The Naturalness Principle and Expectations for a Linear Collider

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Abstract.
Naturalness considerations are frequently used to place theoretical upper limits on the masses of new particles in supersymmetric extensions of the standard model. These theoretical limits, in turn, shape our expectations of how likely it is that new physics will be discovered at any particular collider. A significant portion of the literature devoted to naturalness studies apply criteria which do not have a reliable correspondence to fine-tuning. This report discusses the principled method for quantifying naturalness, and presents theoretical bounds on sparticle masses in two scenarios.

I INTRODUCTION

Supersymmetry attempts to solve the naturalness problem by canceling the quadratic divergences that render a light weak-scale unnatural. This is the original and principle motivation for weak-scale supersymmetry. [1] This cancellation of quadratic divergences is imperfect when supersymmetric is broken. As the masses of super-partners increase, delicate cancellations are required to maintain a hierarchy between the weak-scale and the masses of super-partners. Hence, if weak-scale supersymmetry is to satisfy its original and principle motivation, super-partners can not be arbitrarily heavy. [2–4]

The simplest example of a naturalness problem is a light fundamental scalar. Radiative corrections to a fundamental scalar mass are quadratically divergent. In a theory which is assumed to be valid up to some scale Λ, at one-loop a scalar mass is of the form:

$$m_S^2 = -m_0^2 + g^2\Lambda^2 \tag{1}$$

where g is a dimensionless coupling. The lightness of the scalar mass becomes progressively unexplained by our effective theory as $\Lambda/m_S \to \infty$ because increasingly delicate and unexplained cancellation must occur between the two terms listed in the equation above. This unexplained hierarchy is unnatural in the sense that any cancellation we imposed would be destroyed by a minute variation of the Lagrangian parameters. The first attempt to quantify fine-tuning proposed equating a sensitivity parameter with the severity of fine tuning [4]. For a set of "physical" parameters y, and a set of "fundamental" Lagrangian parameters x, the sensitivity function is defined:

$$c(y,x) = \frac{x}{y}\frac{dy}{dx} \quad \text{or} \quad \frac{\delta y}{y} = c\frac{\delta x}{x} \tag{2}$$

For the simple quadratic divergence, the sensitivity function is:

$$c = 2\frac{g^2\Lambda^2}{m_S^2} = 2\frac{g^2}{g^2 - g_c^2} \tag{3}$$

where we define a critical coupling $g_c = m_0/\Lambda$. Although the largeness of the sensitivity function is in good correspondence to incidence of fine-tuning in this example, this prescription does not accurately identify fine-tuning in more complicated scenarios. Consider for example the one hierarchy which we do understand. The proton mass, or more precisely Λ_{QCD} is related to the Plank mass by dimensional transmutation:

$$\Lambda = Me^{-\frac{8\pi^2}{bg^2(M)}} \tag{4}$$

Although the hierarchy Λ_{QCD}/M_{Pl} is quite natural, the value of Λ_{QCD} is a highly sensitive function of the Lagrangian parameters: $c = \frac{16\pi^2}{bg^2}$. Of course the proton mass would have been very sensitive to variations in g if it had been 1 MeV, or 10 GeV, there are no large unexplained cancellations at work. Sensitivity is only useful and meaningful when supplemented with global information about the parameter space of a theory. This, of course, is to be expected since naturalness is a relative concept.

II THE NATURALNESS MEASURE

A principled naturalness measure [2] can be formulated in terms of scale invariant intervals and an assumed metric on parameter space. Consider a set of physical parameters, $y_i(x_1, x_2, \ldots)$, which can be calculated in terms of the fundamental Lagrangian parameters $x_1, x_2, \ldots$. Because any meaningful naturalness measure is equivalent to some assumed metric on the fundamental parameters of a theory we make this theoretical prejudice explicit by introducing a dimensionless measure $f(x)$, so that the probability that a "fundamental" parameter lies within a scale invariant interval $\delta x/x$ is:

$$dP = f(x)\frac{\delta x}{x}, \tag{5}$$

where δx is some infinitesimal range of x centered on x. For a scale invariant distribution, $f(x)$ is of course just a constant. The measure $f(x)$ along with an implied domain for x completely encapsulates the theoretical prejudice inherent in naturalness measures. We would like to know, given that a fundamental parameter has takes a value within some percentage of x, what is the relative likelihood that a physical parameter lies within some percentage of y. The result, can be recast in terms of the ratio of a particular sensitivity, to a particular average sensitivity:

$$\gamma = \frac{c}{\bar{c}} \tag{6}$$

where

$$1/\bar{c} \equiv \langle \rho \rangle = \frac{\int \frac{dx}{x} f(x) c^{-1}(x)}{\int \frac{dx}{x} f(x)} \tag{7}$$

This naturalness principle $\gamma \not\gg 1$ can be stated as: Observable properties of a system should not be *unusually* unstable with respect to minute variations in the fundamental parameters. Note that formally $\gamma \overset{x_+ \to x_-}{\longrightarrow} 1$. A Convergence test for a sufficient domain $\{x_-, x_+\}$ can be achieved for example through

$$\delta_\pm \equiv \mid \frac{x_\pm}{\bar{c}} \frac{d\bar{c}}{dx_\pm} \mid = \mid \pm \hat{f}(x_\pm) \left(1 - \gamma_\pm^{-1}\right) \mid \tag{8}$$

where $\hat{f} = f(x)/\int \frac{dx}{x} f(x)$, and and auxiliary requirement that: $\delta_- \leq \epsilon \quad \text{or} \quad \delta_+ \leq \epsilon$ where ϵ is of order one.

A Electro-weak Symmetry Breaking

Although sensitivity is manifestly not a reliable measure of fine-tuning, several authors have attempted to ignore this issue, and have continued to use sensitivity parameter because of its relative simplicity. The problem with this approach is that the average sensitivity of a particular variable is dependent on the values of other Lagrangian parameters. Although the global normalization of the sensitivity is unimportant in the simple case of a scalar quadratic divergence, electro-weak symmetry breaking is not of this simple form. In the MSSM:

$$m_Z^2 = \frac{(m_1^4 - m_2^4)}{\sqrt{\left(m_1^2 + m_2^2\right)^2 - 4B^2}} - \left(m_1^2 + m_2^2\right) \tag{9}$$

where m_1, m_2 and B themselves have a non-trivial dependence on the fundamental Lagrangian parameters. Moreover, the dependence of these parameters on the dimensionless Lagrangian parameters is exponential. In this case it is known explicitly that sensitivity does not reliably quantify the magnitude of fine-tuning [2]. The naturalness of electro-weak symmetry breaking in gravitationally mediated and gauge mediation scenarios has been assessed using the naturalness measure γ. The results are presented in Figs. 1,2 with markers indicating $\gamma = 2, 5, 10$. The vertical lines extend up to values of $\gamma = 10, 20$ respectively.

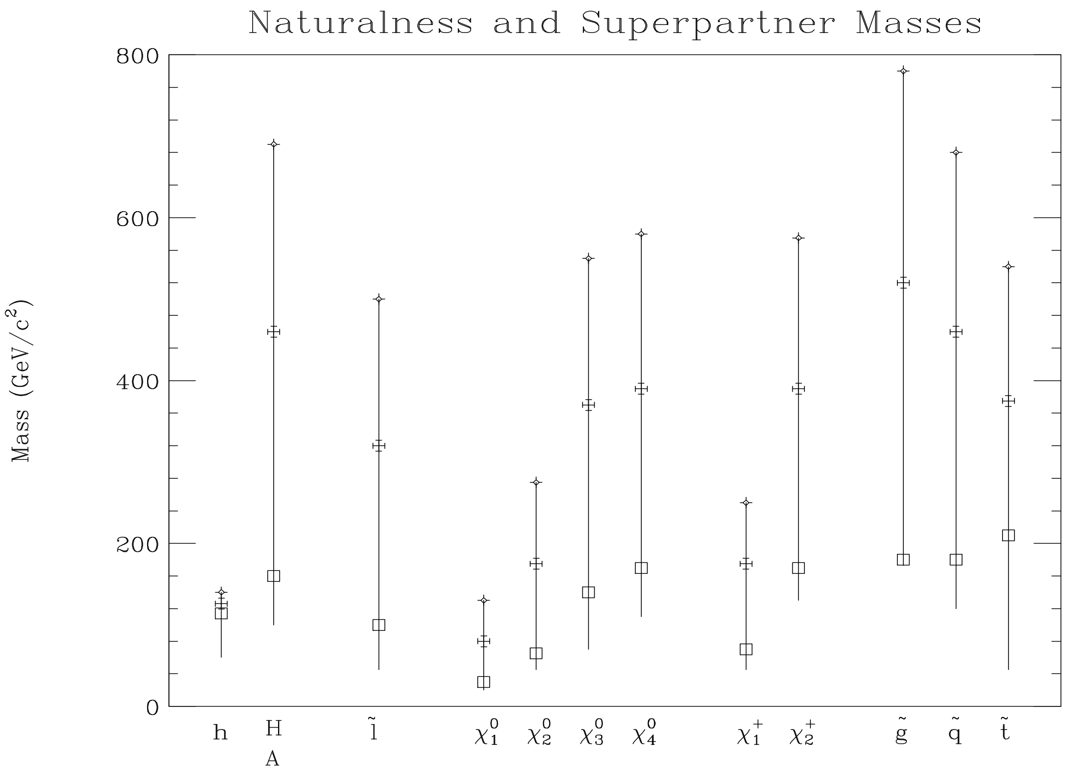

FIGURE 1. Natural values in mSUGRA

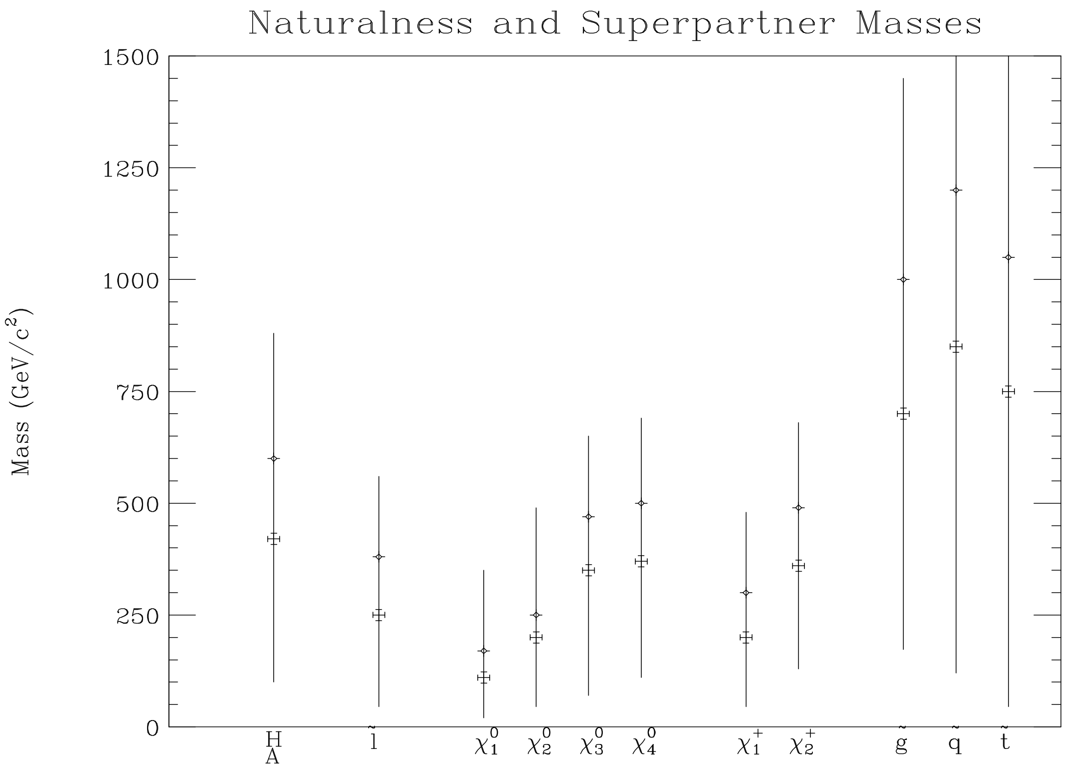

FIGURE 2. Natural values in gauge mediated scenario

III SUMMARY AND CONCLUDING REMARKS

Principled naturalness criteria, can play a role in shaping our expectations about what the tenable ranges of masses for new particles are. Indeed, in the absence of direct evidence of an accessible scales above $v = 246$ GeV, they are one of the few tools we have to place an upper limit to on the scales of new physics. However, even principled criteria like the one discussed in this report should be understood in the context of the theoretical prejudice inherent in any naturalness measure. Any claims about the expected values of sparticle masses which are based on unreliable fine-tuning criteria should be treated with extreme skepticism. In most cases they should be disregarded. The principled criteria discussed here does not support the assertion that multi-TeV third generation squarks are compatible with the absence of fine-tuning, contrary to claims in the literature. Naturalness indicates that supersymmetry, in whatever form, if relevant to the weak-scale, should provide a multitude of sparticles which are kinematically accessible at a 1 TeV NLC. However, naturalness arguments do not guarantee that the entire spectrum of superpartners would be kinematically accessible at a 1 TeV NLC.

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