A-B-Cs of Sun-Synchronous Orbit Mission Design

Ronald J. Boain

The sun-synchronous-orbit (SS-O) is one of the most commonly used forms of earth orbit for science missions. Historical examples of missions which use the SS-O include NIMBUS, TIROS, COBE, SME, LANDSAT, and others. More recent earth science missions based on the SS-O are Terra, EO-1, and Aqua. And even now there are several future missions that will be launched into a SS-O within the next few years: Aura, CloudSat, CALIPSO, Aquarius, and OCO. The list of past, present, and future earth orbiting sun-synchronous missions is long and impressive. Given the widespread utility of the SS-O, it is worthwhile to review characteristics that make the SS-O so useful and therefore desirable for scientific application. It might also be useful to describe the process of how one goes about selecting the mission parameters defining the SS-O mission design.

The primary reasons for the frequent utility of the SS-O are that it readily provides desirable orbital characteristics that satisfy key mission requirements for science missions. Since the orbital inclination is nearly polar (96.5° - 102.5°), the SS-O provides global coverage at all latitudes (with the exception of just a few degrees within the poles). Since the position of the line of node remains roughly fixed with respect to the sun direction, sun lighting conditions along the sunlit groundtrack remain roughly the same throughout the mission. (This property is also important to spacecraft designers in that it provides the mission with a "dark side" to the orbit which often can be used to solve otherwise complex thermal problems.) And because SS-Os can be selected over a wide, desirable range of spacecraft instrument viewing conditions. Another complementary attribute is that altitudes can be selected within this range to provide SS-Os with groundtracks over a fixed interval of days. This repeat groundtrack characteristic is useful to scientists in ensuring that global coverage is complete and repeatable over a designated sampling period desirable to an investigator.

With the utility and desirability of these first-order orbital characteristics, it is easy to see why earth resource and climate scientists, among others, are attracted to SS-Os, at least as an initial consideration for their mission design. And as the record shows many of them do indeed select the SS-O for their mission. So how does one go about defining the best SS-O to meet the needs of a particular scientific investigation? Are there systematic steps that one can go through that enable the mission analyst and system designer to select the specific parameters for a SS-O that most readily meet the mission's needs?

With this in mind, there are several objectives for this paper: First, it is intended to aid scientists, spacecraft designers, and even mission designers in understanding the mechanism within celestial mechanics that enables the SS-O to exist. The paper will define the terms and parameters used to describe a SS-O. It is intended that the paper should also convey an intuitive feel to designers about the orbital geometries for various SS-Os and the naturally occurring second-order variations in the geometry that are

important to mission and system design. With this intuition, the paper should enable one to intelligently go about orbit parameter selection. And lastly, the paper is intended to provide some handy equations and algorithms that can be used to quickly explore the design trade space and assess key properties of the SS-O without having to resort to sophisticated, large mission analysis computer programs.

The Perturbations due to a Non-Spherical Earth

What exactly qualifies an orbit to be labeled sun-synchronous? To answer this question we must first review some basic theory and results from celestial mechanics about the first order perturbations to an earth orbiting satellite driven by the earth's oblateness. Almost all textbooks on the subject [for example Refs. 1 and 2] discuss the theory associated with how an orbit plane is perturbed as a result of the earth's equatorial bulge. This bulge creates an out of plane gravitational force on the orbit causing the orbit to gyroscopically precess. The operative equation describing the rate at which the line of nodes moves owing to this bulge is given by:

$$\dot{\Omega} = -\frac{3}{2}J_2(\frac{a_e}{p})^2 n * \cos(i)$$

where $p = a(1 - e^2)$ is the orbit parameter (the semi-latus rectum), $n = \sqrt{\mu/a^3}$ is the mean motion, and *i* is the inclination. The constant J_2 is the zonal harmonic coefficient, with a value for earth equal to 0.001 082 63. Thus, the nodal rate of precession is a function of the three classic orbital elements, namely, the semi-major axis (*a*), the eccentricity (*e*), and the inclination (*i*). (a_e in the equation is the equatorial radius of the earth.) Moreover, we see from the equation that for inclinations < 90° the node regresses, i.e., moves clockwise as seen from the north, and for inclination > 90° the nodal motion is prograde, i.e., counter-clockwise from the north. With this equation and by the proper selection of the semi-major axis, eccentricity, and inclination, we can cause the orbit plane to precess/regress at different rates along the equator.

Fortunately, most earth science missions prefer to use a circular or near circular orbit (i.e., frozen orbit), thereby pre-determining the value for eccentricity to be zero. Therefore, for the analyses in this paper, the orbital eccentricity will be assumed to be zero or approximately zero such that eccentricity may be ignored as a variable in the future applications of the equation. And with this value fixed, the precession rate of the node is reduced to depending only on the inclination and the orbital altitude, $(h = a - a_e)$, through altitude's dependence on semi-major axis.

This relation defines the nodal regression rate but, by itself, does not help us in defining the condition for an orbit being sun-synchronous. What is needed is a specification of the nodal rate that gives the property already alluded to as maintaining the geometry of the nodes roughly fixed with respect to the sun. But this equation does tell us that if we can

specify a value for the precession rate, Ω , then the inclination for sun-synchronous behavior can be determined given the orbit's altitude (*h*), or vice-versa.

Determining the Precession Rate, $\boldsymbol{\Omega}$

If we think of the earth's motion about the sun as circular with a period of one year, the rate of revolution would be constant and would given by $(360^{\circ}/365.242199 \text{ days} =)$ 0.9856 deg/day. As another approximation, if we further consider the earth's polar axis to be perpendicular to the earth's orbit plane (as represented in Fig. 1), then the line joining the center of the earth and the sun would be perpendicular to the direction of motion. And because the earth's orbit is posigrade this motion would be in a clockwise sense as seen from the north pole. Next for the geometry shown in Fig. 1, the plane defined by the line joining the earth to the sun and the earth's polar axis, on the sunlit side, would be the solar meridian. (For the approximation at hand, the sun's position would be directly over the equator all year long.) Also, for all points north and south along this meridian, the local solar time would be 12:00 noon. For points west, the local time would be *ante meridiem* (a.m.); similarly for points east, the local time would be *post meridiem* (p.m.). Clearly, 6:00 a.m. would correspond to being on the terminator reckoned to the west of the solar meridian and vice versa for 6:00 p.m.

If we now consider a satellite with its ascending node positioned as shown in the figure, we can label the angle between this node and the direction to the sun as the Mean Local Time (MLT) of the ascending node. For an arbitrary orbit, the MLT would be continually changing owing to the regression/precession phenomenon described above. If, however, we were able to precess the line of nodes at exactly the earth's orbital rate around the sun, the geometry shown in the figure would be preserved and the MLT angle

with respect to the sun would remain fixed. The selection of earth's orbit rate for Ω would, therefore, achieve the desired result of maintaining the orbit's geometry with respect to the sun fixed and the sun-lighting conditions along the orbit's groundtrack the same. Hence, the condition for an orbit to be sun-synchronous is that precession rate is equal to the earth's Mean Motion, and then with the specification of the orbit's altitude, the inclination is automatically defined by the equation for orbital precession above. Figure 2 below shows how the orbital altitude and inclination for SS-Os are paired, i.e., select one and the other is determined, over an altitude range of general interest to earth scientists.



Now in reality the earth's orbit is not circular. Moreover, the earth's polar axis is inclined with respect to the ecliptic plane. Therefore, the idealized description of the behavior for SS-Os is not totally correct. The earth's eccentric orbit results in the MLT for the SS-O having an annual variation of +3.5/-4.1 degrees relative to the Mean Solar Meridian. As will be discussed in the paper, this is a consequence of the Equation of Time, i.e., the difference between Mean Solar Time and Apparent Solar Time. Then due to the obliquity of the earth to the ecliptic, the declination of the sun's apparent position moves north and south 23.4 degrees relative to the equator over the course of a year. This introduces additional changes to the geometry between the sun and the SS-O plane. As an interesting result of the paper, all of this changing geometry can be characterized and quantified by use of the annalemma. The annalemma is the function relating the sun's Equation of Time and its declination with the calendar day of year as a parameter. [It is sometimes seen graphically depicted on world globes as a figure-eight, usually placed somewhere over the Pacific Ocean.] Thus, a graph of it can be represented in a coordinate system fixed with the origin at the intersection of the equator and the Mean Solar Meridian. And since the annalemma changes very little from year to year, it provides a convenient means of computing the sun's angular distance from an orbit plane.

For SS-Os, the annalemma can be used to evaluate the annual variation of the sun's direction relative to the orbit plane, i.e., the beta-angle. By judicious choice of a rotating coordinate system, it is possible to graphically visualize the geometry between the orbit plane and the annalemma. Then by visual inspection, one can determine the time of year

at which the annalemma is closest to the orbit plane. This allows with good accuracy the evaluation of the beta-angle for those dates. From the beta-angle variation, the maximum and minimum eclipse times are also conveniently computed (as will be shown in the paper).

Most importantly, the paper will discuss a recommended set of steps for selecting a SS-O appropriate for a particular mission's needs, starting with an examination of the trades involving the orbital altitude. Orbital altitude is clearly a good place to begin because it trades with so many other parameters of interest to a mission designer. For example, altitude relates to other important system parameters such as the launch vehicle performance, the instrument coverage, the instrument performance sensitivity, the ambient environment, the duration of telecommunications opportunities, and the orbit's lifetime. It also relates to the groundtrack repeat cycle interval and the spacing between consecutive and adjacent groundtracks. A simple algorithm will be provided that enables the mission analyst to compute with hand calculator the options available to him/her for achieving a repeat groundtrack in 8 or 9 days, as an example, and for ensuring that the distance between any two groundtracks within the cycle is less than 350 km, as another example.

In conclusion, the paper will not reveal any new theory or discovery about SS-Os. Rather it will provide, hopefully, an easily read treatment of the A-B-Cs of sun-synchronous orbit mission design, including handy algorithms useful in the quick evaluation of the key orbit parameters for such missions.

References:

1) Richard H. Battin, An Introduction to the Mathematics and Methods of Astrodynamics, AIAA Educational Series, AIAA, 1987

2) J.M.A. Danby, Fundamentals of Celestial Mechanics, Willmann-Bell, Inc. 1989

Figure 1. Approximate Earth/Sun Geometry

