11. THE CKM QUARK-MIXING MATRIX

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11.1. Introduction

The masses and mixings of quarks have a common origin in the Standard Model (SM). They arise from the Yukawa interactions with the Higgs condensate,

$$\mathcal{L}_Y = -Y^d_{ij}\,\overline{Q^I_{Li}}\,\phi\, d^I_{Rj} - Y^u_{ij}\,\overline{Q^I_{Li}}\,\epsilon\,\phi^* u^I_{Rj} + \text{h.c.}, \tag{11.1}$$

where $Y^{u,d}$ are 3×3 complex matrices, ϕ is the Higgs field, i,j are generation labels, and ϵ is the 2×2 antisymmetric tensor. Q^I_L are left-handed quark doublets, and d^I_R and u^I_R are right-handed down- and up-type quark singlets, respectively, in the weak-eigenstate basis. When ϕ acquires a vacuum expectation value, $\langle\phi\rangle = (0, v/\sqrt{2})$, Eq. (11.1) yields mass terms for the quarks. The physical states are obtained by diagonalizing $Y^{u,d}$ by four unitary matrices, $V^{u,d}_{L,R}$, as $M^f_{\rm diag} = V^f_L\, Y^f\, V^{f\dagger}_R (v/\sqrt{2})$, $f = u, d$. As a result, the charged-current $W^\pm$ interactions couple to the physical u_{Lj} and d_{Lk} quarks with couplings given by

$$V_{\rm CKM} \equiv V^u_L V^{d\dagger}_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{11.2}$$

This Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2] is a 3×3 unitary matrix. It can be parameterized by three mixing angles and a CP-violating phase. Of the many possible parameterizations, a standard choice has become

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{11.3}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and δ is the KM phase [2] responsible for all CP-violating phenomena in flavor-changing processes in the SM. The angles θ_{ij} can be chosen to lie in the first quadrant, so $s_{ij}, c_{ij} \ge 0$.

It is known experimentally that $s_{13} \ll s_{23} \ll s_{12} \ll 1$, and it is convenient to exhibit this hierarchy using the Wolfenstein parameterization. We define [3–5]

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}\,, \qquad s_{23} = A\lambda^2 = \lambda\left|\frac{V_{cb}}{V_{us}}\right|,$$

$$s_{13}e^{i\delta} = V^*_{ub} = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar\rho + i\bar\eta)]}\,. \tag{11.4}$$

These relations ensure that $\bar\rho + i\bar\eta = -(V_{ud}V^*_{ub})/(V_{cd}V^*_{cb})$ is phase-convention-independent, and the CKM matrix written in terms of λ, A, $\bar\rho$, and $\bar\eta$ is unitary to all orders in λ. The definitions of $\bar\rho, \bar\eta$ reproduce all approximate results in the literature. For example, $\bar\rho = \rho(1 - \lambda^2/2 + \ldots)$ and we can write $V_{\rm CKM}$ to $\mathcal{O}(\lambda^4)$ either in terms of $\bar\rho, \bar\eta$ or, traditionally,

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,. \tag{11.5}$$

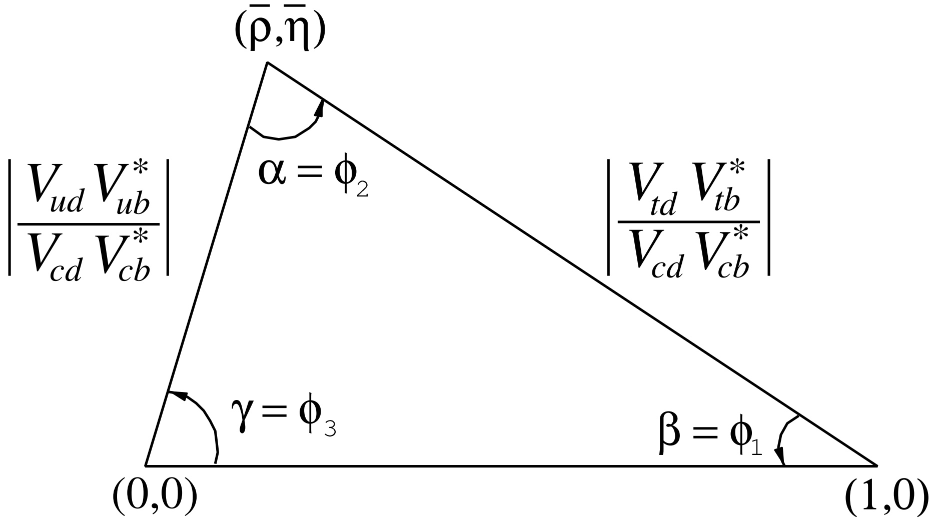

Figure 11.1: Sketch of the unitarity triangle.

The CKM matrix elements are fundamental parameters of the SM, so their precise determination is important. The unitarity of the CKM matrix imposes $\sum_i V_{ij}V_{ik}^* = \delta_{jk}$ and $\sum_j V_{ij}V_{kj}^* = \delta_{ik}$. The six vanishing combinations can be represented as triangles in a complex plane, of which the ones obtained by taking scalar products of neighboring rows or columns are nearly degenerate. The areas of all triangles are the same, half of the Jarlskog invariant, J [6], which is a phase-convention-independent measure of CP violation, defined by $\mathrm{Im}\left[V_{ij}V_{kl}V_{il}^*V_{kj}^*\right] = J\sum_{m,n}\varepsilon_{ikm}\varepsilon_{jln}$.

The most commonly used unitarity triangle arises from

$$V_{ud}\,V_{ub}^* + V_{cd}\,V_{cb}^* + V_{td}\,V_{tb}^* = 0\,, \tag{11.6}$$

by dividing each side by the best-known one, $V_{cd}V_{cb}^*$ (see Fig. 1). Its vertices are exactly $(0,0)$, $(1,0)$, and, due to the definition in Eq. (11.4), $(\bar\rho,\bar\eta)$. An important goal of flavor physics is to overconstrain the CKM elements, and many measurements can be conveniently displayed and compared in the $\bar\rho,\bar\eta$ plane.

Processes dominated by loop contributions in the SM are sensitive to new physics, and can be used to extract CKM elements only if the SM is assumed. In Sec. 11.2 and 11.3, we describe such measurements assuming the SM, and discuss implications for new physics in Sec. 11.5.

11.2. Magnitudes of CKM elements

11.2.1. $|V_{ud}|$:

The most precise determination of $|V_{ud}|$ comes from the study of superallowed $0^+ \to 0^+$ nuclear beta decays, which are pure vector transitions. Taking the average of the nine most precise determinations [7] yields [8]

$$|V_{ud}| = 0.97418 \pm 0.00027. \tag{11.7}$$

The error is dominated by theoretical uncertainties stemming from nuclear Coulomb distortions and radiative corrections. A precise determination of $|V_{ud}|$ is also obtained from the measurement of the neutron lifetime. The theoretical uncertainties are very small, but the determination is limited by the knowledge of the ratio of the axial-vector and vector couplings, $g_A = G_A/G_V$ [8]. The PIBETA experiment [9] has improved the measurement of the $\pi^+ \to \pi^0 e^+ \nu$ branching ratio to 0.6%, and quote $|V_{ud}| = 0.9728 \pm 0.0030$, in agreement with the more precise results listed above. The interest in this measurement is that the determination of $|V_{ud}|$ is very clean theoretically, because it is a pure vector transition and is free from nuclear-structure uncertainties.

11.2.2. $|V_{us}|$:

The product of $|V_{us}|$ and the form factor at $q^2 = 0$, $|V_{us}|\, f_+(0)$ have been extracted traditionally from $K_L^0 \to \pi e \nu$ decays in order to avoid isospin-breaking corrections ($\pi^0 - \eta$ mixing) that affect the charged-kaon semileptonic decay, and the complications induced by a second (scalar) form factor present in the muonic decays. The last round of experiments has lead to enough experimental constraints to justify the comparison between different decay modes. Systematic errors related to the experimental quantities, *e.g.*, the lifetime of neutral or charged kaons, and the form factor determinations for electron and muonic decays, differ among decay modes, and the consistency between different determinations enhances the confidence in the final result. For this reason, we follow the prescription [10] to average $K_L^0 \to \pi e \nu$, $K_L^0 \to \pi \mu \nu$, $K^\pm \to \pi^0 e^\pm \nu$, $K^\pm \to \pi^0 \mu^\pm \nu$ and $K_S^0 \to \pi e \nu$. The average of these five decay modes yields $|V_{us}|\, f_+(0) = 0.21668 \pm 0.00045$. Results obtained from each decay mode, and exhaustive references to the experimental data, are listed for instance, inRef. 8. The form factor value $f_+(0) = 0.961 \pm 0.008$ [11] is still broadly accepted [8] and gives

$$|V_{us}| = 0.2255 \pm 0.0019. \tag{11.8}$$

Lattice gauge theory calculations [12] provide results for $f_+(0)$ in agreement with [11], while other calculations [14] differ by as much as 2%.

Other determinations of $|V_{us}|$ involve leptonic kaon decays, hyperon decays, and τ decays. The calculation of the ratio of the kaon and pion decay constants enables one to extract $|V_{us}/V_{ud}|$ from $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$, where (γ) indicates that radiative decays are included [15]. The KLOE measurement of the $K^+ \to \mu^+\nu(\gamma)$ branching ratio [16], combined with the lattice QCD calculation, $f_K/f_\pi = 1.198 \pm 0.010$ [12], leads to $|V_{us}| = 0.2242 \pm 0.0019$, where the accuracy is limited by the knowledge of the ratio of the decay constants. The determination from hyperon decays was updated in [18]. These authors focus on the analysis of the vector form factor, protected from first order $SU(3)$ breaking effects by the Ademollo-Gatto theorem [19], and treat the ratio between the axial and vector form factors g_1/f_1 as experimental input, thus avoiding first order $SU(3)$ breaking effects in the axial-vector contribution. They find $|V_{us}| = 0.2250 \pm 0.0027$, although this does not include an estimate of the theoretical uncertainty due to second-order $SU(3)$-breaking, contrary to Eq. (11.8). Concerning hadronic τ decays to strange particles, the latest determinations based on ALEPH, OPAL, CLEO, and recent *BaBar* and Belle data yields $|V_{us}| = 0.2165 \pm 0.0027$ [20].

11.2.3. $|V_{cd}|$:

The magnitude of V_{cd} can be extracted from semileptonic charm decays if theoretical knowledge of the form factors is available. Three-flavor unquenched lattice QCD calculations for $D \to K\ell\nu$ and $D \to \pi\ell\nu$ have been published [21]. Using these estimates and the average of recent CLEO-c [22] and Belle [23] measurements of $D \to \pi\ell\nu$ decays, one obtains $|V_{cd}| = 0.218 \pm 0.007 \pm 0.023$, where the first uncertainty is experimental, and the second is from the theoretical error of the form factor.

This determination is not yet as precise as the one based on neutrino and antineutrino interactions. The difference of the ratio of double-muon to single-muon production by neutrino and antineutrino beams is proportional to the charm cross section off valence d-quarks, and therefore to $|V_{cd}|^2$ times the average semileptonic branching ratio of charm mesons, $\mathcal{B}_\mu$. The method was used first by CDHS [24] and then by CCFR [25,26] and CHARM II [27]. Averaging these results is complicated, not only because it requires assumptions about the scale of the QCD corrections, but also because $\mathcal{B}_\mu$ is an effective quantity, which depends on the specific neutrino beam characteristics. Given that no new experimental input is available, we quote the average provided in a previous review, $\mathcal{B}_\mu |V_{cd}|^2 = (0.463 \pm 0.034) \times 10^{-2}$ [28]. Analysis cuts make these experiments insensitive to neutrino energies smaller than 30 GeV. Thus, $\mathcal{B}_\mu$ should be computed using only neutrino interactions with visible energy larger than 30 GeV. An appraisal [29] based on charm-production fractions measured in neutrino interactions [30,31] gives $\mathcal{B}_\mu = 0.088 \pm 0.006$. Data from the CHORUS experiment [32] are now sufficiently precise to extract $\mathcal{B}_\mu$ directly, by comparing the number of charm decays with a muon to the total number of charmed hadrons found in the nuclear emulsions. Requiring the visible energy to be larger than 30 GeV, CHORUS finds $\mathcal{B}_\mu = 0.085 \pm 0.009 \pm 0.006$. To extract $|V_{cd}|$, we use the average of these two determinations, $\mathcal{B}_\mu = 0.0873 \pm 0.0052$, and obtain

$$|V_{cd}| = 0.230 \pm 0.011. \tag{11.9}$$

11.2.4. $|V_{cs}|$:

The determination of $|V_{cs}|$ from neutrino and antineutrino scattering suffers from the uncertainty of the s-quark sea content. Measurements sensitive to $|V_{cs}|$ from on-shell $W^\pm$ decays were performed at LEP-2. The branching ratios of the W depend on the six CKM matrix elements involving quarks with masses smaller than M_W. The W branching ratio to each lepton flavor is given by $1/\mathcal{B}(W \to \ell\bar{\nu}_\ell) = 3\big[1 + \sum_{u,c,d,s,b} |V_{ij}|^2\,(1 + \alpha_s(m_W)/\pi)\big]$. The measurement assuming lepton universality, $\mathcal{B}(W \to \ell\bar{\nu}_\ell) = (10.83 \pm 0.07 \pm 0.07)\,\%$ [33], implies $\sum_{u,c,d,s,b} |V_{ij}|^2 = 2.002 \pm 0.027$. This is a precise test of unitarity, but only flavor-tagged measurements determine $|V_{cs}|$. DELPHI measured tagged $W^+ \to c\bar{s}$ decays, obtaining $|V_{cs}| = 0.94^{+0.32}_{-0.26} \pm 0.13$ [34]. Hereafter, the first error is statistical and the second is systematic, unless mentioned otherwise.

The direct determination of $|V_{cs}|$ is possible from semileptonic D or leptonic D_s decays, using unquenched lattice QCD calculations of the semileptonic D form factor or the D_s decay constant. For muonic decays, the average of the recent data from BABAR [35], Belle [36], and CLEO-c [37] gives $\mathcal{B}(D_s^+ \to \mu^+\nu) = (6.26 \pm 0.51) \times 10^{-3}$.

For decays with τ leptons, a measurement from CLEO-c is available: $\mathcal{B}(D_s^+ \to \tau^+\nu) = (6.47 \pm 0.61 \pm 0.26) \times 10^{-2}$ [38]. From each of these two values, determinations of $|V_{cs}|$ can be obtained by using the PDG values for the mass and lifetime of the D_s, the masses of the leptons, and $f_{D_s} = (249 \pm 3 \pm 16)\,\text{MeV}$ [39]. The average of these two determinations gives $|V_{cs}| = 1.07 \pm 0.08$, where the error is dominated by the lattice QCD determination of f_{D_s}. In semileptonic D decays, unquenched lattice QCD calculations have predicted the normalization and the shape (dependence on the invariant mass of the lepton pair, q^2) of the form factors in $D \to K\ell\nu$ and $D \to \pi\ell\nu$ [21]. Using these theoretical results and the average of recent CLEO-c [22], Belle [23] and BaBar [40] measurements of $B \to K\ell\nu$ decays, one obtains $|V_{cs}| = 0.99 \pm 0.01 \pm 0.10$, where the first error is experimental and the second one, which is dominant, is from the theoretical error of the form factor. Averaging the determinations from leptonic and semileptonic decays, we quote

$$|V_{cs}| = 1.04 \pm 0.06. \tag{11.10}$$

11.2.5. $|V_{cb}|$:

This matrix element can be determined from exclusive and inclusive semileptonic decays of B mesons to charm. The inclusive determinations use the semileptonic decay rate measurement, together with the leptonic energy and the hadronic invariant-mass spectra. The theoretical foundation of the calculation is the operator product expansion (OPE) [41,42]. It expresses the total rate and moments of differential energy and invariant-mass spectra as expansions in α_s, and inverse powers of the heavy-quark mass. The dependence on m_b, m_c, and the parameters that occur at subleading order is different for different moments, and a large number of measured moments overconstrains all the parameters, and tests the consistency of the determination. The precise extraction of $|V_{cb}|$ requires using a "threshold" quark mass definition [43,44]. Inclusive measurements have been performed using B mesons from Z^0 decays at LEP, and at e^+e^- machines operated at the $\Upsilon(4S)$. At LEP, the large boost of B mesons from Z^0 allows the determination of the moments throughout phase space, which is not possible otherwise, but the large statistics available at the B factories leads to more precise determinations. An average of the measurements and a compilation of the references are provided by Kowalewski and Mannel [45]: $|V_{cb}| = (41.6 \pm 0.7) \times 10^{-3}$.

Exclusive determinations are based on semileptonic B decays to D and D^*. In the $m_{b,c} \gg \Lambda_{\text{QCD}}$ limit, all form factors are given by a single Isgur-Wise function [46], which depends on the product of the four velocities, $w = v \cdot v'$, and of the initial and final-state hadrons. Heavy-quark symmetry determines the normalization of the rate at $w = 1$, the maximum momentum transfer to the leptons, and $|V_{cb}|$ is obtained from an extrapolation to $w = 1$. The exclusive determination, $|V_{cb}| = (38.6 \pm 1.3) \times 10^{-3}$ [45], is less precise than the inclusive one, because the theoretical uncertainty in the form factor, and the experimental uncertainty in the rate near $w = 1$, are both about 3%. The $|V_{cb}|$ review quotes a combination with a scaled error as [45]

$$|V_{cb}| = (41.2 \pm 1.1) \times 10^{-3}. \tag{11.11}$$

11.2.6. $|V_{ub}|$:

The determination of $|V_{ub}|$ from inclusive $B \to X_u \ell \bar{\nu}$ decay suffers from large $B \to X_c \ell \bar{\nu}$ backgrounds. In most regions of phase space where the charm background is kinematically forbidden, the hadronic physics enters via unknown nonperturbative functions, so-called shape functions. (In contrast, the nonperturbative physics for $|V_{cb}|$ is encoded in a few parameters.) At leading order in $\Lambda_{\rm QCD}/m_b$, there is only one shape function, which can be extracted from the photon energy spectrum in $B \to X_s \gamma$ [47,48], and applied to several spectra in $B \to X_u \ell \bar{\nu}$. The subleading shape functions are modeled in the current calculations. Phase space cuts for which the rate has only subleading dependence on the shape function are also possible [49]. The measurements of both hadronic and leptonic systems are important for effective choice of phase space. A different approach is to extend the measurements deeper into the $B \to X_c \ell \bar{\nu}$ region to reduce the theoretical uncertainties. Analyses of the electron-energy endpoint from CLEO [50], BABAR [51], and Belle [52] quote $B \to X_u e \bar{\nu}$ partial rates for $|\vec{p}_e| \geq 2.0\,\text{GeV}$ and $1.9\,\text{GeV}$, which are well below the charm endpoint. The large and pure $B\overline{B}$ samples at the B factories permit the selection of $B \to X_u \ell \bar{\nu}$ decays in events where the other B is fully reconstructed [53]. With this full-reconstruction tag method, the four momenta of both the leptonic and the hadronic systems can be measured. It also gives access to a wider kinematic region due to improved signal purity.

To extract $|V_{ub}|$ from an exclusive channel, the form factors have to be known. Experimentally, better signal-to-background ratios are offset by smaller yields. The $B \to \pi \ell \bar{\nu}$ branching ratio is now known to 6%. Unquenched lattice QCD calculations of the $B \to \pi \ell \bar{\nu}$ form factor for $q^2 > 16\,\text{GeV}^2$ are available [54,55] and yield $|V_{ub}| = (3.4 \pm 0.2^{+0.6}_{-0.4}) \times 10^{-3}$. Light-cone QCD sum rules are applicable for $q^2 < 14\,\text{GeV}^2$ [56] and yield similar results. The theoretical uncertainties in extracting $|V_{ub}|$ from inclusive and exclusive decays are different. A combination of the determinations is quoted by the V_{cb} and V_{ub} minireview as [45],

$$|V_{ub}| = (3.93 \pm 0.36) \times 10^{-3}, \tag{11.12}$$

which is dominated by the inclusive measurement. Given the theoretical and experimental progress, it will be interesting to see how the inclusive and exclusive determinations develop.

11.2.7. $|V_{td}|$ *and* $|V_{ts}|$:

The CKM elements $|V_{td}|$ and $|V_{ts}|$ cannot be measured from tree-level decays of the top quark, so one has to rely on determinations from B–$\overline{B}$ oscillations mediated by box diagrams with top quarks, or loop-mediated rare K and B decays. Theoretical uncertainties in hadronic effects limit the accuracy of the current determinations. These can be reduced by taking ratios of processes that are equal in the flavor $SU(3)$ limit to determine $|V_{td}/V_{ts}|$.

The mass difference of the two neutral B meson mass eigenstates is very well measured, $\Delta m_d = 0.507 \pm 0.005\,\text{ps}^{-1}$ [57]. For the B_s^0 system, the CDF Collaboration measured $\Delta m_s = (17.77 \pm 0.10 \pm 0.07)\,\text{ps}^{-1}$ [58] with more than 5σ significance (the DØ result [59] is compatible and has about 2σ significance). Using the unquenched lattice QCD calculations [60] $f_{B_d}\sqrt{\widehat{B}_{B_d}} = (223 \pm 8 \pm 16)\,\mathrm{MeV}$, $f_{B_s}\sqrt{\widehat{B}_{B_s}} = (275 \pm 7 \pm 15)\,\mathrm{MeV}$, and assuming $|V_{tb}| = 1$, one finds

$$|V_{td}| = (8.1 \pm 0.6) \times 10^{-3}, \qquad |V_{ts}| = (38.7 \pm 2.3) \times 10^{-3}. \tag{11.13}$$

The uncertainties are dominated by lattice QCD. Several uncertainties are reduced in the calculation of the ratio $\xi = (f_{B_s}\sqrt{\widehat{B}_{B_s}})/(f_{B_d}\sqrt{\widehat{B}_{B_d}}) = 1.23 \pm 0.02 \pm 0.03$, and therefore the constraint on $|V_{td}/V_{ts}|$ from $\Delta m_d/\Delta m_s$ is more reliable theoretically. These provide a new theoretically clean and significantly improved constraint

$$\left|V_{td}/V_{ts}\right| = (0.209 \pm 0.001 \pm 0.006) \times 10^{-3}. \tag{11.14}$$

The inclusive rate $\mathcal{B}(B \to X_s\gamma) = (3.55{\pm}0.26){\times}10^{-4}$ extrapolated to $E_\gamma > 1.6\,\mathrm{GeV}$ [61] is also sensitive to $V_{tb}V_{ts}^*$. In addition to t-quark penguins, a large part of the sensitivity comes from charm contributions proportional to $V_{cb}V_{cs}^*$ via the application of 3×3 CKM unitarity (which is used here; any CKM determination from loop processes necessarily assumes the SM). With the recently completed NNLO calculation of $\mathcal{B}(B \to X_s\gamma)_{E_\gamma > E_0}/\mathcal{B}(B \to X_c e\bar{\nu})$ [62], we obtain $|V_{ts}/V_{cb}| = (1.04 \pm 0.05)$.

A complementary determination of $|V_{td}/V_{ts}|$ is possible from the ratio of $B \to \rho\gamma$ and $K^*\gamma$ rates. The ratio of the neutral modes is theoretically cleaner than that of the charged ones, because the poorly known spectator-interaction contribution is expected to be smaller (W-exchange vs. weak annihilation). For now, we average the charged and neutral rates assuming the isospin symmetry and heavy-quark limit motivated relation, $|V_{td}/V_{ts}|^2 = \xi_\gamma^2\,[\Gamma(B^+ \to \rho^+\gamma) + 2\Gamma(B^0 \to \rho^0\gamma)]/[\Gamma(B^+ \to K^{*+}\gamma) + \Gamma(B^0 \to K^{*0}\gamma)] = (2.96 \pm 0.57)\%$ [61]. Here ξ_γ contains the poorly known hadronic physics. Using $\xi_\gamma = 1.2 \pm 0.2$ [63], and combining the experimental and theoretical errors in quadrature, gives $|V_{td}/V_{ts}| = 0.21 \pm 0.04$.

A theoretically clean determination of $|V_{td}V_{ts}^*|$ is possible from $K^+ \to \pi^+\nu\bar{\nu}$ decay [64]. Experimentally, only three events have been observed [65], and the rate is consistent with the SM with large uncertainties. Much more data are needed for a precision measurement.

11.2.8. $|V_{tb}|$:

The determination of $|V_{tb}|$ from top decays uses the ratio of branching fractions $R = \mathcal{B}(t \to Wb)/\mathcal{B}(t \to Wq) = |V_{tb}|^2/(\sum_q |V_{tq}|^2) = |V_{tb}|^2$, where $q = b, s, d$. The CDF and DØ measurements performed on data collected during Run II of the Tevatron give $|V_{tb}| > 0.78$ [66] and $|V_{tb}| > 0.89$ [67], respectively, at 95% CL. The direct determination of $|V_{tb}|$ without assuming unitarity is possible from the single top-quark-production cross section. From the average cross sections of DØ [68] and CDF [69], $(3.7 \pm 0.8)\,\mathrm{pb}$, the lower limit at 95% CL is obtained to be

$$|V_{tb}| > 0.74\,. \tag{11.15}$$

An attempt at constraining $|V_{tb}|$ from the precision electroweak data was made in [70]. The result, mostly driven by the top-loop contributions to $\Gamma(Z \to b\bar{b})$, gives $|V_{tb}| = 0.77^{+0.18}_{-0.24}$.

11.3. Phases of CKM elements

As can be seen from Fig. 11.1, the angles of the unitarity triangle are

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$$
$$\alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right),$$
$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \quad (11.16)$$

Since CP violation involves phases of CKM elements, many measurements of CP-violating observables can be used to constrain these angles and the $\bar\rho, \bar\eta$ parameters.

11.3.1. *ϵ and ϵ'* :

The measurement of CP violation in K^0–$\overline{K}^0$ mixing, $|\epsilon| = (2.233 \pm 0.015) \times 10^{-3}$ [71], provides important information about the CKM matrix. In the SM [72]

$$|\epsilon| = \frac{G_F^2 f_K^2 m_K m_W^2}{12\sqrt{2}\,\pi^2 \Delta m_K}\, \widehat{B}_K \Big\{\eta_c S(x_c) \mathrm{Im}[(V_{cs}V_{cd}^*)^2]$$
$$+ \eta_t S(x_t) \mathrm{Im}[(V_{ts}V_{td}^*)^2] + 2\eta_{ct} S(x_c, x_t) \mathrm{Im}(V_{cs}V_{cd}^* V_{ts}V_{td}^*)\Big\}, \quad (11.17)$$

where S is an Inami-Lim function [73], $x_q = m_q^2/m_W^2$, and η_i are perturbative QCD corrections. The constraint from ϵ in the $\bar\rho, \bar\eta$ plane is bounded by approximate hyperbolas. The dominant uncertainties are due to the bag parameter, for which we use $\widehat{B}_K = 0.720 \pm 0.039$ from lattice QCD [12], and the parametric uncertainty proportional to $\sigma(A^4)$ from $(V_{ts}V_{td}^*)^2$, which is approximately $\sigma(|V_{cb}|^4)$.

The measurement of ϵ' provides a qualitative test of the CKM mechanism because its nonzero experimental average, $\mathrm{Re}(\epsilon'/\epsilon) = (1.67 \pm 0.23) \times 10^{-3}$ [71], demonstrates the existence of direct CP violation, a prediction of the KM ansatz. While $\mathrm{Re}(\epsilon'/\epsilon) \propto \mathrm{Im}(V_{td}V_{ts}^*)$, this quantity cannot easily be used to extract CKM parameters, because the electromagnetic penguin contributions tend to cancel the gluonic penguins for large m_t [74], thereby significantly increasing the hadronic uncertainties. Most estimates [75–77] can agree with the observed value, indicating that $\bar\eta$ is positive. Progress in lattice QCD, in particular finite-volume calculations [78,79], may eventually provide a determination of the $K \to \pi\pi$ matrix elements.

11.3.2. β / ϕ_1 :

11.3.2.1. *Charmonium modes:*

CP-violation measurements in B-meson decays provide direct information on the angles of the unitarity triangle, shown in Fig. 11.1. These overconstraining measurements serve to improve the determination of the CKM elements, or to reveal effects beyond the SM.

The time-dependent CP asymmetry of neutral B-decays to a final state f common to B^0 and $\overline{B}^0$ is given by [80,81]

$$\mathcal{A}_f = \frac{\Gamma(\overline{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\overline{B}^0(t) \to f) + \Gamma(B^0(t) \to f)} = S_f \sin(\Delta m_d\, t) - C_f \cos(\Delta m_d\, t), \tag{11.18}$$

where

$$S_f = \frac{2\,\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}\,, \qquad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\,, \qquad \lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}\,. \tag{11.19}$$

Here, q/p describes B^0–$\overline{B}^0$ mixing and, to a good approximation in the SM, $q/p = V_{tb}^* V_{td}/V_{tb}V_{td}^* = e^{-2i\beta+\mathcal{O}(\lambda^4)}$ in the usual phase convention. A_f ($\bar{A}_f$) is the amplitude of the $B^0 \to f$ ($\overline{B}^0 \to f$) decay. If f is a CP eigenstate, and amplitudes with one CKM phase dominate the decay, then $|A_f| = |\bar{A}_f|$, $C_f = 0$, and $S_f = \sin(\arg\lambda_f) = \eta_f \sin 2\phi$, where η_f is the CP eigenvalue of f and 2ϕ is the phase difference between the $B^0 \to f$ and $B^0 \to \overline{B}^0 \to f$ decay paths. A contribution of another amplitude to the decay with a different CKM phase makes the value of S_f sensitive to relative strong interaction phases between the decay amplitudes (it also makes $C_f \neq 0$ possible).

The $b \to c\bar{c}s$ decays to CP eigenstates ($B^0 \to$ charmonium $K^0_{S,L}$) are the theoretically cleanest examples, measuring $S_f = -\eta_f \sin 2\beta$. The $b \to sq\bar{q}$ penguin amplitudes have dominantly the same weak phase as the $b \to c\bar{c}s$ tree amplitude. Since only λ^2-suppressed penguin amplitudes introduce a new CP-violating phase, amplitudes with a single weak phase dominate, and we expect $||\bar{A}_{\psi K}/A_{\psi K}| - 1| < 0.01$. The e^+e^- asymmetric-energy B-factory experiments from BABAR [82] and Belle [83] provide precise measurements. The world average is [61]

$$\sin 2\beta = 0.681 \pm 0.025\,. \tag{11.20}$$

This measurement has a four-fold ambiguity in β, which can be resolved by a global fit mentioned in Sec. 11.4. Experimentally, the two-fold ambiguity $\beta \to \pi/2 - \beta$ (but not $\beta \to \pi + \beta$) can be resolved by a time-dependent angular analysis of $B^0 \to J/\psi K^{*0}$ [84,85], or a time-dependent Dalitz plot analysis of $B^0 \to \overline{D}^0 h^0$ ($h^0 = \pi^0, \eta, \omega$) with $\overline{D}^0 \to K^0_S \pi^+\pi^-$ [86,87]. These results indicate that negative $\cos 2\beta$ solutions are very unlikely, in agreement with the global CKM fit result.

The $b \to c\bar{c}d$ mediated transitions, such as $B^0 \to J/\psi\pi^0$ and $B^0 \to D^{(*)+}D^{(*)-}$, also measure approximately $\sin 2\beta$. However, the dominant component of the $b \to d$ penguin amplitude has a different CKM phase ($V_{tb}^* V_{td}$) than the tree amplitude ($V_{cb}^* V_{cd}$), and its magnitudes are of the same order in λ. Therefore, the effect of penguins could be large, resulting in $S_f \neq -\eta_f \sin 2\beta$ and $C_f \neq 0$. These decay modes have also been measured by BABAR and Belle. The world averages [61], $S_{J/\psi\pi^0} = -0.65 \pm 0.18$, $S_{D^+D^-} = -0.75 \pm 0.26$, and $S_{D^{*+}D^{*-}} = -0.67 \pm 0.18$, are consistent with $\sin 2\beta$ obtained from $B^0 \to$ charmonium K^0 decays, and the C_f's are also consistent with zero, although the uncertainties are sizable.

11.3.2.2. *Penguin-dominated modes:*

The $b \to s\bar{q}q$ penguin-dominated decays have the same CKM phase as the $b \to c\bar{c}s$ tree level decays, up to corrections suppressed by λ^2, since $V_{tb}^* V_{ts} = -V_{cb}^* V_{cs}[1 + \mathcal{O}(\lambda^2)]$. Therefore, decays such as $B^0 \to \phi K^0$ and $\eta' K^0$ provide $\sin 2\beta$ measurements in the SM. Any new physics contribution to the amplitude with a different weak phase would give rise to $S_f \neq -\eta_f \sin 2\beta$, and possibly $C_f \neq 0$. Therefore, the main interest in these modes is not simply to measure $\sin 2\beta$, but to search for new physics. Measurements of many other decay modes in this category, such as $B \to \pi^0 K_S^0$, $K_S^0 K_S^0 K_S^0$, *etc.*, have also been performed by BABAR and Belle. The results and their uncertainties are summarized in Fig. 12.3 and Table 12.1 of Ref. 81.

11.3.3. α / ϕ_2 :

Since α is the angle between $V_{tb}^* V_{td}$ and $V_{ub}^* V_{ud}$, only time-dependent CP asymmetries in $b \to u\bar{u}d$-dominated modes can directly measure $\sin 2\alpha$, in contrast to $\sin 2\beta$, where several different transitions can be used. Since $b \to d$ penguin amplitudes have a different CKM phase than $b \to u\bar{u}d$ tree amplitudes, and their magnitudes are of the same order in λ, the penguin contribution can be sizable, and makes the α determination complicated. To date, α has been measured in $B \to \pi\pi$, $\rho\pi$ and $\rho\rho$ decay modes.

11.3.3.1. $B \to \pi\pi$:

It is now experimentally well established that there is a sizable contribution of $b \to d$ penguin amplitudes in $B \to \pi\pi$ decays. Thus, $S_{\pi^+\pi^-}$ in the time-dependent $B^0 \to \pi^+\pi^-$ analysis does not measure $\sin 2\alpha$, but

$$S_{\pi^+\pi^-} = \sqrt{1 - C_{\pi^+\pi^-}^2}\, \sin(2\alpha + 2\Delta\alpha), \tag{11.21}$$

where $2\Delta\alpha$ is the phase difference between $e^{2i\gamma}\bar{A}_{\pi^+\pi^-}$ and $A_{\pi^+\pi^-}$. The value of $\Delta\alpha$, hence α, can be extracted using the isospin relations among the amplitudes of $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$, and $B^+ \to \pi^+\pi^0$ decays [88],

$$\frac{1}{\sqrt{2}}\, A_{\pi^+\pi^-} + A_{\pi^0\pi^0} - A_{\pi^+\pi^0} = 0, \tag{11.22}$$

and a similar one for the $\bar{A}_{\pi\pi}$'s. This method utilizes the fact that a pair of pions from $B \to \pi\pi$ decay must be in a zero angular momentum state, and, because of Bose statistics, they must have even isospin. Consequently, $\pi^0\pi^\pm$ is in a pure isospin-2 state, while the penguin amplitudes only contribute to the isospin-0 final state. The latter does not hold for the electroweak penguin amplitudes, but their effect is expected to be small. The isospin analysis uses the world averages [61] $S_{\pi^+\pi^-} = -0.61 \pm 0.08$, $C_{\pi^+\pi^-} = -0.38 \pm 0.07$, the branching fractions of all three modes, and the direct CP asymmetry $C_{\pi^0\pi^0} = -0.48^{+0.31}_{-0.32}$. This analysis leads to 16 mirror solutions for $0 \le \alpha < 2\pi$. Because of this, and the sizable experimental error of the $B^0 \to \pi^0\pi^0$ rate and CP asymmetry, only a loose constraint on α can be obtained at present [89], $0^\circ < \alpha < 7^\circ$, $83^\circ < \alpha < 103^\circ$, $118^\circ < \alpha < 152^\circ$, and $167^\circ < \alpha < 180^\circ$ at 68% CL.

11.3.3.2. *$B \to \rho\rho$:*

The decay $B^0 \to \rho^+\rho^-$ contains two vector mesons in the final state, which in general is a mixture of CP-even and CP-odd components. Therefore, it was thought that extracting α from this mode would be complicated.

However, the longitudinal polarization fractions (f_L) in $B^+ \to \rho^+\rho^0$ and $B^0 \to \rho^+\rho^-$ decays were measured to be close to unity [90], which implies that the final states are almost purely CP-even. Furthermore, $\mathcal{B}(B^0 \to \rho^0\rho^0) = (0.86 \pm 0.28) \times 10^{-6}$ is much smaller than $\mathcal{B}(B^0 \to \rho^+\rho^-) = (24.2^{+3.1}_{-3.2}) \times 10^{-6}$ and $\mathcal{B}(B^+ \to \rho^+\rho^0) = (18.2 \pm 3.0) \times 10^{-6}$ [61], which implies that the effect of the penguin diagrams is small. The isospin analysis using the world averages, $S_{\rho^+\rho^-} = -0.05 \pm 0.17$ and $C_{\rho^+\rho^-} = -0.06 \pm 0.13$ [61], together with newly measured time-dependent CP asymmetry, $S_{\rho^0\rho^0} = -0.5 \pm 0.9$ and $C_{\rho^0\rho^0} = -0.4 \pm 0.9$ [91], and the above-mentioned branching fractions, gives $\alpha = (87^{+10}_{-12})^\circ$ [89], with a mirror solution at $3\pi/2 - \alpha$. A possible small violation of Eq. (11.22) due to the finite width of the ρ [92] is neglected.

11.3.3.3. *$B \to \rho\pi$:*

The final state in $B^0 \to \rho^+\pi^-$ decay is not a CP eigenstate, but this decay proceeds via the same quark-level diagrams as $B^0 \to \pi^+\pi^-$, and both B^0 and $\overline{B}^0$ can decay to $\rho^+\pi^-$. Consequently, mixing-induced CP violations can occur in four decay amplitudes, $B^0 \to \rho^\pm\pi^\mp$ and $\overline{B}^0 \to \rho^\pm\pi^\mp$. The measurements of CP violation parameters for these decays, where $B^0 \to \pi^+\pi^-\pi^0$ decays are treated as quasi-two-body decays, have been made both by *BABAR* [93] and the Belle [94]. However, the isospin analysis is rather complicated, and no significant model-independent constraint on α has been obtained.

The time-dependent Dalitz plot analysis of $B^0 \to \pi^+\pi^-\pi^0$ decays permits the extraction of α with a single discrete ambiguity, $\alpha \to \alpha + \pi$, since one knows the variation of the strong phases in the interference regions of the $\rho^+\pi^-$, $\rho^-\pi^+$, and $\rho^0\pi^0$ amplitudes in the Dalitz plot [95]. The combination of Belle [96] and *BABAR* [97] measurements gives $\alpha = (120^{+11}_{-7})^\circ$ [89]. This constraint is still moderate, and there are also solutions around 30° and 90° within 2σ significance level.

Combining the above-mentioned three decay modes [89], α is constrained as

$$\alpha = (88^{+6}_{-5})^\circ. \tag{11.23}$$

A different statistical approach [98] gives similar constraint from the combination of these measurements.

11.3.4. γ / ϕ_3 :

By virtue of Eq. (11.16), γ does not depend on CKM elements involving the top quark, so it can be measured in tree-level B decays. This is an important distinction from the measurements of α and β, and implies that the direct measurements of γ are unlikely to be affected by physics beyond the SM.

11.3.4.1. $B^\pm \to DK^\pm$:

The interference of $B^- \to D^0 K^-$ ($b \to c\bar{u}s$) and $B^- \to \overline{D}^0 K^-$ ($b \to u\bar{c}s$) transitions can be studied in final states accessible in both D^0 and $\overline{D}^0$ decays [80]. In principle, it is possible to extract the B and D decay amplitudes, the relative strong phases, and the weak phase γ from the data.

A practical complication is that the precision depends sensitively on the ratio of the interfering amplitudes

$$r_B = \left| A(B^- \to \overline{D}^0 K^-) / A(B^- \to D^0 K^-) \right| , \tag{11.24}$$

which is around $0.1 - 0.2$. The original GLW method [99,100] considers D decays to CP eigenstates, such as $B^\pm \to D^{(*)}_{CP}(\to \pi^+\pi^-)K^{\pm(*)}$. To alleviate the smallness of r_B and make the interfering amplitudes (which are products of the B and D decay amplitudes) comparable in magnitude, the ADS method [101] considers final states where Cabibbo-allowed $\overline{D}^0$ and doubly-Cabibbo-suppressed D^0 decays interfere. Extensive measurements have been made by the B factories using both methods [102].

It was realized that both D^0 and $\overline{D}^0$ have large branching fractions to certain three-body final states, such as $K_S\pi^+\pi^-$, and the analysis can be optimized by studying the Dalitz plot-dependence of the interferences [103,104]. The best present determination of γ comes from this method. Belle [105] and BABAR [106] obtained $\gamma = 53^{+15}_{-18} \pm 3 \pm 9°$ and $\gamma = 92 \pm 41 \pm 11 \pm 12°$, respectively, where the last uncertainty is due to the D-decay modeling. The error is sensitive to the central value of the amplitude ratio r_B (and r_B^* for the D^*K mode), for which Belle found somewhat larger central values than BABAR. The same values of $r_B^{(*)}$ enter the ADS analyses, and the data can be combined to fit for $r_B^{(*)}$ and γ. The D^0–$\overline{D}^0$-mixing has been neglected in all measurements, but its effect on γ is far below the present experimental accuracy [107], unless D^0–$\overline{D}^0$-mixing is due to CP-violating new physics, in which case it could be included in the analysis [108].

Combining the GLW, ADS, and Dalitz analyses [89], γ is constrained as

$$\gamma = (77^{+30}_{-32})° . \tag{11.25}$$

The likelihood function of γ is not Gaussian, and the 95% CL range is $26° < \gamma < 130°$. Similar results are found in [98]. The error on γ increased from the previous review [109] because of a decrease of $r_B^{(*)}$.

11.3.4.2. $B^0 \to D^{(*)\pm}\pi^\mp$:

The interference of $b \to u$ and $b \to c$ transitions can be studied in $\overline{B}^0 \to D^{(*)+}\pi^-$ ($b \to c\bar{u}d$) and $\overline{B}^0 \to B^0 \to D^{(*)+}\pi^-$ ($\bar{b} \to \bar{u}c\bar{d}$) decays and their CP conjugates, since both B^0 and $\overline{B}^0$ decay to $D^{(*)\pm}\pi^\mp$ (or $D^\pm\rho^\mp$, *etc.*). Since there are only tree and no penguin contributions to these decays, in principle, it is possible to extract from the four time-dependent rates the magnitudes of the two hadronic amplitudes, their relative strong phase, and the weak phase between the two-decay paths, which is $2\beta + \gamma$.

A complication is that the ratio of the interfering amplitudes is very small, $r_{D\pi} = A(B^0 \to D^+\pi^-)/A(\overline{B}^0 \to D^+\pi^-) = \mathcal{O}(0.01)$ (and similarly for $r_{D^*\pi}$ and $r_{D\rho}$), and therefore it has not been possible to measure it. To obtain $2\beta + \gamma$, $SU(3)$ flavor symmetry and dynamical assumptions have been used to relate $A(\overline{B}^0 \to D^-\pi^+)$ to $A(\overline{B}^0 \to D_s^-\pi^+)$, so this measurement is not model-independent at present.Combining the $D^\pm\pi^\mp$ $D^{*\pm}\pi^\mp$ and $D^\pm\rho^\mp$ measurements [110] gives $\sin(2\beta + \gamma) > 0.59$ at 68% CL [89], consistent with the previously discussed results for β and γ. The amplitude ratio is much larger in the analogous $B_s^0 \to D_s^\pm K^\mp$ decays, so it will be possible at LHCb to measure it and model-independently extract $\gamma - 2\beta_s$ [111] (where $\beta_s = \arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*)$ is related to the phase of B_s mixing).

11.4. Global fit in the Standard Model

Using the independently measured CKM elements mentioned in the previous sections, the unitarity of the CKM matrix can be checked. We obtain $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0011$ (1st row), $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.136 \pm 0.125$ (2nd row), $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1.002 \pm 0.005$ (1st column), and $|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.134 \pm 0.125$ (2nd column), respectively. The uncertainties in the second row and column are dominated by that of $|V_{cs}|$. For the second row, a more stringent check is obtained from the measurement of $\sum_{u,c,d,s,b} |V_{ij}|^2$ in Sec. 11.2.4 minus the sum in the first row above: $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.002 \pm 0.027$. These provide strong tests of the unitarity of the CKM matrix. The sum of the three angles of the unitarity triangle, $\alpha + \beta + \gamma = (186^{+31}_{-32})^\circ$, is also consistent with the SM expectation.

The CKM matrix elements can be most precisely determined by a global fit that uses all available measurements and imposes the SM constraints (*i.e.*, three generation unitarity). The fit must also use theory predictions for hadronic matrix elements, which sometimes have significant uncertainties. There are several approaches to combining the experimental data. CKMfitter [89,5] and Ref. 112 (which develops [113,114] further) use frequentist statistics, while UTfit [98,115] uses a Bayesian approach. These approaches provide similar results.

The constraints implied by the unitarity of the three generation CKM matrix significantly reduce the allowed range of some of the CKM elements. The fit for the Wolfenstein parameters defined in Eq. (11.4) gives

$$\begin{aligned} \lambda &= 0.2257^{+0.0009}_{-0.0010}\,, \qquad & A &= 0.814^{+0.021}_{-0.022}\,, \\ \bar{\rho} &= 0.135^{+0.031}_{-0.016}\,, \qquad & \bar{\eta} &= 0.349^{+0.015}_{-0.017}\,, \end{aligned} \tag{11.26}$$

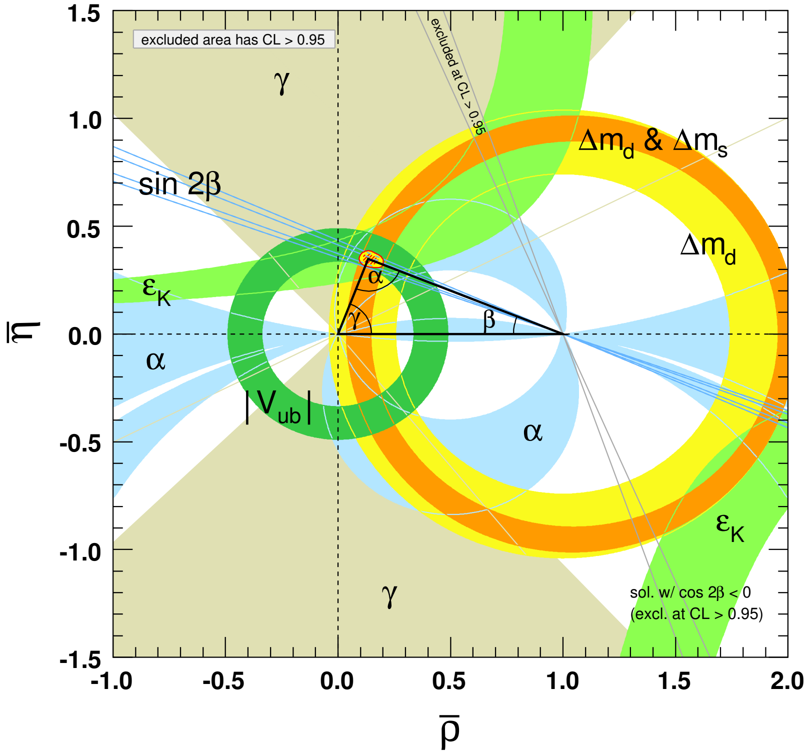

Figure 11.2: Constraints on the $\bar\rho, \bar\eta$ plane. The shaded areas have 95% CL. See full-color version on color pages at end of book.

These values are obtained using the method of Refs. [5,89], and the prescription of Refs. [98,115] gives similar results [116]. The fit results for the magnitudes of all nine CKM elements are

$$V_{\rm CKM} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}, \qquad (11.27)$$

and the Jarlskog invariant is $J = (3.05^{+0.19}_{-0.20}) \times 10^{-5}$.

Fig. 11.2 illustrates the constraints on the $\bar\rho, \bar\eta$ plane from various measurements and the global fit result. The shaded 95% CL regions all overlap consistently around the global fit region, though the consistency of $|V_{ub}/V_{cb}|$ and $\sin 2\beta$ is not very good.

11.5. Implications beyond the SM

If the constraints of the SM are lifted, K, B, and D decays and mixings are described by many more parameters than just the four CKM parameters and the W, Z, and quark masses. The most general effective Lagrangian at lowest order contains around a hundred flavor changing operators, and the observable effects of interactions at the weak scale or above are encoded in their coefficients. For example, Δm_d, $\Gamma(B \to \rho\gamma)$, and $\Gamma(B \to X_d \ell^+ \ell^-)$ are all proportional to $|V_{td} V_{tb}^*|^2$ in the SM, however, they may receive unrelated contributions from new physics. Similar to the measurements of $\sin 2\beta$ from tree- and loop-dominated modes, such overconstraining measurements of the magnitudes and phases of CKM elements provide excellent sensitivity to new physics. Another very clean test of the SM can come from future measurements of $K_L^0 \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$. These loop-induced rare decays are sensitive to new physics, and will allow a determination of β independent of its value measured in B decays [117].

Not all CP-violating measurements can be interpreted as constraints on the $\bar{\rho}, \bar{\eta}$ plane. Besides the angles in Eq. (11.16), it is also useful to define $\beta_s = \arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*)$, which is the small, λ^2-suppressed, angle of a "squashed" unitarity triangle obtained by taking the scalar product of the second and third columns. The angle β_s can be measured via time-dependent CP violation in $B_s^0 \to J/\psi\phi$, similar to β in $B^0 \to J/\psi K^0$. Checking if β_s agrees with its SM prediction, $\beta_s = 0.019 \pm 0.001$ [118,89], is an equally important test of the theory. The first flavor-tagged time-dependent CP-asymmetry measurements of $B_s^0 \to J/\psi\phi$ decay appeared recently [119], disfavoring large negative $\sin 2\beta_s$ values.

In the kaon sector, both CP-violating observables ϵ and ϵ' are tiny, so models in which all sources of CP violation are small were viable before the B-factory measurements. Since the measurement of $\sin 2\beta$, we know that CP violation can be an $\mathcal{O}(1)$ effect, and it is only flavor mixing that is suppressed between the three quark generations. Thus, many models with spontaneous CP violation are excluded. Model-independent statements for the constraints imposed by the CKM measurements on new physics are hard to make, because most models that allow for new flavor physics contain a large number of new parameters. For example, the flavor sector of the MSSM contains 69 CP-conserving parameters and 41 CP-violating phases (*i.e.*, 40 new ones) [120].

In a large class of models, the unitarity of the CKM matrix is maintained, and the dominant new-physics effect is a contribution to the B^0–$\overline{B}^0$-mixing amplitude [121], which can be parameterized as $M_{12} = M_{12}^{\rm SM}\,(1 + h_d e^{i\sigma_d})$. While the constraints on h_d and σ_d are significant (before the measurements of γ and α they were not), new physics with a generic weak phase may still contribute to M_{12} at order 20% of the SM. Measurements unimportant for the SM-CKM fit, such as the CP asymmetry in semileptonic decays, play a role in constraining such extensions of the SM [118]. Similar results for the constraints on new physics in K^0 and B_s^0 mixing are discussed in Refs. [115,122].

The CKM elements are fundamental parameters, so they should be measured as precisely as possible. The overconstraining measurements of CP asymmetries, mixing, semileptonic, and rare decays have started to severely constrain the magnitudes and phases of possible new physics contributions to flavor-changing interactions. When new particles are observed at the LHC, it will be important to know the flavor parameters as precisely as possible to understand the underlying physics.

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