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Top Quark Pair Production at Higher Energies at the First Muon Collider *

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Abstract

For a proper theoretical description of the top quark pair production cross section at the First Muon Collider in the kinematic regime away from the threshold the complete mass and energy dependence has to be taken into account. Two-loop calculations for the photon mediated total cross section are reviewed.

For experiments at the Z pole quark mass effects in the total hadronic cross section $\sigma(e^+e^- \to \text{hadrons})$ can often be neglected in the first approximation and treated perturbatively in an expansion in the ratio M^2_{quark}/s, where $\sqrt{s}$ is the c.m. energy. The situation is quite different for top quark pair production at the First Muon Collider, where c.m. energies up to 500 GeV are planned. Here, the $t\bar{t}$ threshold energy[1] $(\sqrt{s})_{thr} = 2M_t \approx 350$ GeV and the c.m. energies are comparable in size and an expansion in M_t^2/s is not adequate. In the immediate vicinity of the threshold point conventional multi-loop perturbation theory breaks down and a resummation of Coulomb terms $\sim (C_F\,\alpha_s\,\pi/\beta)^n$, $n = 0, 1, 2, \ldots$ ($\beta = (1-4M_t^2/s)^{1/2}$ being the c.m. velocity of the top quarks), has to be carried out to all orders in the strong coupling (see [1, 2]). This complication is avoided if only energies sufficiently above the threshold point are considered. As a rule of thumb one can take the relation $C_F\alpha_s\pi/\beta < 2$ as a requirement to avoid the complication of the threshold regime [3]. For $t\bar{t}$ production this means that conventional multi-loop methods can be used for c.m. energies which are at least 10 GeV above the threshold point.

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[1]Throughout this talk M_t is understood as the top quark pole mass. The constants $C_A = 3$, $C_F = 4/3$ and $T = 1/2$ denote SU(3) group theoretical factors and α is the electromagnetic coupling.

In this talk I report on developments in the past two years in the determination of two-loop $\mathcal{O}(\alpha_s^2)$ QCD corrections to the total $t\bar{t}$ production cross section where the full dependence on the top quark mass has been taken into account. Since the results for the production of $t\bar{t}$ pairs through the axial-vector current are not completed yet,[2] only the photon-mediated contributions are considered. Therefore, the result presented here cannot be considered as absolute predictions for $t\bar{t}$ production at the First Muon Collider. Rather they should be taken as an indication for the size and importance of all the $\mathcal{O}(\alpha_s^2)$ contributions. In particular, they can illustrate the stability of the results for variations in the renormizalization scale and their sensitivity on the strong coupling and the top quark mass. Assuming the notation and conventions used in [3], the total photon-mediated $t\bar{t}$ cross section normalized to the point cross section $\sigma_{\rm pt} = 4\pi\alpha^2/3s$ can be cast into the form,

$$
\begin{aligned}
R \, = \, \frac{\sigma(e^+e^- \to \gamma^* \to t\bar{t})}{\sigma_{\rm pt}} \, = \, Q_t^2 \, \bigg[\, R^{(0)} + \Big(\frac{\alpha_s^{(5)}(\mu^2)}{\pi}\Big) \, C_F \, R^{(1)} \\
+ \Big(\frac{\alpha_s^{(5)}(\mu^2)}{\pi}\Big)^2 \, \Big(C_F^2 \, R_A^{(2)} + C_F \, C_A \, R_{NA}^{(2)} + C_F \, T \, n_l \, R_l^{(2)} + C_F \, T \, R_F^{(2)} \Big) \bigg] \, . \qquad (1)
\end{aligned}
$$

$R^{(0)}$ an $R^{(1)}$ denote the well known expressions for the Born cross section and the one-loop [6] corrections. $\alpha_s(\mu^2)$ is the strong coupling in the $\overline{\rm MS}$ scheme at the scale μ for $n_l = 5$ light flavors. The Abelian and non-Abelian parts $R_A^{(2)}$ and $R_{NA}^{(2)}$ come from two gluon exchange two-loop diagrams and have been calculated numerically in [7, 8, 9] using Padè approximation methods. The uncertainties coming from possible choices of different Padè approximants can be ignored for practical purposes. Handy approximation formulae for $R_A^{(2)}$ and $R_{NA}^{(2)}$ which reduce to the correct analytic expressions in the high energy and the small velocity limit can be found in [8]. $R_l^{(2)}$ and $R_F^{(2)}$ originate from one gluon exchange diagrams with the insertion of a massless and a top quark loop, respectively. $R_l^{(2)}$ also contains the effects from real radiation of a massless quark pair through gluon radiation off one of the top quarks. It should be noted that both, the real and virtual radiation of massless quark pairs, have to be taken into account to render the $\mathcal{O}(\alpha_s^2)$ contributions in $R_l^{(2)}$ infrared finite. $R_l^{(2)}$ and $R_F^{(2)}$ have been calculated in closed analytic form in [10, 11] using dispersion relation techniques for the virtual corrections and integration of the four-body phase space for the real corrections. In [9] and [11] also the complete set of "non-singlet" axial-vector current induced $\mathcal{O}(\alpha_s^2)$ corrections can be found. The $\mathcal{O}(\alpha_s^2)$ contributions coming from secondary radiation of $t\bar{t}$ pairs through gluon radiation off massless quarks are numerically small and can be neglected [12]. To fix our notation explicitly I present the complete form of the Born and one-loop contributions,

$$
\begin{aligned}
R^{(0)} & = \frac{3}{2}\,\beta\,(3-\beta^2)\,, \\
R^{(1)} & = 3\,\bigg\{\,\frac{(3-\beta^2)\,\,(1+\beta^2)}{2}\,\Big[\,2\,{\rm Li}_2(p) + {\rm Li}_2(p^2) + \ln p\,\Big(2\,\ln(1-p) + \ln(1+p)\Big)\Big] \\
& \qquad - \beta\,(3-\beta^2)\,\Big(2\,\ln(1-p) + \ln(1+p)\Big) \\
& \qquad - \frac{(1-\beta)\,\,(33-39\,\beta-17\,\beta^2+7\,\beta^3)}{16}\,\ln p + \frac{3\,\beta\,\,(5-3\,\beta^2)}{8}\,\bigg\}\,, \qquad (2)
\end{aligned}
$$

[2] Two-loop corrections for the total Z-mediated production cross section have been calculated recently in an expansion in M_t^2/s up to order $(M_t^2/s)^6$ [4]. These results, however, have only limited applicability for the First Muon Collider, in particular for lower energies [5].

where $p = (1-\beta)/(1+\beta)$. In Fig. 1 R is plotted versus $\sqrt{s}$ including successively higher orders in α_s. The $\mathcal{O}(\alpha_s^2)$ cross section is displayed for different choices of the renormalization scale. The stability under renormalization scale variations has been improved by also including the known three-loop $\mathcal{O}(\alpha_s^3)$ in the large momentum expansion [13, 14, 15]. In Fig. 2(a) the sensitivity of R on the strong coupling and in Fig. 2(b) on the top quark mass are illustrated.

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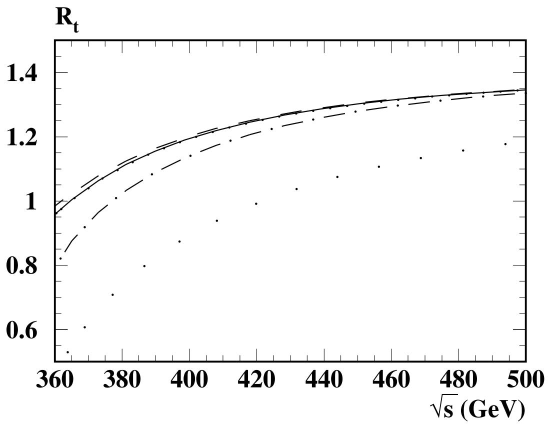


Figure 1: The total normalized photon-mediated cross section at the two-loop level versus $\sqrt{s}$ for the renormalization scales $\mu = M_t$ (dashed), $\mu = 2M_t$ (solid) and $\mu = \sqrt{s}$ (dotted line), $M_t = 175$ GeV and $\alpha_s^{(5)}(M_z) = 0.118$. For comparison also the Born (wide dots) and the one-loop cross section for $\mu = 2M_t$ (dashed-dotted line) are displayed. To improve the stability under renormalization scale variations the known three-loop $\mathcal{O}(\alpha_s^3)$ in the large momentum expansion have been added to the two-loop cross section. The plot is taken from [3].

References

[1] M.S. Berger, these proceedings and hep-ph/9712486.

[2] A.H. Hoang, these proceedings and hep-ph/9801273.

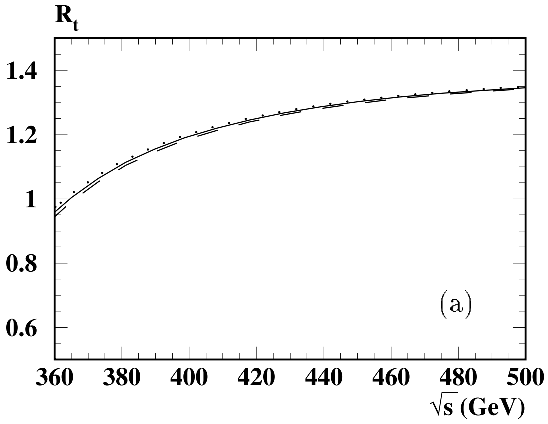
[3] K.G. Chetyrkin, A.H. Hoang, J.H. Kühn, M. Steinhauser and T. Teubner, University of Karlsruhe Report No. TTP97-42 and hep-ph/9711327, to be published in *Z. Phys.* **C**.

[4] R. Harlander and M. Steinhauser, University of Karlsruhe Report No. TTP97-40 and hep-ph/9711413.

[5] K.G. Chetyrkin, R. Harlander, J.H. Kühn and M. Steinhauser, *Nucl. Phys.* **B 503** (1997) 339.

[6] G. Källen and A. Sabry, *K. Dan. Vidensk. Selsk. Mat.-Fys. Medd.* **29** (1955) No. 17,1.

[7] K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, *Phys. Lett.* **B 371** (1996) 93.



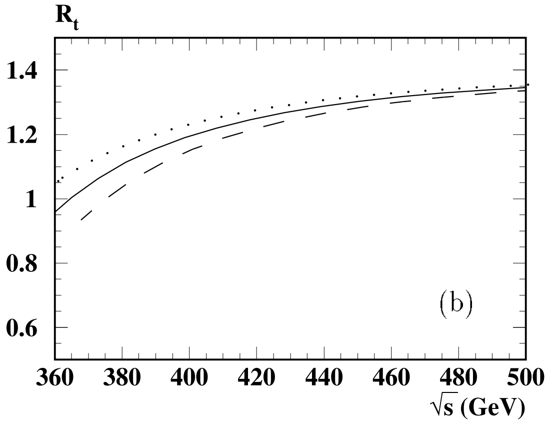


Figure 2: Sensitivity of the total normalized photon-mediated cross section at the two-loop level on variations of the strong coupling and the top quark mass. In (a) the solid, dashed and dotted lines correspond to $\alpha_s^{(5)}(M_z) = 0.118, 0.115$ and 0.121, respectively, for $M_t = 175$ GeV. In (b) the solid, dashed and dotted lines correspond to $M_t = 175, 180$ and 170 GeV, respectively, for $\alpha_s^{(5)}(M_z) = 0.118$. For both plots the choice $\mu = 2M_t$ has been made. The plots are taken from [3].

[8] K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, *Nucl. Phys.* **B 482** (1996) 213.

[9] K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, *Nucl. Phys.* **B 505** (1997) 40.

[10] A.H. Hoang, J.H. Kühn and T. Teubner, *Nucl. Phys.* **B 452** (1995) 173.

[11] A.H. Hoang, and T. Teubner, University of California, San Diego Report No. UCSD/PTH 97-16 and hep-ph/9707496, to be published in *Nucl. Phys.* **B**

[12] K.G. Chetyrkin, A.H. Hoang, J.H. Kühn, M. Steinhauser and T. Teubner, in the proceedings of the workshop *Physics with e^+e^- Linear Colliders*, DESY-96-123D and hep-ph/9711327

[13] S.G. Gorishny, A.L. Kataev and S.A. Larin, *Phys. Lett.* **B 259** (1991) 144;
L.R. Surguladze and M.A. Samuel, *Phys. Rev. Lett.* **66** (1991) 560; (E) ibid., 2416;
K.G. Chetyrkin, *Phys. Lett.* **B 391** (1997) 402.

[14] K.G. Chetyrkin and J.H. Kühn, *Phys. Lett.* **B 248** (1990) 359.

[15] K.G. Chetyrkin and J.H. Kühn, *Phys. Lett.* **B 406** (1997) 102.