Simulations of GL instability

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first, reproduction of earlier numerical simulations of GL instability. then, do new stuff with cosmological constant and charges. I will first summarize lessons learned during my visit to Korea in March 2006. I propose working on GL instability with numerical methods and I will apply lessons in this problem first. ...

PACS numbers:

Korea lessons

- When providing results, always estimate relative scale of errors of primary functions being evolved. This should be done with focus on the SOLUTION behavior, NOT derived quantities at least initially. $\delta \sim f_h/(f_h - f_{2h})$
- Implement INDEPENDENT RESIDUAL EVALUATOR as soon as possible. this is acting on the Finite Difference Approximation $G_{\mu\nu} = 0$?. in vaccum it is equivalently $R_{\mu\nu} = 0$?. Perhaps use maple to do this! (cf. /example/adm_free/maple).
- OK: coordinate system (r, θ, ϕ, z) and to obtain eqn of motion (ok this is what we want) given metric in general $n+1$ dimension, use maple to calculate ADM or BSSN equation...?!
- Implement flow method for AHF...

Other lessons to consider:

- It's 2D problem. So use cartoon??? or since it's only for r,"z", maybe cartesian grid should be fine? but no wait it's spherically symmetric metric....

EQUATIONS

We will consider 5-dim cases. What I need is a way to start from some generic form of metric and obtain evolution equations. Perhaps I will use maple to achieve that... (I think Matt's tensor package does that for 4-d.) Now it's 4+1 formalism.

For 4+1 dim., general metric would be

$$ds^2 = (-\alpha^2 + g_{AB}\beta^A\beta^B)dt^2 + 2g_{AB}\beta^A dx^B dt + g_{AB}dx^A dx^B \tag{1}$$

where $A, B = 1, 2, 3, 4$. Since we are going to separate out the 5th dimension, we will explicitly separate out the 4th space dimension in the line element.

$$\begin{aligned}
ds^2 \quad &= (-\alpha^2 + g_{44}\beta^4\beta^4 + 2g_{i4}\beta^i\beta^4 + g_{ij}\beta^i\beta^j)dt^2 \\
&\quad +2(g_{44}\beta^4 dx^4 dt + 2g_{i4}\beta^i dx^4 dt + g_{ij}\beta^i dx^j dt) \\
&\quad +(g_{44}dx^4dx^4 + 2g_{i4}dx^i dx^4 + g_{ij}dx^i dx^j) \\
&= (-\alpha^2 + g_{ij}\beta^i\beta^j)dt^2 + 2g_{ij}\beta^i dx^j dt + g_{ij}dx^i dx^j \\
&\quad +(g_{44}\beta^4\beta^4 + 2g_{i4}\beta^i\beta^4)dt^2 \\
&\quad +2(g_{44}\beta^4 dx^4 dt + 2g_{i4}\beta^i dx^4 dt) \\
&\quad +(g_{44}dx^4 dx^4 + 2g_{i4}dx^i dx^4)
\end{aligned} \tag{2}$$

where $i, j = 1, 2, 3$. We are going to consider the situation where the 4-dimensional subspace ($i = 1, 2, 3$) has spherical symmetry. Most general form of the metric for spherically symmetric 4-d subpace is, in spherical coordinate, (r, θ, ϕ),

$$\begin{aligned} ds^2_{(4)} &= (-\alpha^2 + g_{rr}\beta^r\beta^r)dt^2 + 2g_{rr}\beta^r dtdr + g_{rr}dr^2 \\ &+r^2(d\theta^2 + \sin\theta^2 d\phi^2) \end{aligned} \tag{3}$$

with $\beta^\theta = \beta^\phi = 0$. Putting the eqn. (3) into eqn (2) (and we also restrict ourselves (why??) to the case where $g_{\theta 4} = g_{\phi 4} = 0$.) gives

$$\begin{aligned} ds^2 &= (-\alpha^2 + g_{rr}\beta^r\beta^r)dt^2 + 2g_{rr}\beta^r dtdr + g_{rr}dr^2 \\ &+r^2(d\theta^2 + \sin\theta^2 d\phi^2) \\ &+(g_{44}\beta^4\beta^4 + 2g_{r4}\beta^r\beta^4)dt^2 \\ &+2(g_{44}\beta^4 dx^4 dt + 2g_{r4}\beta^r dx^4 dt) \\ &+(g_{44}dx^4 dx^4 + 2g_{r4}dx^r dx^4) \end{aligned} \tag{4}$$

Rearranging the terms again,

$$\begin{aligned} ds^2 &= (-\alpha^2 + g_{ab}\beta^a\beta^b)dt^2 + 2g_{ab}\beta^a dx^b dt \\ &+g_{ab}dx^a dx^b + r^2(d\theta^2 + \sin\theta^2 d\phi^2) \end{aligned} \tag{5}$$

where $a, b = (r, 4)$ and (adapting the notation $4 \to z$ to indicate the 5th dimension) $\alpha \equiv \alpha(r, z, t), \beta^a \equiv \beta^a(r, z, t), g_{ab} \equiv g_{ab}(r, z, t)$.

But let me clarify something here before we proceed. Eqn. (3) already assumed that the coordinate is $(r, \theta, \phi) \equiv (r_{AREAL}, \theta, \phi)$, i.e. it's written in AREAL coordinate as is true in Eqn. (5).

In other words, Eqn. (5) should be written, to be precise, as

$$\begin{aligned} ds^2 &= (-\alpha^2 + g_{ab}\beta^a\beta^b)dt^2 + 2g_{ab}\beta^a dx^b dt \\ &+g_{ab}dx^a dx^b + r^2_{AREAL}(d\theta^2 + \sin\theta^2 d\phi^2) \end{aligned} \tag{6}$$

with $a, b = (r_{AREAL}, z)$ and $\alpha \equiv \alpha(r_{AREAL}, z, t), \beta^a \equiv \beta^a(r_{AREAL}, z, t), g_{ab} \equiv g_{ab}(r_{AREAL}, z, t)$. Again, this identification is coordinate choice $(r = r_{AREAL})$

However, in general, we should use, the following form

$$\begin{aligned} ds^2 &= (-\alpha^2 + g_{ab}\beta^a\beta^b)dt^2 + 2g_{ab}\beta^a dx^b dt \\ &+g_{ab}dx^a dx^b + \gamma_\Omega(d\theta^2 + \sin\theta^2 d\phi^2) \end{aligned} \tag{7}$$

with $a, b = (r, z)$ and $\alpha \equiv \alpha(r, z, t), \beta^a \equiv \beta^a(r, z, t), g_{ab} \equiv g_{ab}(r, z, t), \gamma_\Omega \equiv \gamma_\Omega(r, z, t)$. Indeed, this form was used by Choptuik et al paper (their eqn. (1)). Now the reason they chosen this form, I believe, is because of their coordinate choices. Their choices are NOT compatible with (radial) areal coordinate. They use, $\alpha = \alpha_{BS}, \beta^z = \beta^z_{BS} = 0$ with some choice of β^r.

So to summarize the evolution and gauge variables are $\{\alpha, \beta^r, \beta^z, g_{rr}, g_{rz}, g_{zz}, \gamma_\Omega(\equiv g_{tt}??)\}$ and in addition, in ADM formalism we have $\{k_{rr}, k_{rz}, k_{zz}, k_{tt}\}$.

INITIAL DATA

Now I will skip over to the initial data consideration (and will get back to evolution later). We like to reproduce Choptuik et al. They used perturnbed black string as ID. I assume their metric at $t = 0$ is

$$\begin{aligned} ds^2_{BS} &= -(1 - 2M/r)dt^2 + 4M/rdrdt + (1 + 2M/r)dr^2 + dz^2 \\ &+ r^2 g_{\theta\theta}d\theta^2 + r^2\sin^4\theta g_{\phi\phi}d\phi^2 \end{aligned} \tag{8}$$

and since they choose $g_{\theta\theta} = g_{\phi\phi}sin^2\theta$,

$$\begin{aligned} ds^2_{BS} &= -(1 - 2M/r)dt^2 + 4M/rdrdt + (1 + 2M/r)dr^2 + dz^2 \\ &+ r^2 g_{\theta\theta}d\theta^2 + r^2\sin^2\theta g_{\theta\theta}d\phi^2 \\ ds^2_{BS} &= -(1 - 2M/r)dt^2 + 4M/rdrdt + (1 + 2M/r)dr^2 + dz^2 \\ &+ r^2 g_{\theta\theta}(d\theta^2 + \sin^2\theta d\phi^2) \end{aligned} \tag{9}$$

where, Choptuik et al choose, $g_{\theta\theta}(t=0,r,z) = 1 + A\sin(z\frac{2\pi q}{L})e^{-(r-r_0)^2/\delta_r^2}$. They say given this $g_{\theta\theta}$, solve for $g_{rr}, k_{rr}, k_{\theta\theta}$. That means their form of metric at $t=0$ is (need to check, YES IT IS! May 4, 2006)

$$\begin{aligned} ds^2_{BS} &= -(1-2M/r)dt^2 + 4M/rdrdt + g_{rr}dr^2 + dz^2 \\ &+ r^2 g_{\theta\theta}(d\theta^2 + \sin^2\theta d\phi^2) \end{aligned} \tag{10}$$

where $g_{rr} \equiv g_{rr}(r,z), g_{\theta\theta} \equiv g_{\theta\theta}(r,z)$.

So non-zero metric elements are $(r,r),(z,z),(\theta,\theta),(\phi,\phi)$ out of which only two are variables $(r,r),(\theta\theta)$. Non-zero gauge functions are β^r, α. So what is their choice of extrinsic curvature. It seems that only non-zero extrinsic curvature elments are $(r,r),(\theta\theta)$ and everything else is zero??

Ok if the above statements are true, then, we should write down constraint equations to solve and solve them!

GH references: (**don't erase!!**) gr-qc/0512093.

ADS BLACK STRING

Now I am going to try to understand what GK did with AdS Black Strings because that is the new things we will do.

Gungwon says that the solution for asymptotically AdS spacetime.

—– Initial data in our case, as you can see in the attached file, is the (un)perturbed AdS black string metric is as follows;

$$ds^2 = H^{-2}[-fdt^2 + \frac{dr^2}{f} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + dz^2]$$

where $H = H(z)$ and $f(r)$ is given in the file.

$$H(z) = \frac{l_4}{l_5}\sin\frac{\mathrm{z}}{\mathrm{l}_4}$$
$$f(r) = 1 - \frac{r_0}{r} - \frac{\Lambda_4}{3}r^2$$

what is the meaning of r_0?

Now we define some coordinate system which is not 'singular' in the vicinity of the horizon. Let

$$dr^* = \frac{dr}{f}, dv = dt + dr^*, d\tilde{t} = dv - dr$$

Then we have

$$dt = d\tilde{t} + \frac{f-1}{f}dr$$

Thus, the bs metric above can be written as, in $(\tilde{t}, r)$ coordinate,

$$\begin{aligned} ds^2 = H^{-2}[&-fd\tilde{t}^2 + 2(1-f)drd\tilde{t} + (2-f)dr^2 \\ &+r^2(d\theta^2 + \sin^2\theta d\phi^2) + dz^2] \end{aligned} \tag{11}$$

By renaming $\tilde{t} \to t$, we finally have the unperturbed metric in the so-called ingoing Eddington-Finkelstein coordinates. (I do not like this terminology though.)

$$\begin{aligned} ds^2 = H^{-2}[&-fdt^2 + 2(1-f)drdt + (2-f)dr^2 \\ &+r^2(d\theta^2 + \sin^2\theta d\phi^2) + dz^2] \end{aligned} \tag{12}$$

In the case of $H = 1, r_0 = 2M, and\Lambda_4 = 0$, we recover the Schwarzschild bs metric in their paper.

END GUNGWON —–

regarding perturbed AdS BS initial data Gungwon says —– For the perturbed initial data in our case things do not seem to change much. We need to check if the warping factor affects the analysis in the modified constraint equations below. I'll assume no change for the moment. We may use the form you mentioned above, but with the warping factor.

$$\begin{aligned} ds^2_{BS} = H^{-2}[&-fdt^2 + 2(1-f)drdt + g_{rr}dr^2 \\ &+r^2 g_{\theta\theta}(d\theta^2 + \sin^2\theta d\phi^2) + dz^2] \end{aligned} \tag{13}$$

Now we may try the same form for the initial $g_{\theta\theta}$ as in the Choptuik et. al., and solve the modified constraints equations due to the five-dimensional cosmological constant $\Lambda(\equiv \Lambda_4)$ in order to obtain the full set of initial data, including $g_{rr}, k_{rr}, k_{\theta\theta}$. **We are ready to solve the constraint equations now!** (empha is Dale's)

Note: we may try the form in which the z-dependent warping factor is absorbed already. So,

$$\begin{aligned} ds^2_{BS} = &-H^{-2}fdt^2 + 2H^{-2}(1-f)drdt + g_{rr}dr^2 \\ &+r^2 g_{\theta\theta}(d\theta^2 + \sin^2\theta d\phi^2) + H^{-2}dz^2. \end{aligned} \tag{14}$$

I do not know which one is better at the present. **END GUNGWON** —–

OR because, equations for $g_{rr}, k_{rr}, k_{\theta\theta}$ are not all elliptic equations, I propose alternative form

$$\begin{aligned} ds^2_{BS} = H^{-2}[&-fdt^2 + 2\beta^r drdt + 2\beta^z dzdt + g_{rr}dr^2 \\ &+r^2 g_{\theta\theta}(d\theta^2 + \sin^2\theta d\phi^2) + dz^2] \end{aligned} \tag{15}$$

Then, I think, I end up with elliptic equations for g_{rr}, β^r, β^z by solving HC, MC_r, MC_z constraint equations. You want to double-check, but principal parts are

$$\begin{aligned} &\frac{\partial^2}{\partial z^2} g_{rr}(HC) \\ \frac{\partial^2}{\partial z^2}\beta^r, &\frac{\partial^2}{\partial z^2} g_{rr}(MC_r) \\ \frac{\partial^2}{\partial r^2}\beta^z, &\frac{\partial^2}{\partial z^2} g_{rr}(MC_z) \end{aligned}$$

- **To generate codes for ID and Evolution, I use MWC's tensor package (refer to FP's various maple scripts and understand what Frans did with all_nd_common maple script—need to understand dimensional reduction... effective stress energy tensor from the dimensional reduction is**

$$.... \tag{16}$$

)

*gl_ic_v2 and gl scripts now run. Progress so far:

- Examined, all_ic_gen, in gh3d. And understanding how two basic routines (A) LOP ($L[f] = 0$) (B) relaxation routine are implemented.
- CREATEd my MG routine mg_sup_blst.f (similar to mg_sup.f) in prepartion to implement AdS BLST ID. In the process got an idea what is needed from maple.
- **go back to gl_ic maple script to output HC, MC eqn. or LOP and RELAX to files.**
- (1) Implement ID elliptic solver in PARMD structure using maple-generated codes: see gh3d example!
  – gh3d_MG_residual (mg_sup_) where residuals cal'ed psi_res, etc.
  – gh3d_MG_relax (mg_sup_) where one relaxation sweep is done.
  – gh3d_L_op (mg_sup_) where Lop is calculated.
  – note that the same routine (mg_sup_) is called with first argument tells which action to take in the routine.

- (2) move my-applications (bos2d, adm_free if not more) out of examples/ directory and put into pamr_apps... (also I modified BStar ID part)
- Note there is a slight modification to HC/MC equations due to Λ_4.
- check (later-1) whether AdS BS is soln to the EE. (need to know the meaning of all the constants appearing in the functions H, f).

MORE ON IVP

first of all, let's work out (3+1)D case for conformal transformation.
let's assume $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$. (here $i, j = 1, 2, 3$.) then we have

$$\Gamma^i_{jk} = \tilde{\Gamma}^i_{jk} + 2\psi^{-1}(\delta^i_k \partial_j \psi + \delta^i_j \partial_k \psi - \tilde{\gamma}^{il}\tilde{\gamma}_{jk}\partial_l \psi) \tag{17}$$

Note that the dimensionality comes in because of trace of δ^μ_μ, etc.

$$\begin{aligned} R_{\sigma\nu} \equiv R^\rho_{\sigma\rho\nu} &= \tilde{R}^\rho_{\sigma\rho\nu} + 2\psi^{-1}\left[(2-\delta^b_b)\tilde{\nabla}_\nu\tilde{\nabla}_\sigma\psi - \tilde{\gamma}_{\nu\sigma}\tilde{\nabla}^a\tilde{\nabla}_a\psi\right] \\ &+\psi^{-2}\left[(6\delta^b_b - 12)\tilde{\nabla}_\nu\psi\tilde{\nabla}_\sigma\psi + (10-4\delta^b_b)\tilde{\gamma}_{\nu\sigma}\tilde{\nabla}^b\psi\tilde{\nabla}_b\psi\right] \\ R_{\sigma\nu} &= \tilde{R}_{\sigma\nu} + 2\psi^{-1}\left[(2-\delta^b_b)\tilde{\nabla}_\nu\tilde{\nabla}_\sigma\psi - \tilde{\gamma}_{\nu\sigma}\tilde{\nabla}^a\tilde{\nabla}_a\psi\right] \\ &+\psi^{-2}\left[(6\delta^b_b - 12)\tilde{\nabla}_\nu\psi\tilde{\nabla}_\sigma\psi + (10-4\delta^b_b)\tilde{\gamma}_{\nu\sigma}\tilde{\nabla}^b\psi\tilde{\nabla}_b\psi\right] \end{aligned} \tag{18}$$

let's take 3-dimensional case where $\delta^b_b = 3$. Then,

$$\begin{aligned} R_{\sigma\nu} &= \tilde{R}_{\sigma\nu} + 2\psi^{-1}(-\tilde{\nabla}_\nu\tilde{\nabla}_\sigma\psi - \tilde{\gamma}_{\nu\sigma}\tilde{\nabla}^a\tilde{\nabla}_a\psi) \\ &+ \psi^{-2}(6\tilde{\nabla}_\nu\tilde{\nabla}_\sigma\psi - 2\tilde{\gamma}_{\nu\sigma}\tilde{\nabla}^b\psi\tilde{\nabla}_b\psi) \end{aligned}$$

$$\begin{aligned} R &= \gamma^{\sigma\nu}R_{\sigma\nu} \\ &= \psi^{-4}\tilde{\gamma}^{\sigma\nu}\tilde{R}_{\sigma\nu} + \psi^{-4}\tilde{\gamma}^{\sigma\nu}\left[2\psi^{-1}(99) + \psi^{-2}(99)\right] \\ R &= \psi^{-4}\tilde{\gamma}^{\sigma\nu}\tilde{R}_{\sigma\nu} + \psi^{-4}\left[2\psi^{-1}(-\tilde{\nabla}^a\tilde{\nabla}_a\psi - \delta^\sigma_\sigma\tilde{\nabla}^a\tilde{\nabla}_a\psi) + \psi^{-2}(6\tilde{\nabla}^\sigma\psi\tilde{\nabla}_\sigma\psi - 2\delta^\sigma_\sigma\tilde{\nabla}^b\psi\tilde{\nabla}_b\psi)\right] \end{aligned}$$

Then, this confirms the celebrated York's conformal decomposition—

$$R = \psi^{-4}\tilde{R} - 8\psi^{-5}\tilde{\nabla}^a\tilde{\nabla}_a\psi$$

now going back to (4+1)D black string context, for n-dimension

$$R = \psi^{-4}\tilde{R} + 4\psi^{-5}(1-n)\tilde{\nabla}^a\tilde{\nabla}_a\psi + \psi^{-6}(16n - 12 - 4n^2)\tilde{\nabla}^b\psi\tilde{\nabla}_b\psi$$

plug in $n = 4$ for (4+1)D.

$$^{(4)}R = \psi^{-44}\tilde{R} - 12\psi^{-5}\tilde{\nabla}^a\tilde{\nabla}_a\psi - 12\psi^{-6}\tilde{\nabla}^b\psi\tilde{\nabla}_b\psi$$

So if I were to choose, time-symmetric initial data, i.e. $K_{ij} \equiv= 0$, then I am ready to solve HCE. The equation to solve is

$$\tilde{\nabla}^a\tilde{\nabla}_a\psi + \psi^{-1}\tilde{\nabla}^b\psi\tilde{\nabla}_b\psi - \frac{1}{12}\tilde{R}\psi = 0 \tag{19}$$

later stuff: introdusing non-time-symmetric initial data, in n-dimension,

$$K_{ij} \equiv A_{ij} + \frac{1}{n}\gamma_{ij}\mathrm{tr(K)}$$

we can, for example, choose maximal slice ($K = 0$)... In terms of conformal split what choice should we make? like

$$A^{ij} = \psi^{-10}\tilde{A}^{ij} \tag{20}$$

first I need to know whether "10" is dimension depedent quantity?

INITIAL DATA SETTING: LICHNEROWICZ-YORK IN 2-DIMENSIONS

The regular conformal relationship:

$$g_{ij} = \phi^4 \tilde{g}_{ij}. \tag{21}$$

Hamiltonian Constraint

The relation between the two Ricci scalars can be written as (see Wald, appendix D):

$$R = \tilde{R}\phi^4 - 2\phi^4\tilde{g}^{kl}\tilde{\nabla}_k\tilde{\nabla}_l \ln\phi^{-2} = \tilde{R}\phi^4 + 4\phi^4\tilde{g}^{kl}\tilde{\nabla}_k\frac{\tilde{\nabla}_l\phi}{\phi} \tag{22}$$

Further manipulations

$$R = \tilde{R}\phi^{-4} - 4\phi^{-5}\tilde{\nabla}^2\phi + 4\phi^{-6}\left(\tilde{\nabla}^k\phi\right)\left(\tilde{\nabla}_k\phi\right) \tag{23}$$

Now we the trace of the metric is 2.

$$K_{ij} = A_{ij} + \frac{1}{2}g_{ij}K \tag{24}$$

The Hamiltonian constraint,

$$R + K^2 - K_{ij}K^{ij} = 16\pi\rho, \tag{25}$$

can be writen as:

$$R + \frac{1}{2}K^2 - A_{ij}A^{ij} = 16\pi\rho. \tag{26}$$

where we have used:

$$K^{ij} = A^{ij} + \frac{1}{2}g^{ij}K \tag{27}$$

Note that we have the following relations (following Marronetti & Matzner):

$$A^{ij} \ = \ \phi^{-8}\left(\tilde{A}^{ij} + \left(\tilde{l}W\right)^{ij}\right) \tag{28}$$

$$A_{ij} \ = \ \left(\tilde{A}_{ij} + \left(\tilde{l}W\right)_{ij}\right), \tag{29}$$

where

$$\left(\tilde{l}W\right)^{ij} = \tilde{\nabla}^i W^j + \tilde{\nabla}^j W^i - \tilde{g}^{ij}\tilde{\nabla}_k W^k. \tag{30}$$

It results on:

$$\tilde{R}\phi^{-4} - 4\phi^{-5}\tilde{\nabla}^2\phi + 4\phi^{-6}\left(\tilde{\nabla}^k\phi\right)\left(\tilde{\nabla}_k\phi\right) + \frac{1}{2}K^2 - \phi^{-8}\left(\tilde{A}_{ij} + \left(\tilde{l}W\right)_{ij}\right)\left(\tilde{A}^{ij} + \left(\tilde{l}W\right)^{ij}\right) = 16\pi\rho \tag{31}$$

Or after manipulation:

$$4\tilde{\nabla}^2\phi - \tilde{R}\phi - 4\phi^{-1}\tilde{\nabla}^k\phi\tilde{\nabla}_k\phi - \frac{1}{2}\phi^5K^2 + \phi^{-3}\left(\tilde{A}_{ij} + \left(\tilde{l}W\right)_{ij}\right)\left(\tilde{A}^{ij} + \left(\tilde{l}W\right)^{ij}\right) + 16\pi\phi^{-3}\tilde{\rho} = 0 \tag{32}$$

where $\rho = \phi^{-8}\tilde{\rho}$.

Momentum Constraint

$$\nabla_i K^{ij} - \nabla^j K = 8\pi j^j \tag{33}$$

which can be writen as:

$$\nabla_i A^{ij} - \frac{1}{2}\nabla^j K = 8\pi j^j \tag{34}$$

Using the fact that $\nabla^j K = \phi^{-4}\tilde{\nabla}^j K$ and defining $j^j = \phi^{-8}\tilde{j}^j$:

$$\nabla_i A^{ij} - \frac{1}{2}\phi^{-4}\tilde{\nabla}^j K = 8\pi\phi^{-8}\tilde{j}^j \tag{35}$$

with

$$\left(\tilde{l}W\right)^{ij} = \tilde{\nabla}^i W^j + \tilde{\nabla}^j W^i - \tilde{g}^{ij}\tilde{\nabla}_k W^k \tag{36}$$

which implies:

$$\tilde{\nabla}_j\left(\tilde{l}W\right)^{ij} = \tilde{\nabla}^2 W^i + \tilde{R}^i{}_j W^j \tag{37}$$

The momentum equation results in:

$$\phi^{-8}\left[\tilde{\nabla}^2 W^i + \tilde{R}^i{}_j W^j\right] - \frac{1}{2}\phi^{-4}\tilde{\nabla}^j K - 8\pi\phi^{-8}\tilde{j}^j + \phi^{-8}\tilde{\nabla}_j\tilde{A}^{ij} = 0 \tag{38}$$

where we have used the fact that $\nabla_j A^{ij} = \phi^{-8}\tilde{\nabla}_j\left(\tilde{A}^{ij} + \left(\tilde{l}W\right)^{ij}\right)$. After a little bit of manipulation:

$$\tilde{\nabla}^2 W^i + \tilde{R}^i{}_j W^j - \frac{1}{2}\phi^4\tilde{\nabla}^j K - 8\pi\tilde{j}^j + \tilde{\nabla}_j\tilde{A}^{ij} = 0 \tag{39}$$

RESULTS

ACKNOWLEDGEMENTS

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[1] See, for example, recent reviews on numerical relativity, L. Lehner, Numerical Relativity: A Review, Class. Quant. Grav., **18**, R25 (2001); T. W. Baumgarte and S. L. Shapiro, Numerical Relativity and Compact Binaries, Phys. Rep. **376**, 41 (2003).
[2] T. Hirayama and G. Kang, Phys. Rev. **D64**, 064010 (2001).