Transverse Flow at Ultrarelativistic Energies

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Abstract

Within the framework of a phenomenological hydrodynamical model, the relationship of the freeze-out temperature parameter T_f and the transverse flow velocity $v_\perp$ is discussed. It is found that the experimental single-particle distributions are not very sensitive to the exact form of the assumed velocity profile, but that there is a definite anticorrelation between T_f and the average $v_\perp$.

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Hydrodynamical models have been successful in describing the space-time evolution of heavy-ion collisions from energies of a few hundred MeV per nucleon at the LBL Bevelac and GSI SIS to ultrarelativistic energies of a few tens and hundreds of GeV per nucleon at the BNL AGS and CERN SPS [1]- [9]. The experimentally measured transverse momentum distributions [9]- [12] are well reproduced by hydrodynamical calculations in which the collective degrees of freedom, such as the freeze-out temperature and transverse velocity are unambiguously identified. In addition, in two-particle interferometry analysis, the hydrodynamical behavior that was predicted by several theory groups [13]- [15] has also been observed in experiments NA35 [16] and NA44 [17].

A distinct advantage of hydrodynamical approaches lies in their universality. The macroscopic characteristics of hydrodynamical systems are independent of the underlying kinematics that led to the assumed local equilibrium. For this reason, the incredibly complex systems formed by heavy-ion collisions can be reasonably well described by using just a few hydrodynamical parameters.

A disadvantage of hydrodynamics, however, is that quantities that are not directly measurable, such as energy density ρ_ϵ, pressure P, velocity fields v^μ, etc., must be used as the inputs. Measurable quantities, like one-particle momentum distributions and two-particle correlation functions, are then straightforwardly calculated. Quantitative comparisons of these predicted distributions with experimentally measured ones often lead to better sets of inputs for the model calculations. In this way, some understanding, at least within the language of the model, is thus attained.

In this paper, we use a phenomenological hydrodynamic model [8] with several velocity profiles to fit the one-particle transverse momentum spectra of pions, kaons and protons measured from both 200A·GeV/c S+S and 158A·GeV/c Pb+Pb central collisions [12]. Since the velocity profile that we use as input is ultimately connected with the particle phase-space density profile that carries the important information of collision dynamics, we hope to learn whether the one-particle transverse momentum distributions are sensitive to the velocity profile, and therefore to the phase-space density profile.[1]

Recent results on one-particle transverse momentum spectra of heavy-ion collisions at the CERN SPS energies can be found in Refs. [10]- [12]. The transverse momentum distribution inverse slope parameters as a function of particle mass are summarized in Fig. 2 of Ref. [10] for three symmetric colliding systems. In each case, we used only data that were close to mid-rapidity $y_{cm} \approx 3$. The common feature in these spectra is that the inverse slope parameter increases with particle mass. Since transverse collective flow leads to larger increases in the inverse slope parameter for larger mass particles, such an observed increasing suggests that transverse flow may have developed in heavy-ion collisions. In addition, as can be seen in the figure, the dependence is even stronger for heavier incoming nuclei. Note that the absence of a mass dependence in p+p collisions almost certainly indicates the absence of collective flow in those systems.

According to the phenomenological hydro model presented in [8], the invariant cross section can be expressed as:

$$\frac{1}{m_T}\frac{dN}{dm_T} \propto \int_0^{R_{\max}} r dr m_T I_0(\frac{p_T \sinh\rho}{T_f}) K_1(\frac{m_T \cosh\rho}{T_f}), \tag{1}$$

where I_0, K_1 are modified Bessel functions and $\rho = \tanh^{-1} v_\perp(r)$. Here T_f and $m_T = \sqrt{p_T^2 + m^2}$ are the freeze-out temperature and the transverse mass of the particle. The transverse velocity profile $v_\perp$ is parameterized by the surface velocity $v_{\perp,\max}$:

$$v_\perp(r) = v_{\perp,\max} \left(r/R_{\max}\right)^\alpha . \tag{2}$$

In Fig. 1(a), velocity profiles are shown with $\alpha = 0.5$, 1, and 2. Velocity profiles extracted from the transport model RQMD (v1.08) [18,19] are shown in Fig. 1(b).[2] The parametraiza-

[1]Only mid-rapidity one-particle transverse momentum distributions [12] are used in this study. However, this approach is rather general and it does not depend on any specific data set as long as the analysis is self-consistent.

[2]Only particles within the rapidity window $\mid y - 3 \mid \leq 1$ were used in constructing the profile.

tion of RQMD velocity profile is:

$$v_{\perp} = \frac{v_{\perp,\max}}{1 + exp(-a/(r-b))}. \tag{3}$$

With the parameters are: ($a = 0.87, b = 3.7$) and ($a = 0.6, b = 6.5$) for S+S and Pb+Pb collision, respectively. For each velocity profile, one-particle p_T distributions can be calculated as a function of T_f and $v_{\perp,\max}$. Theoretical distributions for pions, kaons, and protons can then be compared to experimental data, and an overall χ^2 for simultaneous fits to these particles can be constructed.[3] Contour maps of χ^2 as a function of T_f and $v_{\perp,\max}$ are shown in Fig. 2, where the top panel is for S+S and bottom is for the Pb+Pb collisions. Figs. 2(a)-(d) represent the results using velocity profiles of Eq. (2) with $\alpha = 0.5$, 1, 2, and of RQMD, respectively. The valley in each plot indicates the most probable combination of T_f and $v_{\perp,\max}$.

Since the data can be fit equally well by each of the four transverse flow profiles discussed above, it is impossible to rule out any of these four strictly on the basis of shape of the one-particle distribution data. Moreover, for any given flow profile, the fitting procedure does not result in a single best temperature and flow velocity, rather they identify acceptable ranges in both of these quantities. However, higher temperatures imply lower flow velocities and vice versa. Note that only light particle (pions, kaons, and protons) distributions are used in this study. It has been pointed out in Ref. [20] that the heavy clusters could be sensitive to the velocity profile. A similar analysis, with a linear flow velocity profile only, has been performed [21] for pion, kaon (including K_s^0), proton, and Λ particles measured in the NA49 experiment [22,23]. The study leads to the freeze-out temperature and averaged flow velocity to be T = 120 MeV and $v_T^{aver} = 0.43$c, respectively. This is in agreement with our result although we could not identify an unique minimum in the temperature and collective flow velocity map.

[3] To avoid the influence of resonance decays, the pion fits start at $m_T - mass \geq 250$ MeV [10]. Otherwise, all fits start from zero.

This relationship between T_f and $v_\perp$ can be made more general by looking at average flow velocities rather than maximum flow velocities. The first three flow profiles are defined by Eq. (2). The most naive way to take an average of $v_\perp(r)$ is to look strictly at the transverse geometry of the models and define

$$\langle v_\perp \rangle = \frac{\int_0^{R_{\max}} r\, dr\, v_\perp(r)}{\int_0^{R_{\max}} r\, dr} = \left(\frac{2}{2+\alpha}\right) v_{\perp,\max}\,. \tag{4}$$

A more sophisticated averaging can be achieved by incorporating not only the transverse geometry but also the phase-space density of particles at a given radius:

$$\langle v_\perp \rangle = \frac{\int d^2p_\perp \int_0^{R_{\max}} r\, dr\, v_\perp(r)\, I_0(\gamma_\perp(r) v_\perp(r) p_\perp/T_f) K_1(\gamma_\perp(r) m_\perp/T_f)}{\int d^2p_\perp \int_0^{R_{\max}} r\, dr\, I_0(\gamma_\perp(r) v_\perp(r) p_\perp/T_f) K_1(\gamma_\perp(r) m_\perp/T_f)}. \tag{5}$$

According to this definition, $\langle v_\perp \rangle / v_{\perp,\max}$ is actually a function of T_f and $v_{\perp,\max}$. However for the values of these parameters that best fit the data, $\langle v_\perp \rangle / v_{\perp,\max}$ can be well approximated by the constants $0.82 \pm 3\%$, $0.70 \pm 5\%$, and $0.60 \pm 10\%$ for $\alpha = 0.5$, 1, and 2, respectively. Notice that for small values of α, the naive geometrical average of Eq. (3) is almost identical to the more sophisticated average of Eq. (4), but that the difference between these averages becomes more pronounced as α gets larger.

Based on the best fits to the one-particle distribution data, it is now possible to write down explicit relationships between the freeze-out temperatures and average transverse flow velocities of each of the reactions considered. Namely for temperatures within the range 100 MeV $< T_f <$ 150 MeV,

$$\begin{aligned} \text{for} \quad \mathrm{S+S:} \quad & \langle v_\perp \rangle / c \simeq 0.72 - 0.0026 \times T/MeV \\ \text{for} \quad \mathrm{Pb+Pb:} \quad & \langle v_\perp \rangle / c \simeq 0.77 - 0.0035 \times T/MeV. \end{aligned} \tag{6}$$

For example, if other data or considerations [9] led one to expect a freeze-out temperature of 140 MeV,[4] then the data considered here would identify average flow velocities for S+S and Pb+Pb of roughly $0.28c$ and $0.41c$, respectively.

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[4]From inspection of Fig. 2 of Ref. [10], one might conclude that the inverse slopes converge to a value near pion mass at the limit of light particle mass [24].

Perhaps the most promising avenue for pinning down temperatures and flow velocities from the ranges given above is by studying two-particle correlations in conjunction with one-particle distributions. This point has been discussed earlier in Refs. [15,25]. Analytical models [26,27] similar to those discussed in this paper show that the dependence of the "side" radius on the average transverse momentum of the pair is governed by the parameter $(\langle v_\perp \rangle^2/T_f)$. [5] In this case, measurement of that parameter combined with Eq. (5) would be enough to unambiguously determine both T_f and $\langle v_\perp \rangle$.

Using a transport model, *e.g.* RQMD, it is also possible to evaluate the average transverse flow velocity:

$$\langle v_\perp \rangle = \int_0^{R_0} n(r) v_\perp(r) dr, \tag{7}$$

where $n(r)$ is the normalized particle density distribution at freeze-out and $R_0 = 20$ and 25 fm for S+S and Pb+Pb collisions, respectively. While the local temperatures [28] are found to be 140 MeV for both colliding systems, the overall average transverse flow velocities are $\langle v_\perp \rangle = 0.31c$ and $0.43c$ for S+S and Pb+Pb, respectively. This result is in good agreement with the above thermal model analysis.

The advantage of using a microscopic transport model here is that one does not need the assumption of thermal equilibrium. A large amount of secondary scattering guarantees the thermalization of hadrons from heavy-ion collisions [18,28]. Furthermore, the narrow freeze-out hypersurface used in hydrodynamical calculations becomes unnecessary, thus avoiding the problematic model dependence [28,29] of the sharp transition from strongly interacting hydrodynamical matter to free streaming hadrons. The drawbacks of using a microscopic model like RQMD, however, are that many cross sections (some of them not measured) are used and that the calculation itself can be very massive. Moreover, it is not always straightforward to grasp the essentials from the calculated results.

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[5] On the other hand, in Refs. [15,25], the authors speculate that such m_T dependence in the "side" radius is connected to the temperature profile of the fireball.

Note that the fireball produced in heavy-ion collisions contains a finite number of particles within a limited volume for a limited amount of time. Since many hydrodynamic formulae are derived for systems where the mean free path of a particle is much smaller than the characteristic size of the system, one should keep in mind that there are many features of hadronic fireballs that are not well described by hydrodynamics. Nevertheless, hydrodynamic parameters can be very useful for providing a global view of the dynamics of these collisions.

In summary, using a hydrodynamical model, we have studied the relationship between freeze-out temperature T_f and transverse flow velocity $v_\perp$ from one-particle p_T distributions of both S+S and Pb+Pb collisions at CERN SPS energies. The best fits to the data show a strong anticorrelation between T_f and $\langle v_\perp \rangle$, which appears to be more or less independent of the particular velocity profile used. Although the best values for T_f and $\langle v_\perp \rangle$ cannot be determined from one-particle distributions alone, combined fits to one- and two-particle distributions may aid in clearing up this ambiguity.

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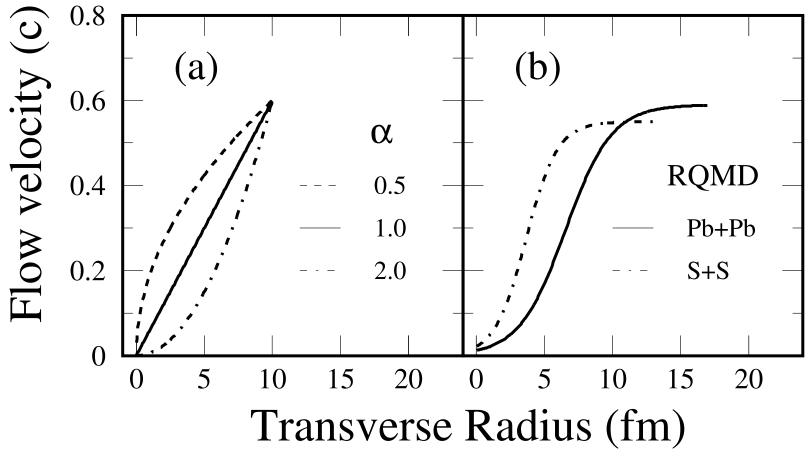
Figure Captions

Figure 1 Transverse flow velocity profile: (a) $v_\perp(r) = v_{\perp,\max}(r/R_{\max})^\alpha$, $\alpha = 0.5, 1$, and 2; (b) extracted from the microscopic transport model RQMD [18,19].

Figure 2 χ^2 fitting map of surface (maximum) transverse flow velocity vs temperature: (top panel) Pb+Pb collisions; (bottom panel) S+S collisions. Labels (a), (b), and (c) are for $\alpha = 0.5, 1$, and 2 of Eq. (2), and (d) is for the RQMD velocity profile [see Fig.1 (b)]. The minimum χ^2/NDF in each of the plots is between 1.0 and 3.0.



Flow velocity (c)
0.8
0.6
0.4
0.2
0
(a)
α
0.5
1.0
2.0
(b)
RQMD
Pb+Pb
S+S
0
5
10
15
20
Transverse Radius (fm)

Fig. 1

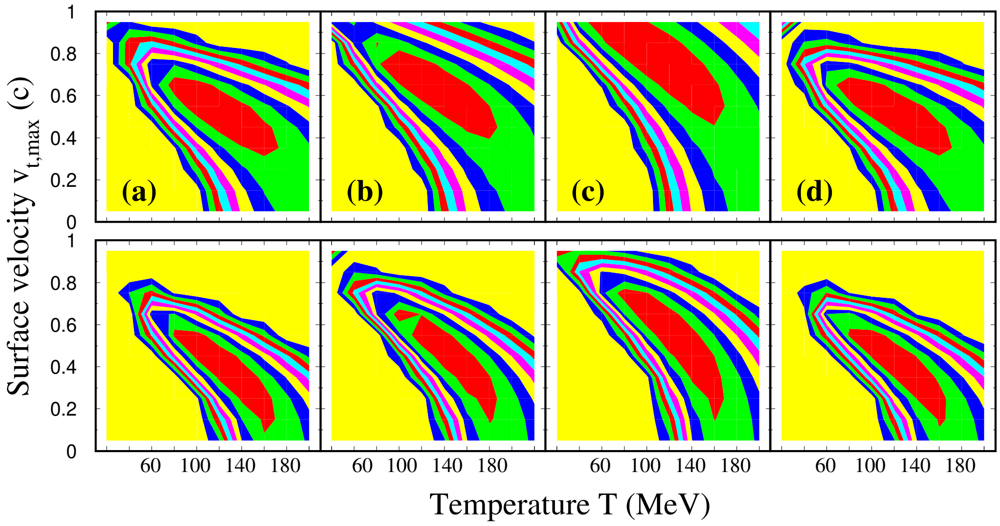


Fig. 2