Errors in the J_3 part of nutation series

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Abstract

This paper points out that the nutation terms of Zhu and Groten (1989) due to the tidal potential n degree 3 are erroneous. Correct values are deduced hero and they coincide very well with those give in Kinoshita and Souchay (1990), These errors explain the discrepancies between the evaluation of th theories of Zhu and Groten (1989) and Kinoshita and Souchay (1990), which can reach values up t 165 μas. Also the two leading nutation terms due to the tidal potential of degree 4 are given. Finall some of the computer programs (NUTC. F, KSV_1994. F) for evaluating the J_3 nutation terms are als erroneous.

1. Introduction

Duo to the high accuracy of Very Long Boa.line Interferometry (VLBI), present theoretical nutatio models should consider amplitudes of the order of a few μas (= micro arcseconds). Thin especiall applies to rigid Earth models which serve as a basis for more complicated and realistic Earth model including elastic and anelastic effects of the mantle, effects from outer and inner core, oceans and th atmosphere etc. (ZMOA Herring (1990), Mathews et al. (1991 a,b)). At this level of accuracy nutatio terms due to the J_3 coefficient of the Earth should be taken into account. Values for them differ in th two most recent nutation series, Kinoshita and Souchay (1990) and Zhu and Groten (1989), by a facto of up to two. These difference were reported by several authors (e.g. Souchay (1993), Williams (1994), but so far no explanation has been given. It is shown here that they can be attributed to two errors in the computation method of Zhu and Groten (1989).

2. Method of computation

The Computation method applied here is based on the tesseral part of the tidal potential and its relationship to the nutation angles. This article takes advantage of a new expansion of the (luni-solar) tidal potential which was recently given by Hartmann and Wenzel (1995). Although the tidal potentials used by Zhu and Groten (1989) are sufficiently accurate for the purpose, here the newer one is chosen duo to its much simpler structure, Using them expansion coefficients the derivation of corresponding nutation coefficients is straightforward (see also Hartmann rim-l Soffel (1 994)) and can be explained by the following steps.

1) The classical formula for the tidal potential W_{tidal} (see e.g. Merits and Müller (1087)) reads:

$$W_{tidal} = GM_b \sum_{\ell=2}^{\ell=\infty} \sum_{m=0}^{m=\ell} \frac{r^\ell}{r_b^{\ell+1}} \frac{1}{2\ell+1} \overline{P}_{\ell m}(\cos\theta) \overline{P}_{\ell m}(\cos\Theta_b(t)) \cos\left[m(\lambda - \Lambda_b(t))\right] \quad . \tag{1}$$

Here, $\overline{P}_{\ell m}$ denote the fully normalised Legendre functions of degree ℓ and order m. r, θ, λ and r_b, Θ_b, Λ_b are the geocentric spherical coordinates of a station on the Earth and the tide generating body, respectively. GM_b is the product of the gravitational constant and the mass of the specific body (Moon, Sun, planets). Equation (1) is evaluated numerically using the standard ephemeris DE200 from JPL and transformed into the following Poisson series: (see Hartmann and Wenzel (1995) for further details)

$$W_{tidal} = \sum_{l=2}^{l=\ell} \sum_{m=0}^{m=\ell} \left(\frac{r}{a}\right)^\ell \overline{P}_{\ell m}(\cos\theta) \sum_i \left[C_i^{\ell m}(t) \cos(\arg_i(t)) + S_i^{\ell m}(t) \sin(\arg_i(t)) \right] \tag{2}$$

The parameter a is taken to be the semi-major axis of the Earth (a: = 6378138.3 m in the IERS 1992 Standards, McCarthy (1992)), t is the time reckoned from the epoch J2000 in Julian, centuries. It should be mentioned that the linear time dependent amplitudes $C_i^{\ell m}$ and $S_i^{\ell m}$ have the dimension m^2/s^2. The arguments $\arg_i(t)$ refer to epoch J2000 and they ate taken to be integer combinations of the Doodson variables τ, s, h, p, NJ, p_s (or equivalently of the Delaunay arguments l_M, l_S, F, D, Ω) and the mean longitude λ_j (t) of the planets. For the purposes here it is sufficient to regard them as linear functions of time

$$\arg_i(t) = \phi_i + \omega_i t \quad . \tag{3}$$

2) It is known (see e.g. Melchior and Georis (1968) or Moritz and Müller (1987)) that the torque N acting on an axisymmetric Earth model is directly proportional to the tesseral part ($m = 1$) of the tidal potential. Denoting the moments of inertia of the Earth by $A = B$ and C, one has to lowest approximation

$$N_{l=2} = \frac{\sqrt{15}}{a^4} \begin{pmatrix} (C-A)\sum_i \left[S_i^{21}\cos(\phi_i + \omega_i t) - C_i^{21}\sin(\phi_i + \omega_i t)\right] \\ -(C-A)\sum_i \left[C_i^{21}\cos(\phi_i + \omega_i t) + S_i^{21}\sin(\phi_i + \omega_i t)\right] \\ 0 \end{pmatrix} \quad . \tag{A}$$

The higher order parts of the torque can be obtained by the replacements (factor and tidal amplitudes) given in ,'Cl'able 1. This also applies to all other formulas given below. The J_l values of the Earth were taken from DE245 and are based on the GEM-T3S model by Lerch et al. (1994)).

TABLE 1: Replacements to obtain the higher order nutation terms

degree	factor	tidal amplitudes	J_l (in 10^{-6})
2	$\sqrt{15}$	S_i^{21}, C_i^{21}	1082.626
3	$\sqrt{42}\,J_3/J_2$	S_i^{31}, C_i^{31}	−2.533
4	$\sqrt{90}\,J_4/J_2$	S_i^{41}, C_i^{41}	−1.010

Note that Zhu and Groten (1989) apply only a factor J_3/J_2. Hence, by means of their simplification they miss a factor of $\sqrt{42}/\sqrt{15}$.

3) Because of the smallness of the nutation terms considered here a distinction between effects for angular momentum axis, rotation axis and figure axis is not made. Assuming again our model Earth to be approximately axisymmetric we proceed therefore with the Poisson equations in the form

$$\sin\varepsilon_0 \cdot \frac{d}{dt}\Delta\psi \approx +\frac{N_x}{C\omega} \quad , \qquad \frac{d}{dt}\Delta\varepsilon \approx +\frac{N_y}{C\omega} \quad , \tag{5}$$

where $\Delta\psi$ and $\Delta\varepsilon$ denote the nutation angles in longitude and obliquity, respectively. ε_0 denotes the obliquity, which takes according to DE200 (see Standish (1962)) the value $23°26'21''.412$ at epoch J2000 C is the largest principle moment of inertia and for the mean value of the Earth's angular velocity ω we take 7,292116 x 10-a rad/sec.

4) The introduction of the time harmonic tidal development in the torque allows the integration of equations (5). Rearranging the sum over the tidal amplitudes in pairs (thus the sum running only up to $i/2$) with frequencies $\Delta\omega_{\pm i} \equiv \pm(\omega_i - \omega)$, which form one nutation term, one gets for degree $\ell = 2$:

$$\Delta\psi = +\frac{E_2}{\sin\varepsilon_0}\sum_{i/2}\frac{\omega}{\Delta\omega_{+i}}\left[\left(C_i^{21} - C_{-i}^{21}\right)\cos(\Phi_i + \Delta\omega_{+i}t) + \left(S_i^{21} + S_{-i}^{21}\right)\sin(\Phi_i + \Delta\omega_{+i}t)\right] \tag{6a}$$

$$\Delta\varepsilon = -E_2\sum_{i/2}\frac{\omega}{\Delta\omega_{+i}}\left[\left(C_i^{21} + C_{-i}^{21}\right)\sin(\Phi_i + \Delta\omega_{+i}t) - \left(S_i^{21} - S_{-i}^{21}\right)\cos(\Phi_i + \Delta\omega_{+i}t)\right] \tag{6b}$$

The scale factor E_2 is (approximately) given by

$$E_2 \approx \sqrt{15}\,\frac{H_{dyn}}{a^2\,\omega^2} \tag{7}$$

and depends on the dimensionless dynamical ellipticity $H_{dyn} \equiv (2C - A - B)/(2C) \equiv J_2 M_\oplus a^2/C$ taken from Kinoshita and Souchay (1990), $H_{dyn} \approx 0.0032740$, so that $E_2 \approx 12{,}091\,\mu as$. For the corresponding quantities E_3 ($\ell = 3$) and E_4 ($\ell = 4$) we obtain -47, s96 μas and $-44.207\,\mu as$, respectively. The results presented below were obtained from numerically evaluating expressions (6a) and (6b). For the even degrees of the tidal potential ($\ell = 2,4$) only the sine-amplitudes S_{21}, S_{41} are relevant while for $\ell = 3$ the quantity C_{31} is most important. (The other tidal amplitudes arise because of planetary effects and therefore their magnitudes are very small.) Thus for $\ell = 2$ and $\ell = 4$ $\Delta\psi$ is proportional to the sum and $\Delta\varepsilon$ to the difference of the two tidal amplitudes constituting one nutation term, while for $\ell = 3$ it is just the other way around. This has not been taken into account by Zhu and Groten (1989). Probably they relied on the *paper* of Melchior and Georis (1968) and theorem 2 therein which is also wrong in that context. Moreover for $\ell = 9$ the nutation in longitude / obliquity is proportional to the cos / sin of the nutation argument which is again opposite to the case of $\ell = 2$. Note, that some of the computer programs (e.g. NUTC.F for the Kinoshita and Souchay (1990) series, KSV_1004.F for the recent nutation series based on VLBI data) for evaluating the J_3 nutation terms are also erroneous since they use the interchanged trigonometric functions (namely those for the $\ell = 2$). The cos / sin dependence of the J_3 longitude / obliquity nutation is correctly given in Kinoshita and Souchay's (1990) text, but it is noted (Souchay and Kinoshita (1995)) that it is incorrectly listed as sin / cos in their final tables 28 and 29.

It should be pointed out that - although the method could be applied in principle to the main-nutation terms due to J_2 too - the results will not be satisfying due to the various assumptions and approximations made above. However, results using a more elaborate computation method will be presented in a subsequent paper.

S) It is easy to derive also values for the precession and obliquity rate! due to these parts of the tidal potential. In that case $\Delta\omega$ vanishes and one obtains

$$\frac{d}{dt}\Delta\psi = +\frac{E_3\,\omega}{\sin\varepsilon_0} S^{31}(\omega_i = \omega) \quad (8a)$$

$$\frac{d}{dt}\Delta\epsilon = -E_3\,\omega\, C^{31}(\omega_i = \omega) \quad . \quad (8b)$$

Numerical values are given in Table 5.

3. Results

In Table 2 all nutation terms due to the J_3 coefficient are given (HWS = this paper, KS Kinoshita and Souchay (1990), ZG = Zhu and Groten (1989)). Eleven amplitudes due to the Moon are larger than 1 μas. No nutation terms due to the sun arise since the solar tidal potential of degree 3 is already very small.

TABLE 2: Nutation due to the J_3 coefficient (in μas; cos for $\Delta\psi$, sin for $\Delta\epsilon$)

l_M	l_S	F	D	Ω	period (d)	$\Delta\psi_{HWS}$	$\Delta\psi_{KS}$	$\Delta\psi_{ZG}$	$\Delta\epsilon_{HWS}$	$\Delta\epsilon_{KS}$	$\Delta\epsilon_{ZG}$
–1	u	1	0	3	65502.276	–6.8	–6.0		-2.7		
-1	0	1	0	2	8159.136	36.6	36.0	30.0	17.6	19.0	-10.0
-1	0	1	0	1	3231.496	–104.0	–105.0	-150.0	–89.0	–89.0	30.0
–1	0	1	0	0	2190.360	33.2	33.0		0.0		
1	0	1	–2	1	19s.680	-1.2			–1.0		
0	0	1	0	2	27.432	3,0			1.4		
0	0	1	0	1	27.S22	-16.2	–16.0	-20.0	–13.8	–14.0	
0	0	1	0	0	27,212	7.5	8.0		0.0		
1	0	1	0	1	1.3.719	-1.3			-1.1		
0	0	3	0	3	0.107	–2.6			–1.1		
o	0..,	s	o	2	9.095	-1.0			-0.5		

Using the same computation method one obtains for the two largest nutation terms due to J_4 values given in Table 3.

TABLE 3: Nutation due to the J_4 coefficient (in μas; sin for $\Delta\psi$, cos for $\Delta\epsilon$)

l_M	l_S	F	D	Ω	period (d)	$\Delta\psi_{HWS}$	$\Delta\epsilon_{HWS}$
0	0	0	0	1	6798.384	-0,7	6.8
0	0	0	0	2	3S99.192	0.6	-0.3

For nutation terms due to J_4 Kinoshita and Souchay (1990) mention the .0.7 μas longitude term, but no the larger obliquity term which slightly exceeds their 5 μas threshold. In Figure 1 and 2 we show fo the time interval 1950 -2050 the differences between the evaluation of our nutation series due to J_3 an those of Kinoshita and Souchay (1990) and Zhu and Groten (1989), respectively. The values for the rm J maximum differences in μas are given in Table 4.

TABLE 4: Differences in μas

	$\delta\Delta\psi$		$\delta\Delta\epsilon$	
	rms	max	rms	max
HWS-KS	3.3	9.9	2.6	8.2
HWS-ZG	4107	93.6	86.8	165.2
J4	0,6	1.1	4.8	7.1

Table 5 gives precession and obliquity rates due to J_3 and J_4. Tho precession rate due to J4 is in good agreement with the theoretical equation given by Kinoshita (1977) and the value of 2 600 $\mu as/J.cy$ given in Kinoshita and Souchay (1990). The rates from J_3 are really ● xceedingly long period terms duo to the motion of the perihelion of the earth's orbit compared to the equinox.

TABLE 5: Precession and obliquity rates in $\mu as/J.cy$

degree	body	$\Delta\dot{\psi}$	$\Delta\dot{\epsilon}$
3	Moon	−5.7	−1.1
	Sun	-2.6	-0.5
	Total	−8.4	−1.6
4	Moon	2S1S.3	0.0

4. Conclusions

Using au independent procedure for computing the nutations from the earth's J_3, a conflict between the series of Kinoshita and Souchay (1990) and Zhu and Groten (1989) hag been resolved. The former series is correct and is extended to smaller coefficients here, Given the accuracy of modern observations, the nutations due to J_3 should be included in computations of the earth's orientation in space.

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FIGURE 1: Differences for $\Delta\psi$ (in μas)

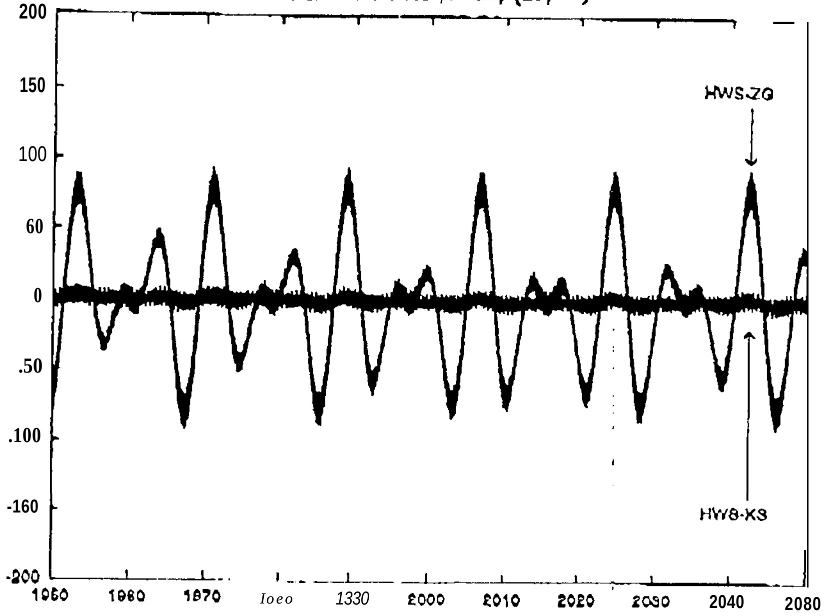

FIGURE 2: Differences for $\Delta\epsilon$ (in μas)