1. Stratiform Cloud Parameterization Overview

The stratiform cloud parameterization calculates the sources and sinks of three tracer fields: stratiform cloud liquid, q_l; cloud ice, q_i; and the fraction of the grid box which is saturated, q_a. Although q_a might casually be termed the cloud fraction, strictly speaking it is the fraction of the grid volume whose vapor content is at or exceeds water vapor saturation. In terms of equations:

$$\frac{\partial q_l}{\partial t} = A_l + S_l + C_l \ , \tag{1}$$

$$\frac{\partial q_i}{\partial t} = A_i + S_i + C_i \ , \tag{2}$$

$$\frac{\partial q_a}{\partial t} = A_a + S_a + C_a \ , \tag{3}$$

where A indicates advection by the large-scale fields of a tracer, S indicates its parameterized sources and sinks from stratiform (or large-scale processes), and C indicates its parameterized sources and sinks from cumulus convection.

The large-scale and convective sources and sinks of the tracers closely follow the parameterization of Tiedtke (1993), hereafter T93. The microphysical source/sinks of cloud liquid and ice closely follow the bulk microphysics of Rotstayn (1997, 2000), hereafter R97 and R00. Novel features of this parameterization include: (a) a coupling of the T93 cloud fraction equation with a more sophisticated 2 phase bulk microphysics scheme (T93 has a single tracer for the sum of cloud liquid and ice) and (b) a separate treatment of the diagnostic precipitation fluxes for the portion of the fluxes that are in unsaturated and saturated air (Jakob and Klein 2000).

The physical processes represented and the transfer of water substance between various species are displayed in Fig. 1. Large-scale or grid-resolved ascent (descent) can lead to condensation or deposition (evaporation or sublimation) of cloud liquid and ice. Furthermore, detrainment from the updrafts that are parameterized within the convection parameterization may contribute to cloud liquid or cloud ice. In addition, liquid and ice may be advected downward by the compensating subsidence that accompanies the convective updrafts. The microphysical processes represented include the conversion of cloud liquid to rain via autoconversion and accretion; the freezing of cloud liquid into ice through homogenous freezing, riming, and the Bergeron pro-

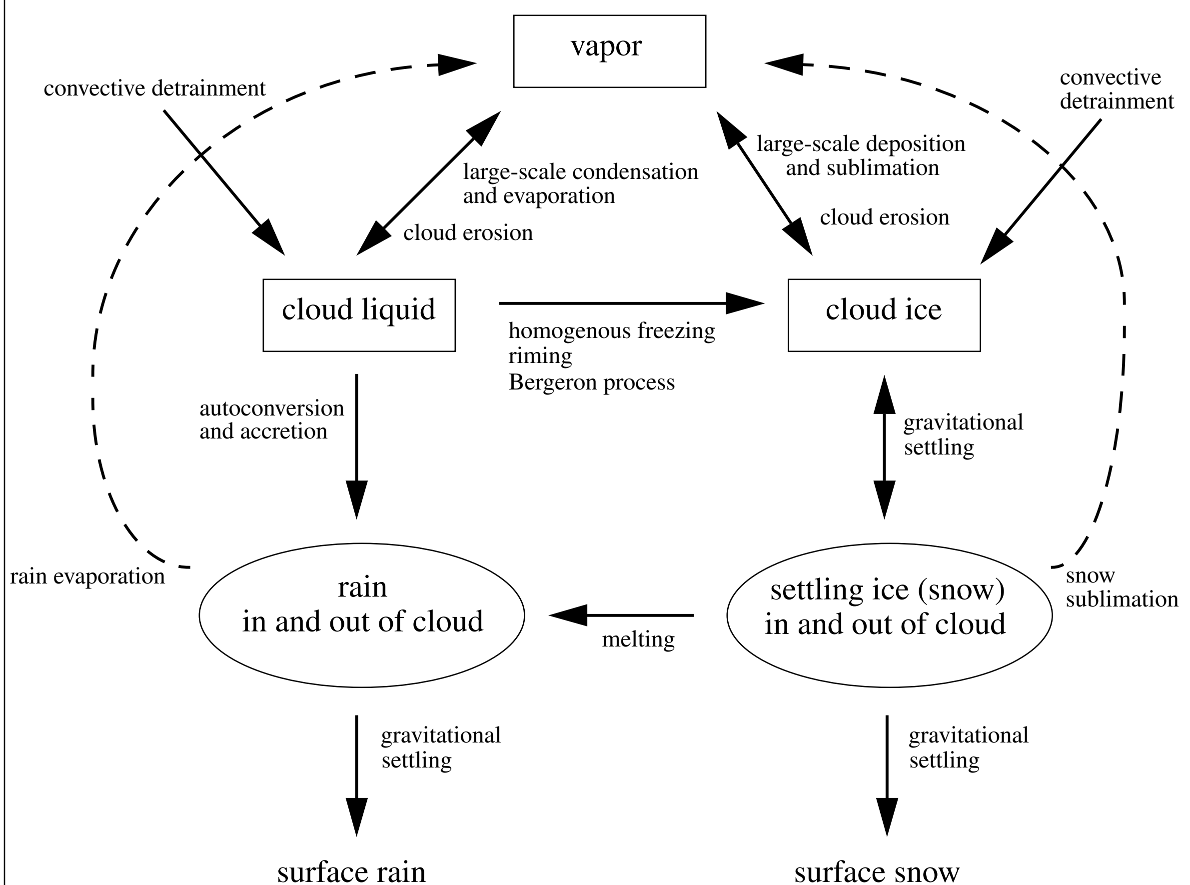

Figure 1. Schematic diagram indicating the hydrologic cycle of the stratiform clouds. Quantities contained within squares are prognostic species (tracers) whereas quantities in ovals indicating diagnostic variables (i.e. time tendencies are neglected). Arrows indicate the cloud macrophysical and microphysical processes represented in the model. Advection of cloud liquid and ice by the grid resolved motion and vertical advection of cloud liquid and ice by the subsidence which compensates for convective updrafts are not indicated.

cess; gravitational settling of ice crystals; and the evaporation and sublimation of rain and snow.

The processes represented in the equation for the saturated mass fraction (cloud fraction) are displayed in Fig. 2. Saturated volume is increased by large-scale condensation or deposition, radiative cooling, turbulence, and by convective detrainment. Cloud fraction is advected by the resolved circulation of the model and vertically advected by the subsidence which compensates for the convective updrafts. Following T93 cloud fraction decreases through mixing with dry air in the grid box (termed cloud erosion) or when all liquid and ice is absent in the grid box.

Principal constants of this parameterization are presented in Table 1. These include relative humidity thresholds for large-scale condensation and precipitation evaporation, cloud droplet thresholds for rain to be formed, and the number density of cloud droplets for land and ocean liquid clouds. Within the convection scheme key parameters are the precipitation efficiencies of shallow and deep convective updrafts. Processes which are highly uncertain include the formulation of ice microphysical processes, in particular the rates of riming and ice crystal sublimation.

Details of the parameterization are presented below.

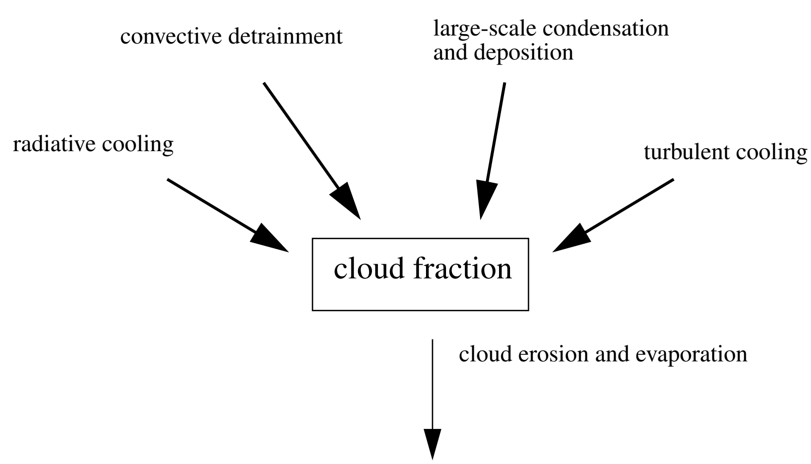

Figure 2. Parameterized sources and sinks of the cloud fraction equation. Advection by the grid resolved motion and vertical advection by the subsidence which compensates for the convective updrafts are not indicated.

Constant	Meaning	Value
U_{00}	Threshold relative humidity above which large-scale condensation may occur	0.8
r_d^{au}	Threshold cloud droplet radius above which autoconversion may occur	7 μm
N^{land} and N^{ocean}	Number density of cloud droplets over land and over ocean	100, 300 cm^{-3}
U_{evap}	Threshold relative humidity above which precipitation does not evaporate	1.0
r_p^{deep} and r_p^{shal}	Precipitation efficieny for deep and shallow convective updrafts	0.95, 0.5
p^{deep} and p^{shal}	Pressure boundaries to the deep and shallow cloud regimes	500, 700 hPa
K	Cloud erosion constant	$2x10^{-5}$ sec^{-1}

Table 1. Principal constants of the parameterization.

2. The Parameterization

a. Phase changes due to large-scale processes

Large-scale vertical motion may responsible for the formation or dissipation of a stratiform cloud. Physically, vertical motion through adiabatic cooling or warming lowers or raises the saturation specific humidity, q_s, and when the vapor content of the air, q_v, exceeds or falls beneath q_s large-scale condensation or evaporation of cloud condensate is assumed to occur. In addition, radiation or turbulence may lower or raise q_s. The rate at which q_s changes due to large-scale vertical motion plus radiative and turbulent heating is calculated as:

$$\left.\frac{\partial q_s}{\partial t}\right|_{ls} = \frac{\left.\frac{\partial q_s}{\partial t}\right|_{dry}}{1 + \left(q_a + 0.5\Delta t_{cld}\left.\frac{\partial q_a}{\partial t}\right|_{ls}\right)\frac{L}{c_p}\left.\frac{\partial q_s}{\partial T}\right|_p}, \tag{4}$$

where $\left.\frac{\partial q_s}{\partial T}\right|_p$ is the temperature derivative of q_s at constant pressure, $\left.\frac{\partial q_a}{\partial t}\right|_{ls}$ is the time tendency of cloud fraction over the time step Δt_{cld} due to large-scale processes (see (7) below), and c_p is the specific heat of air at constant pressure. L is a latent heat equal to the latent heat of vaporization, L_v, if $T > 0°C$, the latent heat of sublimation, L_s, if $T < -20°C$, and a linearly interpolated value in the intermediate temperature range. This functional form matches that used to derive q_s from the saturation specific humidity above liquid, q_{sl}, and the saturation specific humidity above ice, q_{si}.

In (4), $\left.\frac{\partial q_s}{\partial t}\right|_{dry}$ is the time tendency of q_s if no condensation occurred in the grid box:

$$\left.\frac{\partial q_s}{\partial t}\right|_{dry} = \left(\frac{(\omega + gM_c)}{\rho c_p} + \left.\frac{\partial T}{\partial t}\right|_{turb} + \left.\frac{\partial T}{\partial t}\right|_{rad}\right) \times \left.\frac{\partial q_s}{\partial T}\right|_p, \tag{5}$$

where ω is the grid-resolved vertical pressure velocity, g is the acceleration due to gravity, M_c is the cumulus mass flux, and $\left.\frac{\partial T}{\partial t}\right|_{turb}$ and $\left.\frac{\partial T}{\partial t}\right|_{rad}$ are the temperature tendencies from turbulence and radiation respectively. In (5), $\omega + gM_c$ is the vertical pressure velocity in the large-scale environment outside of the convective updrafts and downdrafts.

The use of (4) with (5) is simpler and less computationally expensive to use than the iterative adjustment of Haltiner and Williams (1980) used by T93. The iterative adjustment is needed only because of the non-linearity of the saturation vapor pressure, which is ignored here. For typical vertical ascent rates of a GCM, the differences of this method to the adjustment routine are less than 5% at 220K and less than 1% at 290K.

If q_s is decreasing, new cloud formation is permitted to occur when $U = q_v/q_s$ exceeds a critical relative humidity, U_{00}. It is assumed that in the region of air outside of clouds, $1 - q_a$, q_v is evenly distributed over the range [$q_{v,clr} - (q_s - q_{v,clr}), q_s$], where $q_{v,clr}$ is the mean vapor content of the unsaturated portion of the grid volume. $q_{v,clr}$ is calculated from the mean vapor content of the grid volume, q_v, using:

$$q_v = q_a \cdot q_s + (1 - q_a) \cdot q_{v,clr} . \tag{6}$$

Given the assumed distribution of vapor in the unsaturated portion of the grid box, the increase in saturated volume fraction due to a decrease in q_s is then:

$$\left.\frac{\partial q_a}{\partial t}\right|_{ls} = \begin{pmatrix} \dfrac{-\left.\dfrac{\partial q_s}{\partial t}\right|_{ls} \times (1 - q_a)^2}{2(q_s - q_v)} & \text{if } \left.\dfrac{\partial q_s}{\partial t}\right|_{ls} < 0. \text{ and } U \geq U_{00} \\ 0 & \text{otherwise} \end{pmatrix} . \tag{7}$$

Note that (7) differs from equation 25 of T93 because equation 25 of T93 is incorrect. In cases of decreasing q_s with $U < U_{00}$ or increasing q_s, it is assumed $\left.\frac{\partial q_a}{\partial t}\right|_{ls} = 0$.

Decreasing (increasing) q_s leads to condensation (evaporation) of cloud condensate. These phase changes are parameterized in a way consistent with the assumptions that lead to (7) as:

$$\left.\frac{\partial}{\partial t}(q_l + q_i)\right|_{ls} = -\left.\frac{\partial q_s}{\partial t}\right|_{ls} \times \left(q_a + \frac{\Delta t_{cld}}{2}\left.\frac{\partial q_a}{\partial t}\right|_{ls}\right). \qquad (8)$$

In (8), the first term in the parentheses on the right hand side is the condensation or evaporation in pre-existing cloud where as the second term is the condensation in the cloud newly formed during the timestep Δt_{cld}. Altough this term appears to be second order, it can be nearly as large as the condensation in the pre-existing cloud.

Because cloud liquid and ice are treated separately in this scheme, a method is needed to approtion the large-scale condensation or evaporation to the various phases. In the case of condensation, it is assumed all condensation occurs as liquid for temperatures above -40˚C and as ice for temperatures below -40˚C (R00). The relatively low temperatures at which liquid can form reflects the fact that at temperatures greater than -40˚C ice condensation nuclei are extremely rare in comparison to cloud liquid drop nuclei such that liquid is formed in advance of ice formation for clouds at these temperatures. The Bergeron process may subsequently convert this liquid to ice. In the case of large-scale evaporation of mixed phase clouds, the liquid present is assumed to evaporate in advance of the cloud ice as the saturation vapor pressure over liquid droplets is greater than that over ice (R00).

b. Cloud erosion

Cloud is assumed to eroded by horizontal mixing with unsaturated air, if present, in the same grid box. The mixing is assumed to decrease the cloud fraction but keep the incloud value of cloud condensate unchanged. This mixing is parameterized as in T93 by:

$$\left.\frac{\partial q_a}{\partial t}\right|_{eros} = -Kq_a(q_s - q_v)q_a/(q_l + q_i)\,, \tag{9}$$

$$\left.\frac{\partial q_l}{\partial t}\right|_{eros} = -Kq_a(q_s - q_v)q_l/(q_l + q_i)\,, \tag{10}$$

$$\left.\frac{\partial q_i}{\partial t}\right|_{eros} = -Kq_a(q_s - q_v)q_i/(q_l + q_i)\,, \tag{11}$$

where is K an arbitrary rate constant.

c. Convective sources and sinks of stratiform clouds

Detrainment from convective updrafts is a major source of cloud condensate and saturated mass. The convection scheme used is Relaxed-Arakawa-Schubert (RAS) (Moorthi and Suarez 1992). In RAS, a fraction r_p^j of the water vapor condensed in convective updrafts is assumed to become precipitation mass. This fraction is assumed to be function of the detrainment pressure p^j of updraft type j, such that updrafts which detrain at high and low levels of the atmosphere have fixed precipitation efficiencies r_p^{deep} and r_p^{shal}, respectively. At intermediate p^j, r_p^j is linearly interpolated in pressure:

$$r_p^j = \begin{bmatrix} r_p^{deep} & \text{if } p^j < p^{deep} \\ r_p^{deep} \cdot \left(\dfrac{p^{shal} - p^j}{p^{shal} - p^{deep}}\right) + r_p^{shal} \cdot \left(\dfrac{p^j - p^{shal}}{p^{shal} - p^{deep}}\right) & \text{if } p^{shal} \geq p^j \geq p^{deep} \\ r_p^{shal} & \text{if } p^j > p^{shal} \end{bmatrix}. \tag{12}$$

These fractions, r_p^{deep} and r_p^{shal}, are considered to be tunable parameters of the combined stratiform and convection parameterizations. In original RAS, the fraction of vapor condensed in the updraft which does not precipitate instantaneously evaporates at the level of detrainment. Here, this is assumption is changed such that all of the non-precipitated condensed vapor is a source of stratiform cloud liquid or ice. Although a microphysical model should be used to determine the amount of liquid and ice that results from vapor condensed in the updraft, the crude assumption that if the near surface temperature is less than the freezing point of water then the condensed vapor is ice is used.

With this specification of convective updraft microphysics, the convective source of cloud liquid or ice is:

$$C_{l,i} = \sum_j \left(\frac{D^j(\hat{q}^j_{l,i} - q_{l,i})}{\rho} + gM^j_c \frac{\partial q_{l,i}}{\partial p} \right) \qquad (13)$$

where D^j is the mass detrainment rate from the jth convective updraft, and $\hat{q}^j_{l,i}$ is the liquid or ice specific humidity in the updraft at the point of detrainment. The summation in (13) is over all convective updraft types. Here, $\hat{q}^j_{l,i}$ includes both the non-precipitated fraction of the vapor condensed into the updraft as well as stratiform cloud liquid or ice entrained into the updraft from levels beneath the detrainment level. Entraining stratiform cloud liquid or ice into the convective updraft is necessary so that the compensating subsidence of cloud liquid (i.e. the second term on the right hand side of (13)) does not violate column integrated cloud liquid conservation.

Note that convective detrainment also increases the saturated volume fraction if the detrained mass replaces unsaturated air. It is assumed that at the detrainment level, the probability that that detrained mass replaces unsaturated air is assumed to be $1 - q_a$. With this assumption the convective source of cloud fraction is:

$$C_a = \sum_j \left(\frac{D^j(1-q_a)}{\rho} + gM_c^j \frac{\partial q_a}{\partial p} \right). \tag{14}$$

Following standard RAS procedure, after the computation for updraft type j, the stratiform tracer variables are updated with the convective sources from updraft j. In this way, the subsequent updraft type acts on the stratiform tracer variables modified by the convective sources from previous updrafts.

d. Liquid phase microphysical processes

Cloud liquid can be converted to rain through the creation of raindrops from collisions and coalescences of cloud droplets (autoconversion), and through the collection of cloud droplets by raindrops (accretion). In the formulation of R97, the time rate of change of grid mean liquid from autoconversion is parameterized as:

$$\left.\frac{\partial q_l}{\partial t}\right|_{au} = -q_a \times \left(\frac{0.104 g \rho^{4/3} E_{c,au}}{\mu (N\rho_l)^{1/3}} \right) \times (q_l/q_a)^{7/3} \times H(r_d - r_d^{au}). \tag{15}$$

In (15), μ is the dynamic viscosity of air (fixed at 1.717 x 10^{-5} kg m^{-1} s^{-1}), $E_{c,au}$ is the mean collection efficiency for the autoconversion process (fixed at 0.55), ρ_l is the density of pure liquid, and N is the number of cloud droplets per unit volume. The leading factor of q_a accounts for the fact that the process only occurs in a fraction of the volume where the liquid exists. The other factor of q_a accounts for the fact that where liquid exists its mean local concentration is q_l/q_a.

The use of the Heavyside function, H, in (15) indicates that only if the mean volume radius of cloud drops, r_d, exceeds a critical drop radius, r_d^{au}, does autoconversion occur. r_d is calculated from:

$$\rho q_l / q_a = 4\pi N \rho_l r_d^3 / 3 . \tag{16}$$

The number density of cloud droplets N is specified as one fixed value over land, N^{land}, and another fixed value over the ocean, N^{ocean}. This reflects the fact that cloud drop numbers tend to significantly higher over land where more cloud condensation nuclei are present.

However, as pointed out in R97, the use of (15) in GCMs is problematic because the time scale for which q_l falls beneath the threshold for autoconversion is in most cases shorter than the time step between calls to the stratiform cloud routines, Δt_{cld}. To avoid undershooting the threshold for autoconversion, the rate of decrease of q_l is therefore limited to that that would decrease q_l to the threshold:

$$MAX\left(-\left.\frac{\partial q_l}{\partial t}\right|_{au}\right) = \ln\left(\frac{\rho q_l / q_a}{4\pi N \rho_l (r_d^{au})^3 / 3}\right) \times \frac{q_l}{\Delta t_{cld}} . \tag{17}$$

The use of the natural logarithm in (17) guarantees with the numerical method used (section 2j) that if autoconversion is the only process acting, that at the end of Δt_{cld}, the stratiform cloud time step, $\rho q_l / q_a = 4\pi N \rho_l (r_d^{au})^3 / 3$. In practice this limiter is almost always invoked such that the rate calculated from (15) is almost never used. Indeed one can show that if Δt_{cld} is greater than a critical value Δt_{crit}, then (15) is never used. This critical value is a function of r_d^{au} and N, and is displayed in Fig. 3. For typical r_d^{au} of 10 μ, one can see that the limiter is always invoked for typical Δt_{cld} of 1000-4000 s.

The parameterization of accretion is derived by integrating the continous collection equation over an assumed Marshall-Palmer distribution for the number distribution of rain drops. Generalizing the equation in R97 for the assumptions of the current parameterization regarding the overlap of clouds and precipitation (section 2g) leads to:

$$\left.\frac{\partial q_l}{\partial t}\right|_{acc} = -a_{rain}^{cld} \times 65.8 E_{c,acc} (R_{rain}^{cld} / \rho_l a_{rain}^{cld})^{7/9} \times (q_l / q_a), \qquad (18)$$

where R_{rain}^{cld} is the grid mean flux of rain entering the grid box from above that enters saturated air. The portion of the grid box that this occurs in is a_{rain}^{cld}. $E_{c,acc}$ is the collection efficiency between rain drops and cloud droplets which is parameterized as:

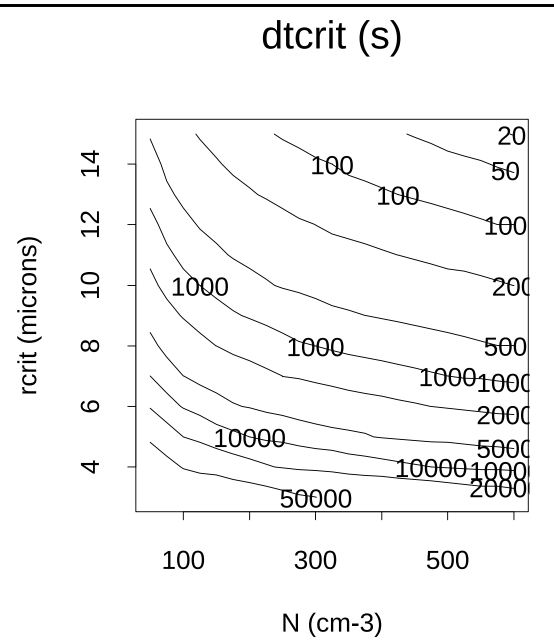

Figure 3. Critical value of the time step for which the autoconversion limiter is always invoked. This value is a function of N, the number of cloud droplets per unit volume, and r_d^{au}, the critical mean cloud drop radius above which autoconversion occurs.

$$E_{c,acc} = r_d^2/(r_d^2 + 20.5\mu^2). \tag{19}$$

This is a reasonable fit to the data in Table 8.2 of Rogers and Yau (1988) for raindrop sizes in the range 1 to 4 mm and cloud droplets in the range 3 to 12 μ.

e. Ice phase microphysical processes

Cloud ice is considered to fall relative to the large-scale vertical motion with a fall speed parameterized by Heymsfield and Donner (1990):

$$V_f = 3.29(\rho q_i/q_a)^{0.16}. \tag{20}$$

With ice crystals falling at this speed, the sink of cloud ice due to gravitational settling is:

$$\left.\frac{\partial q_i}{\partial t}\right|_{gr} = -\frac{\partial}{\partial p}\{q_a \times \rho g V_f \times (q_i/q_a)\}. \tag{21}$$

With this formulation, settling ice that falls into the saturated portion of the grid box from above can increase cloud ice.

Given typical vertical resolution in GCMs, a large fraction of the cloud ice falls out of the layer in a typical Δt_{cld}. To avoid results which are dependent on the timestep used, the numerical integration of (21) must be carefully handled (R97). The numerical integration methods of section 2j guarantee a result equivalent to equation B.10 of R97.

f. Conversions between liquid and ice phases

In this parameterization, cloud liquid can be converted to cloud ice through three processes: homogenous freezing, riming, and the Bergeron process. Homogenous freezing of cloud droplets occurs when the temperature is less than -40˚C. If cloud liquid is present at these temperatures, then all of the liquid is converted to ice within the time step Δt_{cld}. This process is the main means by which the detrained liquid from convective updrafts freezes.

Riming occurs when the falling ice collides and coalesces with cloud droplets. By assuming that the number distribution of settling ice follows a Marshall-Palmer distribution with slope factor λ_f, R97 derives an expression for the time rate of change of riming. Given the assumptions of the current parameterization for the overlap of clouds and precipitation, this rate is expressed as:

$$\left.\frac{\partial q_l}{\partial t}\right|_{rim} = -a_{snow}^{cld} \times \lambda_f E_{c,\,rim}(R_{snow}^{cld}/2\rho_i a_{snow}^{cld}) \times (q_l/q_a)\,, \tag{22}$$

where ρ_i is the assumed density of falling ice crystals (fixed at 0.1 kg m^{-3}), R_{snow}^{cld} is the grid mean flux of settling ice entering the grid box from above that enters saturated air. The portion of the grid box that this occurs in is a_{snow}^{cld}. $E_{c,\,rim}$ is the collection efficiency for the riming process (fixed at 0.7). Following R97, λ_f is parameterized as a function of temperature as:

$$\lambda_f = 1.6{\times}10^{3.+0.023(276.16\text{K}\ -T)}. \tag{23}$$

Where liquid and ice coexist within the same volume, there is preferential condensation onto the ice crystals at the expense of the supercooled droplets (i.e., the Bergeron process). This occurs because the saturation vapor pressure over ice, e_{si}, is lower than the saturation vapor pressure over liquid, e_{sl}. The growth of an ice crystal due to the Bergeron process is according to R00:

$$\left.\frac{\partial q_l}{\partial t}\right|_{berg} = -\frac{q_a \times (N_i/\rho)^{2/3} \times 7.8 \times (MAX(q_i/q_a, M_{i0}N_i/\rho))^{1/3}}{(\rho_i)^{2/3} \times (A+B)} \tag{24}$$

where N_i is the number of ice nuclei per unit volume, M_{i0} is the mass (10^{-12} kg) of an initial crystals assumed to always be present, and ρ_i is the mass density of pristine ice crystals (700 kg m^{-3}). In addition, $A = (L_v/K_aT)\cdot((L_v/R_vT)-1)$ and $B = R_vT/\chi e_s$, where K_a is the thermal conductivity of air, χ is the diffusivity of water vapor in air, and R_v is the gas constant for water vapor. The ice nuclei density, N_i, is parameterized according to Meyers et. al (1992) under the assumption that the air is at liquid water saturation:

$$N_i = 1000 \exp\left[12.96 \frac{(e_{sl} - e_{si})}{e_{si}} - 0.639\right], \tag{25}$$

where e_{sl} is the saturation vapor pressure over liquid and e_{si} is the saturation vapor pressure over ice.

In addition to these processes which convert cloud liquid into ice, cloud ice can be converted to rain when it melts. It is assumed that cloud ice is converted to rain because if an ice crystal of dimension ~100 μm with mass density of 100 kg/m^2 melts it will become a droplet of size ~50 μm which is clearly a drizzle size drop. Ice crystals at temperatures near freezing are assumed to be this large, consistent with the assumed particle size-temperature relationship.

g. Treatment of precipitation fluxes

The vertical transport of rain and settling ice is accomplished by a separate accounting of the amount of rain or settling ice that is within saturated volume (i.e. 'in-cloud') (R^{cld}_{rain} or R^{cld}_{snow}) and is not evaporating and that that is within unsaturated air (i.e. 'out-of-cloud') (R^{clr}_{rain} or R^{clr}_{snow}) and hence evaporating (Jakob and Klein 2000). These precipitation rates are treated as diagnostic fluxes following Ghan and Easter (1992) and have units of kg condensate m^{-2} s^{-1}. To determine the amount of and area covered by each flux, three assumptions are made: (a) the saturated volume completely fills the grid box in the vertical such that q_a represents the area fraction of the grid box covered by saturated mass, (b) saturated mass in adjacent levels overlaps maximally, and (c) precipitation maximally overlaps with saturated mass. These overlap assumptions are consistent with that assumed by the radiation scheme. From these assumptions, the fraction of the area of the grid box covered by these fluxes (a^{cld}_{rain}, a^{cld}_{snow}, a^{clr}_{rain} and a^{clr}_{snow}) can be determined.

The critical part is to determine what amount of the in-cloud precipitating flux falls into cloud of the levels below and what amount of the out-of-cloud precipitating flux that falls into cloud of the levels below. With use of the overlap assumptions detailed above, the fraction of the area in which in-cloud rain falls into clear air of the level below, $\Delta a_{cld \to clr}$, is $MAX(a^{cld}_{rain} - q^{k+1}_a, 0)$, where q^{k+1}_a is the cloud fraction of level below. The fraction of the area in which out-of-cloud rain falls into cloudy air of the level below, $\Delta a_{clr \to cld}$, is $MIN(a^{clr}_{rain}, MAX(0., q^{k+1}_a - q^k_a))$. If it is assumed that the precipitation flux evenly covers the area ascribed to it, then the amount of precipitation that falls from cloud in the current level to clear air in the level below, $\Delta R_{cld \to clr}$, and the amount of precipitation that falls from clear air in the current level into cloud of the level below, $\Delta R_{clr \to cld}$, is:

$$\Delta R_{cld \to clr} = (\Delta a_{cld \to clr} / a^{cld}_{rain}) \times R^{cld}_{rain}, \text{ and} \tag{26}$$

$$\Delta R_{clr \to cld} = (\Delta a_{clr \to cld} / a^{clr}_{rain}) \times R^{clr}_{rain}. \tag{27}$$

Rain evaporation is parameterized following R97 by integrating the growth equation of a droplet over the assumed Marshall-Palmer distribution of raindrops. In terms of the assumptions applied here, the first equation following equation B.9 of R97 reduces to:

$$\left.\frac{\partial q_v}{\partial t}\right|_{rainevap} = \frac{a^{clr}_{rain} \times 5.7 \times 10^4 (R^{clr}_{rain} / \rho_l a^{clr}_{rain})^{11/18} \times (q_s - q_{v,\,clr})}{(\rho)^{1/2} q_s (A + B)}. \tag{28}$$

The amount of rain which evaporates in a given time step is numerically calculated following the time-centred method of R97. This amount is then limited to the amount of rain available to evaporate (R_{rain}^{clr}), and the amount that would raise the relative humidity in the evaporating portion of the grid box up to a critical relative humidity, U_{evap}, above which precipitation evaporation does not occur. This limiting amount is:

$$\Delta q_v\big|_{maxevap} = \frac{a_{rain}^{clr} \times (U_{evap} q_s - q_{v,\,clr})}{\left(1 + (U_{evap}(1-q_a) + q_a)\left(\frac{L}{c_p}\right)\frac{\partial q_s}{\partial T}\bigg|_p\right)}. \tag{29}$$

Although ideally $U_{evap} = 1.$, because of neglected processes in the GCM it may be necessary to use a lower U_{evap}. For example, the evaporation of stratiform precipitation in convective regions rarely saturates subcloud air because of the presence of unsaturated downdrafts. Because this version of RAS does not parameterize downdrafts then it is unreasonable to allow precipitation evaporation to saturate the sub-cloud layer.

Following R97, the sublimation of settling ice is:

$$\begin{aligned}\frac{\partial q_v}{\partial t}\bigg|_{snowsub} = \quad & a_{snow}^{clr} \times 4(R_{snow}^{clr}/3.29 a_{snow}^{clr})^{1.16} \times (q_s - q_{v,\,clr}) \times (\pi\rho\rho_i q_s(A+B))^{-1} \\ & \times [0.65\lambda_f^2 + 200(\rho\lambda_f^3)^{1/2}(R_{snow}^{clr}/a_{snow}^{clr})^{1/14.5}]\end{aligned} \quad . \tag{30}$$

As in rain evaporation, the amount of sublimation is limited to the amount of ice available (R_{snow}^{clr}) and to the amount that would raise the relative humidity in the sublimating portion of the grid box to U_{evap}.

Melting of ice settling outside of cloud occurs when the settling ice falls into a layer with temperature warmer than 2˚C (Mason 1971). The amount of melting, $\Delta R_{snow}^{clr}\big|_{melt}$, is limited to that that would cool the grid box to 2˚C.

h. Energy and moisture balances

For all the processes included in the parameterization, the total change in the tracer variables, water vapor and temperature due to stratiform processes is:

$$S_l = \left.\frac{\partial q_l}{\partial t}\right|_{ls} + \left.\frac{\partial q_l}{\partial t}\right|_{eros} + \left.\frac{\partial q_l}{\partial t}\right|_{au} + \left.\frac{\partial q_l}{\partial t}\right|_{acc} + \left.\frac{\partial q_l}{\partial t}\right|_{fr} + \left.\frac{\partial q_l}{\partial t}\right|_{rim} + \left.\frac{\partial q_l}{\partial t}\right|_{berg}, \qquad (31)$$

$$S_i = \left.\frac{\partial q_i}{\partial t}\right|_{ls} + \left.\frac{\partial q_i}{\partial t}\right|_{eros} + \left.\frac{\partial q_i}{\partial t}\right|_{gr} + \left.\frac{\partial q_i}{\partial t}\right|_{melt} - \left.\frac{\partial q_l}{\partial t}\right|_{fr} - \left.\frac{\partial q_l}{\partial t}\right|_{rim} - \left.\frac{\partial q_l}{\partial t}\right|_{berg}, \qquad (32)$$

$$S_a = \left.\frac{\partial q_a}{\partial t}\right|_{ls} + \left.\frac{\partial q_a}{\partial t}\right|_{eros} + \left.\frac{\partial q_a}{\partial t}\right|_{diss}, \qquad (33)$$

$$S_v = -\left.\frac{\partial q_l}{\partial t}\right|_{ls} - \left.\frac{\partial q_i}{\partial t}\right|_{ls} - \left.\frac{\partial q_l}{\partial t}\right|_{eros} - \left.\frac{\partial q_i}{\partial t}\right|_{eros} + \left.\frac{\partial q_v}{\partial t}\right|_{rainevap} + \left.\frac{\partial q_v}{\partial t}\right|_{snowsub}, \qquad (34)$$

$$\begin{aligned} c_p S_T = {} & L_v\left(\left.\frac{\partial q_l}{\partial t}\right|_{ls} + \left.\frac{\partial q_l}{\partial t}\right|_{eros} - \left.\frac{\partial q_v}{\partial t}\right|_{rainevap}\right) + L_s\left(\left.\frac{\partial q_i}{\partial t}\right|_{ls} + \left.\frac{\partial q_i}{\partial t}\right|_{eros} - \left.\frac{\partial q_v}{\partial t}\right|_{snowsub}\right), \qquad (35) \\ & + L_f\left(\frac{\left.\Delta R^{clr}_{snow}\right|_{melt}}{(\Delta p/g)} + \left.\frac{\partial q_i}{\partial t}\right|_{melt} - \left.\frac{\partial q_l}{\partial t}\right|_{fr} - \left.\frac{\partial q_l}{\partial t}\right|_{rim} - \left.\frac{\partial q_l}{\partial t}\right|_{berg}\right) \end{aligned}$$

where L_f is the latent heat of fusion, $\left.\frac{\partial q_l}{\partial t}\right|_{fr}$ is the rate of homogenous freezing, and $\left.\frac{\partial q_a}{\partial t}\right|_{diss}$ is the dissipation of cloud fraction. Following T93, this is equal to $-q_a/\Delta t_{cld}$ where and when q_l and q_i equal zero.

From these equations one can show that the column integrated total moisture and energy budgets from stratiform processes are:

$$\left.(R^{cld}_{rain} + R^{clr}_{rain} + R^{cld}_{snow} + R^{clr}_{snow})\right|_{surface} = -\sum (S_v + S_l + S_i) \cdot (\Delta p/g), \qquad (36)$$

$$\sum c_p S_T \cdot (\Delta p/g) = L_v\left([\sum S_l \cdot (\Delta p/g)] + (R^{cld}_{rain} + R^{clr}_{rain})\Big|_{surface} \right) , \quad (37)$$
$$L_s\left([\sum S_i \cdot (\Delta p/g)] + (R^{cld}_{snow} + R^{clr}_{snow})\Big|_{surface} \right)$$

where the summation occurs over vertical levels.

For reference, the column integrated total moisture and energy budgets from the convective processes are:

$$R^C_{rain}\Big|_{surface} + R^C_{snow}\Big|_{surface} = -\sum (C_v + C_l + C_i) \cdot (\Delta p/g) \quad (38)$$

$$\sum c_p C_T \cdot (\Delta p/g) = L_v\left([\sum C_l \cdot (\Delta p/g)] + R^C_{rain}\Big|_{surface} \right) , \quad (39)$$
$$L_s\left([\sum C_i \cdot (\Delta p/g)] + R^C_{snow}\Big|_{surface} \right)$$

where is the $R^C_{rain,\,snow}\Big|_{surface}$ is the surface convective rain or snow rate.

i. Cloud radiative properties

Cloud albedo and transmittance in the shortwave and near-infrared bands are computed with a delta-Eddington solution. Shortwave cloud radiative properties for liquid and ice clouds are specified from Slingo (1989) and Ebert and Curry (1992), respectively. For liquid clouds, the effective particle radius is set equal to $k \cdot r_d$, where k is ratio of effective radius to mean volume radius and is dependent on airmass type (continental or maritime) (Martin et al. 1994). In the case of single layer liquid clouds, the liquid water specific humidity typically increases linearly with height above cloud base such that the specific humidity at the top of the cloud is twice the mean specific humidity over the cloud . These conditions occur typically in stratocumulus clouds and it has been observed for these clouds that the effective radius for the cloud as a whole is between 80 and 100% of the cloud top effective radius (Brenguier et al. 2000). Following these observations, the effective radius for single layer liquid or mixed phase clouds in the GCM is taken to be 90% of the effective radius deduced from the cloud top liquid water specific humidity. For ice the effective particle radius is a taken from a temperature dependent table (Donner et al. 1997) whereby the effective radius increases from 15 μm at temperatures less than -55˚C to 92 μm at temperatures greater than -25˚C. This function is based upon data of Heymsfield and Platt (1984) and includes the effect of enhanced scattering due to ice crystals smaller than the minimum size measured by the aircraft instruments. The longwave emissivity of liquid clouds is parameterized with a mass absorption coefficient of 140 m^2 kg^{-1}. The longwave emissivity of ice clouds is parameterized according to Ebert and Curry (1992). For the radiative properties of mixed phase clouds, the method of Rockel et al. (1991) is used to combine the radiative properties of liquid and ice. The radiative properties of ice outside of cloud and rain are neglected as it is assumed that the particle sizes are too large to have a significant radiative effect. However, millimeter wavelength cloud radar observations indicate that ice in unsaturated air has a significant effect on the radiation budget suggesting that in the future the radiative properties of ice outside of cloud will need to be treated (Hogan et al. 2000).

Clouds are assumed to be horizontally inhomogeneous such that the radiative impact is different from the impact of a horizontally homogeneous cloud (Cahalan et al. 1994). This effect is crudely parameterized by multiplying the visible and near-infrared optical depths by 0.8, consistent with the results of large-eddy simulations of stratocumulus (Kogan et al. 1995).

It is assumed that clouds which fill adjacent vertical levels are assumed to be maximally overlapped whereas clouds not connected vertically are assumed to be randomly overlapped. The radiation code being used requires that the radiative properties of multilevel maximally overlapped clouds which vary with level in the cloud be reduced to a single set of properties. In doing so, the following steps were taken. The effective radii for liquid and ice were taken as the mass-weighted effective radii. The cloud fraction for the combined cloud was taken as the maximum cloud fraction of all the levels encompassing the cloud. The liquid and ice water paths for the combined cloud equalled the column integrated liquid and ice divided by the cloud fraction for the combined cloud. Finally, the vertical extent of the cloud was modified so that the top and bottom of the multilayer cloud were at the levels with the maximum exposure from above and below the cloud respectively.

j. Method of numerical integration

With several of the microphysical processes having very rapid time scales (e.g. autoconversion and ice settling), it is important to have a numerical integration of the tracer equations which yields results that are not highly dependent upon the time step, Δt_{cld}, used. This is accomplished by analytic integration of tracer equations in the order of cloud fraction, cloud liquid and cloud ice. The method of analytic integration follows that detailed in section 2g of T93. However, the total conversion of liquid or ice per time step (equation 40 of T93) must be apportioned to each process for proper conservation of tracer and energy. To do this, the amount of conversion for the *i*th process is equal to the fraction of the conversion rate contributed by that process (i.e. $D_i / \sum D_i$ in terminology of T93) times the total conversion rate per time step (equation 40 of T93).

k. Correction for physically unreasonable states

Due to numerical errors in detrainment or advection resulting from the coarse vertical and horizontal grid, occasionally a grid box may be supersaturated. Under the assumption that the temperature is constant in the grid box and that $q_v \leq q_s$ everywhere in the grid box, the excess vapor is condensed and added to the precipitation fluxes. Note that because these conditions are assumed to result from numerical errors, the excess vapor is not added to the cloud field.

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