BNL 13/3/2000

# Polarized PDF's at the advent of RHIC

Marco Stratmann

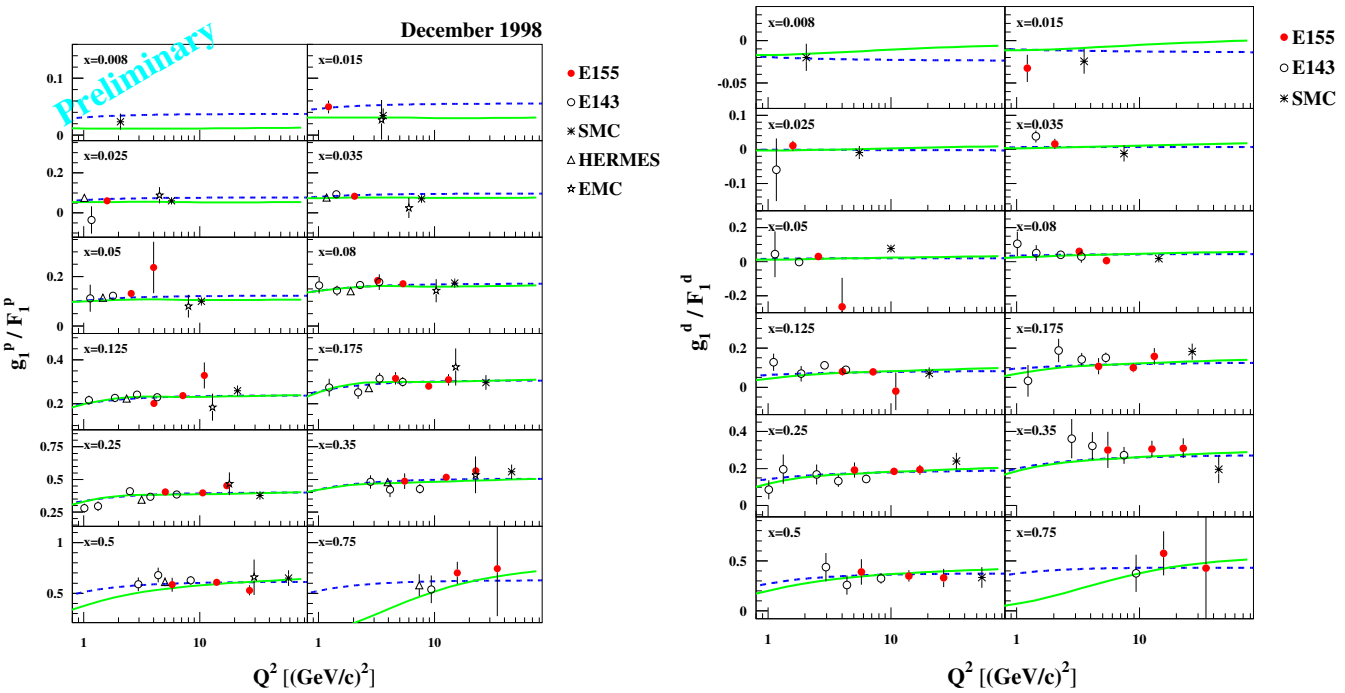
(Regensburg)

- Motivation: Towards a 'global' NLO analysis:
  - Available data
  - Status of QCD calculations
- Some theoretical issues/problems:
  - What can we learn from *inclusive* data *alone*?
  - Should one analyze $g_1$ or $A_1$?
  - Scheme dependence & transformations
  - Constraints: 'positivity', $F$ and $D$ values, ...
- NLO QCD analyses: overview & comparison
  - Available NLO analyses
  - GRSV update
- 'Beyond DIS': SU(2) breaking $(\Delta\bar{u} - \Delta\bar{d})$
  - Theoretical/phenomenological models
  - Impact of existing/upcoming HERMES data
  - 'Dynamical' SU(2) breaking in NLO QCD

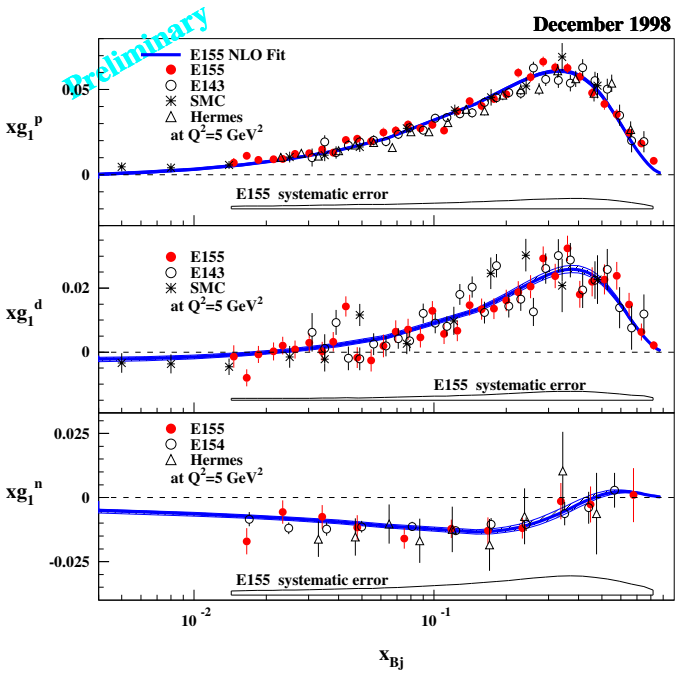
- Available data

## (I) Inclusive DIS data:

For the spin asymmetry $A_1(x, Q^2)$ ...

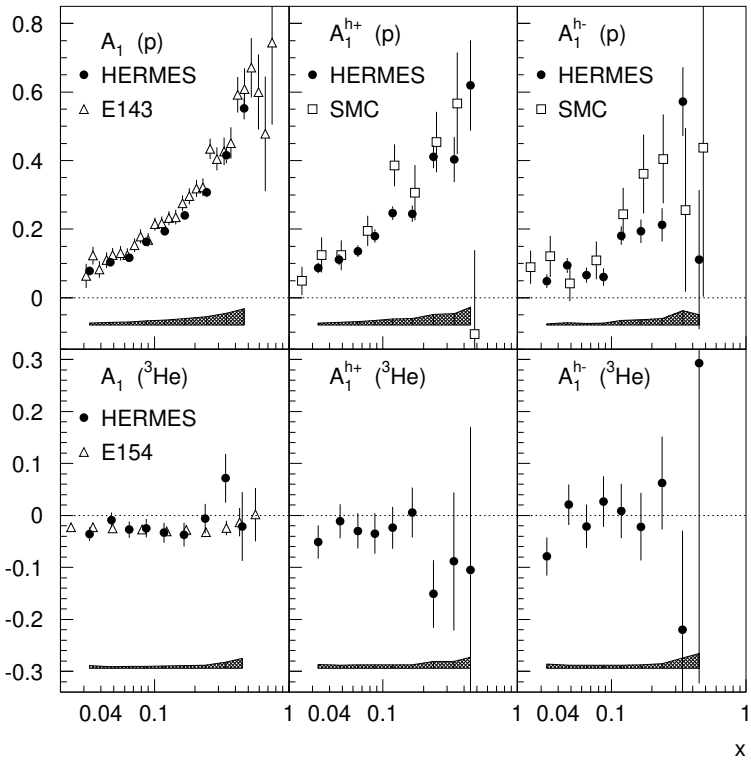



... or for $g_1(x, Q^2)$ ($\simeq A_1 \cdot F_1$)




(figures taken from E155)

## (II) Semi-inclusive DIS data:




## (III) Other data:

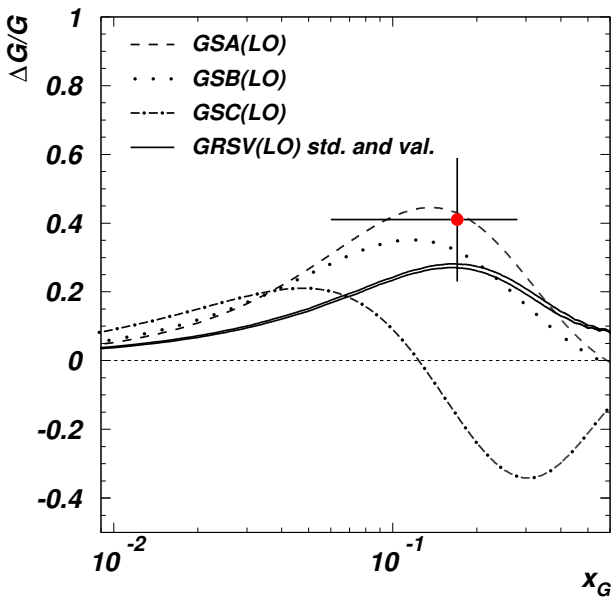
HERMES:

high-$p_T$ hadron pairs

many theor. uncertainties

⇝ only *some* indication that $\Delta g$ is *perhaps* sizeable

(NOT more!)




(figures taken from HERMES)

## (IV) Upcoming data:

HERMES: final $p$ data and more precise $d$ data (DIS and SIDIS)

E155: proton DIS data

# • Status of QCD calculations

Past years have seen a lot of theoretical progress ...

| | | | |
|---|---|---|---|
| evolution kernels | $\Delta P_{ij}$ (spacelike) | NLO | Mertig, van Neerven; Vogelsang |
| | $\Delta P_{i\gamma}$ (photon) | NLO | (trivial) Vogelsang, MS |
| | $\Delta P_{ij}$ (timelike) | NLO | Vogelsang, MS |
| DIS | $\vec{e}\vec{p} \to X$ | NLO | Kodaira et al.; Ratcliffe; Bodwin, Qiu |
| | | NNLO | van Neerven, Zijlstra |
| SIDIS | $\vec{e}\vec{p} \to H^{\pm}X$ | NLO | Chiappetta et al.; De Florian et al. |
| | $\vec{e}p \to \vec{H}^{\pm}X$ | NLO | De Florian et al.; MS |
| | $e\vec{p} \to \vec{H}^{\pm}X$ | NLO | De Florian et al.; MS |
| prompt photons | $\vec{p}\vec{p} \to \gamma X$ | NLO | Gordon, Vogelsang; Contogouris, et al. |
| | $\vec{p}\vec{p} \to \gamma\gamma X$ | NLO | Coriano, Gordon |
| | $\vec{e}\vec{p} \to \gamma X$ | NLO | Contogouris, et al. |
| | $p\vec{p} \to \vec{\gamma} X$ | NLO | Vogelsang |
| jets | $\vec{p}\vec{p} \to \text{jet(s)} + X$ | NLO | De Florian, et al. |
| | $\vec{\gamma}\vec{p} \to \text{jet(s)} + X$ | NLO | De Florian, Frixione |
| | $\vec{e}\vec{p} \to \text{jet(s)} + X$ | NLO | Mirkes, Willfahrt |
| photon + jet | $\vec{p}\vec{p} \to \gamma + \text{jet} + X$ | NLO | Gordon |
| | $\vec{\gamma}\vec{p} \to \gamma + \text{jet} + X$ | NLO | Gordon |
| photon + charm | $\vec{p}\vec{p} \to \gamma + c + X$ | NLO | ($m_c = 0$) Berger, et al. |
| heavy quarks | $\vec{p}\vec{p} \to Q\bar{Q}X$ | LO | Contogouris et al. |
| | | (NLO) | soon! Bojak, MS |
| | $\vec{\gamma}\vec{p} \to Q\bar{Q}X$ | NLO | Bojak, MS |
| | $\vec{e}\vec{p} \to Q\bar{Q}X$ | LO | Watson |
| | | (NLO) | soon! Smith |
| Drell-Yan | $\vec{p}\vec{p} \to \gamma^* X$ | NLO | Weber; Gehrmann |
| | | (NNLO) | (parts) Cheng, et al. |
| vector bosons | $\vec{p}\vec{p} \to (Z^0, W^{\pm})X$ | NLO | Weber; Gehrmann |
| | $p\vec{p} \to (Z^0, W^{\pm})X$ | NLO | Weber; Gehrmann |

plus improvements due to resummations, etc.

$\rightsquigarrow$ ready for a 'global' QCD analysis - only data are still lacking ...

- What can we learn from DIS data alone?

(see also discussion in Leader et al.)

In NLO QCD we have

$$g_1^{N=\frac{p}{n}}(x,Q^2) = \left[\pm\frac{1}{12}\Delta q_3^{\rm NS} + \frac{1}{36}\Delta q_8^{\rm NS} + \frac{1}{9}\Delta\Sigma\right] \otimes \left(1+\frac{\alpha_s}{2\pi}\Delta C_q\right) + \sum_q e_q^2 \frac{\alpha_s}{2\pi}\Delta g \otimes \Delta C_g$$

$\rightsquigarrow$ from perfect data (+ *assuming* QCD) we would get:

| | |
|---|---|
| $\Delta q_3$ | from $g_1^p - g_1^n$ |
| $\Delta q_8,\ \Delta\Sigma,\ \Delta g$ | from $g_1^p + g_1^n$ due to different $Q^2$-evolution |

however, in 'real life' we only get *some* information on

$$\Delta q_3,\ \Delta q_8,\ \Delta\Sigma \ \leftrightarrow\ \Delta q + \Delta\bar{q} \text{ (q=u,d,s)}$$

(no lever-arm in $Q^2$ to *really* study scaling violations $\leftrightarrow$ $\Delta g$)

Note that $\Delta s + \Delta\bar{s} = \frac{1}{3}(\Delta\Sigma - \Delta q_8)$ is fixed by DIS

but we cannot decide if the sea is SU(3) or not!

$\rightsquigarrow$ a transformation with arbitrary functions $f$ and $g$:

Vogelsang, MS

$$\begin{aligned}\Delta\bar{u}' &= \Delta\bar{u} + f\\ \Delta u' &= \Delta u - f\\ \Delta\bar{d}' &= \Delta\bar{d} + g\\ \Delta d' &= \Delta d - g\end{aligned} \quad\rightsquigarrow\quad \Delta\bar{u}' - \Delta\bar{d}' = f - g$$

(assuming $\Delta\bar{u} = \Delta\bar{d}$)

leaves $g_1$ and Bjorken-sum invariant!

$\rightsquigarrow$ relevance of semi-incl. HERMES and $W^\pm$ RHIC data

- Should one analyze $g_1$ or $A_1$?

In principle it shouldn't matter ...

measured: $A_\parallel = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = D(A_1 + \eta A_2)$

$$\rightsquigarrow\ g_1 = \underbrace{\frac{F_2}{2x(1+R)}}_{=F_1}(A_1 + \gamma A_2) \simeq F_1 A_1 \qquad (A_2 \sim g_2 \text{ small})$$

($R = \sigma_L/\sigma_T$ and $\eta$, $\gamma$ only depend on kinematics)

but the measured $x$, $Q^2$ range is tricky ...

$g_1$ analysis:

exp. extraction of $g_1$ requires knowledge of $F_1$
(in principle no problem since $F_2$ and $R$ are measured)

Experience from unpol. fits: CTEQ, GRV, MRST
leading twist description of $F_2$ fails at low $Q^2$ and/or high $x$
$\rightsquigarrow$ impose cuts ($Q^2 \gtrsim 4$, $W^2 \gtrsim 10$ GeV$^2$) to obtain twist-2 PDF's

but we cannot afford *such* 'safe' cuts for $g_1$ yet
(would loose 'small'-$x$ $\rightsquigarrow$ usually only $Q^2 \gtrsim 1$, $W^2 \gtrsim 4$ GeV$^2$ used)

possible/required improvements for a $g_1$ analysis:

- include target mass corrections (a la Georgi, Politzer)
  not only a factor $\sim x^2 M^2/Q^2$ $\rightsquigarrow$ possible but tedious
  (see, e.g., Ridolfi, Piccione; Blümlein, Tkabladze)
- higher twist effects a la $g_1 = g_1^{\mathrm{LT}}(1 + f(x)/Q^2)$
- wait for NNLO kernels (*may* eat up some HT)
- wait for high-energy (collider) data

$A_1$ analysis:

- higher twists may cancel in ratio
  (*of course*, there is no proof for that ...)
- experimental uncertainties should cancel in ratio
  (not *really* relevant since $F_1$ comes from *other* exp. anyway)

$\rightsquigarrow$ usually a leading twist ansatz for $g_1$ is used $(*)$

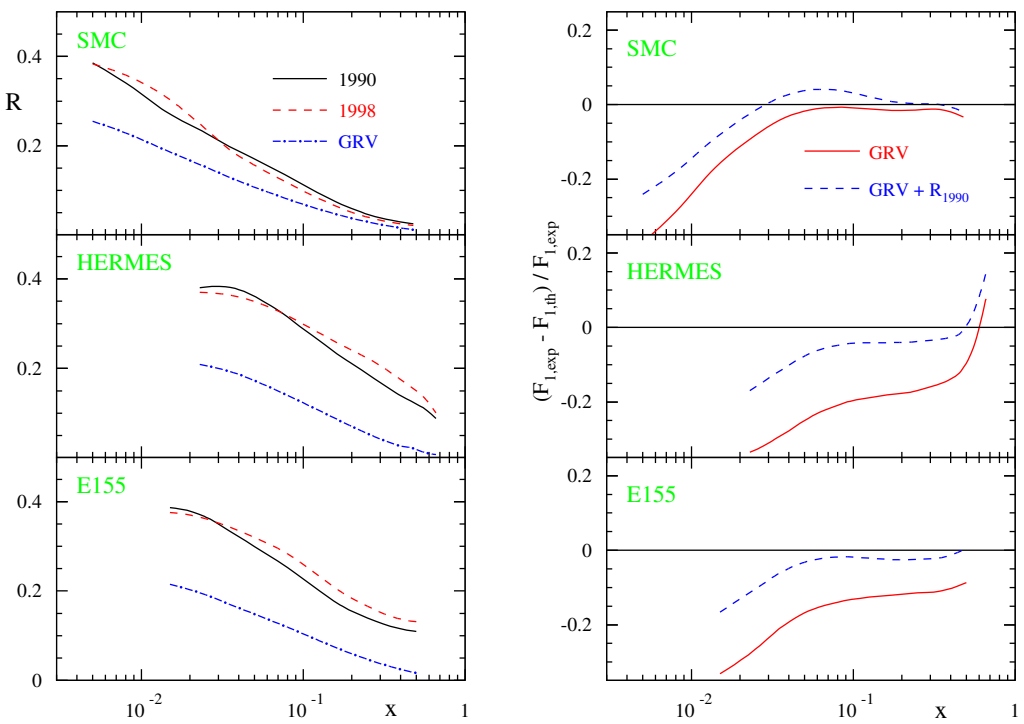
but we need *some* $F_1$ to extract $\Delta f$'s – 3 options:

(1) $F_2^{\mathrm{QCD}}$ and $R_{\mathrm{QCD}}$ (if you *believe* in HT cancellation)

(2) $F_2^{\mathrm{QCD}}$ and $R_{\mathrm{exp}}$ (since we know that $R_{\mathrm{QCD}} \neq R_{\mathrm{exp}}$)

(3) $F_2^{\mathrm{exp}}$ and $R_{\mathrm{exp}}$ (also $F_2^{\mathrm{QCD}} \neq F_2^{\mathrm{exp}}$ at relevant $x$, $Q^2$ )

(2)/(3): improved/good $F_1$ descrip. $\rightsquigarrow$ simul. 'fit' of $A_1$ and $g_1$




but: $F_2^{\mathrm{exp}}$ and $R_{\mathrm{exp}}$ contain $1/Q^2$ ($\leftrightarrow$ HT)

$\rightsquigarrow$ use of $(*)$ inconsistent for (2,3) as in $g_1$ analysis!

$\rightsquigarrow$ both methods have *similar* problems

Is there a *fully* consistent procedure ???

- Scheme dependence & transformations

Observables should not depend on schemes

$\rightsquigarrow$ simple relations between different schemes

Derivation of scheme transformation rules:

Idea: express $\mathrm{d}g_1/\mathrm{d}\ln Q^2$ in terms of $g_1$

(unpol. case: Glück, Reya)

$g_1$ measurable $\rightsquigarrow$ coeff. must not depend on schemes

$\rightsquigarrow$ $\Delta\hat{P}^{(1)} - \frac{\beta_0}{2}\Delta\hat{C} + [\Delta\hat{C}, \Delta\hat{P}^{(0)}]$ invariant

Changes in $\Delta C_{q,g}$ compensated by $Q^2$-evol. of $\Delta f$ !!

Explicitly for the PDF's:

$$\begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}_B = \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}_A + \frac{\alpha_s}{2\pi} \begin{pmatrix} \Delta z_{qq} & \Delta z_{qg} \\ \Delta z_{gq} & \Delta z_{gg} \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}_A$$

where $\quad \Delta C_q|_B = \Delta C_q|_A - \Delta z_{qq}$
$\Delta C_g|_B = \Delta C_g|_A - \Delta z_{qg} \qquad (*)$

Note:

$\Delta z_{gq}$ and $\Delta z_{gg}$ not specified by $(*)$

$\rightsquigarrow$ transformation of $\Delta g$ (and $\Delta\hat{P}^{(1)}$) not fixed by $(*)$

(Different in unpol. case: momentum sum rule constraint
$\rightsquigarrow$ 2nd moments of $\Delta z_{gq}$ and $\Delta z_{gg}$ fixed!)

- Schemes used in polarized NLO analyses

choice of scheme in analyses is a matter of taste

$\overline{\text{MS}}$ used in GRSV; Gehrmann, Stirling (GS); De Florian et al. (DSS); Asym. Anal. Collab. (AAC)

convenient

- $\Delta P_{ij}^{(1)}$ fully known + NLO cross sections usually in $\overline{\text{MS}}$

Main features:

- $1^{\text{st}}$ moment of $\Delta\Sigma$ is $Q^2$ dependent (LO: conserved)
- $1^{\text{st}}$ moment of $\Delta C_g$ vanishes

Important:

polarized $\overline{\text{MS}}$ requires a $\Delta z_{qq} \neq 0$ due to $\gamma_5$ treatment a la HVBM (see Vogelsang; Mertig, van Neerven)

$\Delta z_{qq}$ already included in final $\overline{\text{MS}}$ expressions for $\Delta P_{ij}^{(1)}$ and $\Delta C_q$

consistent $\gamma_5$'s give same results $\rightsquigarrow$ no ambiguity/family of $\overline{\text{MS}}$'s

Other -chiral invariant- schemes: Carlitz, Collins, Mueller; Efremov, Teryaev; Altarelli, Ross

Idea: take $\overline{\text{MS}}$ and reshuffle $\Delta\Sigma \overset{\Delta z_{qg}}{\leftrightarrow} \Delta C_g$ such that

$$\Delta P_{qq}^{n=1} = 0 \rightsquigarrow \Delta\Sigma^{n=1}(Q^2) = \text{const as in LO !}$$

Relation to $\overline{\text{MS}}$ densities:

$$\begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}_{\text{chiral}} = \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}_{\overline{\text{MS}}} + \frac{\alpha_s}{2\pi} \begin{pmatrix} 0 & \Delta z_{qg} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}_{\overline{\text{MS}}}$$

$\rightsquigarrow$ $\Delta g$ and non-singlet unchanged w.r.t. $\overline{\text{MS}}$

Two variants of 'chiral' schemes are in use:

(differ only in choice of $\Delta z_{qg}$; same $1^{\text{st}}$ moment)

## AB

Altarelli, Ball, Forte, Ridolfi (ABFR)

$x$ independent choice: $\Delta z_{qg}^{\text{ABFR}} = N_f$

## JET

Leader, Stamenov, Sidorov (LSS)
(see also Cheng; Müller, Teryaev)

*full* $x$ dependence of 'anomaly': $\Delta z_{qg}^{\text{JET}} = 2N_f(1-x)$

In practice:

scheme independence $\rightsquigarrow$ $\chi^2$ of $g_1$ fit should be invariant

However: cancellation of scheme dependence not perfect in NLO $\rightsquigarrow$ residual terms $\sim \mathcal{O}(\alpha_s^2)$

Simple check: (suggested by LSS)

| | | |
|---|---|---|
| $1^{\text{st}}$ step | NLO fit in scheme A | $\rightarrow \Delta f_A$ |
| $2^{\text{nd}}$ step | NLO fit in scheme B | $\rightarrow \Delta f_B$ |
| $3^{\text{rd}}$ step | scheme A $\overset{\text{transf. rules}}{\rightarrow}$ scheme B | $\rightarrow \Delta f_B$ |

(2) and (3) should give the same PDF's: $\approx$ OK LSS

But one should not forget that all
statements about NLO PDF's are scheme dependent

e.g. 'things' like *...quarks carry only* 20% *of the spin...*
must be supplemented by *...in the XX scheme*

## • Constraints: $F$, $D$, 'positivity' and all that

• 'positivity': $|\Delta\sigma| \leq \sigma$

LO: implies $|\Delta q| \leq q$ ($\leftrightarrow$ naive parton model)

and $|\Delta g| \leq g$ (from $gg \to H$ where $\Delta\hat{\sigma} = \hat{\sigma}$)

Altarelli, Forte, Ridolfi

NLO: $(\Delta)\hat{\sigma}$ and $(\Delta)f$ are not physical

$\rightsquigarrow$ bounds receive proc.-dep. NLO corrections

e.g. for NLO analysis of DIS (and $gg \to H$) AFR

▷ bounds less restrictive at small $x$; corrections small at large $x$

in practice: $|\Delta f| \leq f$ also in NLO (GRSV,GS,DSS, ... )

▷ bounds relevant in fits only at large $x$

▷ num. respected by $\overline{\text{MS}}$ evol. ($\Delta P_{qq}^{(1)} > P_{qq}^{(1)}$ as $x \to 1$ irrelevant)

▷ 'experience': $|\Delta\sigma| \leq \sigma$ OK for known NLO $\overline{\text{MS}}$ processes

• $1^{\text{st}}$ moments, sum rules

· $\Delta q_3(Q^2) = F + D = g_A$ (based on isospin SU(2) $\leftrightarrow$ Bj.-sum) ✓

· $\Delta q_8(Q^2) = 3F - D$ (if hyperons = SU(3) octet) ×

(relation questionable (Lipkin); new test (KTeV): $\Xi^0 \to \Sigma^+ e\bar{\nu}$)

· no momentum sum rule, etc. $\rightsquigarrow$ no relation $\Delta q$'s $\leftrightarrow$ $\Delta g$ ×

• ansatz for polarized PDF's

· form of ansatz $\leftrightarrow$ theor. bias in fit (difficult to quantify)

e.g. $\Delta f \sim x^\alpha (1-x)^\beta f$ $\leftrightarrow$ no oscillations allowed (e.g. in GRSV)

$\rightsquigarrow$ statements about error $\delta f$ on $\Delta f(Q^2)$ difficult

e.g. $\Delta g(x, Q_0^2) = N_g\, x^\alpha\, g \rightsquigarrow \delta g$ small, $\Delta g(Q^2)$ sizeable LSS

but extra $(1-x)^\beta$, etc. possible $\rightsquigarrow \delta g$ large, $\Delta g(Q^2)$ perhaps small

· 'Helicity retention' $(*)$: $\Delta f_v \to f_v$ as $x \to 1$ Brodsky et al.

$\rightsquigarrow$ $\Delta d_v$ must change sign as $x \to 1$ !?, but $(*)$ $\not\leftrightarrow$ pert. QCD ×

- ‘Market view’: NLO QCD analyses

unpolarized:

lots and lots of data; 3 groups: CTEQ, GRV, MRST

polarized: sparse data; ever growing number (!) of fits:

| | |
|---|---|
| GRSV | Glück, Reya, Stratmann, Vogelsang |
| GS | Gehrmann, Stirling |
| ABFR | Altarelli, Ball, Forte, Ridolfi |
| DSS | De Florian, Sampayo, Sassot |
| SMC | SM Collaboration |
| E154 | E154 Collaboration |
| E155 | E155 Collaboration |
| LSS | Leader, Sidorov, Stamenov |
| BBPSS | Bourrely, Buccella, Pisanti, Santorelli, Soffer |
| GGR | Gordon, Goshtasbpour, Ramsey |
| TBK | Tatur, Bartelski, Kurzela |
| GGI | Ghosh, Gupta, Indumathi |
| AAC | Asymmetry Analysis Collaboration |

(highly likely that I still missed some)

Useful/necessary future project:

all groups use *different* methods to solve DGLAP eqs.

$\rightsquigarrow$ detailed comparison of evolution codes is useful

Unpol.: systematic checks using ‘toy inputs’ and $\alpha_s$ values
Blümlein, Riemersma, Botje, Pascaud, Zomer, van Neerven, Vogt

painful, but triggered resolution of small bug in MRST code!

$\rightsquigarrow$ most codes (except CTEQ) now agree on ‘per mille level’

Anyway, let’s have a closer look at all these fits ...