3 SUSY Lagrangians Part II

3.1 SUSY Yang-Mills

$$\delta_{\text{gauge}} A^a_\mu = -\partial_\mu \Lambda^a + g f^{abc} A^b_\mu \Lambda^c \tag{3.1}$$
$$\delta_{\text{gauge}} \lambda^a = g f^{abc} \lambda^b \Lambda^c \tag{3.2}$$

where Λ^a is an infinitesimal gauge transformation parameter, g is the gauge coupling, and f^{abc} are the antisymmetric structure constants

$$\begin{array}{ccc} & \text{Off Shell} & \text{On Shell} \\ A^a_\mu & 3\,\text{d.o.f.} & 2\,\text{d.o.f.} \\ \lambda_\alpha, \lambda^\dagger_{\dot\alpha} & 4\,\text{d.o.f.} & 2\,\text{d.o.f.} \end{array} \tag{3.3}$$

Book-keeping trick: add a real auxillary boson field D

$$\begin{array}{ccc} & \text{Off Shell} & \text{On Shell} \\ D & 1\,\text{d.o.f.} & 0\,\text{d.o.f.} \end{array} \tag{3.4}$$

$$\mathcal{L}_{\text{SYM}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i \lambda^{\dagger a} \overline{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a \tag{3.5}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu \tag{3.6}$$

and

$$D_\mu \lambda^a = \partial_\mu \lambda^a - g f^{abc} A^b_\mu \lambda^c \tag{3.7}$$

$$\delta A^a_\mu = -\frac{1}{\sqrt{2}} \left[\epsilon^\dagger \overline{\sigma}_\mu \lambda^a + \lambda^{\dagger a} \overline{\sigma}_\mu \epsilon \right] \tag{3.8}$$
$$\delta \lambda^a_\alpha = -\frac{i}{2\sqrt{2}} (\sigma^\mu \overline{\sigma}^\nu \epsilon)_\alpha \, F^a_{\mu\nu} + \frac{1}{\sqrt{2}} \epsilon_\alpha \, D^a \tag{3.9}$$
$$\delta \lambda^{\dagger a}_{\alpha} = \frac{i}{2\sqrt{2}} (\epsilon^\dagger \overline{\sigma}^\nu \sigma^\mu)_\alpha \, F^a_{\nu\mu} + \frac{1}{\sqrt{2}} \epsilon^\dagger_\alpha \, D^a \tag{3.10}$$
$$\delta D^a = \frac{-i}{\sqrt{2}} \left[\epsilon^\dagger \overline{\sigma}^\mu D_\mu \lambda^a - D_\mu \lambda^{\dagger a} \overline{\sigma}^\mu \epsilon \right]. \tag{3.11}$$

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)D_\mu X^a \tag{3.12}$$

for $X^a = F^a_{\mu\nu}$, λ^a, $\lambda^{\dagger a}$, D^a, this requires the identities:

$$\xi\sigma^\mu\overline{\sigma}^\nu\chi = \chi\sigma^\nu\overline{\sigma}^\mu\xi = (\chi^\dagger\overline{\sigma}^\nu\sigma^\mu\xi^\dagger)^* = (\xi^\dagger\overline{\sigma}^\mu\sigma^\nu\chi^\dagger)^*; \tag{3.13}$$

$$\overline{\sigma}^\mu\sigma^\nu\overline{\sigma}^\rho = -\eta^{\mu\rho}\overline{\sigma}^\nu + \eta^{\nu\rho}\overline{\sigma}^\mu + \eta^{\mu\nu}\overline{\sigma}^\rho + i\epsilon^{\mu\nu\rho\kappa}\overline{\sigma}_\kappa; \tag{3.14}$$

$$\sigma^\mu_{\alpha\dot{\alpha}}\overline{\sigma}^{\dot{\beta}\beta}_\mu = 2\delta^\beta_\alpha\delta^{\dot{\beta}}_{\dot{\alpha}}. \tag{3.15}$$

3.2 SUSY Gauge Theories

Add chiral supermultiplets in a gauge representaion with hermitian matrices $(T^a)_i{}^j$

$$[T^a, T^b] = if^{abc}T^c. \tag{3.16}$$

$$\delta_{\text{gauge}}X_j = ig\Lambda^a(T^aX)_j \tag{3.17}$$

for $X_j = \phi_j, \psi_j, F_j$

covariant derivatives:

$$D_\mu\phi_j = \partial_\mu\phi_j + igA^a_\mu(T^a\phi)_j \tag{3.18}$$

$$D_\mu\phi^{*j} = \partial_\mu\phi^{*j} - igA^a_\mu(\phi^*T^a)^j \tag{3.19}$$

$$D_\mu\psi_j = \partial_\mu\psi_j + igA^a_\mu(T^a\psi)_j. \tag{3.20}$$

New renormalizable interactions

$$(\phi^*T^a\psi)\lambda^a, \qquad \lambda^{\dagger a}(\psi^\dagger T^a\phi) \qquad \text{and} \qquad (\phi^*T^a\phi)D^a. \tag{3.21}$$

$$\delta\phi_j = \epsilon\psi_j \tag{3.22}$$

$$\delta(\psi_j)_\alpha = -i(\sigma^\mu\epsilon^\dagger)_\alpha\, D_\mu\phi_j + \epsilon_\alpha F_j \tag{3.23}$$

$$\delta F_j = -i\epsilon^\dagger\overline{\sigma}^\mu D_\mu\psi_j \;+\; \sqrt{2}g(T^a\phi)_j\,\epsilon^\dagger\lambda^{\dagger a}. \tag{3.24}$$

$$\begin{aligned}\mathcal{L} \;=\; & \mathcal{L}_{\text{SYM}} + \mathcal{L}_{\text{WZ}} \\ & -\sqrt{2}g\left[(\phi^*T^a\psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a\phi)\right] \\ & +g(\phi^*T^a\phi)D^a.\end{aligned} \tag{3.25}$$

Here $\mathcal{L}_{\text{WZ}}$ means the chiral supermultiplet lagrangian in Eq. 2.58, but with gauge-covariant derivatives, For the theory to be gauge invariant we must have a gauge invariant superpotential i.e.

$$\delta_{\text{gauge}} W \propto W^i (T^a \phi)_i = 0. \tag{3.26}$$

D^a equation of motion:

$$D^a = -g(\phi^* T^a \phi). \tag{3.27}$$

thus the scalar potential is:

$$V(\phi, \phi^*) = F^{*i} F_i + \frac{1}{2} D^a D^a = W_i^* W^i + \frac{1}{2} g^2 (\phi^* T^a \phi)^2. \tag{3.28}$$

$$V(\phi, \phi^*) \geq 0 \tag{3.29}$$

We have the following types of Feynman veritces:

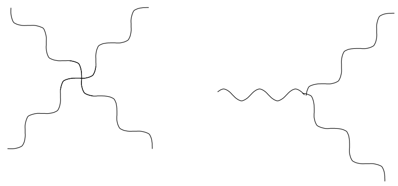

Figure 1: Yang-Mills interactions.

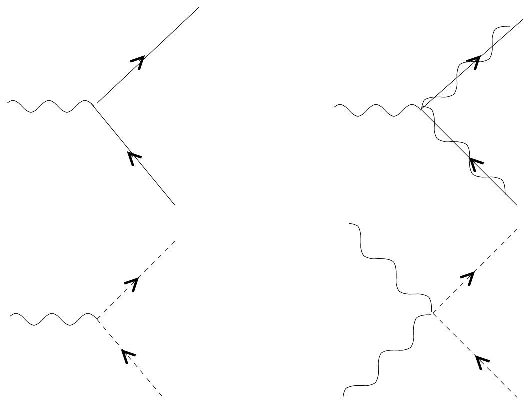

Figure 2: Interactions required by gauge invariance.

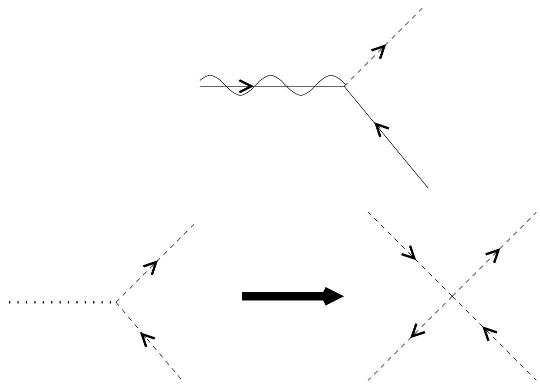

Figure 3: Additional interactions required by gauge invariance and supersymmetry: gaugino-fermion-scalar coupling and (scalar)^2 auxillary field coupling which become a quartic coupling after integrating out the auxilary field.

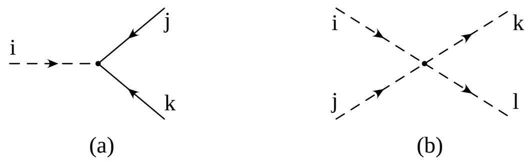

Figure 4: The dimensionless non-gauge interaction vertices in a supersymmetric theory: (a) scalar-fermion-fermion Yukawa interaction y^{ijk}, (b) quartic scalar interaction $y^{ijn}y^*_{kln}$.

using the Noether theorem we find the conserved supercurrent:

$$\begin{aligned} J^\mu_\alpha &= (\sigma^\nu\overline{\sigma}^\mu\psi_i)_\alpha\, D_\nu\phi^{*i} - i(\sigma^\mu\psi^{\dagger i})_\alpha\, W^*_i \\ &\quad - \frac{1}{2\sqrt{2}}(\sigma^\nu\overline{\sigma}^\rho\sigma^\mu\lambda^{\dagger a})_\alpha\, F^a_{\nu\rho} - \frac{i}{\sqrt{2}} g\phi^* T^a\phi\, (\sigma^\mu\lambda^{\dagger a})_\alpha, \end{aligned} \tag{3.30}$$

References

[1] S. Martin, "A Supersymmetry Primer", hep-ph/9709356.

[2] J. Wess and B. Zumino, *Nucl. Phys.* **B78**, 1 (1974).

[3] J. Wess and B. Zumino, *Nucl. Phys.* **B79**, 413 (1974).

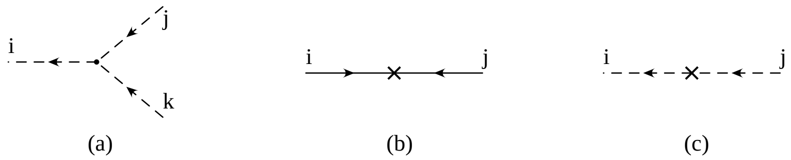

Figure 5: Supersymmetric dimensionful couplings: (a) (scalar)^3 interaction vertex $M^*_{in} y^{jkn}$, (b) fermion mass term M^{ij}, (c) scalar (mass)^2 term $M^*_{ik} M^{kj}$.