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Simplified Treatment of Collective Instabilities in Matter

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By exploiting the simplicity emerging when the temperature is small in comparison with the Fermi energy, it is possible to treat the collective modes in unstable nuclear matter on a nearly analytical form. Some of the key results are outlined and illustrated.

1. INTRODUCTION

Catastrophic phenomena occur throughout the natural world, including nuclear physics. A simple and well-known example is the irreversible division of a fissionable nucleus into two receding fragments, whose sizes and velocities have stochastic values. In nuclear collisions at intermediate energies a large number of qualitatively different channels are open and catastrophic processes are prevalent. In order to describe the dynamics of such processes, it is necessary to incorporate the occurrence of branchings in the evolution of the system, thus allowing a given state to effectively choose between a variety of future evolutions, each one leading to a different appearance of the system.

The nuclear Boltzmann-Langevin model [1] gives a semi-classical description of the nuclear system in terms of its reduced one-particle phase-space density $f(\boldsymbol{r},\boldsymbol{p})$ whose evolution in time is governed by the combined action of the effective one-body field $h[f]$ and the residual Pauli-blocked two-body collisions between individual nucleons, $\dot{f} = I[f]$. The collisionless, self-consistent Vlasov evolution on the left is modified by the dissipative average effect of the collisions, $\bar{I}[f]$, and the diffusive effect of the fluctuating remainder $\delta I[f]$ arising from the stochastic character of the individual nucleon-nucleon collisions. The corresponding distribution of phase-space densities, $\phi[f]$, is amenable to a Fokker-Planck transport treatment [2], in which the dissipative and fluctuating effects are described by the corresponding transport coefficients, the drift coefficient $V[f](\boldsymbol{r},\boldsymbol{p})$ and the diffusion coefficient $D[f](\boldsymbol{r},\boldsymbol{p};\boldsymbol{r}',\boldsymbol{p}')$.

2. COLLECTIVE MODES IN UNSTABLE MATTER

On this basis, a formal analysis was made of the onset of instabilities in dilute nuclear matter in which collective modes are agitated by the stochastic collisions and then exponentially amplified by the self-consistent field [3]. Moreover, simple approximate expressions were recently derived for the transport coefficients [4], thus facilitating analytical studies as well as significantly reducing the numerical effort associated with simulating the transport problem. Taking advantage of those approximate results of, the present study revisits the problem of nuclear matter in the spinodal zone, for the purpose of deriving simple expressions for the key quantities, so that survey calculations are facilitated.

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Instabilities in uniform matter are conveniently discussed in terms of the Landau parameter $F_0 = (3\rho/2\epsilon_F)(\partial h/\partial\rho)$. When harmonic modes in matter are considered, this quantity generalizes to $F_0(k) \equiv (3\rho/2\epsilon_F)(\partial h_k/\partial\rho)$, involving the appropriate Fourier component of the self-consistent response. Further generalization is useful at finite T,

$$F_T(k) \equiv \phi_0 \frac{\rho}{T}\frac{\partial h_k}{\partial\rho} \approx F_0(k)[1 - \frac{\pi^2}{12}(\frac{T}{\epsilon_F})^2] , \qquad (1)$$

employing the Sommerfeld expansion of the Fermi-surface moment ϕ_0 [5]. The dispersion relation for the collective growth time $t_k \equiv m/kP_F\gamma_k$ is then to a good approximation given by $1 = F_T(k)(\gamma_k \arctan(1/\gamma_k) - 1)$, which can readily be solved by iteration:

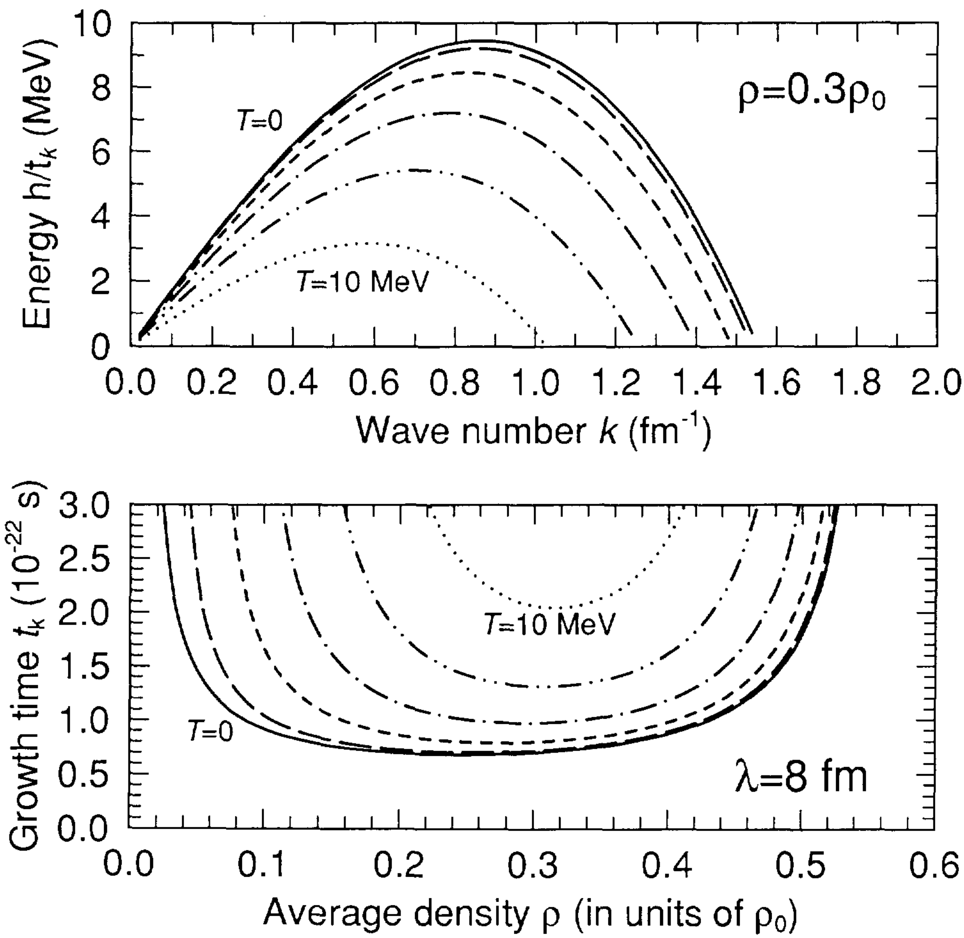

Figure 1: Characteristic times for the collective modes in unstable matter, based on the generalized Seyler-Blanchard model [6] with χ=1 for the effective Hamiltonian $h[\rho](\boldsymbol{r},\boldsymbol{p})$.

3. COLLECTIVE DYNAMICS

For small amplitudes, the dynamcial problem separates according to the wave number of the undulations. For each wave vector $\mathbf{k}$, there are two exponentially evolving conjugate collective modes, one increasing and the other decreasing. The collective motion may then be expanded on the associated eigencomponents [3],

$$f_{\mathbf{k}}^{\pm}(\boldsymbol{p}) = \frac{\partial h_k}{\partial \rho} \frac{\mathbf{k}\cdot\boldsymbol{p}}{\mathbf{k}\cdot\boldsymbol{p} \mp im/t_k} \frac{\partial f^0}{\partial \epsilon} \ . \tag{2}$$

The ensemble average of the corresponding amplitudes vanishes, $\prec A_{\mathbf{k}}^{\nu}(t) \succ = 0$. However, each individual system develops stochastically, and the typical magnitude of the amplitudes and their correlation are most conveniently described by the covariances $\sigma_k^{\nu\nu'}(t) = \prec A_{\mathbf{k}}^{\nu}(t)^* A_{\mathbf{k}}^{\nu'}(t) \succ$. In the linear regime, these quantities evolve as follows [3],

$$\frac{d}{dt}\sigma_k^{\nu\nu'} = 2\mathcal{D}_k^{\nu\nu'} + \frac{\nu+\nu'}{t_k}\sigma_k^{\nu\nu'} \quad : \quad \sigma_k^{\nu\nu'}(t) = 2\mathcal{D}_k^{\nu\nu'} \mathrm{e}^{(\nu+\nu')t/t_k} \int_0^t dt' \ \mathrm{e}^{-(\nu+\nu')t'/t_k} \ . \tag{3}$$

The source terms $\mathcal{D}_k^{\nu\nu'}$ are responsible for agitating the collective modes and they can be obtained from the general diffusion coefficients by on orthogonal projection [3],

$$\mathcal{D}_k^{\nu_1\nu_2} = \sum_{\nu_1'\nu_2'} o_k^{\nu_1\nu_1'} \Delta_k^{\nu_1'\nu_2'} o_k^{\nu_1\nu_1'} \approx \frac{1}{t_0}\frac{9}{5}F_T(k)^2 \frac{\rho T}{\epsilon_F}\left[(\mathcal{F}_k^{-1})^{\nu\nu'} + \frac{5}{6}(\mathcal{F}_k^{-1}\mathcal{G}_k\mathcal{F}_k^{-1})^{\nu\nu'}\right] \ . \tag{4}$$

In the general expression, $o_k^{\nu\nu'}$ is the inverse of the 2×2 overlap matrix for the eigenmodes,

$$(o_k^{-1})^{\nu\nu'} \equiv g\int \frac{d\boldsymbol{p}}{h^3} \ f_{\mathbf{k}}^{\nu}(\boldsymbol{p})^* f_{\mathbf{k}}^{\nu'}(\boldsymbol{p}) \ \approx \ \frac{1}{9}\frac{1}{F_T(k)} \frac{\epsilon_F}{\rho T} \ \mathcal{F}_k^{\nu\nu'} \ , \tag{5}$$

where the approximate result exploits the peaking of the integrand in the Fermi surface, and $\Delta_k^{\nu\nu'}$ is the matrix element of the diffusion coefficient with respect to the eigenmodes.

The approximate formulas have been derived by utilizing the results of [4] and recently derived expressions for Fermi-surface moments generalized to higher degree [7]. Then the 2×2 matrices $\mathcal{F}_k$ and $\mathcal{G}_k$ are given as elementary angular averages over $\mu = \hat{\boldsymbol{p}}\cdot\hat{\mathbf{k}}$,

$$\mathcal{F}_k^{\nu\nu'} \ \equiv \ < \frac{\mu}{\mu + i\nu\gamma_k} \ \frac{\mu}{\mu - i\nu'\gamma_k} > \approx \begin{cases} -F_T(k)^{-1} & (\nu = \nu') \\ -2F_T(k)^{-1} - (1+\gamma_k^2)^{-1} & (\nu \neq \nu') \end{cases} , \tag{6}$$

$$\mathcal{G}_k^{\nu\nu'} \ \equiv \ << \frac{\mu_1}{\mu_1 + i\nu\gamma_k} \ G_{12} \ \frac{\mu_2}{\mu_2 - i\nu'\gamma_k} >> \approx \ -\frac{1+3\nu\nu'\gamma_k^2}{F_T(k)^2} \ . \tag{7}$$

The function $G(\theta_{12})$ expresses the angular dependence of the non-diagonal part of the diffusion coefficient [4], and the above approximation for $\mathcal{G}_k$ emerges in the minimal model, which includes only those correlations that are dictated by conservation laws. Furthermore, $t_0^{-1} \approx \rho\sigma_0 V_F(\pi T/\epsilon_F)^2/[1 + (\pi T/\epsilon_F)^2]$ governs the overall agitation rate.

The resulting evolution is then described by the variance $\sigma_k(t) = \sum_{\nu\nu'}\sigma_k^{\nu\nu'}(t)$, measuring the spectral distribution of the density fluctuations. The relatively slow initial growth, $\sigma_k \approx 4(\mathcal{D}_k^{++} + \mathcal{D}_k^{+-})t$, is soon replaced by an explosive behavior, $\sigma_k \approx \mathcal{D}_k^{++}\exp(2t/t_k)$.

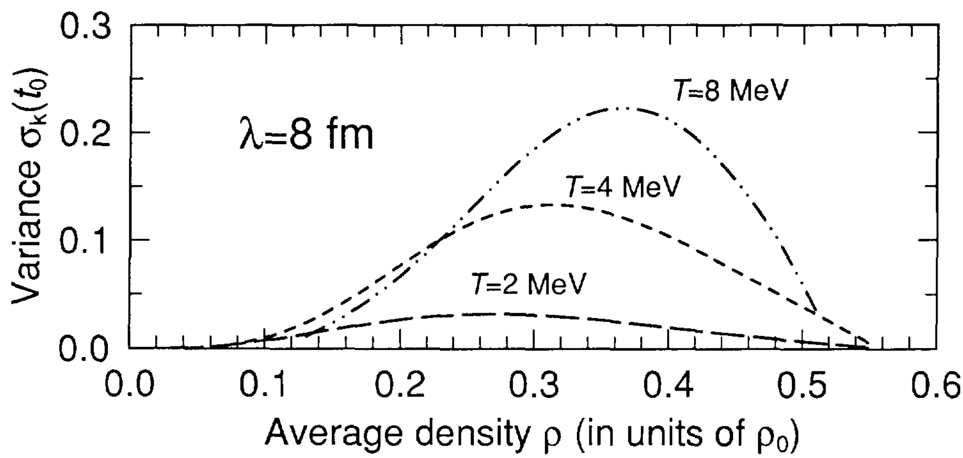

Figure 2: The magnitude of the variance σ_k of density fluctuations having the wave length $\lambda = 2\pi/k = 8$ fm (near which the most rapid growth occurs), after a given time $t_0 = 10^{-22}$ s, as a function of the average density ρ and for specified temperatures T.

4. CONCLUSION

The above analytical approximations facilitate the analysis of the onset of fragmentation in the spinodal zone of the phase diagram. Both the source terms for the fluctuations, $\mathcal{D}_k^{\nu\nu'}$, and the amplification times t_k can be easily obtained. Several additional results are also useful, including the expansion of the angular quantities on complex Legendre polynomials, which is helpful for understanding the multipolarity properties of the BL model.

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