# Elastic Form Factors of the Nucleon: Experimental Results 

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## Electron-Nucleon Elastic Scattering



Nucleon vertex: $\Gamma_{\mu}\left(p^{\prime}, p\right)=\underbrace{F_{1}\left(Q^{2}\right)}_{\text {Dirac }} \gamma_{\mu}+\frac{i \kappa_{p}}{2 M_{p}} \underbrace{F_{2}\left(Q^{2}\right)}_{\text {Pauli }} \sigma_{\mu \nu} q^{\nu}$
$F_{1}$ is the helicity conserving and $F_{2}$ is helicity non-conserving.

$$
\begin{gathered}
G_{E}\left(Q^{2}\right)=F_{1}\left(Q^{2}\right)-\kappa_{N} \tau F_{2}\left(Q^{2}\right) \quad \tau=\frac{Q^{2}}{4 M_{N}} \\
G_{M}\left(Q^{2}\right)=F_{1}\left(Q^{2}\right)+\kappa_{N} F_{2}\left(Q^{2}\right)
\end{gathered}
$$

At $Q^{2}=0$

$$
G_{M p}=2.79, G_{M n}=-1.91 \text { and } G_{E p}=1, G_{E n}=0
$$

Extract $G_{E}$ and $G_{M}$ from:

- $N\left(e, e^{\prime}\right)$ Cross-section measurements
- $\vec{N}\left(\vec{e}, e^{\prime}\right) N$ Beam-target Asymmetries
- $N\left(\vec{e}, e^{\prime}\right) \vec{N}$ Recoil polarization


## Proton $\mathrm{G}_{E}$ and $\mathrm{G}_{M(\text { before } \sim 1990)}$



- ep elastic cross section

$$
\begin{gathered}
\frac{\sigma_{r}}{\mu^{2} G_{D}^{2}}=\frac{d \sigma}{d \Omega} \frac{(1+\tau) \epsilon}{\tau \sigma_{M o t t}}=\frac{\epsilon}{\tau}\left(\frac{G_{E}}{\mu G_{D}}\right)^{2}+\left(\frac{G_{M}}{\mu G_{D}}\right)^{2} \\
G_{D}=\left(1+Q^{2} / .71\right)^{-2}
\end{gathered}
$$

- $Q^{2}>1 \mathrm{GeV}^{2}$ error on $\mathrm{G}_{E}^{p}$ grows.
- $\mathrm{G}_{E}^{p}$ becomes a smaller fraction of $\sigma$
- At $\mathrm{Q}^{2}=5, \mathrm{G}_{E}^{p}$ maximum $8 \%$ contribution to $\sigma$ (assuming $\mu \mathrm{G}_{E}^{p} / \mathrm{G}_{M}^{p}=1$ )


## Neutron $\mathrm{G}_{M}$ ( before ~1990)



- Define a reduced cross-section:

$$
\sigma_{R}=\epsilon(1+\tau) \frac{\sigma\left(E, E^{\prime}, \theta\right)}{\sigma_{M o t t}}=R_{T}+\epsilon R_{L}
$$

- In PWIA : $R_{T} \propto\left(G_{M}^{n}\right)^{2}+\left(G_{M}^{p}\right)^{2}$ and $R_{L} \propto\left(G_{E}^{n}\right)^{2}+\left(G_{E}^{p}\right)^{2}$
- Difficulties:
- Subtraction of large proton contribution
- Sensitive to deuteron model. In particular :
* Final-State Interactions
* Meson Exchange Currents
* Relativistic corrections.

Neutron $\mathrm{G}_{E}$ ( before $\sim 1990$ )


- Elastic ed: $\sigma=\sigma_{M o t t}\left[A\left(Q^{2}\right)+B\left(Q^{2}\right) \tan ^{2}\left(\frac{\theta}{2}\right)\right]$ with:
- $A\left(Q^{2}\right)=F_{C}^{2}\left(Q^{2}\right)+\frac{8}{9} \tau^{2} F_{Q}^{2}\left(Q^{2}\right)+\frac{2}{3} \tau F_{M}^{2}\left(Q^{2}\right)$
- $B\left(Q^{2}\right)=\frac{4}{3} \tau(1+\tau) F_{M}^{2}\left(Q^{2}\right)$
- Extract $G_{E}^{n}$ using deuteron model but very sensitive to NN potential.
- Elastic $d\left(e, e^{\prime}\right) \vec{d}$ reaction to measure $t_{20}$, the tensor polarization.
- $t_{20} \propto F_{C}, F_{M}$, and $F_{Q} . \Rightarrow$ Extract all 3 form factors.
- $F_{Q}$ is insensitive to the deuteron model $\Rightarrow \mathrm{G}_{E}^{n}$


## Developments

- Need make coincidence measurements
$\Rightarrow$ continuous beam accelerators like JLab and MAMI
- Need to measure spin observables
$\Rightarrow$ High beam polarization (70-80\%) at high currents ( $80 \mu \mathrm{~A}$ )
$\Rightarrow$ Recoil polarization measurements possible
$\Rightarrow$ Development of polarized ${ }^{3} \mathrm{He},{ }^{2} \mathrm{H}$ and ${ }^{1} \mathrm{H}$ targets
$\rightarrow$ Beam-Target asymmetry measurement possible
- Need to improve theory of ${ }^{3} \mathrm{He}\left(e, e^{\prime}\right),{ }^{3} \mathrm{He}\left(e, e^{\prime}\right) n,{ }^{2} \mathrm{H}\left(e, e^{\prime} n\right)$ and ${ }^{2} \mathrm{H}\left(e, e^{\prime} p\right)$
$\Rightarrow$ Determine kinematics which reduce sensitivity to nuclear effects
$\Rightarrow$ Determine which observables are sensitive to form factors
$\Rightarrow$ Use model to extract form factors


## $\mathrm{G}_{M}^{n}$ from Quasi-free $d\left(e, e^{\prime} n p\right)$



- Measure ratio $R_{\text {meas }}=\frac{\sigma\left(e, e^{\prime} n\right)}{\sigma\left(e, e^{\prime} p\right)}$
- Proton and neutron detected in same detector simultaneously.
- Need to know absolute neutron detection efficiency.
* Bonn used $p\left(\gamma, \pi^{+}\right) n$ in - situ
* NIKHEF and Mainz used $p(n, p) n$ with tagged neutron beam at PSI.
- Use model to determine $\delta R \Rightarrow$ the deviation from $R_{P W I A}$.
- Sensitivity to deuteron model cancels in ratio. $\delta R \approx 10 \%$.
- $R_{P W I A}=R_{\text {meas }}-\delta R$
- $\mathrm{G}_{M}^{n}$ is extracted knowing $\mathrm{G}_{E}^{n}, \mathrm{G}_{M}^{p}$ and , $\mathrm{G}_{E}^{p}$


## $\mathrm{G}_{M}^{n}$ from Quasi-free ${ }^{3} \overrightarrow{H e}\left(\vec{e}, e^{\prime}\right)$

- $10 \mu \mathrm{~A}$ polarized electron beam with $P_{B}=75 \%$ and spin flipped at 30 hZ .
- Target polarization, $P_{T}=30 \%$.

Simultaneously measure elastic ${ }^{3} \overrightarrow{H e}\left(\vec{e}, e^{\prime}\right)$ to monitor $P_{T} \cdot P_{B}$

- Align the target spin along the $q$ vector and measure $A_{T}=\frac{\sigma^{+}-\sigma^{+}}{\sigma^{+}+\sigma^{+}}$
- $A_{T}$ sensitive to $\mathrm{G}_{M}^{n}$. Use full three-body non-relativistic Fadeev calculation of $A_{T}$ and $\mathrm{G}_{M}^{n}$ modified within the model until agreement with data.



## Neutron Magnetic Form Factor



- Agreement between $\mathrm{JLab}{ }^{3} \overrightarrow{H e}\left(\vec{e}, e^{\prime}\right)$ and Mainz results
- Data has been taken in Hall B at JLab with CLAS, a large acceptance detector.
- Deuteron and Proton target simultaneously
- Continuous $Q^{2}$ coverage from 0.3 to $5 \mathrm{GeV}^{2}$.
- Error bars 3-10\%
- $p\left(e, e^{\prime} n\right) \pi^{+}$to determined neutron effi ciency


## $G_{E}^{n}$ from ${ }^{3} \overrightarrow{\mathrm{He}}\left(\overrightarrow{\mathrm{e}}, \mathrm{e}^{\prime} \mathrm{n}\right)$

${ }^{3} \overrightarrow{\mathrm{He}}\left(\overrightarrow{\mathrm{e}}, \mathrm{e}^{\prime} \mathrm{n}\right)$ at quasi-free kinematic
$\Rightarrow$ best approximation to free $\overrightarrow{\mathrm{n}}\left(\overrightarrow{\mathrm{e}}, \mathrm{e}^{\prime} \mathrm{n}\right)$.

$$
A=P_{B} P_{T} V \frac{a \sin \theta \cos \phi G_{E}^{n} G_{M}^{n}+b \cos \theta\left(G_{M}^{n}\right)^{2}}{c\left(G_{E}^{n}\right)^{2}+d\left(G_{M}^{n}\right)^{2}}
$$

where $\theta$ is the angle of the target neutron's spin relative to the momentum transfer.

When $\theta=90^{\circ}$ :

$$
A=A_{\perp} \propto G_{E}^{n} G_{M}^{n}
$$

To fi rst order when $\theta=0^{\circ}$
$A=A_{\|}$depends only on kinematics.

$$
G_{E}^{n} \approx \frac{b}{a} G_{M}^{n} \frac{\left(P_{B} P_{T} V\right)_{\|}}{\left(P_{B} P_{T} V\right)_{\perp}} \frac{A_{\perp}}{A_{\|}}
$$

## $G_{E}^{n}$ from $\overrightarrow{\mathrm{d}}\left(\overrightarrow{\mathrm{e}}, \mathrm{e}^{\prime} \mathrm{n}\right)$

$$
A_{e d}^{V}=\frac{N^{+}-N^{-}}{N^{+}+N^{-}}=P_{B} P_{T} V \frac{-2 \sqrt{\tau(\tau+1)} \tan \left(\theta_{e} / 2\right) G_{E}^{n} G_{M}^{n}}{G_{E}^{n^{2}}+\tau / \epsilon G_{M}^{n^{2}}}
$$

Extract $\mathrm{G}_{E}^{n}$ from $A_{\text {ed }}^{V}$ :

- Use full model of Arenhovel to predict $A_{e d}^{V}$.
- Modify $\mathrm{G}_{E}^{n}$ to have agreement with the measured $A_{e d}^{V}$






## Recoil Polarization in elastic $e N$



- Helicity independent components are zero.
- Helicity dependent outgoing proton spin components:
- $P_{l}$ is along the proton momentum direction
- $P_{t}$ is in-plane transverse to momentum direction
- $P_{n}$ is out-of-plane transverse to momentum direction $P_{n}=0$

$$
\frac{G_{E}}{G_{M}}=-\frac{P_{t}}{P_{l}} \frac{\left(E_{e}+E_{e^{\prime}}\right)}{2 M} \tan \left(\frac{\theta}{2}\right)
$$



- Outgoing neutrons scatter in $\mathrm{CH}_{2}$ which is the analyzer for the secondary reaction.
- The analyzer can only measure spin components perpendicular to the incoming particle's momentum. $a_{T}=A_{y} P_{x}$
- To measure $P_{l}$ need to precess the neutron spin in a magnetic fi eld so transverse polarization at the $\mathrm{CH}_{2}$ is:

$$
P_{x}=P_{l} \sin \chi+P_{t} \cos \chi
$$

## $\mathrm{d}\left(\overrightarrow{\mathrm{e}}, \mathrm{e}^{\prime} \vec{n}\right)$ at MAMI

$$
\begin{gathered}
a_{T}=P_{B} A_{y}\left(P_{l} \sin \chi+P_{t} \cos \chi\right)=A_{o} \sin \left(\chi-\chi_{\circ}\right) \\
\tan \chi_{\circ}=\frac{P_{B} A_{y} P_{t}}{P_{B} A_{y} P_{l}}
\end{gathered}
$$




- Planned experiment at JLab in Hall A to use ${ }^{3} \overrightarrow{\mathrm{He}}\left(\overrightarrow{\mathrm{e}}, \mathrm{e}^{\prime} \mathrm{n}\right)$ quasi-free reaction to measure $G_{E}^{n}$ to $\mathrm{Q}^{2}=3.4 \mathrm{GeV}^{2}$.

- Both momentum and spin vector precess in the magnet.
- Precession angle, $\chi=\gamma \kappa_{p} \theta_{\text {bending }}$

and in general $P_{N}^{f p}=-P_{L}^{t g t} \sin (\chi)+P_{N}^{t g t} \cos (\chi)$ but for proton, $P_{N}^{t g t}=0$ so

$$
P_{N}^{f p}=-P_{L}^{t g t} \sin (\chi)
$$

- Unlike neutron recoil polarization measure $P_{L}^{t g t}$ and $P_{L}^{t g t}$ separately and simultaneously.


## $\mathrm{G}_{E}^{p} / \mathrm{G}_{M}^{p}$ by Recoil Polarization



$$
\begin{aligned}
N^{ \pm}(\theta, \phi)=N_{0}^{ \pm}(\theta)[1 & +\left[ \pm h A_{c}(\theta) P_{n}^{f p}+b_{i}\right] \cos \phi \\
& \left.+\left[ \pm h A_{c}(\theta) P_{t}^{f p}+a_{i}\right] \sin \phi\right]
\end{aligned}
$$

- $P_{n}^{f p}=-P_{l}^{t g t} \sin (\chi)$ and $P_{t}^{f p}=P_{t}^{t g t}, \frac{G_{E}}{G_{M}} \propto \frac{P_{t}^{f p} \sin (\chi)}{P_{n}^{f p}}$
- $a_{i}$ and $b_{i}$ are the instrumental asymmetries which are eliminated by subtracting $N^{-}$from $N^{+}$.


- $\mathrm{G}_{E} / \mathrm{G}_{M}$ from polarization measurement falls linearly with $\mathrm{Q}^{2}$.
- Disagreement between $\mathrm{G}_{E} / \mathrm{G}_{M}$ extracted from cross section data.


## Systematic problems?

## Unaccounted for physics?

## Recent Rosenbluth measurements



- Global analysis of previous experiments by J. Arrington indicates no inconsistencies between experiments.
- When trying to combine the cross section data and polarization data, the global fit has a larger $\chi^{2}$ indicating that the data are inconsistent with each other.
- New measurements at JLab in Hall C at consistent with previous experiments.

- A dedicated measurement at JLab in Hall A has preliminary results which also agree with previous experiments. Detected the elastically scattered proton instead of electron which has advantages:
- Proton momentum fixed at each $\epsilon$
- Cross section is nearly constant with $\epsilon$
- Reduces size of $\epsilon$-dependent radiative corrections
- Reduces systematic error on beam energy and scattering angle


## Two-photon Contributions

- John Arrington (nuclex0311019) looked at $\frac{\sigma_{e}+p}{\sigma_{e}-p}$ data for $\mathrm{Q}^{2}<2$ but covered wide $\epsilon$ range. Determines a slope of $-(5.7 \pm 1.8) \%$.

- Calculation by Blunden,Melnitchouk and Tjon (prl 99, 122304 (2004)). Only includes nucleon intermediate states.



## Two-photon Contributions



- Chen, Afanasev, Brodsky, Carlson and Vanderhaegen (hep.phro403058) calculates the hard part of the $2 \gamma$ exchange using the "handbag" diagram.

- Limited $\epsilon$ and $Q^{2}$ range since $s,-u, Q^{2} \ll M^{2}$


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## Comparison to Lattice QCD



