DØ Note 2523

The Basics of Supersymmetry

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Preface

This paper serves as a very basic and informal introduction to Supersymmetry. There are some excellent review articles that delve into this topic in detail [1–3], but even with those, it is difficult to prove to oneself that the theory actually delivers what is advertised. Below are the calculations needed to show that the basic aims of SUSY are realized. Much detail is given here with the more mundane derivations presented in the appendices. This paper follows references 1, 3 and notes by Howie Baer [4] on ref. 3.

1 Introduction

Supersymmetry is called a beyond the Standard Model theory, since it describes phenomena over and above Standard Model predictions. Why is it necessary to look beyond the SM?

The Standard Model is our current theory describing elementary particles and their interactions. It agrees remarkably well with experiments, but along with its successes it contains undesirable features. Why must so many parameters (19 in all [5]) be put in by hand? Electroweak symmetry breaking must be performed with the trick of the Higgs mechanism to make the weak gauge bosons massive. Why do there seem to be three generations of leptons and quarks, or for that matter, why do we need more than one or two? All of these puzzles suggest the need for a larger, grander theory that would encompass the Standard Model. Unfortunately, Supersymmetry does not help to solve any of those problems, and in fact it will make some worse as we will see. There is, however, another anomaly within the SM that SUSY is designed to solve completely — the fine tuning problem.

As mentioned, the Higgs mechanism (see refs. 6 and 7 for detailed descriptions) is needed in order to provide the weak gauge bosons with mass. That process requires

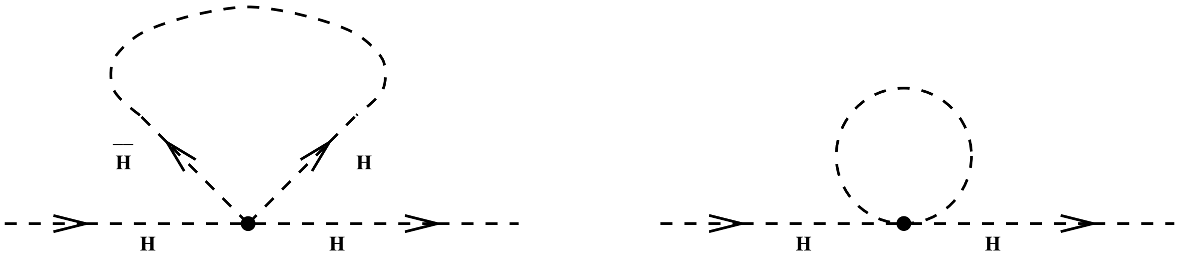

Figure 1: Scalar Higgs scattering with itself, shown in detail left, and more commonly written as on the right.

the introduction of a new boson, the scalar (spin zero) Higgs. Unfortunately, the Standard Model cannot predict the mass of this particle, and worse, it appears that its mass could be infinite. This situation is due to the fact that the part of the Standard Model Lagrangian that describes the Higgs sector contains a self interaction term similar to $\lambda(\phi^\dagger\phi)^2$, where ϕ is the scalar field associated with the Higgs and λ is the strength of the self interaction. The meaning of this term is that a Higgs can scatter by emitting a pair of Higgs particles that annihilate each other, as shown in Fig. 1. Since it represents self interaction and yields a one loop graph, it contributes to the radiative corrections of the Higgs mass or, equivalently, its self energy. The amplitude of this self interaction diagram can be written using Feynman rules (see Table 6.2 in ref. 6) and is proportional to $\int d^4k/(k^2-m^2)$, a quadratically divergent integral. It would seem that the mass of the Higgs diverges! This difficulty can be alleviated by looking beyond the Standard Model. The loop momentum must be integrated to infinity, but perhaps at some high energy scale there would be new degrees of freedom from some grander model that may cut off the loop integral at some finite value, leaving the Higgs mass finite. That idea looks promising, but a problem still remains. The mass of the Higgs with radiative corrections is given to lowest order by,

$$m_H^2 \sim m_0^2 - g^2\Lambda^2 \,, \tag{1.1}$$

where m_H is the Higgs mass that we hope to measure in the future, m_0 is the bare mass, Λ is the scale where new physics is introduced and g is a dimensionless coupling constant of order unity. A natural scale Λ for new physics is the Planck Mass (10^{19} GeV) where gravity can be introduced or the unification scale where the SM running coupling constants converge ($10^{14} - 10^{16}$ GeV). Since the Higgs is involved with electroweak symmetry breaking, it is expected that the mass we would measure be on the order of the weak scale (100 GeV), and thus m_0 would have to be tuned with incredible precision so that the two terms in Eq. (1.1) subtract leaving a few hundred GeV or so left over. This precise adjusting of the Higgs bare mass seems unphysical and is called the fine tuning problem.

There is another way that we can eliminate the quadratic divergence in the correction to the Higgs mass. Say we introduce entirely new diagrams that will cancel the self interaction graph at all energy scales, effectively eliminating the quadratic divergence altogether without any fine tuning. This method is the strategy that Supersymmetry employs. Supersymmetry is a theory that, among other things, gives scalar particles partner fermion particles that conspire to cancel the quadratic divergences, leaving scalar masses finite. Our goal is to look in detail at a simple supersymmetric model to learn the implications of Supersymmetry and see how the theory achieves this cancellation. At the end, we will briefly examine more realistic SUSY theories and learn what experiments have to say on this subject.

2 The Wess and Zumino SUSY model

The simplest supersymmetric model is the one by Wess and Zumino [3, 8, 9]. From it we can derive all of the important aspects of Supersymmetry. Here is their Lagrangian density without interactions,

$$\mathcal{L} = \mathcal{L}_{kinetic} + \mathcal{L}_{mass} \tag{2.1}$$

$$\mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(F^2 + G^2) \tag{2.2}$$

$$\mathcal{L}_{mass} = -m\left[\frac{1}{2}\bar{\psi}\psi + GA + FB\right], \tag{2.3}$$

where A, B, F, and G are real scalar fields (boson like) and ψ is a two degree of freedom Majorana spinor field (fermion like). See appendices A and B for properties of Majorana fields that we will use shortly. Later, we will find that the F and G fields are not independent of A and B, but for simplicity, they will be kept separate for now. Wess and Zumino found that the equations of motion of the fields are not affected when the fields undergo a *supersymmetric transformation*. Remember that transformations can be written as,

$$A \to A' = A + \delta A = A + \bar{\alpha}QA\,, \tag{2.4}$$

where α is the constant parameter of the transformation and Q is the transformation generator. The W-Z supersymmetric transformations for the scalar fields are defined to be,

$$\begin{aligned} \delta A &= i\bar{\alpha}\gamma^5\psi & \delta B &= -\bar{\alpha}\psi \\ \delta F &= -\bar{\alpha}\partial\!\!\!/\psi & \delta G &= -\bar{\alpha}\gamma^5\partial\!\!\!/\psi \end{aligned} \tag{2.5}$$

and the change in the spinor field is,

$$\delta\psi = F\alpha - iG\gamma^5\alpha + (\partial\!\!\!/\gamma^5 A)\alpha + i(\partial\!\!\!/B)\alpha \tag{2.6}$$

These transformations are called supersymmetric since they mix the boson and fermion fields. Note that the α parameter also must be a Majorana spinor since the transformations must not change an object to another type. A spinor must remain a spinor when it is transformed and a scalar must remain a scalar. Let us now verify that these supersymmetric transformations will not alter the equations of motion by showing that the lagrangian density will change only by a total derivative.

2.1 SUSY transformations and the W-Z Lagrangian

The Wess and Zumino supersymmetric transformations are defined in the previous section; Eq. (2.5) and Eq. (2.6). We can show that these transformations merely change the lagrangian by a total derivative. Remember that the lagrangian transforms as $\mathcal{L} \rightarrow \mathcal{L} + \delta\mathcal{L}$. Starting with the kinetic part, we have

$$\delta\mathcal{L}_{kin} = \frac{1}{2}\delta\left[(\partial_\mu A)^2\right] + \frac{1}{2}\delta\left[(\partial_\mu B)^2\right] + \frac{1}{2}\delta\left[\bar{\psi}\partial\!\!\!/\psi\right] + \frac{1}{2}\delta\left[F^2 + G^2\right] . \tag{2.7}$$

Going term by term, we first see that,

$$\begin{aligned}\frac{1}{2}\delta\left[(\partial_\mu A)^2\right] &= \partial^\mu A(\partial_\mu \delta A) = \partial^\mu A \partial_\mu\left(i\bar{\alpha}\gamma_5\psi\right) \\ &= i(\partial^\mu A)\bar{\alpha}\gamma_5\partial_\mu\psi .\end{aligned} \tag{2.8}$$

We can rewrite this expression after noting that,

$$i\partial_\mu\left[(\partial^\mu A)\bar{\alpha}\gamma_5\psi\right] = i(\Box A)\bar{\alpha}\gamma_5\psi + i(\partial^\mu A)\bar{\alpha}\gamma_5\partial_\mu\psi , \tag{2.9}$$

where $\Box = \partial_\mu\partial^\mu$ is the d'Alambertian operator. Thus,

$$\frac{1}{2}\delta\left[(\partial_\mu A)^2\right] = i\partial_\mu\left[(\partial^\mu A)\bar{\alpha}\gamma_5\psi\right] - i(\Box A)\bar{\alpha}\gamma_5\psi . \tag{2.10}$$

Similarly for B,

$$\frac{1}{2}\delta\left[(\partial_\mu B)^2\right] = -\partial_\mu\left[(\partial^\mu B)\bar{\alpha}\psi\right] + (\Box B)\bar{\alpha}\psi . \tag{2.11}$$

Now, for the fermion field term we have

$$\frac{1}{2}\delta\left[\bar{\psi}\partial\!\!\!/\psi\right] = \frac{1}{2}(\delta\bar{\psi})\partial\!\!\!/\psi + \frac{1}{2}\bar{\psi}\partial\!\!\!/(\delta\psi) . \tag{2.12}$$

Note that,

$$\partial^\mu\left[(\delta\bar{\psi})\gamma_\mu\psi\right] = (\partial^\mu\delta\bar{\psi})\gamma_\mu\psi + (\delta\bar{\psi})\gamma_\mu\partial^\mu\psi \tag{2.13}$$

so the first term in Eq. (2.12) can be written

$$\frac{1}{2}(\delta\bar{\psi})\partial\!\!\!/\psi = \frac{1}{2}\partial^\mu\left[(\delta\bar{\psi})\gamma_\mu\psi\right] - \frac{1}{2}(\partial^\mu\delta\bar{\psi})\gamma_\mu\psi . \tag{2.14}$$

Therefore, the fermion field transformation becomes,

$$\frac{1}{2}\delta\left[\bar{\psi}\partial\!\!\!/\psi\right] = \frac{1}{2}\partial^\mu\left[(\delta\bar{\psi})\gamma_\mu\psi\right] - \frac{1}{2}(\partial^\mu\delta\bar{\psi})\gamma_\mu\psi + \frac{1}{2}\bar{\psi}\partial\!\!\!/(\delta\psi)\,. \tag{2.15}$$

We can make this expression even more simple if we manipulate the second term above using Eq. (B.3),

$$-\frac{1}{2}\left[\partial^\mu(\delta\bar{\psi})\right]\gamma_\mu\psi = \frac{1}{2}\bar{\psi}\gamma_\mu\left[\partial^\mu(\delta\psi)\right] = \frac{1}{2}\bar{\psi}\partial\!\!\!/\delta\psi\,. \tag{2.16}$$

This quantity joins with the third term and we finally see

$$\frac{1}{2}\delta\left[\bar{\psi}\partial\!\!\!/\psi\right] = \frac{1}{2}\partial^\mu\left[(\delta\bar{\psi})\gamma_\mu\psi\right] + \bar{\psi}\partial\!\!\!/(\delta\psi)\,. \tag{2.17}$$

We will expand the $\delta\psi$ in a moment. The change in the F and G terms is straight forward,

$$\frac{1}{2}\delta\left[F^2 + G^2\right] = F\delta F + G\delta G = -iF\bar{\alpha}\partial\!\!\!/\psi - G\bar{\alpha}\gamma_5\partial\!\!\!/\psi\,. \tag{2.18}$$

Thus far, we have three total derivatives: one from the first term in Eq. (2.10), another from the first term in Eq. (2.11) and the last from the first term of the fermion transformation in Eq. (2.17). There is still more work to do to make more total derivatives apparent.

Let us now expand the $\delta\psi$ in the last term of Eq. (2.17). Using the fermion field transformation in Eq. (2.6), we see

$$\bar{\psi}\partial\!\!\!/\delta\psi = \bar{\psi}\left[(\partial\!\!\!/F)\alpha - i(\partial\!\!\!/G)\gamma_5\alpha + \gamma_5(\Box A)\alpha + i(\Box B)\alpha\right]\,. \tag{2.19}$$

Remember that $a\!\!\!/a\!\!\!/ = a^2$, so $\partial\!\!\!/\partial\!\!\!/ = \Box$. The change in the kinetic part of the lagrangian density is now as follows, (the circled numbers will be used to refer to terms underneath them),

$$\begin{aligned}\delta\mathcal{L}_{kin} = &-i(\overset{①}{\Box A})\bar{\alpha}\gamma_5\psi + (\overset{②}{\Box B})\bar{\alpha}\psi + i\bar{\psi}\left[(\overset{③}{\partial\!\!\!/F})\alpha - i(\overset{④}{\partial\!\!\!/G})\gamma_5\alpha + \gamma_5(\overset{⑤}{\Box A})\alpha + i(\overset{⑥}{\Box B})\alpha\right]\\ &-i\overset{⑦}{F}\bar{\alpha}\partial\!\!\!/\psi - \overset{⑧}{G}\bar{\alpha}\gamma_5\partial\!\!\!/\psi + \text{total derivatives}\,.\end{aligned} \tag{2.20}$$

We can reduce this expression down to only total derivatives by performing a few manipulations:

- The first term (①) of Eq. (2.20) can be written using Eq. (B.2) as,

$$-i(\Box A)\bar{\alpha}\gamma_5\psi = -i(\Box A)\bar{\psi}\gamma_5\alpha\,, \tag{2.21}$$

  which cancels term ⑤.

- Similarly for term ②, using Eq. (B.1) we can write,

$$(\Box B)\bar{\alpha}\psi = \bar{\psi}(\Box B)\alpha \,, \tag{2.22}$$

which cancels term ⑥.

- Now notice that using Eq. (B.3), term ③ can be written as,

$$i\bar{\psi}(\partial\!\!\!/ F)\alpha = i\bar{\psi}\gamma^\mu(\partial_\mu F)\alpha = -i\bar{\alpha}\gamma^\mu(\partial_\mu F)\psi \,. \tag{2.23}$$

This expression along with ⑦ yields,

$$-i\bar{\alpha}\gamma^\mu(\partial_\mu F)\psi - iF\bar{\alpha}\gamma^\mu\partial_\mu\psi = -i\partial_\mu\left[\bar{\alpha}\gamma^\mu F\psi\right] \,. \tag{2.24}$$

- Furthermore, for term ④ we write with the help of Eq. (B.4),

$$\bar{\psi}(\partial\!\!\!/ G)\gamma_5\alpha = \bar{\alpha}\gamma^\mu(\partial_\mu G)\gamma_5\psi = -\bar{\alpha}\gamma_5\gamma^\mu(\partial_\mu G)\psi \,, \tag{2.25}$$

and along with ⑧ we obtain,

$$-\bar{\alpha}\gamma_5\gamma^\mu(\partial_\mu G)\psi - G\bar{\alpha}\gamma_5\partial\!\!\!/\psi = -\partial_\mu\left[\bar{\alpha}\gamma_5\gamma^\mu G\psi\right] \,. \tag{2.26}$$

After all of those manipulations, we see that,

$$\begin{aligned}\delta\mathcal{L}_{kin} &= -\partial_\mu\left[i\bar{\alpha}\gamma^\mu F\psi + \bar{\alpha}\gamma_5\gamma^\mu G\psi\right] + \text{total derivatives} \\ &= \text{total derivative!}\end{aligned} \tag{2.27}$$

The kinetic part of the Lagrangian changes by a total derivative. Appendix C describes the derivation to show that the change in the mass portion of the Lagrangian is also merely a total derivative. Now remember that the action is the integral of the lagrangian density. Therefore,

$$\delta S = \int d^4x\, \delta\mathcal{L} = \int d(\text{total derivative}) = 0 \,, \tag{2.28}$$

since the integral is just a surface term. Thus, the equations of motion do not change because the action does not change, and so the supersymmetric transformations leave the equations of motion and physics invariant.

2.2 Space-time transformations

We have shown that the supersymmetric transformations leave the equations of motions unchanged, but what do these transformations represent? Remember the form for a transformation,

$$A \to A' = A + \delta A = A + \bar{\alpha}QA \,. \tag{2.29}$$

From Eq. (2.5) and (2.6), we see that the generator of the transformation, Q, is an operator that transforms a spinor particle into a scalar particle and vice versa. That is to say, Q alters the spin of the particle by $\pm 1/2$, and thus changes fermions into bosons and bosons into fermions. Let us now determine the anticommutation relation for Q. We will find that it represents an important phenomenon.

Begin by examining two successive transformations of the A scalar field (the following will work for any of the fields):

$$\delta_2\delta_1 A = \delta_2(i\bar{\alpha}_1\gamma_5\psi) = i\bar{\alpha}_1\gamma_5(F\alpha_2 - iG\gamma_5\alpha_2 + \partial\!\!\!/\gamma_5 A\alpha_2 + i\partial\!\!\!/ B\alpha_2) \tag{2.30}$$

and similarly,

$$\delta_1\delta_2 A = i\bar{\alpha}_2\gamma_5(F\alpha_1 - iG\gamma_5\alpha_1 + \partial\!\!\!/\gamma_5 A\alpha_1 + i\partial\!\!\!/ B\alpha_1)\,. \tag{2.31}$$

When these two expressions are subtracted, nearly all of the terms cancel except for the $i\bar{\alpha}\gamma_5(\partial\!\!\!/\gamma_5 A)\alpha$ terms which *add* together; see Eq. (B.3). They give,

$$(\delta_2\delta_1 - \delta_1\delta_2)A = -2i\bar{\alpha}_1\alpha_2\partial\!\!\!/ A\,. \tag{2.32}$$

We now do the same to the generators,

$$(\delta_2\delta_1 - \delta_1\delta_2)A = (\bar{\alpha}_2 Q\bar{\alpha}_1 Q - \bar{\alpha}_1 Q\bar{\alpha}_2 Q)A\,. \tag{2.33}$$

Manipulating with the help of Eq. (B.1), going to components of the spinors and then comparing with Eq. (2.32) we obtain,

$$\begin{aligned}(\delta_2\delta_1 - \delta_1\delta_2)A &= (\bar{Q}\alpha_2\bar{\alpha}_1 Q - \bar{\alpha}_1 Q\bar{Q}\alpha_2)A \\ &= (\bar{Q}_b\alpha_{2_b}\bar{\alpha}_{1_a}Q_a - \bar{\alpha}_{1_a}Q_a\bar{Q}_b\alpha_{2_b})A \\ &= -\bar{\alpha}_{1_a}\alpha_{2_b}(\bar{Q}_bQ_a + Q_a\bar{Q}_b)A \\ &= -\bar{\alpha}_{1_a}\alpha_{2_b}\left\{Q_a, \bar{Q}_b\right\}A = -2i\bar{\alpha}_{1_a}\alpha_{2_b}(\partial\!\!\!/ A)_{ab}\,.\end{aligned} \tag{2.34}$$

Therefore, we find the anticommutation relation between supersymmetric transformation generators to be,

$$\left\{Q_a, \bar{Q}_b\right\} = 2i(\gamma_\mu\partial^\mu)_{ab} = 2(\gamma_\mu P^\mu)_{ab}\,, \tag{2.35}$$

where P^μ represents the translation generators of the Poincaré group (Lorentz boosts and rotations). Since the transformations are involved with space–time transformations, Supersymmetry is called a space-time symmetry. This distinction is important since it differs from the internal symmetries of the particles, such as charge and lepton number. The generator of a supersymmetric transformation acting on a field will not change any of those quantities, so a fermion transformed into a boson under the W-Z

Supersymmetry model retains its charge, mass, lepton number, etc. The only characteristic that changes is its spin.

We have shown that the equations of motion for the Wess–Zumino Lagrangian density are invariant under supersymmetric transformations. Furthermore, the generator of these transformations acts to turn bosons into fermions and fermions into bosons, leaving all other internal properties of the particles unchanged. Thus in Supersymmetry, each particle has a *super–partner* which looks identical to the original particle but differs in spin by $\pm 1/2$, or more technically, every degree of freedom of a field (scalars have one, Majorana spinors have two) has a super–partner (so the two scalars are the super–partners of the spinor field).

2.3 Interaction part of the lagrangian

We are now nearly ready to show how Supersymmetry solves the problem of quadratic divergences in loop corrections to scalar bosons, which is the reason why Supersymmetry is an important beyond the Standard Model theory. First, we need an interaction term in the lagrangian density that again leaves the equations of motion invariant under supersymmetric transformations. Wess and Zumino determined such an expression [9] with certain properties that will be useful later:

$$\mathcal{L}_{int} = g\left[G(A^2 - B^2) + 2FAB - i\bar{\psi}(A - \gamma_5 B)\psi\right] . \tag{2.36}$$

It is necessary that the interactions have the same coupling constant g so that the fermion loop will cancel the scalar boson loops, as will see shortly. Combining the interaction term with the kinetic and mass parts, the total W-Z Lagrangian density is,

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(F^2 + G^2)\\ &- m\left[\frac{1}{2}\bar{\psi}\psi + GA + FB\right]\\ &+ gG(A^2 - B^2) + 2gFAB - ig\bar{\psi}(A - \gamma_5 B)\psi .\end{aligned} \tag{2.37}$$

Using the equations of motion, we will see that the F and G fields are not independent. Remember that the Euler–Lagrange equations of motion for a field are,

$$\frac{\partial\mathcal{L}}{\partial(\partial^\mu F)} - \frac{\partial\mathcal{L}}{\partial F} = 0 , \tag{2.38}$$

and therefore the F and G fields can be written in terms of A and B,

$$F = mA - g(A^2 - B^2) \tag{2.39}$$
$$G = mB - 2gAB . \tag{2.40}$$

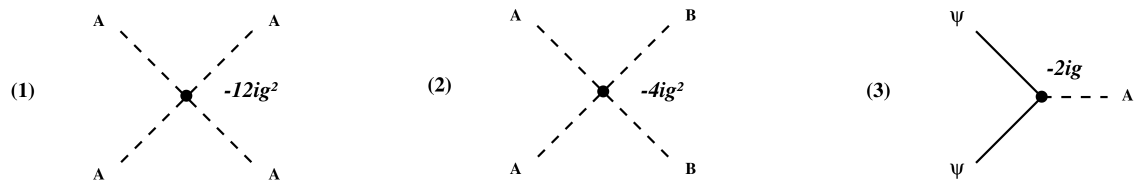

Figure 2: Some interactions given by the Wess–Zumino Lagrangian and their vertex factors.

Figure 3: Propagators for scalar (dashed line) and spinor (solid line) fields.

These expressions simplify Eq. (2.37),

$$\mathcal{L} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \frac{1}{2}m\bar{\psi}\psi - \frac{1}{2}m^2A^2 - \frac{1}{2}m^2B^2$$
$$+ mgA(A^2+B^2) - \frac{1}{2}g^2(A^2 \overset{①}{+} B^2)^2 - ig\overset{②}{\bar{\psi}}A\psi + ig\bar{\psi}\gamma_5 B\psi\,. \tag{2.41}$$

The lagrangian describes many interactions, but only three will be important for resolving the fine tuning problem. Those interactions are shown as Feynman diagrams in Fig. 2 along with their vertex factors. The vertex factors can be read directly from the lagrangian, remembering to include a factor representing the number of ways like lines can be arranged. The first diagram comes from term ① in Eq. (2.41) after squaring A^2. There are $4! = 24$ ways those lines can be arranged, so hence the factor of $\frac{24}{2} = 12$ for the vertex. Term ① also supplies the second interaction shown in the figure after one again expands the quadratic and examines the $2A^2B^2$ part. Finally, the third diagram is given by term ②. The needed propagators from this theory happen to be the same as given by QED [6] and are shown in Fig. 3. We will now use these quantities to explain how Supersymmetry eliminates quadratic divergences.

2.4 Cancellation of quadratic divergencies

Now we are able to show how the quadratic divergencies of the scalar field are canceled. The A scalar field will interact with the other fields in the Wess–Zumino theory governed by the lagrangian density of Eq. (2.41). The one loop interactions, which are

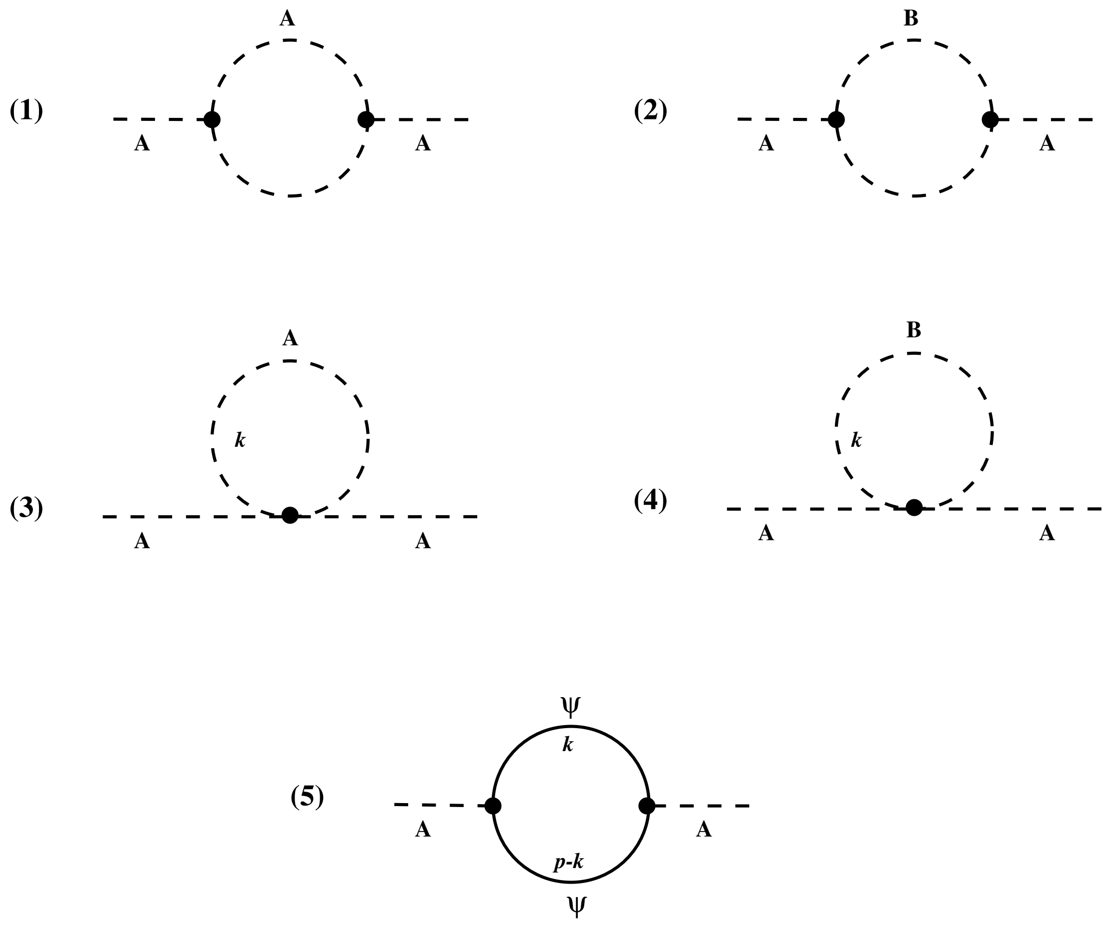

Figure 4: One loop corrections to the A self energy.

corrections to the A self energy and its mass, are shown in Fig. 4. The third diagram looks similar to the correction of the Higgs self energy. Only diagrams 3, 4 and 5 are quadratically divergent, so we will concentrate on those. The amplitude for diagram three can be obtained from the Feynman rules shown in Figs. 2 and 3,

$$-i\mathcal{M}_{(3)} = 6g^2 \int \frac{d^4k}{k^2 - m^2} \,. \tag{2.42}$$

Remember that one integrates over loop momenta, and there is a symmetry factor of 1/2. Similarly, diagram four leads to

$$-i\mathcal{M}_{(4)} = 2g^2 \int \frac{d^4k}{k^2 - m^2} \,, \tag{2.43}$$

so

$$-\, i\mathcal{M}_{(3)+(4)} = 8g^2 \int \frac{d^4k}{k^2 - m^2} \,. \tag{2.44}$$

We see in Eq. 2.44 the quadratic divergence of the scalar mass from single scalar loops. Now we look at the fifth diagram, and, after a little bit of manipulation, we will soon discover why we have gone through all of this hard work:

$$\begin{aligned} -i\mathcal{M}_{(5)} &= (-1)\frac{(-2ig)(-2ig)}{2}\, i^2 \int d^4k \frac{\mathrm{Tr}\,\{(\not{k}+m)[(\not{k}-\not{p})+m]\}}{(k^2-m^2)[(k-p)^2-m^2]} \\ &= -2g^2 \int d^4k \frac{\mathrm{Tr}\,\{(\not{k}+m)[(\not{k}-\not{p})+m]\}}{(k^2-m^2)[(k-p)^2-m^2]} \,. \end{aligned} \tag{2.45}$$

Note the the symmetry factor of 1/2 and the extra factor of (-1) due to the fermion loop. We must now evaluate the trace. Remember that the trace of an odd number of gamma matrices is zero and $\mathrm{Tr}[1] = 4$.

$$\begin{aligned} \mathrm{Tr}\,\{(\not{k}+m)[(\not{k}-\not{p})+m]\} &= \mathrm{Tr}[\not{k}\cdot\not{k} - \not{k}\cdot\not{p} + 2m\not{k} - m\not{p} + m^2] \\ &= 4[k^2 - k\cdot p + m^2] \\ &= 2\left[(k^2-m^2) + [(k-p)^2 - m^2] - p^2 + 4m^2\right] \,. \end{aligned} \tag{2.46}$$

The last line is just a clever expansion of the previous one so that we may write,

$$-i\mathcal{M}_{(5)} = -4g^2 \int d^4k \left(\frac{1}{(k-p)^2 - m^2} + \frac{1}{k^2 - m^2} + \frac{4m^2 - p^2}{(k^2-m^2)[(k-p)^2-m^2]} \right) \,. \tag{2.47}$$

The last term is only logorithmically divergent and can be renormalized. It looks like we now have two more quadratically divergent terms. Adding everything together, we have

$$-i\mathcal{M}_{\text{total}} = 8g^2 \int \frac{d^4k}{k^2 - m^2} - 4g^2 \int \frac{d^4k}{k^2 - m^2} - 4g^2 \int \frac{d^4k}{(k-p)^2 - m^2} + I(p,m) \tag{2.48}$$

where $I(p,m)$ is the logorithmically divergent part. If we shift the integration variable of the third term, $k \to k' = k + p$ (so $dk' = dk$), then the second and third term join and cancel the first, thereby eliminating the quadratic divergence!

It would appear that a particle's mass and its super–partner's mass must be exactly equal to m, and if so then there is a problem. If in some grander SUSY theory the super–partner of the electron has the same mass as the electron itself, then the super–partner should have been discovered, but no supersymmetric particles have been found yet. Luckily, the particle/super–partner masses do not have to be exactly equal. In the case of the Higgs, we expect its mass to be on the order of a few hundred GeV. In a supersymmetric theory, if the masses of the Higgs scalars (we would need two) do not quite equal the mass of its super–partner fermion, the cancellation of the Higgs quadratic divergence will not be exactly complete. However, so long as only a few hundred GeV or so is left over after cancellation, the Higgs mass will still turn out to be finite and reasonable. In fact, the *splitting* between the scalars and the fermion masses can be on the order of a TeV, and the Higgs mass will remain within the electroweak energy scale. The scalar boson masses do not have to be identical to the fermion mass, and since that must be the case to have a realistic theory not immediately ruled out by experiments, Supersymmetry must not be an *exact* symmetry and is *broken* at some scale. Of course we cannot put any random fields into the lagrangian and expect the cancellation to hold even inexactly. As we have shown, only certain fields will work, and terms of those fields are called *soft* breaking terms. A hard breaking term is one that ruins the cancellation of quadratic divergencies to an extent that the Higgs mass remains above the electroweak scale or stays infinite, and thus those terms must not appear in the lagrangian.

2.5 Summary

It is time to look at what we have done. First, we noticed in the Standard Model that self energy corrections to the scalar boson fields (the Higgs) lead to quadratic divergencies. The divergencies can only be eliminated within the SM by tuning the bare mass with incredible precision, hence the fine tuning problem. This feature of the Standard Model is very undesirable, so we look to beyond the Standard Model theories for assistance. The Wess–Zumino Supersymmetry theory provides a toy solution. It contains a new set of transformations that mix bosons and fermions, and its generator will act to change fermions into bosons and vice versa. If we construct a lagrangian immune to the transformations, we find that it must contain a fermion along with two scalar bosons as its super–partner (one for each degree of freedom). This fermion then contributes to the one loop corrections of the scalar self energy and cancels the quadratic divergence. The fine tuning problem is no longer, but the price we pay is that every degree of freedom of a particle's field must have a super–partner.

3 MSSM and SUGRA

Obviously, the Wess–Zumino model is not the complete answer to the fine tuning problem of the Standard Model, since none of the SM model particles are included. The Minimal Supersymmetric Standard Model [1–3] is a more realistic phenomenological theory that extends the Standard Model by including Supersymmetry. The MSSM adds super–partners to the standard model particles in such a way that one adds the fewest number of new particles, or *sparticles* as possible. Remember that the SM fermions are spin $1/2$ particles and have two degrees of freedom (and two more if you count the antiparticles). Thus, they each must have two sparticle partners (plus two antisparticles) called *sfermions* that have spin zero. For example, an electron e will have two *selectrons*, $\tilde{e}_L$ and $\tilde{e}_R$, since the electron may have one of two helicities (left or right handed) and thus has two degrees of freedom. Just as in the Wess–Zumino model, the quadratic divergence of the scalar selectrons will be canceled by the presence of the electron contribution. The quarks are similar and receive scalar *squarks*. The neutrinos have two helicity states (although only one couples to matter in the SM), and thus there are two massive scalar *sneutrinos*.

The partners of gauge particles, called *gauginos*, are more complicated. In the SM, electroweak symmetry $SU(2)\times U(1)$ is spontaneously broken by a single Higgs doublet. For technical reasons [3], the MSSM requires two doublets of Higgs fields representing five massive Higgs particles: two charged scalars $H^{\pm}$, one neutral light scalar h, one neutral heavy scalar H and one neutral pseudoscalar A. These particles exist along with the weak gauge spin one bosons, the $W^{\pm}$ and the Z. The matchings to their sparticles are not so simple. The charged supersymmetric gauge bosons are constructed from a mixture of the partners of the charged SM gauge bosons and Higgses. Therefore, the $W^{\pm}$ (2 charges $\times$ 3 helicities $=$ 6 degrees of freedom) and the $H^{\pm}$ (2 charges $\times$ 1 helicity $=$ 2 d.o.f.) are paired with the spinor *charginos* $\tilde{W}^{\pm}_{1,2}$ (2 particles $\times$ 2 charges $\times$ 2 helicities $=$ 8 degrees of freedom). The charginos may also be called *winos*, and the $\tilde{W}_1$ is defined to be lighter than the $\tilde{W}_2$. For the neutral gauge particles, we have the Z (3 helicities), the massless photon (2 helicities) and the three neutral Higgses (one spin each) for a total of 8 degrees of freedom. Their supersymmetric counterparts are the spin half *neutralinos* or *zinos*, $\tilde{Z}_{1,2,3,4}$ also accounting for a total of eight degrees of freedom (4 particles $\times$ 2 spin). Finally, the spin one gluons are massless and have a total of 16 degrees of freedom (8 colors $\times$ 2 helicities), and thus their partners are the massive spin one-half *gluinos* which also have 16 total degrees of freedom (8 colors $\times$ 2 helicities). The number of degrees of freedom for the SM particles match the d.o.f.

Particle		*Sparticle*	
lepton	ℓ	slepton	$\tilde{\ell}_L, \tilde{\ell}_R$
neutrino	ν	sneutrino	$\tilde{\nu}_L, \tilde{\nu}_R$
quark	q	squark	$\tilde{q}_L, \tilde{q}_R$
gluon	g	gluino	$\tilde{g}$
charged weak boson charged Higgs	$W^\pm$ $H^\pm$	chargino	$\tilde{W}^\pm_{1,2}$
neutral weak boson photon light Higgs heavy Higgs pseudoscalar Higgs	Z γ h H A	neutralino	$\tilde{Z}_{1,2,3,4}$

Table 1: Particle content of the MSSM.

of their super–partners. The MSSM particle content is summarized in Table 1.

The Minimal Supersymmetric Standard Model successfully incorporates Supersymmetry into the Standard Model, but unfortunately it adds an enormous number of new parameters that need to be determined. None of the sparticle masses are predicted, and other parameters are also needed such as new coupling and mixing constants. The usual method that one employs to reduce the number of independent parameters and make the MSSM a tractable theory is to work within the framework of a Grand Unified Theory (GUT). A GUT inspired MSSM relies on some grander theory, such as [10, 11] $SU(5)$, $SU(5) \times U(1)$ (called flipped $SU(5)$), $SO(10)$ or string theories, that live above the Supersymmetry breaking scale. Above the breaking scale, one can determine relations between the parameters and by solving renormalizable group equations (RGEs), the masses of the individual sparticles and their couplings can be calculated at the weak scale. For $SU(5)$, this scheme [3] involves inputting values for the masses of only some of the sparticles and knowing other parameters such as the ratio of the vacuum expectation values of the two Higgs fields (usually represented as $\tan\beta$), a parameter μ describing the mixing of the spinor higgses and other couplings. One advantage of this design is that the masses of the squarks is assumed to be degenerate, except for the mass of the *stop*. Since the top quark is much heavier than the other quarks, it turns out that the stop mass should be very light. The left and right stops mix to form the $\tilde{t}_1$ and the $\tilde{t}_2$, one light and the other heavy. The twelve squark masses have been reduced to a few, but even with this framework, there are still enough independent parameters to make the model cumbersome.

Recently, theorists have started to employ the minimal Supergravity model, abbreviated SUGRA, as another GUT framework for the MSSM [12, 13]. In the SUGRA model, the three gauge couplings from $SU(3)$, $SU(2)$ and $U(1)$ are unified at some enormously large energy scale M_X where gravitational interactions are important. Furthermore, it is assumed that the gravitational interactions are responsible for SUSY breaking. Since those interactions are universal, we need only a few parameters to describe the model:

- m_0, a common mass for all of the scalar sparticles at the M_X scale;
- $m_{1/2}$, a common mass for all of the gaugino sparticles at the M_X scale;
- $\tan\beta$, the ratio of the Higgs doublets vacuum expectation values;
- sign(μ), the Higgsino mixing parameter (only the sign is needed) and;
- A_0, a common trilinear interaction parameter that describes the mixing of the left and right stops at the M_X scale.

Given these parameters along with the masses of the Standard Model particles, the masses and couplings of the sparticles can be determined at the weak scale by solving the RGEs of the model. The SUGRA framework is an especially promising approach for significantly reducing the number of MSSM parameters that one has to input by hand.

4 Experiments

To complete our brief introduction to Supersymmetry, we should look at the results of experiments. After all, it is the experiments that will determine whether or not SUSY is the correct answer to the fine tuning problem. So far, no experiment has seen a supersymmetric particle, and so we only have mass limits. For example, the DØ collaboration at the $\sqrt{s} = 1.8$ TeV Tevatron collider has completed a search for squarks (excluding stop) and gluinos by looking for events with high missing transverse energy and large jet multiplicity. No events were observed above Standard Model backgrounds, and so they set lower mass limits at 95% CL of 212 GeV/c^2 for both squark and gluinos if their masses are equal and 144 GeV/c^2 on the gluino mass if the squarks are heavy [14]. Their mass contour is shown in Fig. 5. Other published limits come from precision electroweak data. From the measurement of the Z width Γ_Z at LEP, a 95% CL limit of approximately 45 GeV/c^2 can be set for the remaining sparticles [15].

5 Conclusions

We have presented a cursury look at Supersymmetry, explaining why it is needed to solve the fine tuning problem and showing how that goal is realized. Since we used the

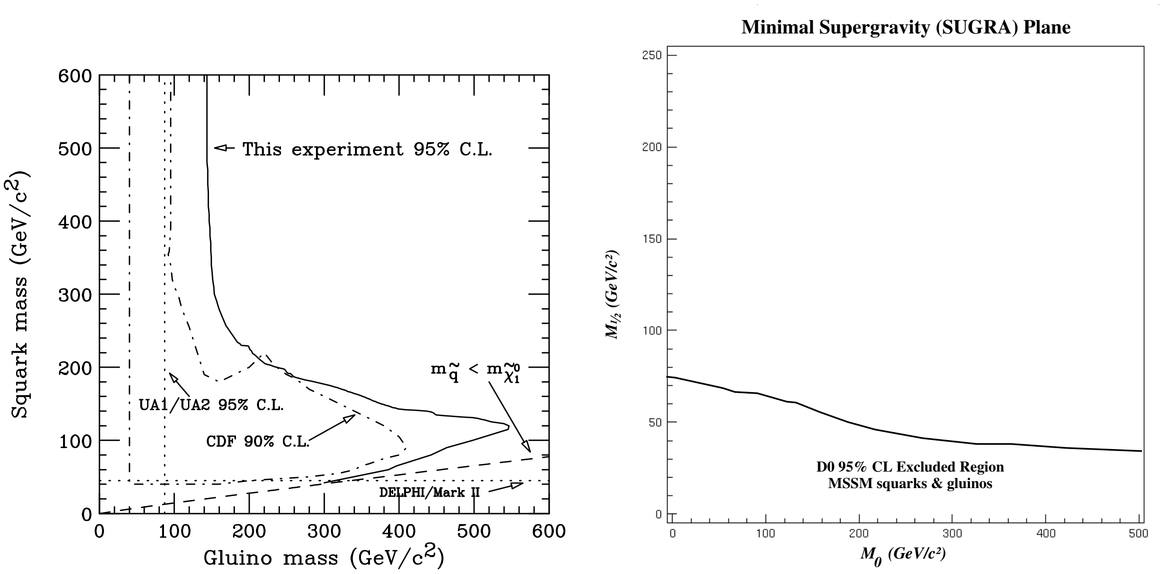

Figure 5: The DØ gluino–squark mass limit contour. Left side is the DØ limit (this experiment) along with previous limits in the gluino–squark mass plane. Below the $m_{\tilde{q}} < m_{\tilde{\chi}^0_1}$ line, the squark becomes the lightest supersymmetric particle and is excluded. The right plot is the same DØ limit translated into the SUGRA $m_0 - m_{1/2}$ plane. The search utilized MSSM parameters of $\tan\beta = 2$, $\mu = -250$ GeV$/c^2$, $m_{H^+} = 500$ GeV$/c^2$ and $m_{\mathrm{top}} = 140$ GeV$/c^2$. The limit is not very sensitive to small changes in these parameters.

toy Wess–Zumino model to work though the details, other more realistic models were discussed as were the so far null results from experiments. Luckily, the search for supersymmetric particles is far from over. With upgrades to the Tevatron, the upcoming LEPII run and the new LHC collider coming on line soon after the year 2000, experimentalists will soon be able to either discover a sparticle or rule out the MSSM entirely. Until then, it appears that Supersymmetry will remain one of the more promising beyond the Standard Model theories.

APPENDIX

A Conjugate relations

In this appendix, we derive some of the conjugate field relations [6] needed for the next appendix. Start with the Dirac equation for an electron in an electromagnetic field,

$$[\gamma^\mu(i\partial_\mu + eA_\mu) - m]\psi = 0\ . \tag{A.1}$$

Similarly, the Dirac equation for a positron in an electromagnetic field is,

$$[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\psi^C = 0\ . \tag{A.2}$$

We seek a simple relation between ψ and ψ^C. Start by taking the complex conjugate of Eq. (A.1),

$$[-\gamma^{\mu *}(i\partial_\mu - eA_\mu) - m]\psi^* = 0\ . \tag{A.3}$$

Now we define a matrix C that satisfies the following relation,

$$-(C\gamma^0)\gamma^{\mu *} = \gamma^\mu(C\gamma^0)\ . \tag{A.4}$$

Then, multiplying Eq. (A.3) on the left by $C\gamma^0$ and using Eq. (A.4) we eventually find,

$$[\gamma^\mu(i\partial_\mu - eA_\mu) - m](C\gamma^0\psi^*) = 0\ . \tag{A.5}$$

Comparing this equation with the Dirac equation above for a positron in an electromagnetic field, we see that

$$\boxed{\psi^C = C\gamma^0\psi^* = C\bar{\psi}^T}\ . \tag{A.6}$$

We can now derive some properties of this C matrix. If we remember that $\gamma^0\gamma^0 = \mathbf{1}$, $\gamma^{\mu^\dagger} = \gamma^0\gamma^\mu\gamma^0$ and then fiddle a bit with Eq. (A.4), we can obtain,

$$\begin{aligned} -(C\gamma^0)\gamma^{\mu^*}(C\gamma^0)^{-1} &= \gamma^\mu \\ -C\gamma^{\mu^T}C^{-1} &= \gamma^\mu\ , \end{aligned}$$

or equivalently,

$$C^{-1}\gamma^\mu C = (-\gamma^\mu)^T\ . \tag{A.7}$$

Now, in the Dirac-Pauli representation of the gamma matrices, we can define the C matrix to be,

$$C = i\gamma^2\gamma^0\ . \tag{A.8}$$

This expression can be checked with Eq. (A.4). We now see that $(\gamma^0 C)^T = (\gamma^0 C)$ and $(C\gamma^0)^2 = \mathbf{1}$. With these relations and Eq. (A.7), we find,

$$C = -C^T = -C^{-1} = -C^\dagger\ . \tag{A.9}$$

Finally, it can be easily shown that C commutes with γ^5, or $[C, \gamma^5] = 0$.

B Majorana spinor relations

We will now prove some relations for Majorana spinors. Remember that a Majorana spinor is one that satisfies $\psi = \psi^C$, so the spinor's associated particle is its own antiparticle.

B.1 $\bar{\psi}\chi = \bar{\chi}\psi$

We wish to show that two Majorana spinors $\bar{\psi}$ and χ commute. Using Eq. (A.6) and (A.9),

$$\begin{aligned} \psi &= \psi^C = C\bar{\psi}^T \\ \psi^T &= \bar{\psi}C^T \\ \bar{\psi} &= \psi^T C \end{aligned}$$

we can write,

$$\begin{aligned} \bar{\psi}\chi &= \psi^T C\chi = \psi_\alpha C_{\alpha\beta}\chi_\beta = -\chi_\beta C_{\alpha\beta}\psi_\alpha \\ &= \chi_\beta C_{\beta\alpha}\psi_\alpha = \chi^T C\psi \\ &= \bar{\chi}\psi\,. \end{aligned} \tag{B.1}$$

Note that we used the fact that spinor components anti-commute (since they represent fermions).

B.2 $\bar{\psi}\gamma^5\chi = \bar{\chi}\gamma^5\psi$

We start with the following,

$$\bar{\psi}\gamma^5\chi = \psi^T C\gamma^5\chi = \psi_\alpha (C\gamma^5)_{\alpha\beta}\chi_\beta = -\chi_\beta (C\gamma^5)_{\alpha\beta}\chi_\beta\,.$$

It is easy to show that

$$C\gamma^5 = ({\gamma^5}^T C^T)^T = (-\gamma^5 C)^T\,.$$

Therefore,

$$\begin{aligned} \bar{\psi}\gamma^5\chi &= \chi_\beta(\gamma^5 C)_{\beta\alpha}\psi_\alpha \\ &= \bar{\chi}\gamma^5\psi\,. \end{aligned} \tag{B.2}$$

B.3 $\bar{\psi}\gamma^\mu\chi = -\bar{\chi}\gamma^\mu\psi$

Begin with,

$$\bar{\psi}\gamma^\mu\chi = \psi^T C\gamma^\mu\chi \,.$$

Use Eq. (A.7) to show,

$$\begin{aligned}\bar{\psi}\gamma^\mu\chi &= -\psi^T CC\gamma^{\mu^T}C^{-1}\chi = -\psi^T\gamma^{\mu^T}C\chi = -\psi_\alpha(\gamma^{\mu^T}C)_{\alpha\beta}\chi_\beta \\ &= \chi_\beta(\gamma^{\mu^T}C)_{\alpha\beta}\psi_\alpha = -\chi_\beta(C\gamma^\mu)_{\beta\alpha}\psi_\alpha \\ &= -\bar{\chi}\gamma^\mu\psi \,.\end{aligned} \tag{B.3}$$

B.4 $\bar{\psi}\gamma^\mu\gamma^5\chi = \bar{\chi}\gamma^\mu\gamma^5\psi$

$$\begin{aligned}\bar{\psi}\gamma^\mu\gamma^5\chi &= \psi^T C\gamma^\mu\gamma^5\chi = -\psi^T CC\gamma^{\mu^T}C^{-1}\gamma^5\chi \\ &= -\psi^T\gamma^{\mu^T}C\gamma^5\chi = -\psi^T\gamma^{\mu^T}\gamma^5 C\chi \\ &= -\chi^T C\gamma^{5^T}\gamma^\mu\psi = -\chi^T C\gamma^5\gamma^\mu\psi = \chi^T C\gamma^\mu\gamma^5\psi \\ &= \bar{\chi}\gamma^\mu\gamma^5\psi\end{aligned} \tag{B.4}$$

C Transformation of $\mathcal{L}_{mass}$

We wish to show how the mass part of the Wess–Zumino Lagrangian density transforms under the supersymmetric transformations given in Eqs. (2.5) and (2.6). Remember that the mass part is given by,

$$\mathcal{L}_{mass} = -m\left[\frac{1}{2}\bar{\psi}\psi + GA + FB\right] \tag{C.1}$$

so

$$\delta\mathcal{L}_{mass} = -m\left[G\delta A + (\delta G)A + F\delta B + (\delta F)B + \frac{1}{2}(\delta\bar{\psi})\psi + \frac{1}{2}\bar{\psi}\delta\psi)\right] \tag{C.2}$$

Note that the last two terms can be combined into $\bar{\psi}\delta\psi$ using Eq. (B.1). Now we expand the terms using the Majorana spinor relations from appendix B,

$$G\delta A = i\bar{\alpha}G\gamma^5\psi = i\bar{\psi}G\gamma^5\alpha \tag{C.3}$$

$$(\delta G)A = -\bar{\alpha}\gamma^5(\partial\!\!\!/\psi)A = (\partial\!\!\!/\bar{\psi})\gamma^5 A\alpha \tag{C.4}$$

$$F\delta B = -\bar{\alpha}F\psi = -\bar{\psi}F\alpha \tag{C.5}$$

$$(\delta F)B = -\bar{\alpha}(\partial\!\!\!/\psi)B = (\partial\!\!\!/\bar{\psi})B\alpha \tag{C.6}$$

$$\bar{\psi}\delta\psi = \bar{\psi}F\alpha - i\bar{\psi}G\gamma^5\alpha + \bar{\psi}(\partial\!\!\!/\gamma^5 A)\alpha + i\bar{\psi}(\partial\!\!\!/B)\alpha \,. \tag{C.7}$$

Note that the change in sign for the $(\delta G)A$ expression comes from the fact that γ^μ and γ^5 were swapped. We see that the $G\delta A$ and the $F\delta B$ terms cancel with terms in $\bar{\psi}\delta\psi$. The remaining terms join together to form,

$$\delta\mathcal{L}_{mass} = -m\left[\partial_\mu(\gamma^\mu\bar{\psi}\gamma^5 A\alpha) + \partial_\mu(\gamma^\mu\bar{\psi}B\alpha)\right] = \text{total derivative}\,. \tag{C.8}$$

The mass part of the W-Z Lagrangian density only changes by a total derivative under supersymmetric transformations, and thus the equations of motion remain unaltered.

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