likelihood procedures that impose the dynamic constraints implied by the forward-looking relation on the instruments set are found to be more reliable than conventional GMM. These "optimal instruments" procedures provide a robust alternative to estimating dynamic macroeconomic relations, and suggest only a limited role for expectational terms.

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The basic framework for macroeconomic analysis has the structure of a simple model consisting of a demand or “IS” equation, an inflation or “AS” equation, and a monetary policy reaction function. Over the years, this model has evolved from the static Keynesian model into a micro-founded rational expectations model in which expectations play a prominent role in the structural equations. Expectations of current and future interest rates affect current aggregate demand, and expectations of current and future aggregate demand affect current inflation. This intertemporal specification of the basic framework, often labeled as “New Keynesian,” purports to preserve the empirical wisdom embodied in the older Keynesian tradition without sacrificing the theoretical insights of modern dynamic macroeconomics.

The extent to which the New Keynesian model is able to replicate key dynamic features of aggregate data, though, remains the subject of much debate. There is a growing consensus that purely forward-looking specifications generate counterfactual dynamics for output and inflation. Some adjustment process must be added to the structural equations in order to match the inertial responses of output and inflation that are apparent in the data. While “hybrid” specifications with both forward and backward looking components seem better suited at characterizing actual dynamics, there is little agreement on the relative role played by expectations (i.e., by the forward-looking component) in the structural equations. Different empirical studies have reached different conclusions concerning the importance of expectations of future interest rates and future demand in determining the dynamics of current output and inflation.[1]

In this study we provide an explanation for the disparate nature of the empirical results on forward-looking demand and inflation relations. As in previous research, we document weak identification in Generalized Method of Moments (GMM) estimation of these macroeconomic relations. Weak instruments lead to GMM point estimates, hypothesis tests, and confidence intervals that are unreliable. In an effort to improve the small sample properties of GMM, we propose a GMM procedure that, instead of instrumenting by means of simple linear projections on the instrument set, uses projections that impose the dynamic constraints implied by the forward-looking relation.

We label this approach to estimating forward-looking relations an “optimal” instruments approach. Any forward-looking relation can be expressed in reduced form, provided that a rational expectations solution exists. The optimal instruments approach explicitly takes into account the constraints placed on the reduced form by the posited structural relation. Conventional GMM estimation, instead, generates instruments simply by means of unconstrained linear projections on the instruments set. This difference in constructing instruments is at the root of the weak identification bias of conventional GMM estimation. In Monte Carlo simulations we show that, in contrast to conventional

[1] For example, Galí and Gertler (1999) argue that expectations about current and future demand pressures are the main determinant of current inflation, with past inflation playing a relatively small role. Others, e.g. Fuhrer (1997), have reached the opposite conclusion that the inertial or backward-looking component, captured by past inflation, is very important in explaining current inflation.

GMM estimation, GMM estimation with optimal instruments produces estimates that are properly centered around the true values.

The optimal instruments approach is typically used in maximum likelihood (ML) estimation of forward-looking relations (see Anderson and Moore 1985). Previous literature has shown that relative to conventional GMM estimation, ML estimation provides small sample estimates that are less biased, more efficient, and dynamically stable. Indeed, in a weak identification context the extent to which ML dominates conventional GMM is striking.[2]

For the appropriate choice of instruments, a maximum likelihood estimator can be expressed as an equivalent instrumental variables estimator. It is then not too surprising that in a weak identification setting GMM estimation with optimal instruments inherits the consistency feature of ML estimation. And in contrast to ML estimation, GMM estimation does not require the assumption of normality of the structural shocks.[3]

Practitioners now have several ways of testing for the presence of weak instruments in GMM estimation. Given the variety of pathologies that GMM exhibit in a weak identification setting, applied researchers should use the available tools to assess whether weak instruments potentially are a problem in a given application. If this is the case, an optimal instruments approach provides a useful alternative to estimating macroeconomic relations with expectational terms. After all, the hallmark of these forward-looking models is precisely to

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[2] See Furer, Moore, and Schuh (1995) and Fuhrer and Rudebusch (2003).

[3] Still, to the extent that such an assumption is satisfied by the data, ML estimates will be more efficient, since maximum likelihood exploits the variance-covariance structure of the shocks.

impose a constrained reduced form that is the rational expectation solution to the relation at hand. The main message of the present work is that optimal instruments are sufficiently strong to properly center the distribution of estimates on the true values in a context where conventional GMM procedures exhibit weak identification.

The rest of the paper proceeds as follows. Section I describes our empirical specification. We consider an Euler equation that allows for both expectational and inertial dynamics. Such a specification has been applied – with modifications that are not crucial for the scope of our analysis – to the estimation of both demand and inflation equations in the previous literature. Euler equations for demand and inflation are isomorphic, and the issues that arise in the estimation of an Euler equation for demand and an Euler equation for inflation are, to a large extent, the same. For this reason, at this stage we consider a specification that, while stylized, is general enough to be cast into both an "IS" and an "AS" framework. While we are ultimately interested in demand and inflation relations, the Euler specification that we consider can be readily extended to many other contexts, for example to inventory or taxation dynamics.

Section II contrasts estimation results for our Euler specification using different estimation techniques in Monte Carlo experiments. We show that the weak-identification bias present in conventional GMM estimation disappears once we use an optimal instruments approach. Estimates obtained by GMM with optimal instruments are comparable to the estimates obtained via maximum likelihood.

In Section III we bring our specification to actual data. We estimate an Euler equation for output and an Euler equation for inflation using optimal instruments, and show that for both relations the estimates indicate a larger inertial component than the one suggested by conventional GMM estimation. In other terms, conventional GMM estimates differ from optimal instruments estimates by giving expectations a more significant role. Since there is evidence that weak identification is an issue in GMM estimation of both the demand and the inflation Euler equations, we are more confident in the estimates generated via optimal instruments procedures. Section IV provides some concluding remarks.

I. Model Specification

The structural relations in the new Keynesian model explicitly represent the dependence of economic decisions upon expectations regarding the future. These relations are derived from the first-order conditions (Euler equations) that characterize optimal behavior of households and firms, and they involve expectations about the future evolution of endogenous variables. The fact that the relations have microeconomic foundations is no guarantee that they are empirically realistic. Indeed, a number of authors have shown that purely forward-looking Euler equations for demand and inflation have a difficult time to match key dynamic features of aggregate data (see, e.g., Estrella and Fuhrer 2002). For this reason, “hybrid” relations have been developed that depart from a purely forward-looking specification to account for the inertial responses of demand and inflation. Traditional explanations of inertia in demand and inflation rely on some form of “backwardness” in spending and price-setting decisions.

In this study, we consider a stylized hybrid Euler equation of the form:

$$z_t = m z_{t-1} + (b - m)\mathrm{E}_t z_{t+1} + g\mathrm{E}_t x_t + e_t \,, \qquad (1)$$

where z and x are the structural variable and the driving process in the equation, respectively, and e is a shock to the equation. The term $\mathrm{E}_t z_{t+1}$ indicates the expectation formed at time t of future z at time $t+1$, and the term $\mathrm{E}_t x_t$ the contemporaneous expectation of x (that is, we allow for the possibility that x_t does not belong to the information set at time t). The parameter m is positive and bounded between 0 and 1, while b is generally taken to be slightly less or equal to 1.

When equation (1) is interpreted as a demand relation, the parameter g is negative, z is a measure of the output gap, and x is a real interest rate. The inertia in the output gap, captured by z_{t-1}, helps to explain the hump-shaped response of the output gap to policy shocks observed in VAR studies. This inertial response is usually attributed to habits in consumption expenditures (Fuhrer 2000) and to adjustment costs in the rate of investment spending (Basu and Kimball 2003). Instead, when equation (1) is interpreted as an aggregate supply relation, the parameter g is positive, z denotes inflation, and x is the output gap or another indicator of the intensity of demand in the economy. Again, the reason for the presence of inflation inertia is largely empirical and is motivated by some form of deviation from an optimizing behavior.[4]

[4] The deviation from the optimizing behavior can take different forms. A popular assumption is that a subset of firms set prices according to a

The crucial element in equation (1) that makes the relation intertemporal is the expectation of future z, $E_t z_{t+1}$. This expectation enters the relation as a shifter, so that changes in $E_t z_{t+1}$ shift the relation between z_t and x_t. According to the structural relation (1), changes in $E_t z_{t+1}$ are driven by changes in expectations about the future path of the driving process x. This can be seen explicitly by iterating equation (1) forward to obtain the following expression:

$$z_t = z_2^{-1} z_{t-1} + g z_2^{-1}(z_1 + z_2)\sum_{i=0}^{\infty} z_1^{i} E_t x_{t+i} + u_t , \qquad (2)$$

where $0 < z_1 < 1$, $z_2 > 1$, and u_t is an error term. The parameters z_1 and z_2 are nonlinear functions of m and b in equation (1). The relation shows that z_t depends on its past (the inertial or backward-looking component, z_{t-1}) and on the present discounted stream of x. In other words, current inflation is affected not only by the current output gap, but also by expectations about future output gaps. Similarly, current demand is affected by the entire term structure of (ex-ante) real interest rates. Other things equal, the smaller m in equation (1), the larger the impact of changes in expectations about the future stream of x on current z.

II. Investigating Estimation with Optimal Instruments

The Euler equation (1) in the previous section does not provide a closed-form solution for z_t. In order to obtain an expression for z in

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backward-looking rule of thumb (Galí and Gertler 1999). Fuhrer and Moore (1995) appeal instead to Buiter and Jewitt's (1985) relative wage hypothesis.

reduced form, it is necessary to specify a law of motion for the driving process x. We assume for the moment that we can write the law of motion for x and other variables affecting x as follows:

$$X_t = AY_{t-1} + h_t \,, \qquad (3)$$

where X_t is a column vector of variables at time *t* that includes x and additional variables ***other than*** z, Y_{t-1} is a column vector of lagged variables, written in first-order form, which includes all the variables in X and z, A is a matrix of coefficients, and *h* a column vector of disturbances. Equation (3) describes the law of motion for x and variables other than z in a vector auto-regressive (VAR) form that allows for potential feedback from lagged z. Given the specification in (3), the reduced form for z can be written as:

$$z_t = b(m, b, g, A)Y_{t-1} + v_t \,, \qquad (4)$$

where b is a row vector of coefficients that depend on the parameters in equations (1) and (3), and *n* is a disturbance term. The vector b is the vector of reduced-form solution coefficients that constitute the unique, stable rational expectation solution to the Euler equation (1) given the auxiliary structure in (3).

In this context, an optimal instrument for z or x is an instrument that is consistent with the posited model reduced-from structure given by equations (3) and (4). Ordering z first in the vector Y and denoting by B a matrix that vertically stacks the vector b in (4) and the matrix A in (3),[5] the optimal time $t-1$ instrument for z_{t+i} (with $i \geq 0$) is given by:

$$\hat{z}^o_{t+i} = e_z B^{i+1} Y_{t-1}, \tag{5}$$

where the row vector e_z has the first element equal to 1 and all other elements equal to zero. Similarly, the optimal time $t-1$ instrument for x_{t+i} will be given by:

$$\hat{x}^o_{t+i} = e_x B^{i+1} Y_{t-1}, \tag{5'}$$

where now the vector e_x has a value of 1 in the same position where x is located in Y, and zero elsewhere.

The optimal instruments in (5) and (5') impose all the constraints placed on the reduced form by the unique and stable closed-form solution to the Euler equation (1), given the auxiliary structure (3). As a result, the coefficients in B are functions of the structural parameters m, b, g, A in (1) and (3). In contrast, conventional GMM estimation forms instruments for z_{t+i} and x_{t+i} simply by means of linear unconstrained projections of these variables on Y_{t-1}.

Note that the closed-form solution for z_t relies on a specific law of motion for x and any other variable that influences x. In other terms, a complete specification of the economic environment is needed in order to perform optimal instruments estimation. While such a task can be in principle daunting, it is still possible to estimate the structural relation (1) via optimal instruments using a data-

[5] That is, the matrix B is written as $B = \left[\frac{b}{A}\right]$.

consistent time series model of the driving process. This means that the matrix A in equation (3), which describes the law of motion for variables other than z, is left unrestricted. The coefficients in the matrix can then be estimated by simple OLS and held fixed in the estimation of (1) with optimal instruments. This general and agnostic way of modeling the driving process in a structural relation avoids the necessity of having a structural equation for each of the variables that bear on the specific relation we want to estimate. Specifying a data-consistent time series model for the driving process also greatly reduces the risk that the estimates for the specific relation we are interested in are driven by misspecification in other relations.

A. Estimation Methodology

We here briefly describe the methods used to estimate the Euler relation (1), and leave details to an appendix. The novel estimation approach to equation (1) we propose in this paper is a GMM procedure with optimal instruments. It is an iterative procedure that updates the optimal instruments at each iteration. The procedure uses an OLS estimate A^{OLS} of the matrix A in equation (3), with the estimate held fixed during the iteration process. The procedure starts with initial values for the parameters in (1). With these initial values and A^{OLS}, we compute the closed-form solution for z and, using expressions (5) and (5′), optimal instruments for z_{t+1} and x_t. The instruments are then used to estimate equation (1) via conventional GMM estimation. With the estimates of (1) and A^{OLS}, we compute a new closed-form solution for z and new optimal instruments for z_{t+1} and x_t. The instruments are then used to generate new estimates of (1) via conventional GMM. Such a process is repeated until the estimates in (1) converge.

The other optimal instruments approach we consider is ML estimation, which has been used in previous literature to estimate Euler relations of the form of (1).[6] The method computes the closed-form solution for z and applies maximum likelihood to the restricted reduced form (4) and the auxiliary structure (3). As with the optimal GMM procedure, we use an OLS estimate A^{OLS} of the matrix A in equation (3) when computing the closed-form solution for z. The likelihood of the solved model can be obtained for any set of parameters under the assumption that the innovations in the model are joint normally distributed with mean zero. If the normality assumption is satisfied, ML estimation will be more efficient than GMM estimation with optimal instruments when the disturbances are correlated across equations.

We compare optimal instruments estimation with conventional GMM estimation. As already emphasized, GMM instruments for the expectational terms in equation (1) without imposing any model structure. In this context, GMM estimation is straightforward because equation (1) is linear in variables and parameters.

B. Monte Carlo Results

In what follows we investigate the behavior of optimal instruments estimation and conventional GMM estimation in a Monte Carlo experiment. We focus on the small sample behavior of these different

[6] See, e.g., Fuhrer and Rudebusch (2003).

estimators under the assumption that the model is correctly specified. Several applications of GMM estimation confront what is known as “weak instruments” or “weak identification,” that is, instruments that are only weakly correlated with the included endogenous variables. When instruments are weak, the sampling distributions of GMM statistics are in general non-normal and conventional GMM point estimates, hypothesis tests, and confidence intervals are unreliable (see, e.g., Stock, Wright, and Yogo 2002). The scope of our Monte Carlo experiment is to ascertain the extent to which optimal instruments methods improve upon conventional GMM estimation. The experiment is performed within a setting that replicates some of the relevant features that the econometrician has to confront when estimating an Euler equation for aggregate demand or inflation on actual data.

Our experiment design consists of estimating the Euler equation (1) augmented by the auxiliary structure (3) in a three-variable setup. We use three variables because the New Keynesian framework, in its simplest form, can be characterized by aggregate demand (expressed in the form of an output gap), inflation, and a short-term interest rate. Moreover, the dynamic interactions between the output gap, inflation, and short-term rates have been explored extensively in the VAR literature.

Using this three-variable setup, we perform two Monte Carlo experiments that differ in the way in which the auxiliary structure (3) is parametrized. We do so in order to cast the two experiments within an “output Euler equation” and an “inflation Euler equation” estimation framework, respectively. In the output Euler equation experiment, z is the output gap and x a real interest rate, while the additional variable is given by inflation. In this case, the auxiliary structure (3) consists of VAR equations for the real interest rate and inflation. In the inflation Euler equation experiment, z is inflation and x the output gap, while the additional variable is given by a nominal interest rate. The auxiliary structure (3) then consists of VAR equations for the output gap and the nominal interest rate.

The parameters for the auxiliary equations are estimated from actual U.S. quarterly data over the period 1966 to 2001. The output gap is the log difference between real GDP and a segmented deterministic linear trend for log real GDP, with breakpoints in 1974 and 1995. Inflation is the log change in the GDP chain-weighted price index, and the nominal interest rate is the federal funds rate. The real interest rate is then given by the difference between the federal funds rate and next-period inflation.

The Euler equation (1) and the auxiliary structure (3) are used to compute 1000 replications of simulated data for a sample size of 180, with shocks drawn from a multivariate normal distribution with variance-covariance matrix equal to the identity matrix, for different values of the parameters in (1). Conventional GMM estimation of equation (1) uses as instruments lags of each of the three variables. We use the minimum lag-length for the instruments that allows to span every realization of the endogenous variables given the assumed data-generating process. The same lag length is used when we estimate the auxiliary structure (3) as an unrestricted VAR in the context of optimal instruments estimation.

Table 1 summarizes the results for the “output Euler equation” Monte Carlo experiment. We set the value of b to 0.98 throughout, and estimate the parameters m and g in the relation. The true value of g is set at –0.5, and we let m take the true values [0.1,0.25,0.5,0.75,0.9]. The top panel of the table reports summary statistics for estimates of m. The true value, m^T, is reported in the second column, followed by the mean estimate, the median estimate, and the standard deviation of the estimate. The bottom panel displays the corresponding estimates of g.

There are two key results that emerge from the table. The first is that conventional GMM estimates of m are biased. GMM understates m by about .18 when the true value is 0.9, and overstates m by about .12 when the true value is 0.1. In other terms, conventional GMM estimates are biased towards 0.5 from either side of 0.5. In addition, conventional GMM estimates of g are biased downward when the true value of m is high. The bias is small for low values of m (i.e., when the forward-looking component becomes more important). Estimates of m obtained via optimal instruments procedures, in contrast, are generally unbiased. In particular, ML estimates are accurate regardless of the value taken by m. GMM with optimal instruments performs equally well except when m is very small (m=0.1). Still, even in that circumstance the mean estimate of m is closer to the actual value than the mean estimate of m obtained by conventional GMM.

The second result apparent in the table is that estimates obtained via optimal instruments are not only more accurate, but also far more efficient than conventional GMM estimates. The standard error for the GMM estimates is usually twice as large as that for optimal instruments estimates. Figures 1-3 illustrate the performance of conventional GMM relative to optimal instruments methods for various parameter values. Figures 1 and 2 show histograms of the parameter distribution of m when m is equal to 0.25 and to 0.75, respectively. Figure 3 shows histograms of g when m is equal to 0.5. Note that even in this case in which the conventional GMM estimate of g is unbiased, the gain in efficiency from using optimal instruments methods is striking.

Table 2 summarizes the results for the “inflation Euler equation” Monte Carlo experiment. In the experiment we set the value of b to 0.98 throughout, and estimate the parameters m and g in the relation. The true value of g is set at 0.10, and we let m take the true values [0.1,0.25,0.5,0.75,0.9]. The table shows that conventional GMM estimates of m exhibit a very pronounced bias. The GMM estimator assigns equal weight to the forward and backward-looking components whenever the true value of m is greater than 0.5. That is, conventional GMM estimates of (1) do not assign a weight of more than one half to the backward looking component on average even when such a component is preponderant. The GMM estimator is somewhat better able at recognizing a specification that places greater weight on the forward-looking component (i.e., $m<0.5$), but the estimates of m are still biased toward 0.5. In addition, conventional GMM estimates of g are biased downward when the true value of m is high, with the bias disappearing as m decreases.

When compared to conventional GMM estimates, optimal instruments estimates are much more accurate. The table shows that ML estimates tend to be well centered across all values taken by m. Estimates obtained via GMM with optimal instruments are very similar to their ML counterparts when m is high. The performance of GMM with optimal instruments deteriorates somewhat for low values of m. Then estimates of g exhibit a downward bias, although estimates of m continue to be much closer to the true value than the corresponding conventional GMM estimates.

It is interesting to note that, for the specification used in this Monte Carlo exercise, when m is high optimal instruments estimates – while unbiased – exhibit about the same degree of dispersion as conventional GMM estimates. Instead, when m is equal to or is less than 0.5, optimal instruments estimates become more efficient than conventional GMM estimates. Figures 4-6 illustrate the performance of conventional GMM relative to optimal instruments methods for various parameter values. Figures 4 and 5 show histograms of the parameter distribution of m when m is equal to 0.25 and to .75, respectively. Figure 6 shows histograms of g when m is equal to 0.5.

Overall, the results of the two Monte Carlo exercises indicate that estimates of the Euler equation (1) obtained using optimal instruments procedures are more precise than conventional GMM methods. Maximum likelihood and GMM with optimal instruments estimates are generally unbiased and tend to behave reliably in a relevant sample size and across a range of values of m.

C. Discussion

The Monte Carlo experiments just described rule out by construction model misspecification. As a result, the difference in estimates obtained by conventional GMM versus optimal instruments methods is driven entirely by finite-sample performance. For valid inference in the context of equation (1), it is necessary to have a strong set of instruments for both $\mathrm{E}_t z_{t+1}$ and $\mathrm{E}_t x_t$.

Since the number of variables to be instrumented is greater than one, simple first-stage *F*-statistics do not provide information about the joint relevance of the instruments. However, Stock and Yogo (2003) have developed a test based on Donald and Cragg's (1993) multivariate version of the *F*-statistic. Specifically, they consider a test of whether the worst-behaved linear combination of the instruments provides sufficient information about the included endogenous variables in the GMM regression. While conservative, this approach is tractable and critical values for the test have been tabulated.

The Stock and Yogo statistic for weak instruments provides evidence that in our Monte Carlo exercises conventional GMM methods suffer from weak identification. If one is willing to accept a bias as high as 20 percent of the inconsistency of ordinary least squares, then it is possible to show that the average Stock and Yogo statistic is always below the appropriate critical value for *any* parameter configuration considered in Tables 1 and 2.[7]

[7] If one is willing to accept a bias as high as 30 percent, then we fail to reject the hypothesis of weak instruments only when m is less than 0.5 in the Monte Carlo experiments of Table 1 (output Euler equation).

In sum, in our Monte Carlo experiments the inclusion of z_{t-1} in equation (1) makes the remaining instruments too weak to produce unbiased estimates of m and g under conventional GMM estimation. Instead, optimal instruments estimation methods, by imposing all the constraints placed on the reduced form by the model, provide sufficiently strong instruments to generate an unbiased distribution of estimates in most circumstances.

III. Empirical Applications

In this section we compare estimates for the Euler equation (1) on actual data using conventional GMM estimation and optimal instruments methods. We estimate both an output Euler equation and an inflation Euler equation. The sample period is 1996:Q1 to 2001:Q4. We use two different measures for the output gap: (i) the deviation of log real GDP from its Hodrick–Prescott (HP) filtered trend; and (ii) the deviation of log real GDP from its segmented deterministic linear trend, with breakpoints in 1974 and 1995. Inflation and interest rates are as defined in the previous section. For the inflation Euler equation, we also consider a system augmented by the inclusion of real unit labor costs in the nonfarm business sector.[8] In this four-variable system, real unit labor costs replace the output gap as the driving process in equation (1).

Conventional GMM estimation is conducted with an instruments set consisting of four lags of each of the endogenous variables plus a

[8] Real unit labor costs are defined as unit labor costs in the nonfarm business sector deflated by the nonfarm business sector implicit price deflator.

constant term.[9] When performing optimal instruments estimation, the unrestricted VAR for the auxiliary structure (3) has lag length of four, also. Table 3 displays estimation results for the output Euler equation. For each estimation method, the table reports two sets of estimates according to the definition of the output gap that is being used. Overall, lagged output appears to be an essential component across all specifications and estimation methods. The estimate of m is one half when using conventional GMM estimation, and it is somewhat higher when optimal instruments methods are used. The real interest rate coefficient estimates are economically minute, and statistical significance is achieved in ML estimation only. Note that ML and optimal instruments GMM estimates are very close, although standard errors for the optimal instruments GMM method are large. These results are similar to the findings of Fuhrer and Rudebusch (2003), who compare conventional GMM and ML estimates for a richer specification of the output Euler equation. Conventional GMM estimates center on a larger forward-looking component than optimal instruments estimates, but the link between output and current and future real interest rates is largely missing.

Table 4 displays estimation results for the inflation Euler equation. The table has entries also for the specification in which real unit labor costs replace the output gap as the driving process in (1).[10] In this case, the instruments set for conventional GMM

[9] We use a Newey-West estimate of the weighting matrix with a lag length of 4.
[10] In a micro-founded setup, the optimal price level is set as a markup over a present discounted value of current and future marginal costs. Thus, the driving process for inflation is better described by real marginal costs. The conditions under which real marginal costs can be well approximated by a

estimation comprises four variables, while the vector X in the auxiliary structure (3) used for the construction of optimal instruments includes real unit labor costs, the output gap, and the federal funds rate. Conventional GMM estimates are not particularly encouraging for this simple specification of the inflation Euler equation. GMM estimates suggest a larger forward-looking component than optimal instruments estimates, but the link between inflation and current and future activity, measured either by the output gap or by real unit labor costs, is either insignificant or has the wrong sign. Optimal instruments estimates have the correct sign for g, although the “demand pressure” coefficient is significant in one instance only. Again, ML estimates and GMM with optimal instruments estimates are extremely close.

Estimates in tables 3 and 4 are suggestive of the potential differences between conventional GMM estimation and estimation with optimal instruments. While the results are specific to the simplified version of the Euler equation we have considered, it is important to note that in this particular context weak identification is a feature of conventional GMM estimation. It is indeed possible to show that the Stock and Yogo test statistic for instrument relevance is well below the critical value in all the conventional GMM estimates reported in tables 3 and 4.[11] Conventional GMM provides only weak instruments for

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measure of the output gap are in fact restrictive. See Galí and Gertler (1999).

[11] Specifically, the estimated Stock and Yogo statistic is always below the appropriate critical value when the bias is no more than 20 percent of the inconsistency of OLS. This weak identification feature has already been noted by Fuhrer and Rudebusch (2003) in the context of estimating an Euler equation for ouput, and by Ma (2002) in the context of estimating an Euler equation for inflation.

z_{t+1} and x_t, and such a feature will continue to persist for more general specifications of (1) unless other variables explaining a higher fraction of the joint variation in z and x can be found.

IV. Conclusions

Structural relations that explicitly represent the dependence of economic decisions upon expectations regarding the future provide the foundations of modern macroeconomic analysis. The degree to which agents are forward-looking has important consequences for the analysis of the character of optimal monetary policy.[12] In this context, the debate about a quantitatively realistic account of the monetary transmission mechanism remains open. Different studies have reached different conclusions about the importance of expectations of future interest rates and demand pressures on the actual dynamics of output and inflation.

This study compares different methods for estimating forward-looking output and inflation equations. Such an exercise is relevant because we suspect that the disparate nature of the extant empirical findings is largely dependent on the estimation methodology. We show that weak identification can be an issue in conventional GMM estimation of output and inflation forward-looking relations. It is thus important to resort to methods that are more reliable than GMM when instruments are weak. We propose a GMM procedure that, instead of

[12] For example, with purely forward-looking specifications, the optimal response to an inflationary cost-push shock usually requires policymakers to initially allow for a spurt in inflation and later induce a period of deflation. In the presence of a large inertial component in inflation, it is instead optimal to bring inflation down gradually without allowing for an initial “overshooting,” and endure a much larger contraction in output.

instrumenting by means of simple linear projections on the instruments set, uses projections that impose the dynamic constraints implied by the forward-looking relation. This “optimal instruments” procedure is similar to maximum likelihood estimation, and provides an alternative to maximum likelihood when the assumption of normality of the structural shocks is not satisfied in the data. In contrast to conventional GMM estimation, we show that both GMM with optimal instruments and maximum likelihood provide instruments that are sufficiently strong to center the parameter distributions on the true values when conventional GMM procedures exhibit weak identification.

Overall, our findings argue in favor of using optimal instruments techniques when estimating output or inflation Euler relations. Optimal instruments methods also provide a tighter test of the Euler relation because they impose a constrained reduced form that is the rational expectations solution to the relation at hand. In so doing, optimal instruments methods exploit the most distinguishing feature of dynamic rational expectations macro models.

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Table 1

Properties of ML and GMM Estimators: Output Euler Equation

$$z_t = \mu z_{t-1} + (0.98 - \mu)\mathrm{E}_t z_{t+1} + \gamma \mathrm{E}_t x_t + \varepsilon_t$$

Panel A. Estimates of μ

Estimation Method	μ^T	Mean($\hat{\mu}$)	Median($\hat{\mu}$)	SE($\hat{\mu}$)
GMM	0.90	0.72	0.72	0.22
GMM	0.75	0.69	0.69	0.18
GMM	0.50	0.51	0.51	0.13
GMM	0.25	0.33	0.33	0.14
GMM	0.10	0.22	0.23	0.16
ML	0.90	0.90	0.90	0.09
ML	0.75	0.76	0.75	0.06
ML	0.50	0.50	0.50	0.04
ML	0.25	0.26	0.26	0.06
ML	0.10	0.13	0.13	0.09
Optimal Inst. GMM	0.90	0.90	0.89	0.09
Optimal Inst. GMM	0.75	0.76	0.75	0.06
Optimal Inst. GMM	0.50	0.50	0.50	0.04
Optimal Inst. GMM	0.25	0.28	0.28	0.05
Optimal Inst. GMM	0.10	0.16	0.16	0.09

Panel B. Estimates of γ

Estimation Method	μ^T	Mean($\hat{\gamma}$)	Median($\hat{\gamma}$)	Median SE($\hat{\gamma}$)
GMM	0.90	-0.31	-0.29	0.25
GMM	0.75	-0.42	-0.41	0.28
GMM	0.50	-0.51	-0.51	0.26
GMM	0.25	-0.56	-0.56	0.21
GMM	0.10	-0.55	-0.54	0.20
ML	0.90	-0.51	-0.49	0.11
ML	0.75	-0.52	-0.51	0.11
ML	0.50	-0.51	-0.50	0.11
ML	0.25	-0.49	-0.48	0.12
ML	0.10	-0.47	-0.47	0.13
Optimal Inst. GMM	0.90	-0.51	-0.49	0.11
Optimal Inst. GMM	0.75	-0.52	-0.50	0.11
Optimal Inst. GMM	0.50	-0.50	-0.49	0.11
Optimal Inst. GMM	0.25	-0.45	-0.44	0.13
Optimal Inst. GMM	0.10	-0.42	-0.42	0.15

Note: The true data generating process has $\mu = \mu^T$, which is displayed in the second column, and a $\gamma = \gamma^T = -0.5$ in all cases.

Table 2

Properties of ML and GMM Estimators: Inflation Euler Equation

$$z_t = m z_{t-1} + (0.98 - m)\mathrm{E}_t z_{t+1} + g \mathrm{E}_t x_t + e_t$$

Panel A. Estimates of m

Estimation Method	m^T	Mean$(\hat{m})$	Median$(\hat{m})$	SE$(\hat{m})$
GMM	0.90	0.50	0.50	0.15
GMM	0.75	0.51	0.51	0.17
GMM	0.50	0.51	0.50	0.16
GMM	0.25	0.38	0.35	0.20
GMM	0.10	0.28	0.25	0.22
ML	0.90	0.88	0.88	0.15
ML	0.75	0.76	0.73	0.16
ML	0.50	0.50	0.50	0.03
ML	0.25	0.26	0.26	0.09
ML	0.10	0.12	0.13	0.06
Optimal Inst. GMM	0.90	0.88	0.89	0.15
Optimal Inst. GMM	0.75	0.76	0.72	0.16
Optimal Inst. GMM	0.50	0.50	0.50	0.03
Optimal Inst. GMM	0.25	0.28	0.28	0.05
Optimal Inst. GMM	0.10	0.16	0.17	0.06

Panel B. Estimates of g

Estimation Method	m^T	Mean$(\hat{g})$	Median$(\hat{g})$	Median SE$(\hat{g})$
GMM	0.90	0.02	0.02	0.05
GMM	0.75	0.03	0.04	0.07
GMM	0.50	0.10	0.10	0.10
GMM	0.25	0.13	0.12	0.09
GMM	0.10	0.13	0.13	0.07
ML	0.90	0.10	0.09	0.07
ML	0.75	0.11	0.09	0.07
ML	0.50	0.10	0.09	0.03
ML	0.25	0.08	0.08	0.04
ML	0.10	0.08	0.08	0.04
Optimal Inst. GMM	0.90	0.10	0.09	0.07
Optimal Inst. GMM	0.75	0.11	0.09	0.07
Optimal Inst. GMM	0.50	0.09	0.09	0.03
Optimal Inst. GMM	0.25	0.06	0.05	0.04
Optimal Inst. GMM	0.10	0.05	0.05	0.04

Note: The true data generating process has $m = m^T$, which is displayed in the second column, and a $g = g^T = 0.1$ in all cases.

Table 3

Estimates of Output Euler Equation: 1966:Q1 to 2001:Q4

$$z_t = m z_{t-1} + (1-m)\mathrm{E}_t z_{t+1} + g\mathrm{E}_t x_t + e_t$$

Estimation Method	Specification	m	$SE(m)$	g	$SE(g)$
GMM	HP	0.5033	0.0350	-0.0068	0.0141
GMM	ST	0.5083	0.0319	-0.0100	0.0136
ML	HP	0.5418	0.0313	-0.0214	0.0079
ML	ST	0.5725	0.0313	-0.0295	0.0089
Optimal Inst. GMM	HP	0.5556	3.1843	-0.0193	0.7374
Optimal Inst. GMM	ST	0.5866	3.47055	-0.0279	0.8691

Note: The specification column provides the output trend procedure. HP is the Hodrick-Prescott filter of log real GDP, and ST is a segmented deterministic linear trend for log real GDP.

Table 4

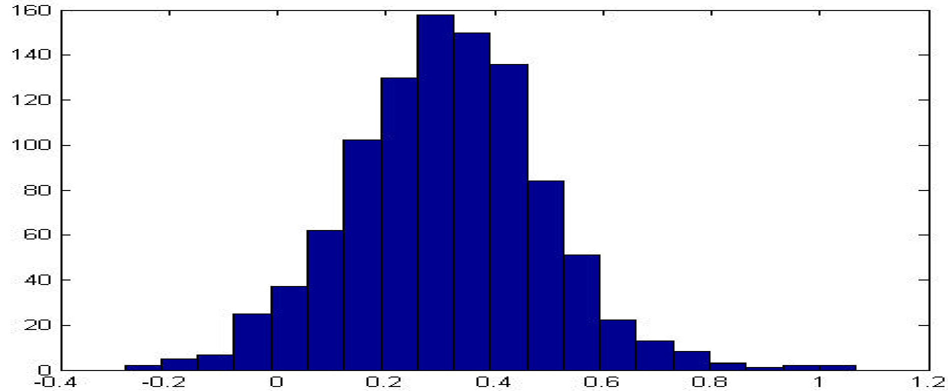
Estimates of Inflation Euler Equation: 1966:Q1 to 2001:Q4

$$z_t = m z_{t-1} + (1-m)\mathrm{E}_t z_{t+1} + g\mathrm{E}_t x_t + e_t$$

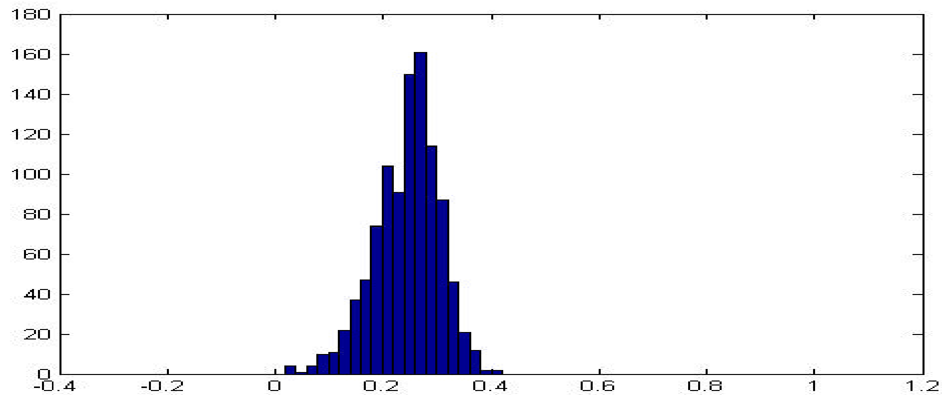
Estimation Method	Specification	m	$SE(m)$	g	$SE(g)$
GMM	HP	0.3024	0.0986	-0.0517	0.0373
GMM	ST	0.3167	0.1054	-0.0296	0.0215
GMM	*rulc*	0.4076	0.0596	0.0293	0.0193
ML	HP	0.5861	0.0304	0.0248	0.0170
ML	ST	0.5906	0.0317	0.0206	0.0117
ML	*rulc*	0.4772	0.0212	0.0335	0.0075
Optimal Inst. GMM	HP	0.5861	3.6211	0.0247	0.5906
Optimal Inst. GMM	ST	0.5868	3.2577	0.0114	0.6041
Optimal Inst. GMM	*rulc*	0.4773	1.4956	0.0334	0.4487

Note: The specification column provides the output trend procedure when the entry is HP or ST. HP is the Hodrick-Prescott filter of log real GDP, and ST is a segmented deterministic linear trend for log real GDP. When the entry is *rulc*, the specification replaces the output gap with real unit labor costs as the driving process in equation (1).

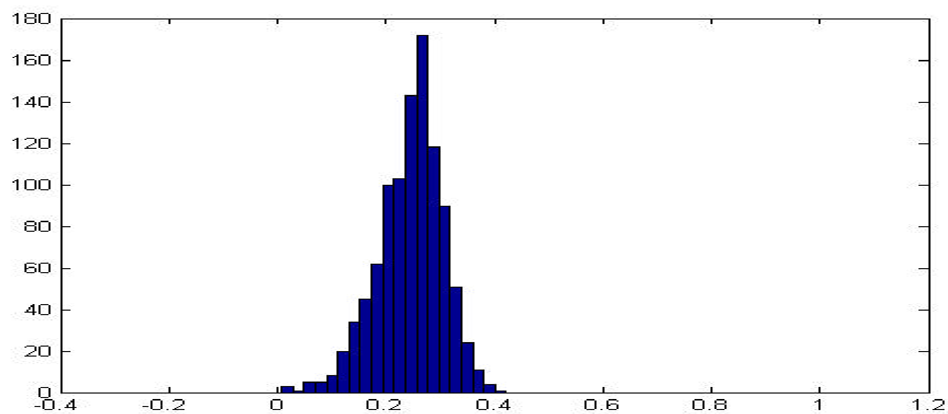
Figure 1
Monte Carlo Parameter Estimates of μ. True $\mu = 0.25$



Conventional GMM Estimation

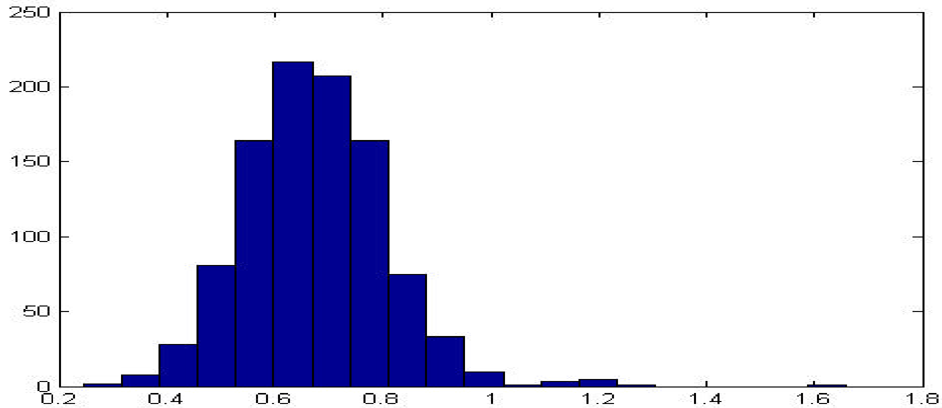


Optimal Instruments GMM Estimation

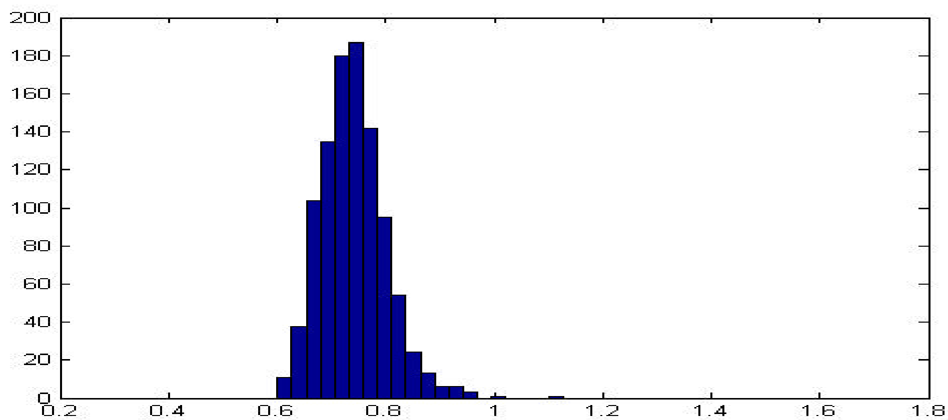


Maximum Likelihood Estimation

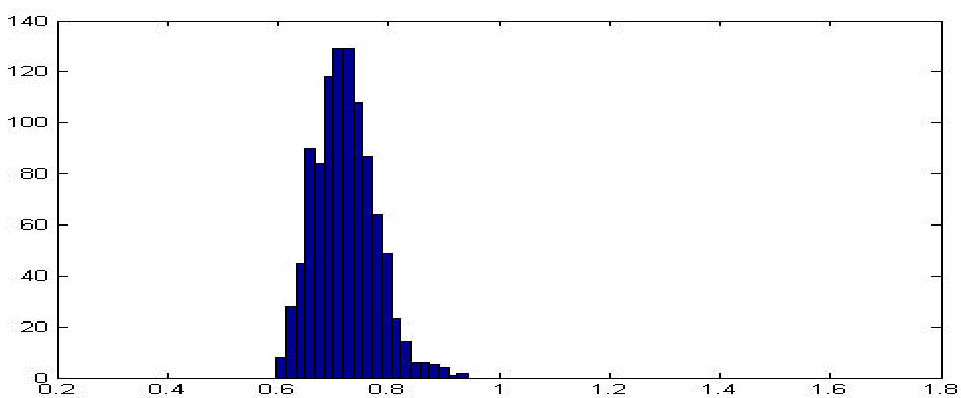
Figure 2
Monte Carlo Parameter Estimates of μ. True μ = 0.75



Conventional GMM Estimation

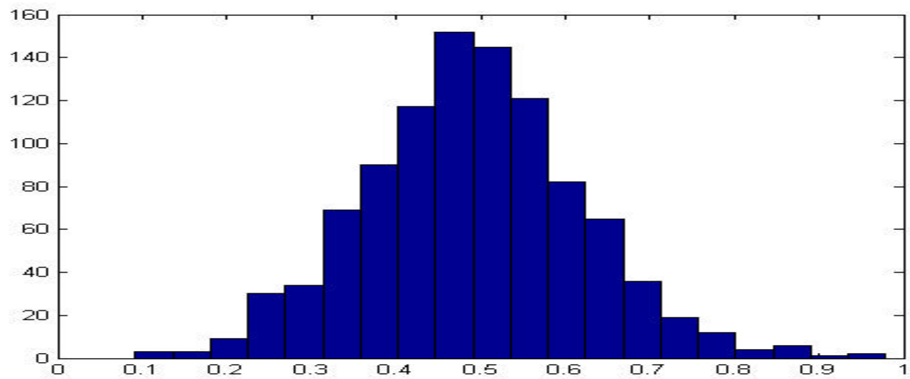


Optimal Instruments GMM Estimation

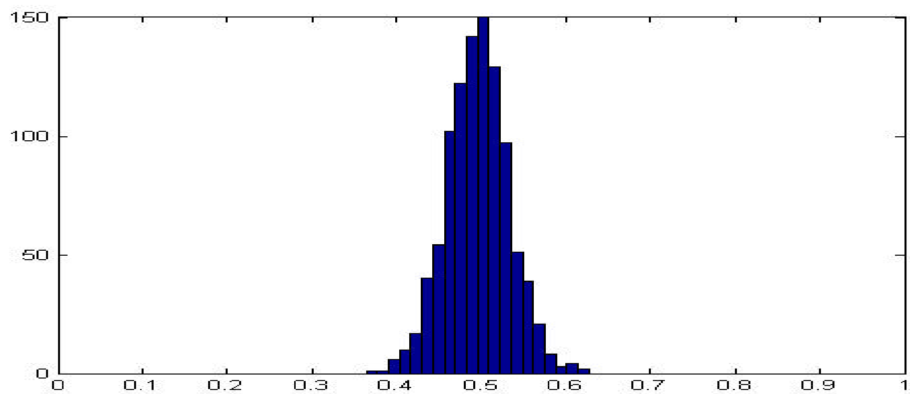


Maximum Likelihood Estimation

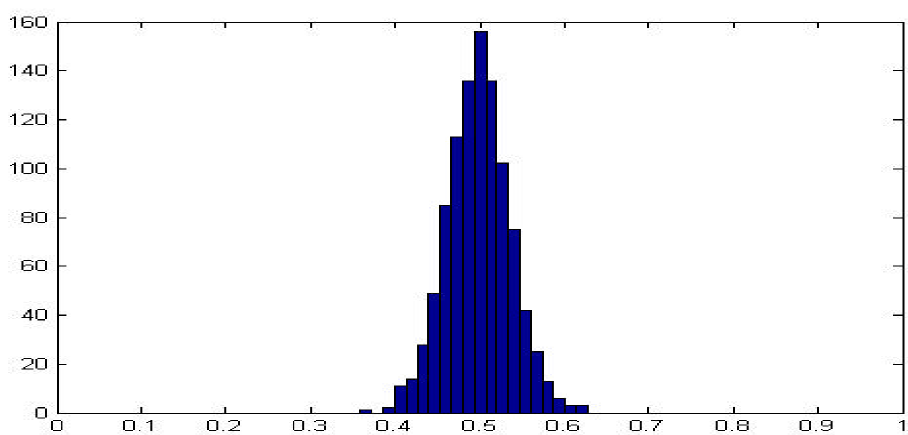
Figure 3
Monte Carlo Parameter Estimates of ?. True ? =μ = 0.50



Conventional GMM Estimation



Optimal Instruments GMM Estimation



Maximum Likelihood Estimation