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Intrabeam Scattering in the Recycler Ring: A Fresh Look

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Abstract

In this report, we estimate the longitudinal and transverse growth rates for the Recycler Ring for a range of relative momentum fraction and a coasting beam of 2×10^{12} antiprotons. The coupling between the horizontal and vertical degrees of freedom is ignored.

Intrabeam Scattering: Recycler Ring

This is coulomb scattering between beam particles inside a given beam - bunch or otherwise. There are two approches in estimating the growth rates:

- Bjorken-Mtingwa Formalism: S-matrix formalism and relativistic quantum mechanical treatment - the diffusion equation follows from Fermi's Golden Rule and conservation laws
- Piwinsky: Relativistic classical coulomb scattering. Following a treatment of Bane (EPAC-2002) for high energy extension/approximation.

Assumptions:

- Assumes no coupling between the vertical and horizontal planes.
- No vertical dispersion.
- The lattice version 22 is used - latest.
- Coasting beam - no bunch structure.

Intrabeam Scattering: Bjorken-Mtingwa Formalism

$$\frac{1}{T_p} = \frac{1}{\sigma_p}\frac{d\sigma_p}{dt}$$

$$\frac{1}{T_x} = \frac{1}{\epsilon_x^{1/2}}\frac{d\epsilon_x^{1/2}}{dt}$$

$$\frac{1}{T_y} = \frac{1}{\epsilon_y^{1/2}}\frac{d\epsilon_y^{1/2}}{dt}$$

$$\frac{1}{T_i} = 4\pi A(log)\langle\int_0^\infty \frac{d\lambda\lambda^{1/2}}{|(L+\lambda I)|^{1/2}}\{Tr(L^i)Tr(\frac{1}{L+\lambda I}) - 3Tr(L^i)(\frac{1}{L+\lambda I})\}\rangle$$

$$A = \frac{r_0^2 cN}{16\sqrt{2\pi^3}C\beta_r^3\gamma^4\epsilon_x\epsilon_y\sigma_p}$$

$$L = L^p + L^x + L^y$$

$$L^p = \left\{\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right\}$$

$$L^x = \left\{\begin{array}{ccc} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2\mathcal{H}_x/\beta_x & 0 \\ 0 & 0 & 0 \end{array}\right\}$$

$$L^y = \left\{\begin{array}{ccc} 0 & 0 & 0 \\ 0 & -\gamma^2\mathcal{H}_y/\beta_y & \gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{array}\right\}$$

$$\mathcal{H} = \frac{\eta^2 + (\beta\eta' - \beta'\eta/2)^2}{\beta}$$

$$\phi = \eta' - \frac{1}{2}\beta'\eta/\beta$$

where the average is taken all over the ring and:

r_0 - classical proton radius

N - the number of particles in the bunch

β_r - Rleativistic v/c

(log) - Coulomb log factor given by $\ln(\frac{2}{\theta_{min}})$

ϵ_i - emittance with i = x, y

β_i, η - lattice beta and dispersion function with i = x, y

C - RR circumference

σ_p - the relative energy spread

One can obtain a useful approximation for high γ ($\approx$: 10 or higher)

$$\frac{\beta_x}{\epsilon_x}, \frac{\beta_y}{\epsilon_y} \;<<\; \frac{\gamma^2\eta^2}{\epsilon_x\beta_x}, \frac{\beta_x\gamma^2\phi^2}{\epsilon_x}, \frac{\gamma^2}{\sigma_p^2}$$

$$\frac{1}{\tau_p} = \pi^2\alpha^2 MN(log)\frac{\gamma}{\Gamma\sigma_p^2}\int_0^\infty d\lambda \frac{\sqrt{\lambda}[2a\lambda + b]}{[\lambda^3 + a\lambda^2 + b\lambda + c]^{3/2}}$$

$$a = \frac{\gamma^2\eta^2}{\epsilon_x\beta_x} + \frac{\gamma^2\phi^2\beta_x}{\epsilon_x} + \frac{\gamma^2}{\sigma_p^2}$$

$$b = [(\frac{\beta_x}{\epsilon_x} + \frac{\beta_y}{\epsilon_y})(\frac{\gamma^2\eta^2}{\epsilon_x\beta_x} + \frac{\gamma^2}{\sigma_p^2}) + \frac{\beta_x\beta_y\gamma^2\phi^2}{\epsilon_x\epsilon_y}]$$

$$c = \frac{\beta_x\beta_y}{\epsilon_x\epsilon_y}(\frac{\gamma^2\eta^2}{\epsilon_x\beta_x} + \frac{\gamma^2}{\sigma_p^2})$$

$$\frac{1}{\tau_x} = \frac{\sigma_p^2}{\tau_p} < [\frac{\eta^2}{\epsilon_x \beta_x} + \frac{\beta_x \phi^2}{\epsilon_a}] >$$

$$\Gamma_{unbunched} = \frac{(2\pi)^{5/2}}{\sqrt{2}} \beta^3 \gamma^3 M^3 \epsilon_x \epsilon_y \sigma_p C$$

Intrabeam Scattering - Piwinsky Formalism

$$\frac{1}{T_p} = A\langle[\frac{\sigma_h^2}{\sigma_p^2} f(a_p, b_p, q)]\rangle$$

$$\frac{1}{T_x} = A\langle[f(\frac{1}{a_p}, \frac{b_p}{a_p}, \frac{q}{a_p}) + \frac{\eta_x^2 \sigma_h^2}{\beta_x \epsilon_x} f(a_p, b_p, q)]\rangle$$

$$\frac{1}{T_y} = A\langle[f(\frac{1}{b_p}, \frac{a_p}{b_p}, \frac{q}{b_p}) + \frac{\eta_y^2 \sigma_h^2}{\beta_y \epsilon_y} f(a_p, b_p, q)]\rangle$$

$$\frac{1}{\sigma_h^2} = \frac{1}{\sigma_p^2} + \frac{\eta_x^2}{\beta_x \epsilon_x} + \frac{\eta_y^2}{\beta_y \epsilon_2}$$

$$a_p = \frac{\sigma_h}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x}}$$

$$b_p = \frac{\sigma_h}{\gamma}\sqrt{\frac{\beta_y}{\epsilon_y}}$$

$$q = \sigma_h \beta \sqrt{\frac{2d}{r_0}}$$

$$f(a_p, b_p, q) = 8\pi \int_0^1 du \frac{1 - 3u^2}{PQ} \{2ln[\frac{q}{2}(\frac{1}{P} + \frac{1}{Q})] - 0.577\}$$

$$P^2 = a_p^2 + (1 - a_p^2)u^2$$

$$Q^2 = b_p^2 + (1 - b_p^2)u^2$$

The above formalism has to be modified for rings with $< \mathcal{H} > \neq < \eta^2/\beta >$. Therefore by replacing $\mathcal{H}$ with η^2/β, we obtain the modified Piwinsky Formalism. Also with proper choice of the maximum impact parameter d, the coulomb logarithms can be made same. Thus we can write:

$$\frac{1}{T_p} \approx \frac{r_0^2 cN(log)\sqrt{2\pi}}{8C\gamma^3(\epsilon_x\epsilon_y)^{3/4}\sigma_p^3}\langle\sigma_H h(a,b)(\beta_x\beta_y)^{-1/4}\rangle$$

$$h(a,b) = \frac{2\sqrt{ab}}{\pi}\int_0^\infty \frac{du}{\sqrt{(a^2+u^2)(b^2+u^2)}}$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\epsilon_x} + \frac{\mathcal{H}_y}{\epsilon_y}$$

Computations Summary

Notes:

- N = 2 $\times 10^{12}$
- $\epsilon_x = 5 - 20$ π-mm-mr (Normalized 95%)
- $\epsilon_y = \epsilon_x$ for simplicity
- Rates are computed as a function $\sigma_p = \Delta/$P for various values of ϵ_x.
- Lattice averaging is done via breaking up the Ring into 1300 parts
- One has to be very careful with numerical integrations as quantities of relative magnitudes of E+24 in the denominator!

Recycler Ring IBS Results

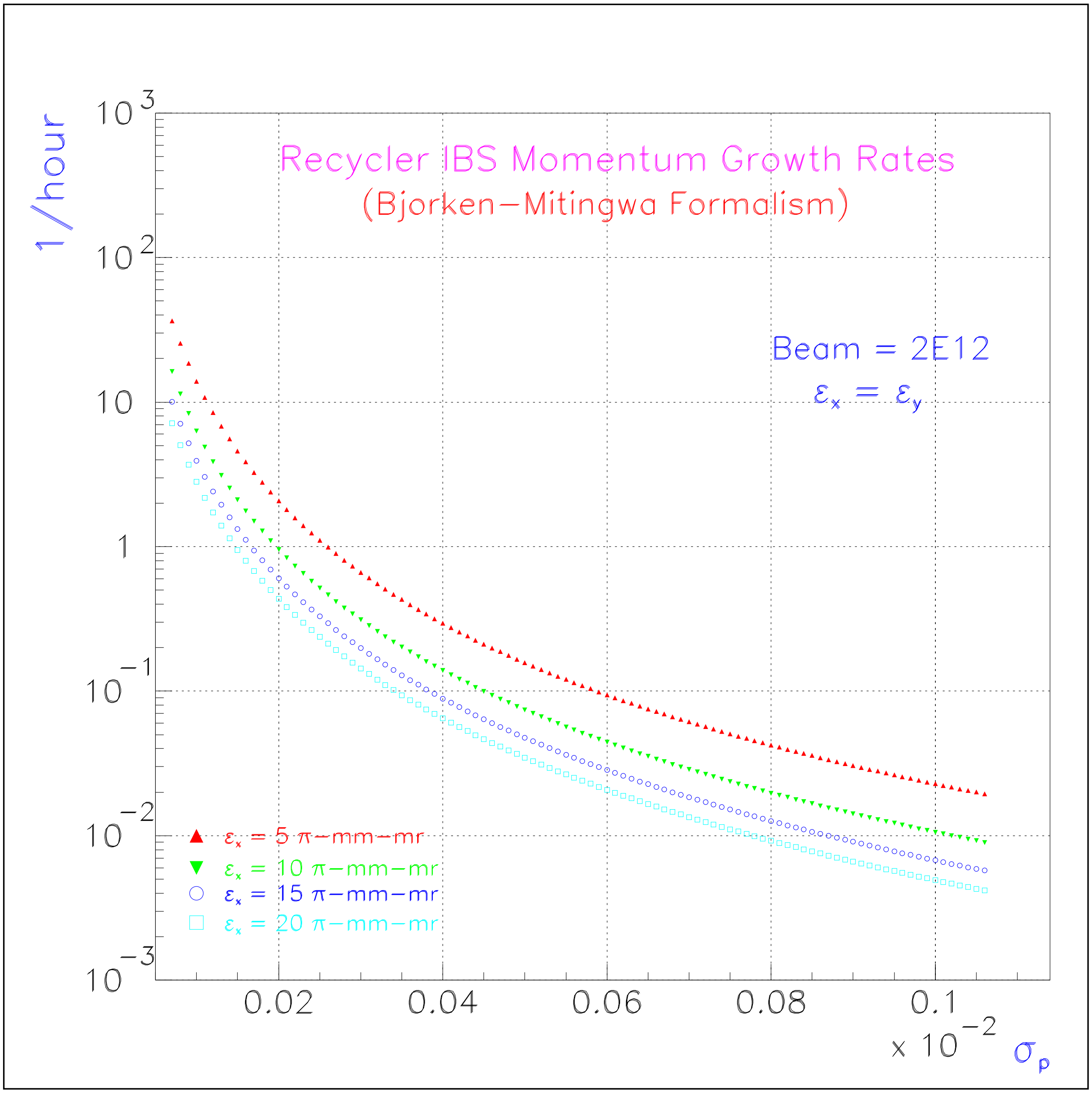
Quantity	ϵ_x [π-mm-mr]	σ_p	**BJM** Formalism	**PWKB** Formalism
Mom. Growth (1/hr)	5.0	0.00025	1.11	1.17
Trans. Growth (1/hr)	5.0	0.00025	0.06	0.06
d(ϵ_x)/dt (π-mm-mr/hr)	5.0	0.00025	0.30	0.32
Mom. Growth (1/hr)	5.0	0.00075	0.05	0.07
Trans. Growth (1/hr)	5.0	0.00075	0.02	0.03
d(ϵ_x)/dt (π-mm-mr/hr)	5.0	0.00075	0.12	0.17
Mom. Growth (1/hr)	10.0	0.00025	0.52	0.45
Trans. Growth (1/hr)	10.0	0.00025	0.01	0.01
d(ϵ_x)/dt (π-mm-mr/hr)	10.0	0.00025	0.14	0.12
Mom. Growth (1/hr)	10.0	0.00075	0.02	0.03
Trans. Growth (1/hr)	10.0	0.00075	0.01	0.01
d(ϵ_x)/dt (π-mm-mr/hr)	10.0	0.00075	0.06	0.08

Beam = 2.0E+12

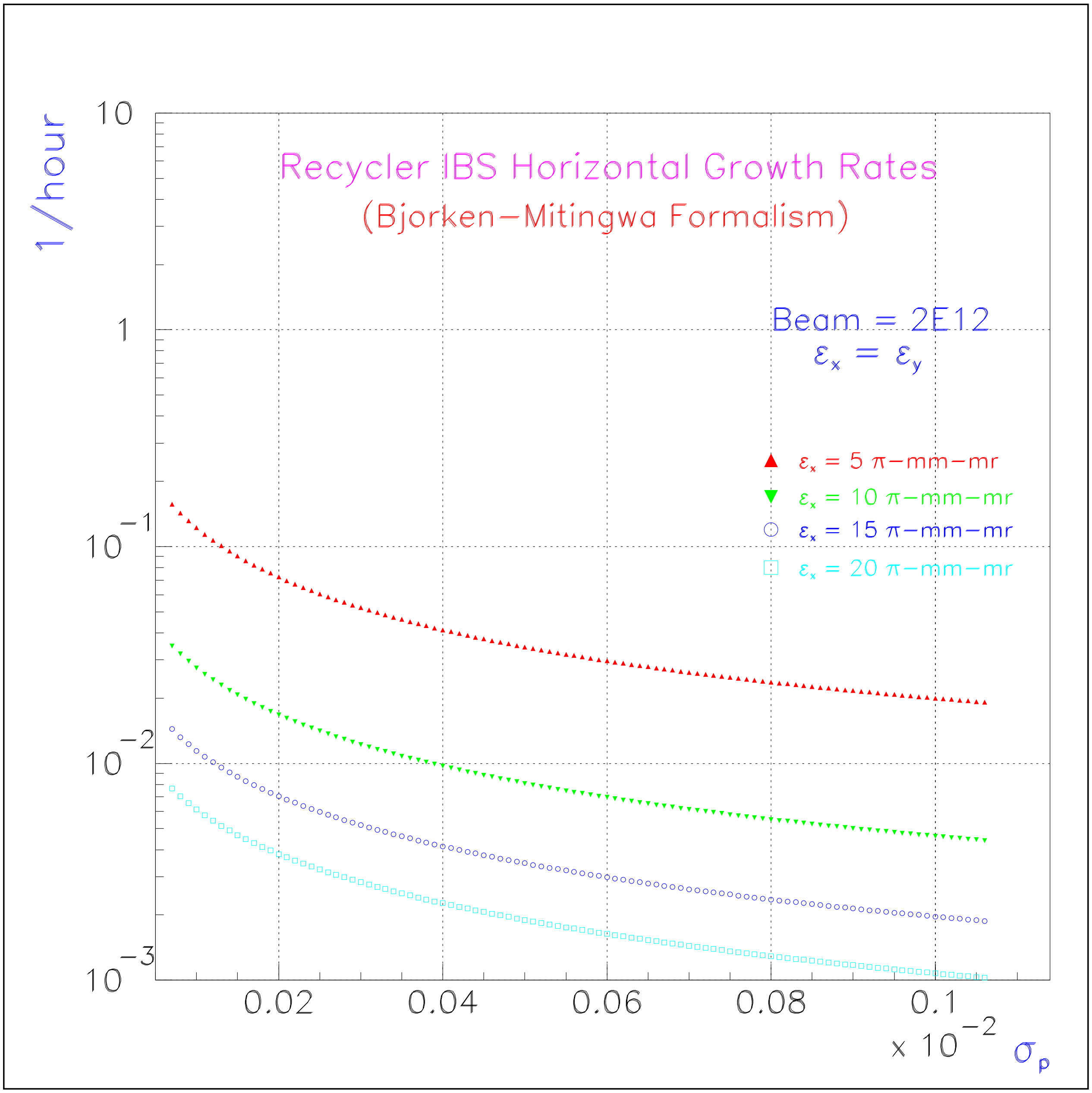
$\epsilon_x = \epsilon_y$

clog = 20.0

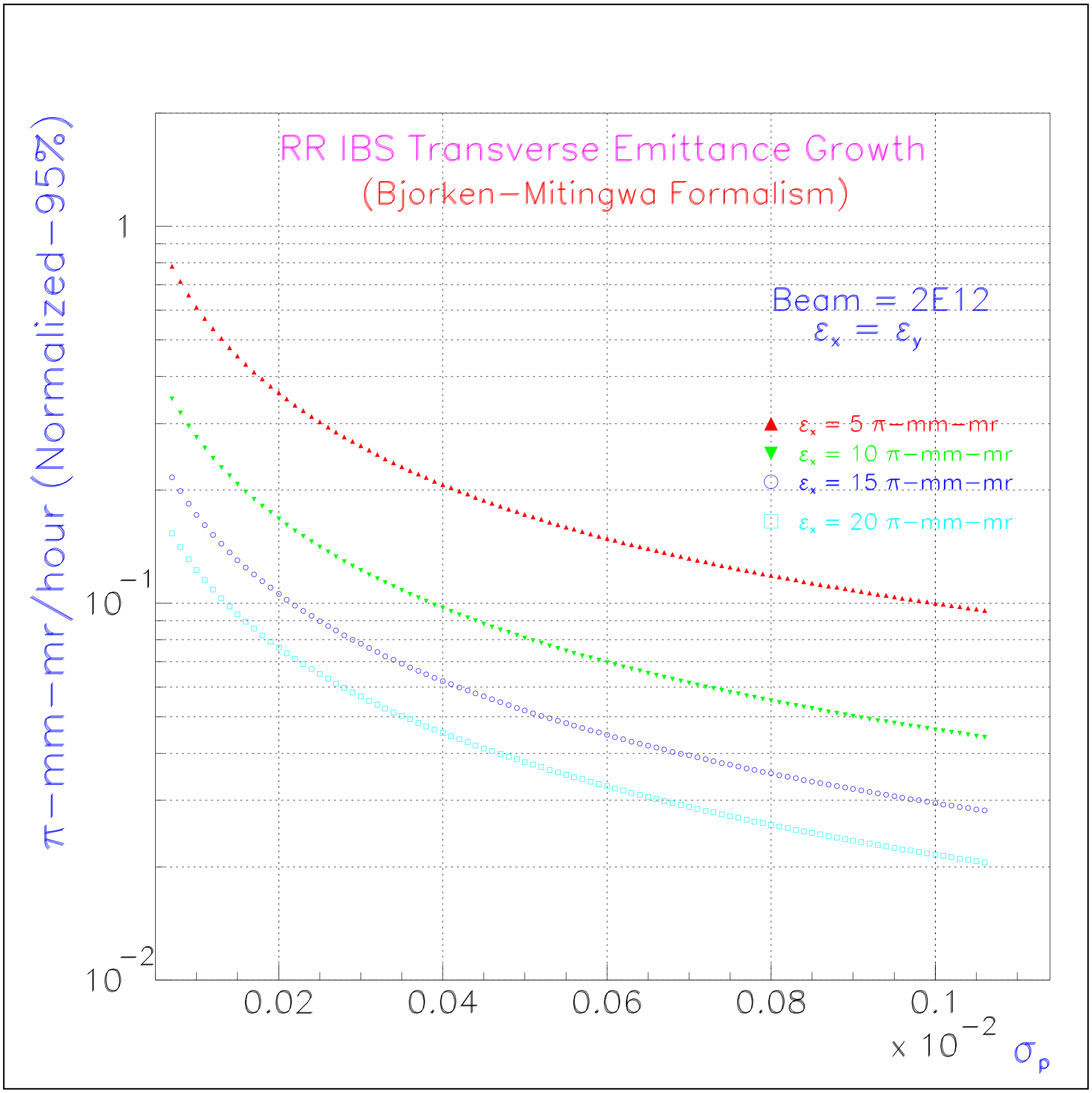
Coasting Beam



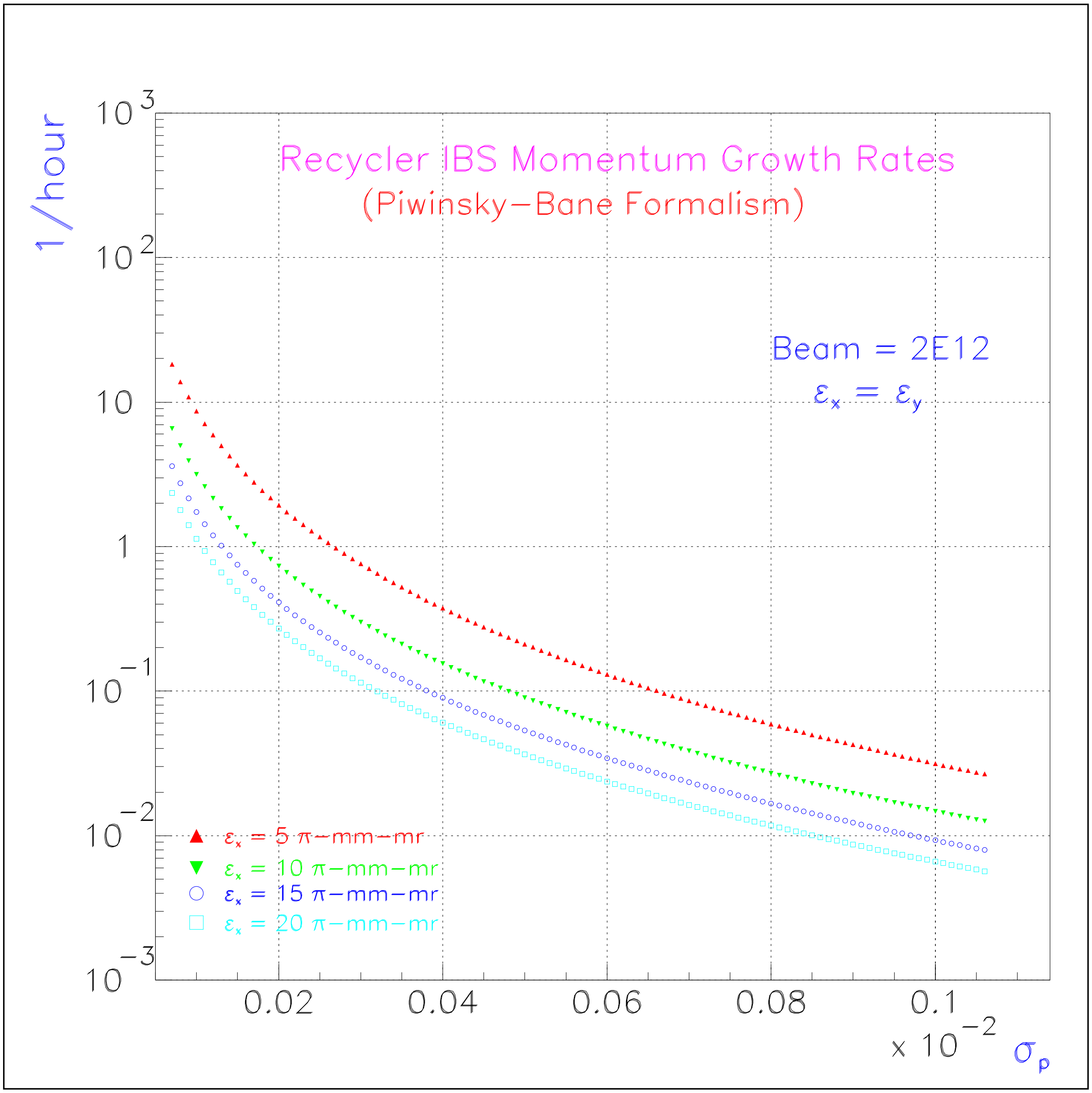
Recycler IBS Momentum Growth Rates
(Bjorken–Mitingwa Formalism)
Beam = 2E12
$\varepsilon_x = \varepsilon_y$
ε_x = 5 π–mm–mr
ε_x = 10 π–mm–mr
ε_x = 15 π–mm–mr
ε_x = 20 π–mm–mr
1/hour
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
0.02
0.04
0.06
0.08
0.1
x 10^{-2}
σ_p



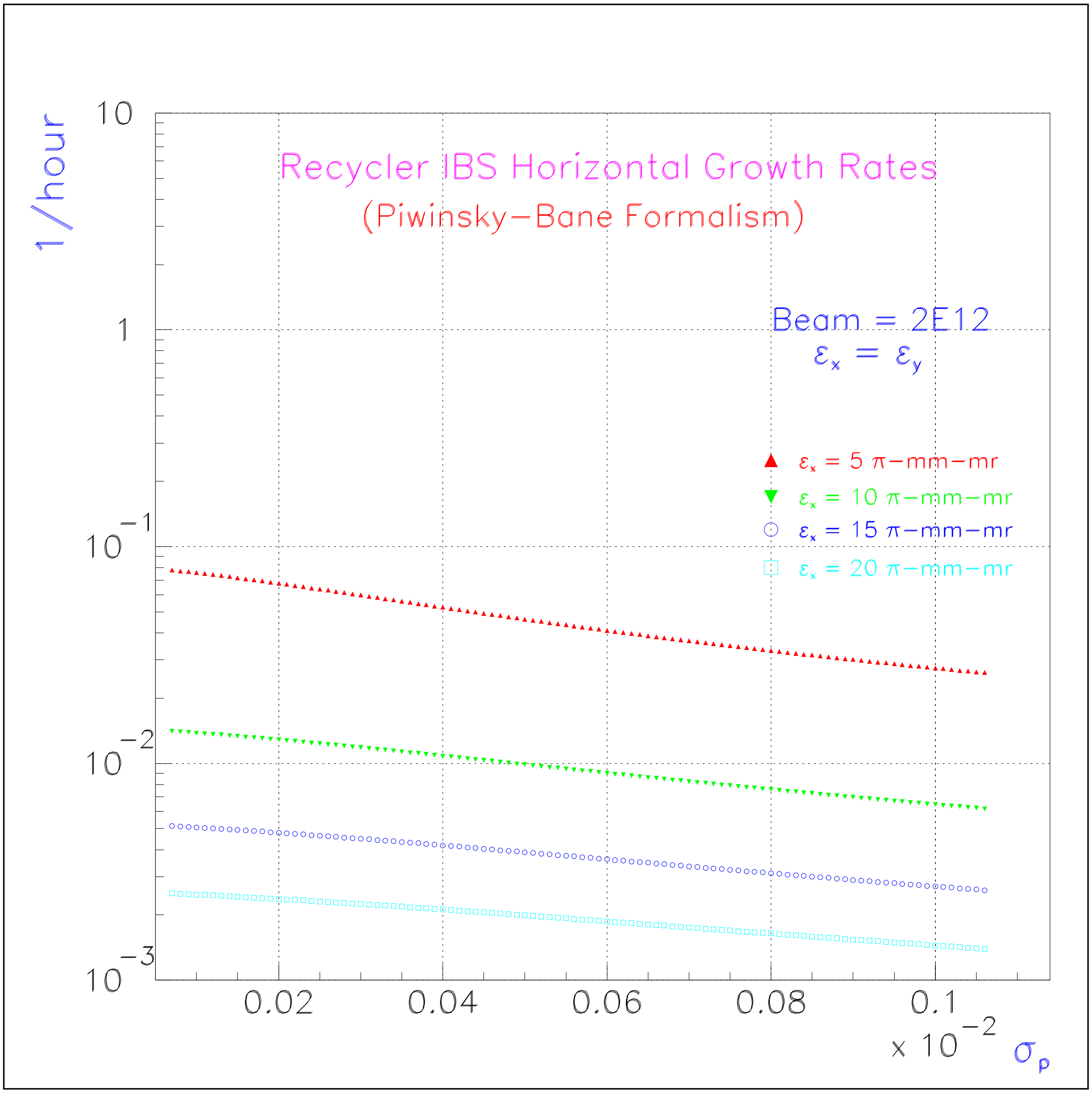
Recycler IBS Horizontal Growth Rates
(Bjorken–Mitingwa Formalism)
Beam = 2E12
$\varepsilon_x = \varepsilon_y$
ε_x = 5 π–mm–mr
ε_x = 10 π–mm–mr
ε_x = 15 π–mm–mr
ε_x = 20 π–mm–mr
1/hour
10
1
10^{-1}
10^{-2}
10^{-3}
0.02
0.04
0.06
0.08
0.1
$\times 10^{-2}$
σ_p



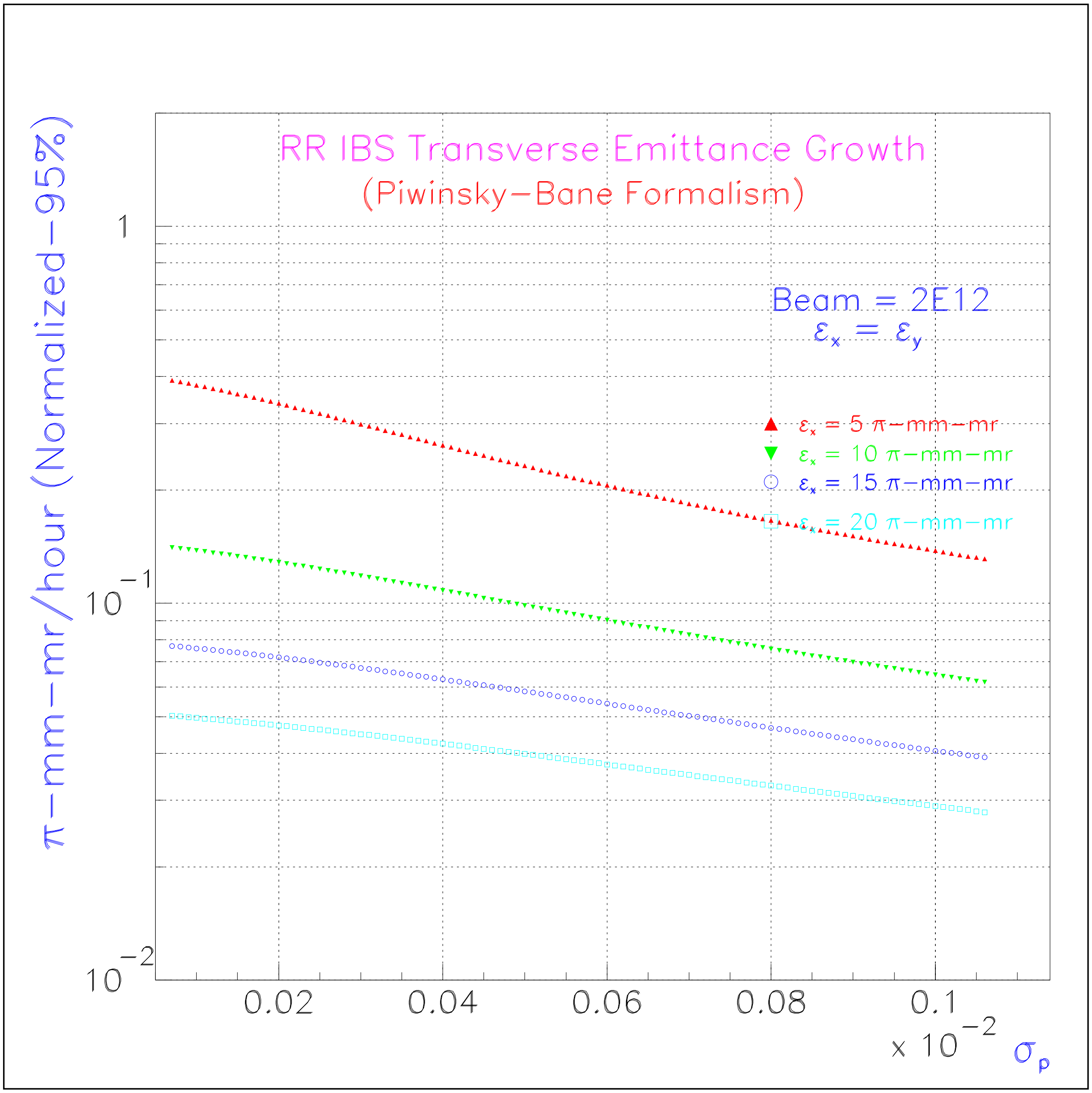
RR IBS Transverse Emittance Growth
(Bjorken−Mitingwa Formalism)
Beam = 2E12
$\varepsilon_x = \varepsilon_y$
ε_x = 5 π−mm−mr
ε_x = 10 π−mm−mr
ε_x = 15 π−mm−mr
ε_x = 20 π−mm−mr
π−mm−mr/hour (Normalized−95%)
1
10^{-1}
10^{-2}
0.02
0.04
0.06
0.08
0.1
$\times 10^{-2}$
σ_p



Recycler IBS Momentum Growth Rates
(Piwinsky–Bane Formalism)
Beam = 2E12
$\varepsilon_x = \varepsilon_y$
1/hour
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
0.02
0.04
0.06
0.08
0.1
$\times 10^{-2}$
σ_p
ε_x = 5 π–mm–mr
ε_x = 10 π–mm–mr
ε_x = 15 π–mm–mr
ε_x = 20 π–mm–mr



Recycler IBS Horizontal Growth Rates
(Piwinsky–Bane Formalism)
Beam = 2E12
$\varepsilon_x = \varepsilon_y$
ε_x = 5 π–mm–mr
ε_x = 10 π–mm–mr
ε_x = 15 π–mm–mr
ε_x = 20 π–mm–mr
1/hour
10
1
10^{-1}
10^{-2}
10^{-3}
0.02
0.04
0.06
0.08
0.1
$\times 10^{-2}$
σ_p



RR IBS Transverse Emittance Growth
(Piwinsky−Bane Formalism)
Beam = 2E12
εx = εy
εx = 5 π−mm−mr
εx = 10 π−mm−mr
εx = 15 π−mm−mr
εx = 20 π−mm−mr
π−mm−mr/hour (Normalized−95%)
1
10−1
10−2
0.02
0.04
0.06
0.08
0.1
x 10−2
σp

Summary and Outlook

We completed a fresh look at the intrabeam scattering rates for the Recycler Ring using Bjorken-Mtingwa formalism and that of Piwinsky. With in the high energy approximation, both results are consistent with each other.

- The transverse emittance growth rate is about 0.12 π-mm-mr/hour for a beam of 2×10^{12} particles, $\epsilon_{tN95\%} = 10$ π-mm-mr, $\sigma_p = 0.00025$.
- The longitudinal growth rate is 0.5/hr - may test the performance of the longitudinal cooling ??
- There are also similar calculations by Pat Colestock and Alexi Burov. The results are comparable.

Looking ahead:

- Look at the final equation of state for the Recycler Ring - all cooling and heating processes included both in transverse and longitudinal degrees of freedom
- Detailed treatment of Ion production and clearing mechanisms for the Recycler Ring.

Acknowledgement

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References

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