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Abstract

In this article we study effects of microbunch cooperative electromagnetic radiation in a bend on transverse beam dynamics. An overtaking radiative interaction between different parts of the bunch results in three major forces variable along the bunch. Longitudinal force leads to energy loss and causes the bunch emittance growth in the bend due to the dispersion effect. Radial force consists of logarithmically large "Talman" centrifugal force and smaller centripetal force. Due to general radius-energy dependence in the bend, the "Talman" force does not affect beam dynamics while the centripetal force leads to projected emittance growth. Finally, radial and vertical focusing forces lead to trajectory distortions which vary along the bunch. These cooperative forces significantly affect the dynamics of short high-populated bunch in bends.

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1 Introduction

Several projects with high charge microbunches are currently under consideration. Examples are modern linac-based FELs and damping rings and bunch compressors for future Linear Colliders. This article deals with effects of cooperative forces due to synchrotron radiation of microbunches in bends. Opposite to the well known collective effects in accelerators where the wake-fields produced by head particles act on the particles behind, the cooperative radiation fields generated at the tail of the bunch overtake its head if the bunch follows a curved trajectory.

The longitudinal effect of microbunch cooperative radiative field was considered in Refs. [1, 2], and an **overtaking** tail-head interaction was analyzed. This interaction induces energy spread along the bunch. Apparently, this effect is accompanied by another unpleasant phenomenon – excitation of the effective transverse horizontal(radial) emittance due to the radius-energy dependence (dispersion) in the bend [1].

Besides that, there are transverse forces of the microbunch radiation. For instance, in a case of coasting beam, the impact of radial "centrifugal space charge force" (CSCF, R.Talman, Ref. [3]) was treated in the Ref.[4] and found to be compensated by the energy shift under the effect of the Coulomb electric field.

This paper deals with transverse forces in the microbunch. The below treatment is based on the perturbative solution of the complete system of particle motion equations. The microbunch collective electromagnetic fields are derived under assumption of no shielding effect due to metallic beam pipe.

In this article we show, that:

–longitudinal overtake force dominates in the radial disruption of the microbunch;

–the Talman's centrifugal force, itself, is extendible on the microbunch case;

–for relativistic particles the centrifugal force effect on radial emittance is cancelled by the energy shift under the effect of the Coulomb part of radial electric field, irrelatively to bending magnetic field configuration;

–there is new radiative kind of radial force, with the same form as CSCF but with lower magnitude (it has no logarithmic factor), with opposite sign, i.e.**centripetal**, but with no compensating partner from the electric field (the residual of the centrifugal effect);

–there are the gradient transverse radiation forces affecting the beam focusing with their own wake functions.

The radiative effects grow rapidly when the bunch length decreases.

Besides this introduction, the article consists of four sections. Precise formulas for the longitudinal and transverse particle motion are derived in Section 2. Using retarded potentials, in Section 3 we find cooperative radiation forces and estimate their effect on the particles. In Section 4 we present estimations of characteristic contributions of the cooperative radiation forces to the increase of beam transverse emittances in the TESLA Test Facility Free Electron Laser (TTF FEL) bunch compressors. Section 5 summarizes results.

2 Basic equations

Let us derive particle dynamics equations for a relativistic ($\beta = v/c \approx 1$) bunch that follows curved trajectory with radius R - see Fig.1. We use the following designations of coordinates:

x for radial, y for vertical (perpendicular to the bending plane OAB), z – along trajectory, $s = z - \beta ct$ is for coordinate inside the moving bunch.

If a particle at the tail of the bunch radiates at the point A then the electromagnetic fields can overtake another particle at the distance s ahead of the source particle, at the point B that satisfies the following condition:

$$s = arc(AB) - |AB| = R\theta - 2R\sin(\theta/2) \approx \frac{\theta^3 R}{24},\ if\,\theta \ll 1. \tag{1}$$

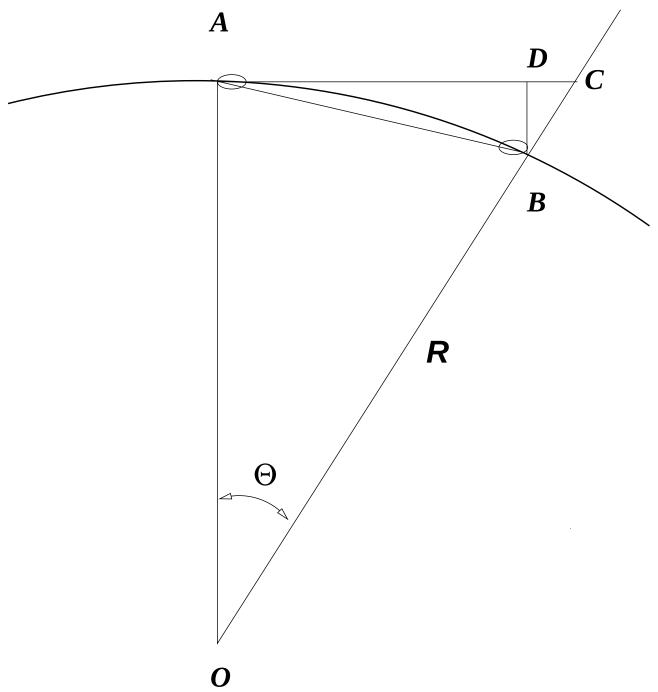

Figure 1: Geometry of the retarding process

This condition determines three important geometrical parameters which play important role in the cooperative radiation interaction process [1]:

- overtaking angle $\theta = 2(3s/R)^{1/3}$,

- overtaking distance $L_o = |AB| = \theta R = 2(3sR^2)^{1/3}$,

- characteristic transverse distance $r_o = |DB| = L_o\theta/2 = 2(9s^2R)^{1/3}$ - see Fig.1.

Under influence of the radiation forces the particle motion nearby an equilibrium trajectory (for a given energy $\mathcal{E}$ and given momentum) can be described by the following equations [5]:

$$y'' + ny + y'\frac{\mathcal{E}'}{\mathcal{E}} = \frac{F_y}{\mathcal{E}}, \tag{2}$$

$$x'' + (K^2 - n)x + x'\frac{\mathcal{E}'}{\mathcal{E}} = K\frac{\Delta\mathcal{E}}{\mathcal{E}} + F_x/\mathcal{E}, \tag{3}$$

$$\mathcal{E}' = e\vec{E}\vec{v}/v_z, \qquad t' = (1 + Kx)/v_z. \tag{4}$$

Here $(') \equiv d/dz,\ K(z) = 1/R$ is the equilibrium orbit curvature, $n(z)$ is the external focusing quadrupole field index, F_x and F_y are the Lorenz force components:

$$\vec{F} = e(\vec{E} + \vec{\beta} \times \vec{B}), \tag{5}$$

and $\vec{E}, \vec{B}$ are electric and magnetic fields, e is the particles' charge. Note, that parametric damping terms of $y'\mathcal{E}',\ x'\mathcal{E}'$ are important only if there is an essential acceleration in the external RF-field, which is not our case.

To work out the perturbation of x and y motion under the effect of the microbunch field, we transform Eqs.(2,3) into equations for complex amplitudes C_x and C_y accordingly to the standard form of unperturbed motion [5]:

$$y = C_y^* f_y + C_y f_y^*, \qquad y' = C_y^* f_y' + c.c., \tag{6}$$

$$x = \Psi\frac{\Delta\mathcal{E}}{\mathcal{E}} + (C_x^* f_x + c.c.), \qquad x' = \Psi'\frac{\Delta\mathcal{E}}{\mathcal{E}} + (C_x^* f_x' + c.c.)\,, \tag{7}$$

here f_x , f_y are the complex solution (i.e. $f = u_1 + iu_2$, where $u_1,\ u_2$ are two independent solutions) of equations

$$f_x'' + (K^2 - n)f_x = 0, \qquad f_y'' + nf_y = 0, \tag{8}$$

with normalization $f_x'f_x^* - c.c. = f_y'f_y^* - c.c. = 2i$. The Floquet functions relate to beta-functions as $|f_{x,y}|^2 = \beta_{x,y}$ and "Courant-Snyder invariant" ϵ relates to $C_{x,y}$ as $\epsilon_{x,y} = 2|C_{x,y}|^2$.

The dispersion function $\Psi(z)$ satisfies the equation

$$\Psi'' + (K^2 - n)\Psi = K. \tag{9}$$

and can be calculated as

$$\Psi(z) = \frac{f_x}{2i}\int\limits_{-\infty}^{z} f_x K\,dz + c.c., \tag{10}$$

with the property

$$\Psi f_x' - \Psi' f_x = -\int\limits_{-\infty}^{z} f_x K\,dz \equiv -\eta(z). \tag{11}$$

From (6), (7) one can find the complex amplitudes as functions of coordinates, velocities and energy:

$$2iC_y = yf_y' - y'f_y; \qquad 2iC_x = xf_x' - x'f_x + \eta\frac{\Delta\mathcal{E}}{\mathcal{E}}. \tag{12}$$

These formulas represent the integrals of unperturbed motion, when $F_x = 0,\ F_y = 0$, and $\Delta\mathcal{E} = const$. Now, taking the time derivative of Eqs. (12) we obtain:

$$2iC_y' = -f_y(F_y/\mathcal{E})(1+Kx) \tag{13}$$

$$2iC_x' = -f_x(F_x/\mathcal{E})(1+Kx) + \eta\mathcal{E}'/\mathcal{E}. \tag{14}$$

The second term in (14) takes into account the perturbation of x - oscillator with respect to "an equilibrium position" $x = \Psi\frac{\Delta\mathcal{E}}{\mathcal{E}}$.

Our next step is to express the microbunch Lorenz force via the electromagnetic potential $(A_o, \vec{A})$, according to general presentation

$$\vec{E} = -\vec{\nabla}A_o - \frac{1}{c}\frac{\partial}{\partial t}\vec{A}, \qquad \vec{B} = \vec{\nabla}\times\vec{A}. \tag{15}$$

We have to expand vector $\vec{A}$ and $\vec{\nabla}$ in terms of co-moving frame:

$$\vec{A} = A_x\vec{e_x} + \vec{A}_y\vec{e_y} + A_z\vec{e_z}; \quad \vec{\nabla} = \vec{e_x}\frac{\partial}{\partial x} + \vec{e_y}\frac{\partial}{\partial y} + \frac{1}{1+Kx}\vec{e_z}\frac{\partial}{\partial z}, \tag{16}$$

with taking into account that

$$\vec{e_z'} = -K\vec{e_x},\ \vec{e_x'} = K\vec{e_z}.$$

The microbunch current will not create the A_y component. Then we have:

$$E_y = -\frac{\partial}{\partial y}A_o, \qquad B_x = \frac{\partial A_z}{\partial y} \tag{17}$$

$$E_x = -\frac{\partial A_o}{\partial x} - \frac{1}{c}\frac{\partial A_x}{\partial t}, \quad B_y = \frac{1}{1+Kx}\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} - \frac{K}{1+Kx}A_z \tag{18}$$

$$E_z = -\frac{\partial A_o}{\partial z} - \frac{1}{c}\frac{\partial A_z}{\partial t}, \quad B_z = -\frac{\partial A_x}{\partial y}. \tag{19}$$

Thus,

$$F_y = -e\frac{\partial}{\partial y}(A_o - \beta A_z) = -\frac{\partial V_o}{\partial y}, \tag{20}$$

$$F_x = -\frac{\partial V_o}{\partial x} - e\frac{dA_x}{cdt} + e\frac{KA_z}{1+Kx}, \tag{21}$$

and

$$\dot{\mathcal{E}} = \frac{\partial}{\partial t}V_o - e\frac{dA_o}{dt}, \tag{22}$$

where the <u>interaction Hamiltonian</u> is $V_o = e(A_o - \beta A_z)$.
After the substitution of Eqs.(20,21,22) into (13,14), we obtain:

$$2i\mathcal{E}C_x' = -f_x\left[-\frac{\partial V_o}{\partial x} - eA_x' + eKA_z\right] + \eta\left[\frac{\partial V_o}{c\partial t} - eA_o'\right]. \tag{23}$$

For further reduction, we reorganize the vector terms in (23) in the way as follows:

$$2i\mathcal{E}C_x' = f_x\left[\frac{\partial V_o}{\partial x} + KV_o\right] - e(\eta A_o)' + e(f_x A_x)' - e f_x' A_x + \eta\frac{\partial}{\partial t}V_o \tag{24}$$

Introducing the displaced amplitudes

$$\hat{C}_x = C_x + e\frac{\eta A_o}{2i\mathcal{E}} - e\frac{f_x A_x}{2i\mathcal{E}}, \qquad \hat{C}_y = C_y, \tag{25}$$

and energy

$$\hat{\mathcal{E}} = \mathcal{E} + eA_o, \tag{26}$$

then we derive the final equations:

$$2i\mathcal{E}\hat{C}_x' = \left(\frac{\eta}{c}\frac{\partial}{\partial t} + f_x\frac{\partial}{\partial x}\right)(1+Kx)V_o - e f_x' A_x, \tag{27}$$

$$\hat{\mathcal{E}}' = \frac{1}{c}\frac{\partial}{\partial t}V_o(1+Kx), \qquad 2i\mathcal{E}\hat{C}_y' = f_y\frac{\partial}{\partial y}V_o. \tag{28}$$

Note, that substitution (25), (26) doesn't change the x coordinate dependence on amplitudes and energy. With respect to x', it is reduced to the simple canonic substitution $x' + eA_x/\mathcal{E}$.

3 Transverse microbunch radiative effects

To calculate the effective transverse forces and perturbation of amplitudes for microbunch in compressor, we use the approach of [1] where it is assumed that the bunch length σ_s satisfies conditions of a "thin" bunch and absence of beampipe shielding:

$$\sigma_\perp\sqrt{\sigma_\perp/R} \ll \sigma_s \ll b\sqrt{b/R}, \tag{29}$$

where $\sigma_\perp$ is transverse bunch size and b is beam pipe size.

The electromagnetic potentials are given by the integrals as follows:

$$A_o = \int\frac{d^3r_1}{c\tau}\rho(\vec{r_1}, t-\tau), \quad \tau \equiv |\vec{r} - \vec{r_1}|/c \tag{30}$$

$$\vec{A} = \int\frac{d^3r_1\vec{\beta}_1}{c\tau}\rho(\vec{r_1}, t-\tau), \tag{31}$$

here $\rho(\vec{r}, t)$ is the space charge density. The retarding distance $c\tau$ for a constant bend radius R can be presented in the form as follows (see Fig. 1):

$$|\vec{r} - \vec{r_1}| = \left[(R+x)^2 + (R+x_1)^2 - 2(R+x)(R+x_1)\cos\frac{\xi}{R} + (y-y_1)^2\right]^{1/2}, \tag{32}$$

where $\xi \equiv z - z_1$.

Following Ref.[1], we may write the expansion:

$$|\vec{r}-\vec{r_1}| \approx \left|\xi - \frac{\xi^3}{24R^2} + \xi\frac{x+x_1}{2R} + \frac{(x-x_1)^2+(y-y_1)^2}{2\xi}\right|. \tag{33}$$

The subject of [1] was to calculate the particle energy change, therefore, the interaction Hamiltonian V_0 was found at $x = x_1 = 0,\ y = y_1 = 0$. In this paper, we derive the effective transverse forces with linear accuracy on x and y. Correspondingly, we have to calculate V_0 with the second order terms and also find the A_x component in linear approximation. As in [1], we neglect the integration over $\xi < 0$, small ultra-relativistic terms $\propto \gamma^{-2}$ and transverse dispersion of τ in denominator of the integrand in (30) and (31). Then we get

$$V_o = Ne^2 \int\limits_0^\infty \frac{\xi d\xi}{2R^2}\left[1 - (\xi\frac{x}{2R} + \frac{x^2+y^2}{2\xi})\frac{1}{c}\frac{\partial}{\partial t} + \frac{1}{2}\xi^2\frac{x^2}{4R^2}\frac{\partial^2}{c^2\partial t^2}\right]\lambda(s - \frac{\xi^3}{24R^2}), \tag{34}$$

$$A_x = \int\limits_0^\infty \frac{d\xi}{R}\left[1 + \xi\frac{x}{2R}\frac{\partial}{\partial s}\right]\lambda(s - \frac{\xi^3}{24R^2}), \tag{35}$$

where $\lambda(s) \equiv \lambda(z - \beta ct)$ is the linear charge distribution along the orbit $\int \lambda(s)ds = 1$. We assume here that charge density ρ is an even function of x_1 and y_1 and ignore the small terms of the order of $\sim \bar{x_1^2}, \bar{y_1^2}$. The linear term $\propto x$ in (34) is simply integrated, while the last term is reduced to be proportional to $\propto \partial/\partial t$ by integration in parts, then we obtain final expression for potential $V_o(x, y, z, t)$

$$V_0 = U(s)(1 + Kx) - F_0(s)x + \frac{1}{2}g(s)(3x^2 + y^2), \tag{36}$$

where

$$U(s) = \frac{2Ne^2}{(3R^2)^{1/3}}\int_0^\infty \frac{ds_1}{s_1^{1/3}}\lambda(s - s_1) \tag{37}$$

$$F_0(s) = -\frac{2Ne^2}{R}\lambda(s) \tag{38}$$

$$g(s) = \frac{Ne^2}{(3R^2)^{2/3}}\frac{\partial}{\partial s}\int_0^\infty \frac{ds_1}{s_1^{2/3}}\lambda(s - s_1) \tag{39}$$

The radial vector potential A_x contributes in C'_x with small terms of the order of $\sim (R^2\sigma_s)^{1/3}/\beta_x$ with respect to $F_0(s)$ and $g(s)$, and, therefore, they could be neglected in further consideration. Then we have

$$2i\mathcal{E}\hat{C}'_x = -\eta\frac{\partial}{\partial s}U(s) - f_x F_0(s) + 3f_x g(s)x \tag{40}$$

$$2i\mathcal{E}\hat{C}'_y = f_y g(s)y \tag{41}$$

$$\hat{\mathcal{E}}' = -\frac{\partial}{\partial s}U(s). \tag{42}$$

Therefore, comparing with initial Eqs.(13,14), one can see that

1. $\partial U(s)/\partial s$ is longitudinal energy loss gradient originally found in [1, 2],
2. $F_0(s)$ is effective centripetal radial force,
3. terms with $g(s)$ describe focusing field distortions in both transverse planes.

All these forces cause emittance growth.

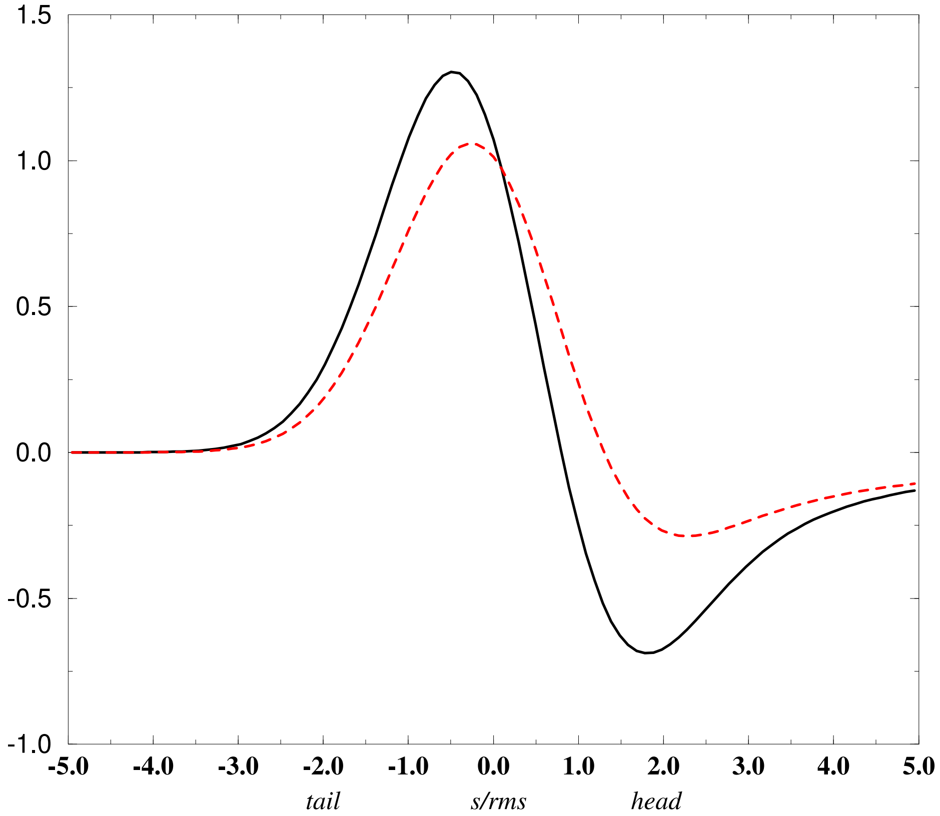

Figure 2: Transverse (solid line) and longitudinal (dashed line) overtake functions $I_0(s), I_1(s)$ vs. coordinate s/σ_s along the Gaussian bunch.

For a bunch with Gaussian linear charge density distribution $\lambda(s) = (1/\sqrt{2\pi}\sigma_s)e^{-s^2/2\sigma^2}$ the energy loss gradient along the bunch is equal to [1]:

$$\hat{\mathcal{E}}' = \frac{d\mathcal{E}}{cdt} = -\frac{2Ne^2}{\sqrt{2\pi}(3R^2\sigma_s^4)^{1/3}} I_0(s/\sigma_s), \tag{43}$$

where the function $I_0(s/\sigma_s)$ is presented by dashed line in Fig.2. As it is qualitatively understood, the bunch head particles get some excess of energy while the tail and center part mostly lose the energy.

Transverse forces within the Gaussian bunch are given by formulae:

$$F_x(s) = -\frac{2Ne^2}{R}\lambda(s) - x\frac{3Ne^2}{\sqrt{2\pi}(9R^4\sigma_s^5)^{1/3}}I_1(s/\sigma_s), \tag{44}$$

$$F_y(s) = -y\frac{Ne^2}{\sqrt{2\pi}(9R^4\sigma_s^5)^{1/3}}I_1(s/\sigma_s), \tag{45}$$

where $I_1(s)$ is shown in Fig.2 by solid line.

One can see that particles at the head of the bunch are defocused by overtaking radiation fields while other particles are focused.

3.1 "Talman force" - why it does not play a role.

The potential V_o we found contains all major effects of cooperative radiation impact on the longitudinal and transverse motion in a constant bend. Let us note that effective radial force $-\partial V_0/\partial x$ is essentially different from the initial force F_x in Eq.(21) – the difference is the term $KA_z/(1+Kx)$. In fact, the later term dominates in F_x and it is the well known "Talman force" [3]. We can derive it directly from (31):

$$A_z = e\int\frac{\beta\cos\xi/R}{c\tau}\rho(\vec{r_1}, t-\tau). \tag{46}$$

To calculate this integral we can apply the series of (33) – as we did for calculation of V_0 – and we get:

$$A_z = e\int\frac{dx\,dy\,d\xi}{\sqrt{\xi^2+\xi_\perp^2}}\rho(\vec{r_1}, t-\tau), \tag{47}$$

where $\xi_\perp^2 = (x-x_1)^2 + (y-y_1)^2$. Evaluation of this integral under condition (29) gives us

$$A_z \simeq Ne\lambda(s)\Big(ln\frac{(R\sigma_s^2)^{2/3}}{\sigma_\perp^2}(1+\sigma_\perp/\sigma_s)\Big), \tag{48}$$

and consequently the centrifugal Talman force is $F_{Talman} = eA_z/R$.

As we found above, the effect of this force on the bunch particles is cancelled by effect of the particle energies deviation under the influence of the transverse electric field, and therefore does not lead to the emittance growth. Similar cancellation effect in the particular case of a coasting beam was found by E.P.Lee [4]. Now we can conclude that it is valid for any relativistic bunch.

Nevertheless, there is a residual effect of centripetal force F_0 (38) which does not contain a logarithmic factor but distorts particle motions. We should note that $F_0 \equiv 0$ for coasting beam (stationary ring current).

4 Example: TTF FEL bunch compressors

Let us apply the obtained results to the TESLA Test Facility Free Electron Laser [6], which requires three bunch compressors in order to shrink the bunch length down to 50 micron rms value. The FEL bunch contains N= 6.2·10^9 electrons with normalized transverse emittance of ϵ = 2 μm.

Table 1: Radiative Tail-Head Effects at the TTF FEL

Parameter	C#2	C#3	Units
E	144	516	MeV
R	1.3	1.3	m
β_x	11	11	m
B	3.7	13.2	kG
σ_z	0.8→0.25	0.25→0.05	mm
L_o	0.32→0.22	0.22→0.13	m
ΔE	0.09	0.63	MeV
$\Delta\epsilon_{dE}$	7	25	μm
B_c^{max}	0.22→0.73	0.73→3.65	G
$\Delta\epsilon_c$	2.2	12.5	μm
f_x^{rad}	168→24	87→6	m
F_x^{rad}/F_{Coul}	0.3→0.65	2.3→6.9	

We compare the radiative longitudinal and transverse effects at the bunch compressors C#2 and C#3. – see Table 1. Vertical rows are marked as compressor number. Each 5-m long compressor consists of four 50-cm-long bending magnets with vertical magnetic field B, which are arranged accordingly to the scheme $[+B] \rightarrow [drift] \rightarrow [-B] \rightarrow [drift] \rightarrow [-B] \rightarrow [drift] \rightarrow [+B]$.

The related parameters are: beam energy in the compressor E, length of each of four magnets L_d, magnetic field B, curvature radius R in the magnets, horizontal beta-function β_x, and the bunch length σ_s which decreases from the entrance to the exit of the compressor.

Then we compute the overtaking length $L_o = 2(3R^2\sigma_s)^{1/3}$ which is found to be less than the magnet length L_d and, therefore, the cooperative effects should take place. The estimated rms value of the energy spread in the bunch ΔE induced by longitudinal tail-head fields is taken from Ref.[1] as well as the corresponding rms emittance increase due to the dispersion in compressor $\Delta\epsilon_{dE}$.

The maximum centripetal field takes place at the center of the Gaussian bunch and can be calculated as :

$$B_c^{max} = -\frac{2Ne}{\sqrt{2\pi}R\sigma_s}. \tag{49}$$

As centripetal force depends on the position along the bunch, then it will induce the normalized emittance growth of the order of:

$$\Delta\epsilon_c = \gamma\beta_x\theta_d^2(\frac{r.m.s.\,B_c}{B})^2, \tag{50}$$

where $\theta_d \simeq 0.38$ rad is the bending angle in each magnet, and $r.m.s.\,B_c \approx 0.28B_c^{max}$ for the Gaussian bunch.

Transverse focusing forces due to cooperative radiation can be characterized by minimum focusing length:

$$f_x^{rad} = \frac{\gamma R^{4/3}\sigma_s^{5/3}}{I_1^{max}3^{1/3}Nr_0L_d}, \tag{51}$$

where $I_1^{max} \approx 1.3$ is the maximum value of the function $I_1(s)$ – see Fig.2.

For comparison, we also present the ratio of this focusing radiative force and the Coulomb expansion force:

$$\frac{F_x^{rad}}{F_{Coul}} \simeq \frac{\gamma^2 I_1^{max}\sigma_r^2}{2R^{4/3}\sigma_s^{2/3}} \tag{52}$$

Finally, Table 1 allows us to make following conclusions for TTF FEL:

1. the emittance growth due to both effects of longitudinal and transverse cooperative radiation forces is several times the design emittance of 2 μm;

2. the emittance growth due to transverse radiative forces is some 2-3 times less than that due to the longitudinal radiative effects;

3. focusing radiative forces are some 7 times the Coulomb expansion force at the exit of the bunch compressor #3 and comparable with strength of the external focusing.

5 Conclusions

We have analyzed the microbunch cooperative synchrotron radiation in bend and found that it essentially influences the microbunch dynamics. First of all, the longitudinal force redistributes radiative energy losses along the bunch, so that head particles are somewhat accelerated by the field radiated by tail particles. That effect can be described by longitudinal *overtake-function* $\mathcal{W}_0'(s)$:

$$\mathcal{W}_0'(s) = -\frac{2}{3^{1/3}R^{2/3}}\frac{1}{s^{1/3}}\frac{\partial}{\partial s}, \tag{53}$$

here $\partial/\partial s$ is an operator of derivative along the longitudinal coordinate s ($s = -\infty$ corresponds to the bunch tail). The energy loss is convolution of $\mathcal{W}_0'$ with linear charge density:

$$\frac{d\mathcal{E}}{cdt} = Ne^2\int_{-\infty}^{s}\mathcal{W}_0'(s-s')\lambda(s')ds'. \tag{54}$$

Therefore, the energy losses originate from derivative of the linear charge density, that is characteristic feature of the effect. Aside of the energy spread along the bunch, the effect leads to radial emittance increase due to dispersion.

The transverse radiative force consists of two components. The smaller term represents focusing forces which are produced by the back part of the bunch:

$$F_{(x,y)} = -(3x, y)Ne^2 \int_{-\infty}^{s} \mathcal{W}_1(s - s')\lambda(s')ds', \tag{55}$$

where transverse overtake-function $\mathcal{W}_1(s)$ is equal to:

$$\mathcal{W}_1(s) = \frac{1}{3^{2/3}R^{4/3}} \frac{1}{s^{2/3}} \frac{\partial}{\partial s}. \tag{56}$$

Note, that the radial focusing gradient is three times the vertical one.

Finally, there is radial centripetal force $F_x^c(s) = -2Ne^2\lambda(s)/R$ which is much bigger than the focusing forces and depends on local charge density $\lambda(s)$. Nevertheless, this force also originates from radiation of the particles behind:

$$F_x^c(s) = -\int_{-\infty}^{s} \mathcal{W}_1^c(s - s')\lambda(s')ds', \tag{57}$$

with purely inductive centripetal overtake-function $\mathcal{W}_1^c(s)$:

$$\mathcal{W}_1^c(s) = \frac{2}{R}\frac{\partial}{\partial s}. \tag{58}$$

The transverse forces also cause the transverse emittance growth. We have found that the combined effects in the TTF FEL bunch compressors can lead to many-fold increase of the initial beam emittance.

Our results are applicable if the characteristic overtaking length $L_o = 2(3\sigma_s R^2)^{1/3}$ is less than the bend length $L_d = R\theta_d$ and if there is no shielding due to metallic vacuum pipe $(\sigma_s^2 R)^{1/3} \ll b$ (for example, the last condition yields $\sigma_s \ll 2$ mm for $R \simeq 2$m and $b \simeq 2$cm). Thorough studies of the beam pipe shielding are underway.

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