A Measure of Sensitivity

The Problem

The Solution

Application

Conclusions

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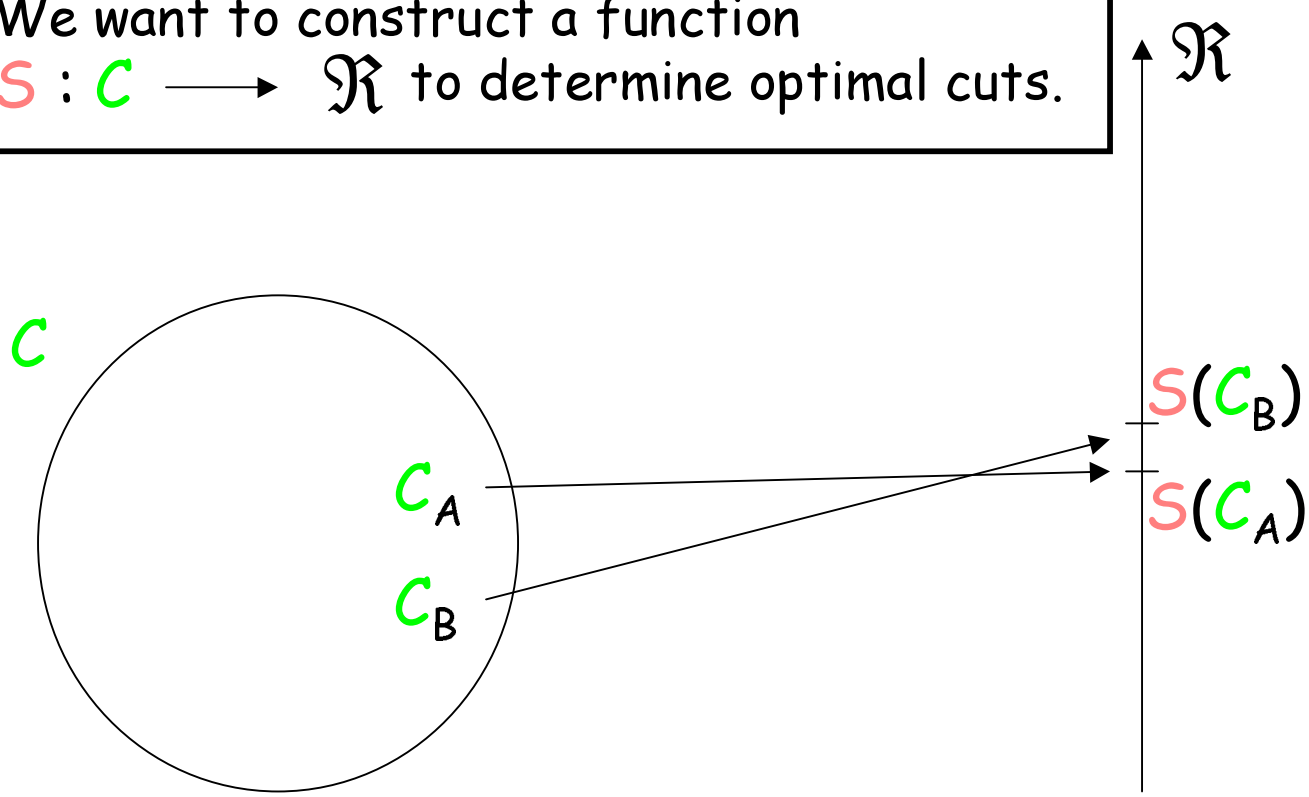
The Problem

Where do you set your cuts?

C = $E_T^{J1} > 30$ GeV
$E_T^{e} > 20$ GeV
trigger1==true

We want to construct a function
$S : C \longrightarrow \Re$ to determine optimal cuts.



Previous Efforts

Notation: s = number of expected signal events
b = number of expected background events

For a discovery analysis, choose among

$$S(C) = s/\sqrt{b} \qquad S(C) = s^{1.15}/b^{0.35}$$

$$S(C) = s/\sqrt{b+\delta b^2} \qquad S(C) = \frac{s}{\sqrt{s+b}}$$

For a limit setting analysis, people use

"maximize s at fixed b = .4"

$$S(C) = \begin{cases} s & b = .4 \\ 0 & b \neq .4 \end{cases}$$

Rationale: - Able to set a good limit if you see 0 events
- 67% chance of seeing 0 events

(Gambling . . .)

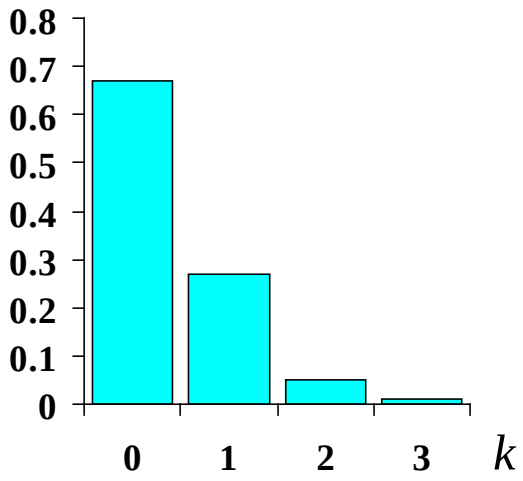
Weighted Gambling

Sensitivity = expected payoff

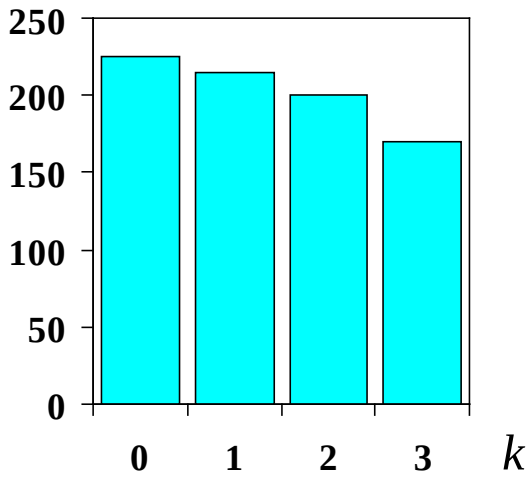
$$S(C) = \hat{M}^{95\%}(C) = \sum_{k=0}^{\infty} P(k \mid b)\, M^{95\%}(k)$$

where $M^{95\%}(k)$ is the 95% CL mass limit obtained if k events are observed

$P(k \mid b = 0.4)$



$M^{95\%}(k)$



A Brief Review

b = # expected background events ---> P(b)

L = integrated luminosity ---> P(L)

ε = efficiency ---> P(ε)

σ = cross section ---> Bayesian prior P(σ)

$s = L \varepsilon \sigma$ is the number of expected signal events

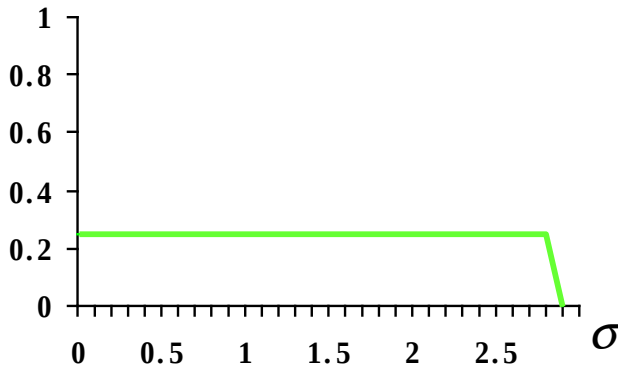
$\mu = s + b$ is the number of expected events

$P(k \mid L, \varepsilon, b, \sigma) = \frac{e^{-\mu} \mu^k}{k!}$ is the probability to see k data events

A Bayesian Approach

Prior distribution

$$P(\sigma)$$



Recall Bayes' Theorem

$$P(A,B) = P(A \mid B)\,P(B) = P(B \mid A)\,P(A)$$

Apply Bayes' Theorem

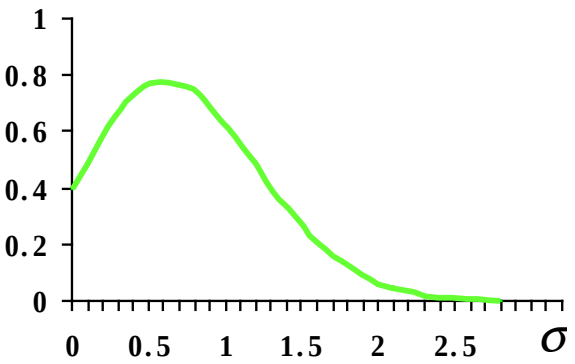
$$P(L,\varepsilon,b,\sigma \mid k) = \frac{P(k \mid L,b,\varepsilon,\sigma)\,P(\sigma)\,P(L)\,P(b)\,P(\varepsilon)}{P(k)}$$

Integrate out Nuisance Parameters

$$P(\sigma \mid k) = \int_0^\infty dL \int_0^1 d\varepsilon \int_0^\infty db\, P(L,\varepsilon,b,\sigma \mid k)$$

Posterior distribution

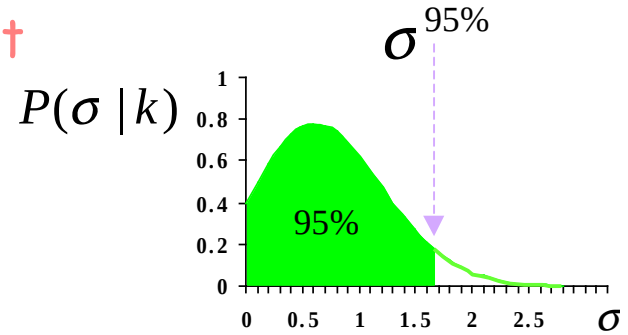
$$P(\sigma \mid k)$$



A Bayesian Approach

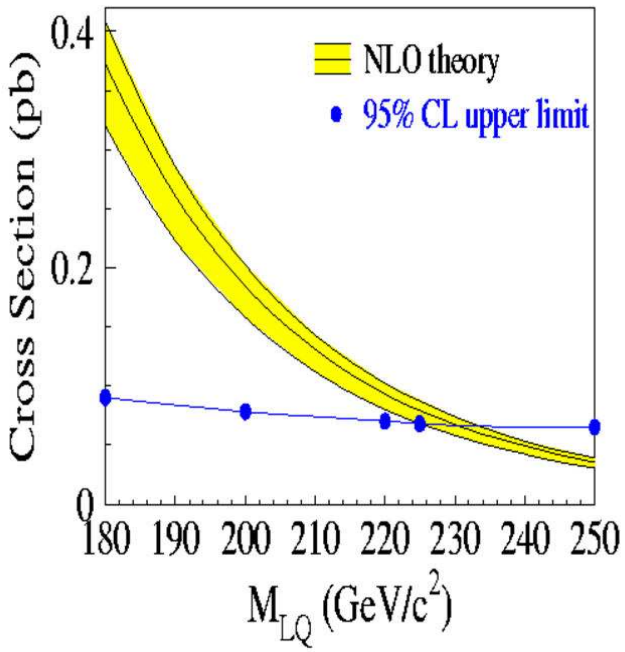
95% CL Cross section Limit

$$0.95 = \int_0^{\sigma^{95\%}} d\sigma\, P(\sigma \mid k)$$



95% CL mass limit $M^{95\%}$

$$\sigma^{95\%} \rightarrow M^{95\%}$$



Sensitivity

$$S = \hat{M}^{95\%} = \int P(b)\, db \sum_{k=0}^{\infty} P(k \mid b)\, M^{95\%}(k)$$

A Frequentist Approach

$s^{95\%}(k)$: The number of expected signal events such that the probability of background fluctuating up to or above signal + expected background is 5%.

$$\sum_{i=0}^{k} \frac{e^{-(s^{95\%}+b)}(s^{95\%}+b)^i}{i!} = (0.05)\sum_{i=0}^{k} \frac{e^{-b}b^i}{i!}$$

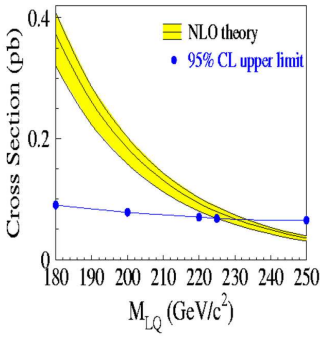
Solve for $s^{95\%}(k)$ numerically.

95% CL cross section limit

$$\sigma^{95\%}(k) = \frac{s^{95\%}(k)}{\hat{s}}\sigma_{\text{theoretical}}$$

95% CL mass limit

$$\sigma^{95\%} \rightarrow M^{95\%}$$

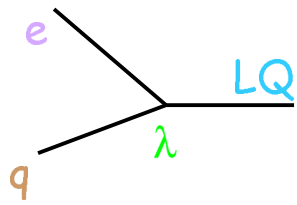


Sensitivity

$$S = \hat{M}^{95\%} = \int P(b)\,db \sum_{k=0}^{\infty} P(k\,|\,b)\,M^{95\%}(k)$$

Leptoquarks decaying to eejj

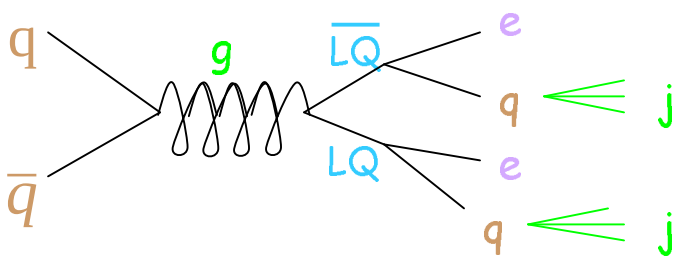
Leptoquarks arise as an attempt to explain the symmetry between leptons and quarks



Leptoquarks carry

SU(3) color
Lepton number
Baryon number
Fractional electric charge

Experimental Signature

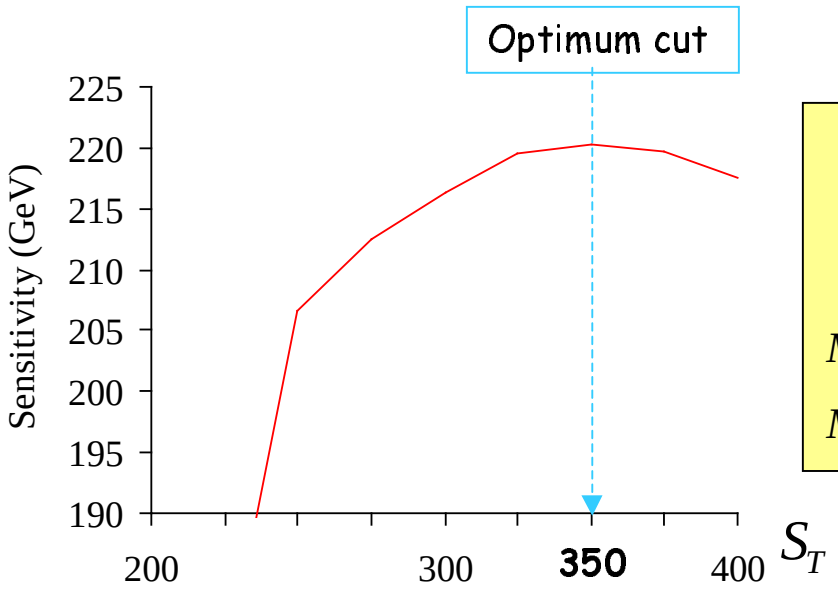


Leptoquarks decaying to eejj

Expect hard leptons and jets, so define energy variables.

Use our measure of sensitivity to choose the cut on S_T.

$$\begin{cases} H_T^{\,e} \equiv \sum E_T^{\,e} \\ H_T^{\,j} \equiv \sum E_T^{\,j} \\ S_T \equiv H_T^{\,e} + H_T^{\,j} \end{cases}$$



$\hat{b} = 0.44$

$k = 0$

$\hat{M}^{95\%} = 220\,GeV$

$M^{95\%} = 225\,GeV$

The punch line:

- We used an optimum cut
- We expect a mass limit of 220 GeV
- We see 0 events in our data, and set a mass limit of 225 GeV.

Conclusions

- Analyses need an explicit measure of sensitivity.

- A natural measure is $S(C)$, the mass limit one expects to set with the set of cuts C.

- We understand the details of calculating $S(C)$, with full incorporation of errors.

- This measure has been applied to a previous 1st generation leptoquark analysis, and works as advertised.

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