The Angular Momentum Problem

or: Why we reverse the Solenoids

In the absence of external forces and energy loss in materials, the angular momentum of a particle, about any defined axis, is conserved.

A non paraxial particle in a fixed solenoidal field, or a paraxial particle in a varying solenoidal field will cross radial field components and experience forces that will change its azimuthal (p_ϕ) and angular ($r\ p_\phi$) momentum. If the field is varying slowly, relative to the particles radius, and there is no material present, then the induced changes in azimuthal momentum are given by:

$$\Delta(p_\phi) \ = \ \frac{e}{2}\ \Delta(B_z\ r)$$

If we add an angular momentum ($p_{\phi\ c}\ r$) equal to the value a particle would have before entering, or after leaving, the solenoidal field, then this addition will be conserved and:

$$p_\phi\ r \ = \ p_{\phi\ c}\ r \ + \ \frac{e}{2}\ \Delta(B_z\ r)\ r$$

This additional term is defined as the "canonical" angular momentum:

$$p_{\phi\ c}\ r \ = \ p_\phi\ r \ - \ \frac{e}{2}\ \Delta(B_z\ r)\ r$$

If material is introduced then, neglecting scattering and straggling, all momenta, both longitudinal and transverse, will be attenuated. Acceleration can be introduced to restore the longitudinal momentum thus leading to a reduction of transverse momenta and cooling. But if this process takes place in a solenoidal field, then besides reducing the random initial transverse momenta, the process will reduce the coherent field-induced transverse momenta, and this will generate an average canonical momentum.

Consider the case of almost complete transverse cooling: all transverse momenta are reduced to near zero leaving the beam streaming parallel to the axis: down the field lines of the solenoid.

$$p_\phi\ r \ = \ 0$$

but

$$p_{\phi\ c}\ r \ = \ p_\phi\ r \ - \ \frac{e}{2}\ \Delta(B_z\ r)\ r \ = \ -\frac{e}{2}\ \Delta(B_z\ r)\ r$$

where the r is the radius in the solenoid where the cooling took place. If the initial emittance is large, r is also large and the resulting angular momentum is large. When the beam exits the solenoid, to enter a quadrupole focused accelerator or collider, then this canonical angular momentum becomes a real angular momentum and represents an effective emittance. In practice, after a few cooling stages, this effective emittance becomes greater than the true emittance and effective cooling ends.

The problem is to find a way to avoid generating, or a way to remove, this canonical angular momentum. Cooling in a quadrupole channel, if strong enough focusing were possible, would not have the problem. Nor is it present in a lithium lens focused channel. This latter alternative is reasonable to be use in the final cooling stages, but it has disadvantages: it must be pulsed, and lithium has relatively more coulomb scattering than the hydrogen that can be used in solenoid channels. We have therefore explored ways of removing this coherent angular momentum in the solenoid case.

We know of only two ways of doing this. The first is to end the solenoidal field on an iron grid. The field lines end on the grid and are carried radially in the iron. Muons that pass through the iron are strongly deflected and lost, but those that pass through the openings emerge with relatively little deflection, and thus relatively little angular momentum. The method only works for low magnetic fields and is not very efficient.

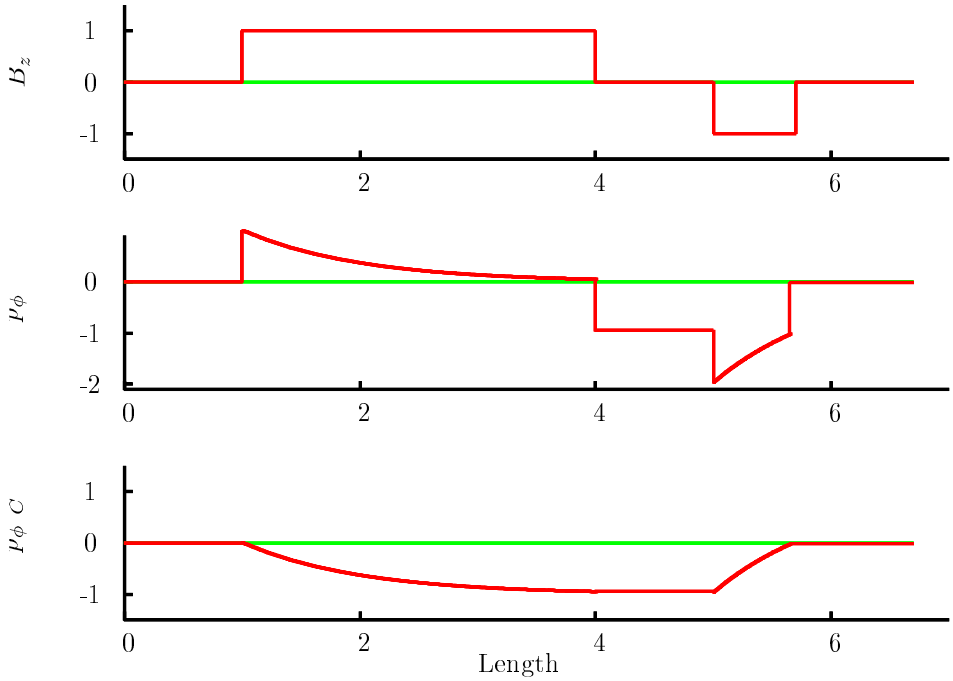

Figure 1. Axial Field, Angular Momentm, and Canonical Angular Momentum, in an Ideal, Single Field Reversal, Solenoid Cooling System.

The second way involves, at the least, applying one further stage of ionization cooling after a field reversal. Consider again the above case of almost complete initial cooling. At the end, after exiting the solenoid, we have real coherent

angular momentum:

$$p_\phi\ r\ =\ p_{\phi\ c}\ r\ =\ -\frac{e}{2}\ \Delta(B_z\ r)\ r$$

If (see figure 1) the beam now enters a solenoid with opposite field $B_{z\ 2} = -B_z$ then the canonical angular momentum remains the same, but the real angular momentum is further increased to exactly twice this value.

$$p_\phi\ r\ =\ -\frac{e}{2}\ \Delta(B_z\ r)\ r + \frac{e}{2}\ \Delta(B_{z\ 2}\ r)\ r\ =\ -2\frac{e}{2}\ \Delta(B_z\ r)\ r$$

If now we introduce material enough to halve the transverse field components then again

$$p_\phi\ r\ =\ -\frac{e}{2}\ \Delta(B_z\ r)\ r$$

but this is now inside the field $-B_z$. The canonical momentum, and the angular momentum on exiting, is now:

$$p_\phi\ r\ =\ -\frac{e}{2}\ \Delta(B_z\ r)\ r - \frac{e}{2}\ \Delta(B_{z\ 2}\ r)\ r\ =\ 0$$

It is instructive to look at the transverse motions in a solenoid of particles with different canonical momenta. Figures 2a, 2b, and 2c show the motions of particles with canonical angular momenta of $-\frac{eB_z r}{2}$, 0 and $+\frac{eB_z r}{2}$, which correspond to real angular momenta of 0,$+\frac{eBr}{2}$, and $+2\ \frac{eBr}{2}$.

Ignoring the values given below the figures we can use these pictures to indicate typical motions at different stages along the above cooling example. Figure 2a can be taken to indicate an initial particle with finite amplitude, but no angular momentum. Figure 2b shows the motion just after entering the field. Figure 2a can also be taken to show the beam after the first cooling, where there is essentially no transverse motion. Figure 2b can also be taken to show the motion after exiting the first solenoid, but the motion is in now the opposite direction. Figure 2c shows it in the reversed solenoid where the real angular momentum is twice that for a zero canonical angular momentum particle. Note that, despite the large transverse momentum, the required aperture is not larger. Figure 2b can be taken to show the motion in this second solenoid after partial cooling. After exiting the motion is as in Figure 2a with no transverse momentum.

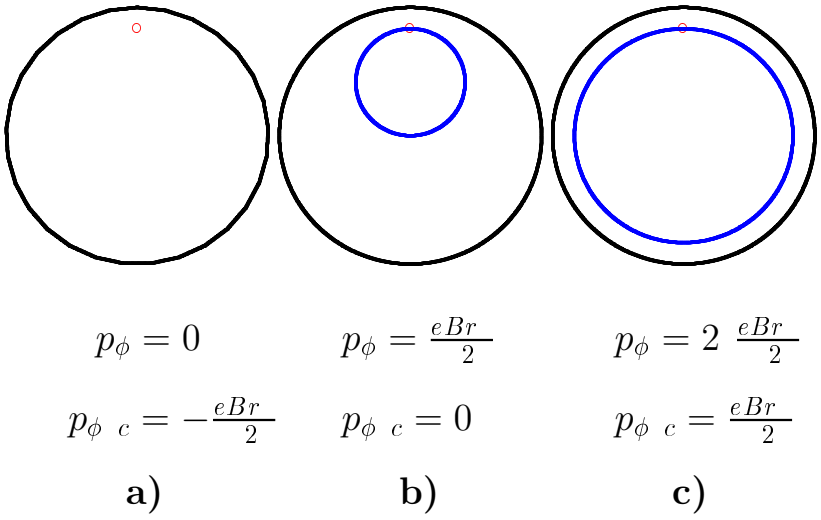

Figure 2. Transverse Motions of particles in a solenoid.

The above is not an efficient way to do it. The second "cooling" is used solely to reduce the coherent angular momentum of an already cold beam. There are other problems: the beam in the first solenoid, though it is cooled, does not experience any reduction of its radius, and the solenoid diameters have to be larger than they would be if the solenoids are reversed more frequently. In addition, simulations suggest that the alternations should have a spacing small compared to the synchrotron wavelength. If not one has difficulties keeping particle in the rf buckets because of large longitudinal velocity changes of some particles at each field reversal.

Figure 3 shows the angular momenta and canonical angular momenta in a simulation of a 1.2 T initial cooling stage. It is seen that while the coherent angular momenta are large, the canonical angular momentum (in red) remains very small.

If there had been no field reversals in this example, then the angular momentum would have been reduced by about a factor of 2 (from 3 m MeV/c to approximately 1.5 m MeV/c) and the resulting canonical momentum of 1.5 m MeV/c would have been approximately equal to the final random angular momenta. In later stages without reversals, the coherent canonical momenta would continue to rise and soon far exceed the falling random values.

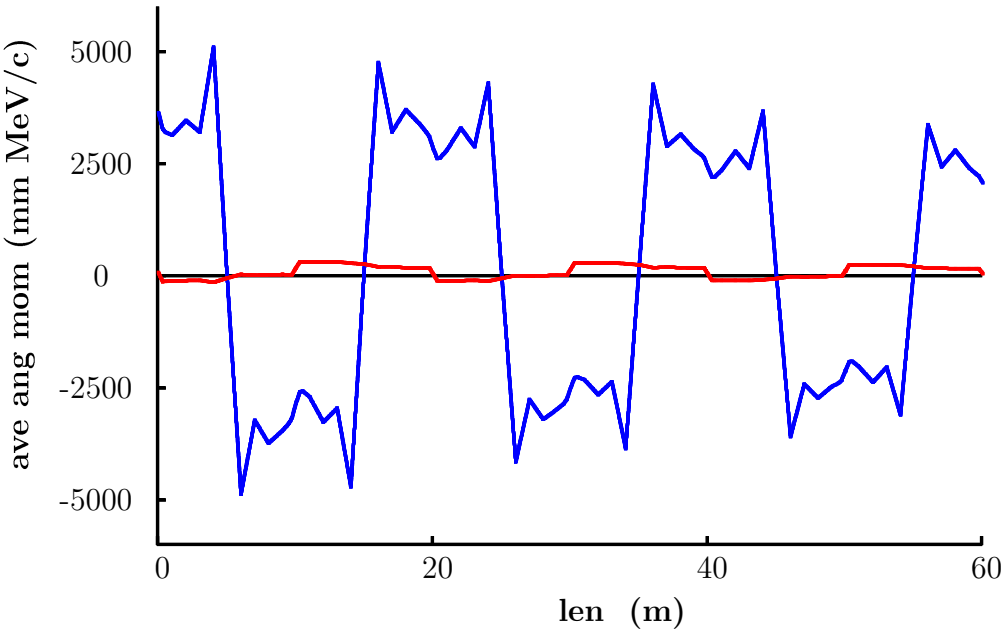

Figure 3. Average Angular momentum (blue), and Canonical Angular Momentum (red) in a 1.2 T Alternating Solenoid Cooling Channel.