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Decoherence of Quantum Fields: Pointer States and Predictability

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Abstract: We study environmentally induced decoherence of an electromagnetic field in a homogeneous, linear, dielectric medium. We derive an independent oscillator model for such an environment, which is sufficiently realistic to encompass essentially all of linear physical optics. Applying the "predictability sieve" to the quantum field, and introducing the concept of a "quantum halo", we recover the familiar dichotomy between background field configurations and photon excitations around them. We are then able to explain why a typical linear environment for the electromagnetic field will effectively render the former classically distinct, but leave the latter fully quantum mechanical. Finally, we suggest how and why quantum matter fields should suffer a very different form of decoherence.

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1 Introduction

Decoherence and environmentally induced superselection[1] have been studied extensively in the system composed of a single harmonic oscillator linearly coupled to a bath of independent oscillators[2, 3, 4]. This system has generally been presented as a conveniently solvable model of value in investigating fundamental problems of principle, such as the issues of dissipation in quantum mechanics[5, 6], or of emergence of classical behaviour in open systems[7, 8]. In this paper we point out that this simple system actually constitutes a realistic description of a quantum electromagnetic field propagating in a linear dielectric medium. The mechanisms of decoherence identified in single oscillator models can therefore be applied straightforwardly to electrodynamics.

The particular aspect of decoherence that we consider is the selection, by the environment and its coupling to the system, of a preferred basis of *pointer states*[1]. We find that the linear interaction of the electromagnetic field with the environment implies that the pointer states of the quantum field are coherent states. While single oscillator models often tend to suggest the interpretation of coherent states as localized particles, in the case of the field they are not localized photon packets at all: the pointer states of the quantum electromagnetic field are in fact background field configurations.

There are also, however, many experiments which reveal the existence of photons; and so we examine decoherence in our model more carefully, to determine how it is that photons can be robust despite propagating through an environment. We are led to associate with every pointer state a *quantum halo* of states that are not effectively distinguished from it by the environment, and we show that excitations of a few photons above a background field are typically states within such a quantum halo.

The paper is organized as follows. The following section presents our model system, and derives a description of a typical dielectric medium as a bath of independent oscillators, from the assumption that such a medium will contain a large number of molecules within a volume on the scale of the smallest electromagnetic wavelength under study. We then specialize considerably to the case of ultraweak Ohmic dissipation at ultrahigh temperatures. In Section II, we take advantage of this simplification to derive several exact results concerning the pointer states of our system. Our third section then discusses quantum halos. Section IV then summarizes our results, and briefly suggests why decoherence may be expected to affect matter fields much differently from linearly coupled systems such as the electromagnetic field.

2 The Model

The system we will study will be an electromagnetic field in 3+1 dimensions. We quantize the field in Coulomb gauge, in a box of linear dimension $2L$, and couple it to molecules composing a dielectric medium inside the box:

$$\mathcal{L}_{\mathcal{S}} = \frac{1}{2}\sum_{s=1}^{2}\sum_{\vec{k}=-\vec{K}}^{\vec{K}}\left[\dot{A}^2_{\vec{k},s} - \omega(k^2)A^2_{\vec{k},s} + g\sum_n \dot{A}_{\vec{k},s}e^{-i\frac{\pi}{L}\vec{k}\cdot\vec{x}_n}j_{n,s}\right] . \tag{1}$$

We let $\vec{k}$ label the Fourier modes and s the polarization states; $\vec{K} \equiv (L\Gamma, L\Gamma, L\Gamma)$, where Γ is an ultraviolet cut-off wave number, above which we consider the gauge field to decouple from the medium (or at least to interact with it in such a way that there will be negligible effect on the field modes below the cut-off). The quantities $j_{n,s}$ represent the electric dipole interaction with molecules located at positions $\vec{x}_n$; g is the coupling strength of this interaction, which is assumed to be the same for all n and to be small.

Turning now to the environment, we will initially assume merely that it consists of a large number of molecules, which interact with each other only via the gauge field coupling presented above, and which are located at the points $\vec{x}_n$. We will neglect the motions of the molecules (with consequences that may be easily remedied, as discussed below), and consider only their internal energies:

$$\hat{H}_{\mathcal{E}} = \sum_n \hat{H}_n \ , \tag{2}$$

where $\hat{H}_n$ have some arbitrary discrete spectra. We will *not* assume that the environment actually consists of independent harmonic oscillators, but instead we will derive the fact that a rather generic environment may be treated as such, in the limit of large N[12]. The N that must be large is the number of molecules within a volume on the cut-off scale; we will therefore require that the number density of the medium satisfy $d >> \Gamma^3$. For an ideal gas at room temperature and atmospheric pressure, $d \simeq 10^3\Gamma^3$ corresponds to a cut-off of electromagnetic modes in the high ultraviolet range ($\lambda \sim 10$ nm). In solids or liquids we might perhaps handle somewhat shorter wavelengths, but our derivation of the independent oscillator model as a large N approximation to a general non-conducting environment must be expected to break down in the X-ray band ($\lambda \sim 1$ nm).

We will treat the medium as an unobserved environment, and describe only the state of the electromagnetic field, using the reduced density matrix formed by tracing over the states of the environmental molecules. If we assume that the initial state is a direct product of field and medium states, then we can obtain the evolution of the reduced density matrix from the path integral propagator

$$\rho[A, A'; t] = \int \mathcal{D}A\mathcal{D}A' \, \rho[A, A'; 0] \, e^{\frac{i}{\hbar}\left(S[A]-S[A']\right)} \, F[A, A'; t] \ , \tag{3}$$

where $F[A, A']$ is the influence functional[12] describing the effect of environmental molecules on the electromagnetic field. With the Hamiltonian (2), and a thermal initial state for all the molecules, the influence functional is given by

$$\begin{aligned} F[A, A'; t] \;&=\; \mathrm{Tr}_{\mathcal{E}}\Big(\mathcal{T} \exp[-\frac{i}{\hbar} \sum_{n,\vec{k},s} \int_0^t dt' \, \dot{A}_{\vec{k},s}(t') e^{-i\frac{\pi}{L}\vec{k}\cdot\vec{x}_n} \hat{\jmath}_{n,s}(t')] \\ &\quad \times \ \exp[-\sum_n \beta_n \hat{H}_n \ \bar{\mathcal{T}} e^{\frac{i}{\hbar}\sum_{n,\vec{k},s}\int_0^t dt' \, \dot{A}'_{\vec{k},s}(t') e^{-i\frac{\pi}{L}\vec{k}\cdot\vec{x}_n} \hat{\jmath}_{n,s}(t')}]\Big) \ , \end{aligned} \tag{4}$$

where $A_{\vec{k},s}(t)$ and $A'_{\vec{k},s}(t)$ are c-numbers in the path integral for the field, but $\hat{\jmath}_{n,s}(t)$ is the dipole moment operator of the nth molecule, in the interaction picture. $\mathcal{T}$ and $\bar{\mathcal{T}}$ denote time-ordering and anti-time-ordering, respectively, while β_n is the usual inverse temperature, which we allow to vary from place to place in the environment. The trace is to be taken over the states of the environment only.

We can now reduce this very general influence functional to the special form of an independent oscillator model, by implementing our large N approximation. We divide the box of volume $8L^3$ into cells of volume $\alpha^3\Gamma^{-3}$, where α is a number much smaller than one. Within the cell C centred at the point $\vec{x}_c$ there will be a large number $N(\vec{x}_c) = d(\vec{x}_c)\alpha^3\Gamma^{-3}$ of molecules.[1] By using time-dependent perturbation theory in the interaction picture, keeping explicitly only terms up to second order in g, and zeroth order in α, we can obtain a simple form for the influence functional for a single cell of dielectric medium:

$$\begin{aligned} F[A,A';t] &= \prod_{\vec{x}_n\in C}\Big(1-\frac{g^2}{\hbar^2}(\sum_j e^{-\beta E_j})^{-1}\sum_{l,l'}|J_{ll'}|^2 \\ &\qquad \times\sum_{\vec{k},s}\int_0^t dt'\int_0^{t'}dt''\, e^{-\beta_c E_l}[\dot{A}_{\vec{k},s}(t')-\dot{A}_{\vec{k},s}(t')] \\ &\qquad\qquad \times[\dot{A}^{*}_{\vec{k},s}(t'')e^{-i\omega_{lm}(t'-t'')}-\dot{A}'^{*}_{\vec{k},s}(t'')e^{i\omega_{lm}(t'-t'')}]\Big)\ . \end{aligned} \tag{5}$$

Here $J_{ll'}$ are unpolarized matrix elements of the dipole moment operator, *i.e.*, we assume unpolarized scattering from individual molecules, so that at the initial time $t=0$

$$\langle E_l|\hat{\jmath}_{n,s}\hat{\jmath}_{n,s'}|E_m\rangle = \delta_{s,s'}\sum_m J_{lm}J_{ml'}\ . \tag{6}$$

There are no terms linear in g, because we take our molecules to have no preferred orientation of their dipole moments: $\langle\hat{\jmath}_{n,s}\rangle = 0$. And we assume that the initial state of the environment is a direct product of single-molecule thermal states, with every molecule in a cell having the same initial temperature $(k_B\beta_c)^{-1}$.

From the last line of (5) we discard all but the leading terms in $N(\vec{x}_c)$, then put all the cells together and smooth out the cell structure by defining interpolated density and inverse temperature fields $d(\vec{x}), \beta(\vec{x})$. We can even allow the molecular composition of the environment to vary from cell to cell, so that the entire form of I_{eff} is spatially dependent as well. We find that the influence functional for the dielectric medium is that of a set of independent harmonic oscillators at every point in the box, (which we can now allow to become infinite):

$$\begin{aligned} F[A,A';t] &= \exp\Big(-\frac{g^2}{2\hbar}\sum_s\int d^3x\, d(x)\int_0^\infty\frac{d\omega}{\omega}\, I(\beta(x),\omega;x) \\ &\qquad \times\int_0^t dt'\int_0^{t'}dt''\,[\dot{A}-\dot{A}]_{t'}([\dot{A}-\dot{A}]_{t''}\coth\frac{\hbar\beta(x)\omega}{2}\cos\omega(t'-t'') \\ &\qquad\qquad -i[\dot{A}+\dot{A}]_{t''}\sin\omega(t'-t''))\Big)\ , \end{aligned} \tag{7}$$

[1]The appearance of α here would seem to lower, perhaps by an order of magnitude, the maximum frequencies up to which our analysis will be accurate. As we will discuss below, however, it is easy to dispense with α, which is only present to ensure that $e^{i\vec{k}\cdot\vec{x}}$ varies negligibly within a cell.

where the spectral density of the effective bath of independent oscillators is the (generally) temperature dependent quantity

$$I(\beta(x),\omega;x)=\frac{4\omega\sinh\frac{\hbar\beta(x)\omega}{2}\sum_{l,m}|J_{lm}(x)|^2\,\delta(\omega-\frac{E_l(x)-E_m(x)}{\hbar})}{\hbar\sum_l e^{-\beta(x)E_l(x)}}\,. \tag{8}$$

This effective environmental model describes physical optics in linear dielectric media, at all frequencies below the cut-off, and for all field strengths below thresholds for current generation. (The combination of Eqns. (3) and (7) is of course equivalent to a master equation for $\rho[A,A';t]$, which at late times will in general approach a Markovian limit. Markovian master equations can indeed be used to examine the kinds of issues with which we are concerned[13], and are a familiar formalism in quantum optics; but our derivation of optical open quantum dynamics from underlying microphysics has led us naturally to a propagator, which already gives us the solution to our model's master equation.)

The failure of our model to describe conductors and non-linear media is clearly due to our neglect of charge motion and higher-order terms in $\frac{1}{N}$, and so our recovery of the rest of optics is not simply a co-incidence. In the important and prevalent cases where free charges and non-linear effects are negligible, our result is indeed physically sound, even though our derivation may have appeared somewhat naive. In particular, our assignment of fixed positions to the molecules is certainly a very crude treatment, especially for gases; but our results can be checked by comparison with a more sophisticated analysis, in which the molecules are treated as an ideal gas whose initial state is described by a grand canonical ensemble. The only additional effects one finds are thermal broadening of the molecular spectra, and a Gaussian cut-off on the effective coupling of field modes with energies on the scale of the temperature (reflecting the smaller number of molecules possessing kinetic energies in this range).

This more sophisticated analysis must assume that the gas is dilute, so that quantum statistics are not significant, as well as that $d\Gamma^{-3} >> 1$. It is worth noting that the fuller analysis does not require $d\alpha^3\Gamma^{-3} >> 1$ for some small α: the delocalization of the molecules will itself smear out the phases $e^{i\vec{k}\cdot\vec{x}_n}$, so that the smallest volume containing very many molecules need only be on the cut-off scale, and not so much smaller still that $e^{i\vec{k}\cdot\vec{x}_n}$ varies negligibly across it. This effect of the fuller treatment can be incorporated in an approach like ours above, by making the x_n into stochastic variables, which fluctuate over distances on the order of Γ^{-1}. In the influence functional, we can then take the ensemble averages of all the locations, and obtain Eqn. (7) even when $\alpha\to 1$. It is thus evident that the delocalization of molecules that obviates α need not be coherent. For solids and liquids delocalization is not so obviously sufficient to eliminate α, but since they are denser, we can retain α and still achieve a cut-off in the high UV range.

Thermal broadening and cut-offs can also be incorporated phenomenologically, and so we have presented the cruder analysis with fixed molecular positions, in order to more clearly make the physical point that large numbers of molecules within a cut-off volume leads to effectively linear behaviour of an environment. (It is also in aid of this demonstration that we have been careful to employ the infra-red regulator L, for if we had assumed from the start a countable number of molecules and a continuum of field modes below any UV cut-off, we could never have achieved the correct high ratio of molecules to modes. In this instance, the IR regulator is not just mathematical pedantry, but is actually necessary to express some important physics.)

Since thermal motion and various sources of dissipation on the molecular excitations will broaden the spectral lines, we will assume that $I(\omega, \beta; x)$ is a continuous function of ω — though it may have sharp peaks around strong absorption lines. This will have the unphysical effect of giving the environment an infinite specific heat capacity, so that radiative heating and cooling will be neglected; but for most optical phenomena, and for the subjects discussed in this paper, this will not be important.

The model we have arrived at encompasses all the physics of reflection and refraction, and absorption. It provides

$$\mathrm{Im}K(\omega, \beta; x) = \frac{\pi g^2}{2\Omega} I(\omega, \beta; x) , \tag{9}$$

where $n(\omega, \beta; x) \equiv \sqrt{K(\omega, \beta; x)}$ is the complex index of refraction. The real part of K is given, as it should be for a linear medium, by the Kramers-Kronig relation

$$K(\omega, \beta; x) = 1 + \frac{2}{\pi} \int_0^\infty d\omega' \, \frac{\omega'}{\omega'^2 - \omega^2} \, \mathrm{Im}K(\omega', \beta; x) , \tag{10}$$

taking the Cauchy principal part of the integral. (The formal derivation of these results is straightforward; the relation between the quantum theory and classical optics will be clarified in the remainder of this paper.) Our model also describes thermal radiation, albeit without heating or cooling of sources and sinks.

Nevertheless, for simplicity in the remainder of this paper we will assume perfect spatial homogeneity. In this limit, the Fourier modes of the field decouple, even though they interact with the environment. Each field mode thus constitutes a harmonic oscillator linearly coupled to its own private bath of independent oscillators, with a continuous spectral density. And so we obtain a conclusion which will allow us to apply the results of many apparently idealized studies of decoherence to a real and important physical phenomenon: electrodynamics in a homogeneous linear dielectric medium is, within the physically tenable assumptions and approximations we have made, a realization of harmonic quantum Brownian motion in the independent oscillator model.

3 Pointer states

Having mapped our field theoretic problem onto the problem of harmonic Brownian motion in an independent oscillator environment, we are now able to determine the pointer states of the field, in a straightforward way. We first review a clear-cut procedure for identifying pointer states: the *predictability sieve*[14, 8]. We extend slightly the argument of Ref. [8], in which certain squeezed states are shown to minimize linear entropy, and also to yield the smallest von Neumann entropy generation among all Gaussian initial states. Here we show that these same states actually minimize von Neumann entropy against unrestricted variations of the initial states.

Pointer states are those states which are preferred by decoherence, in a process that may be termed "environmentally induced superselection". A generic quantum state will tend to evolve into a probabilistic mixture of pointer states. The suppression of quantum interference between these states makes the parameter space of pointer states the natural phase space of the classical limit of the quantum system in question. The predictability sieve identifies the pointer states by demanding that the environmentally induced splitting of a quantum state into non-interfering branches be stable: the branches must not rapidly branch in their turn. A pointer state must remain as pure as possible despite environmental decoherence.

A concrete expression of this requirement is that pointer states minimize the growth of the entropy. We therefore wish to use our propagator (3) to compute the reduced density operator $\hat{\rho}(t)$ that evolves from some pure initial state with wave function ψ_i. From this we will obtain the von Neumann entropy $S(t) = -\mathrm{Tr}\hat{\rho}(\mathrm{t})\ln\hat{\rho}(\mathrm{t})$ of this density operator, as a functional of ψ_i. Extremizing $S(t)$ with respect to variations of ψ_i then identifies those initial states that acquire the least entropy by time t. Since we must ensure that our variations maintain the normalization of the initial state, we must solve the constrained variational problem

$$\mathrm{Tr}\Big[(\ln\hat{\rho}+1)\frac{\delta\hat{\rho}}{\delta\psi_{\mathrm{i}}} = \lambda\psi_{\mathrm{i}}^{*}\Big]\ , \tag{11}$$

for some Lagrange multiplier λ.

In general, entropy evolves in a complicated way during Brownian motion, and this procedure becomes too difficult; but since we are concerned here with decoherence, and not with such other effects as dissipation and thermalization, we select the simple model which has Ohmic spectral density and in which the dissipation rate $\gamma \to 0$. We let the temperature become infinite, such that γT remains finite, and the environmental noise becomes white. In this limit, decoherence for a single oscillator is characterized by the dimensionless quantity

$$D \equiv 8\frac{\gamma k_B T}{\hbar\Omega^2}\ , \tag{12}$$

where Ω is the frequency of the Brownian oscillator — which in our case is a Fourier mode of the quantum field, so that $\Omega = ck$. Since all our field modes decouple, we will first focus on a single mode, and write A and A' without subscripts to refer to its amplitude. (To avoid complex numbers, we will assume that we are discussing Fourier sine and cosine modes, and rectangular polarizations.)

The single-mode part of the density matrix propagator, in this weak coupling, high temperature limit, is

$$\begin{aligned}\rho(A,A';t) &= \frac{\Omega}{2\pi\hbar\sin\Omega t}\int dA_i dA_i'\,\Big(\rho(A_i,A_i';0)\\ &\qquad\times\exp\Big[\frac{i\Omega}{2\hbar\sin\Omega t}[(A^2-A'^2+A_i^2-A_i'^2)\cos\Omega t-2(AA_i-A'A_i')]\Big]\end{aligned}$$

$$\times\ \exp -\frac{\Omega D}{4\hbar\sin^2\Omega t}\Big[((A-A')^2+(A_i-A_i')^2)(\Omega t-\sin\Omega t\cos\Omega t)$$
$$-2(A-A')(A_i-A_i')(\Omega t\cos\Omega t-\sin\Omega t)\Big]\ . \qquad (13)$$

The mixed state density matrix that evolves from any initial squeezed state, according to (13), can be diagonalized explicitly. We present the results for an arbitrary squeezed state in the Appendix; here we quote only a particularly relevant special case, namely the one-parameter family of initial states with $\rho(A,A';0)=\psi(A,\tau)\psi^*(A',\tau)$ for

$$\psi(A,\tau)=Ze^{-\frac{\Omega}{2\hbar}\sigma(\tau)A^2}\ . \qquad (14)$$

Here Z is a normalization constant, and

$$\begin{aligned}|\sigma(\tau)|^2 &= \frac{2\Omega\tau+\sin 2\Omega\tau}{2\Omega\tau-\sin 2\Omega\tau}\\ \mathrm{Im}(\sigma(\tau)) &= \frac{2\sin^2\Omega\tau}{2\Omega\tau-\sin 2\Omega\tau}\ .\end{aligned} \qquad (15)$$

The quantity $|\sigma(\tau)|^{-1}$ is the "squeezing factor" for these states.

For a given final time t, we will consider the initial state $\psi(A,\tau)|_{\tau=t}$. By the final time, this state will have evolved into a state with the density matrix

$$\begin{aligned}\rho(A,A';t) &= \sqrt{\frac{\Omega\mathrm{Re}(\sigma)}{\pi\hbar\Lambda}}\exp -\Big(\frac{\Omega\mathrm{Re}(\sigma)}{4\hbar\Lambda}\Big[(A+A')^2+\Lambda^2(A-A')^2\\ &\qquad -2i(D\sin^2\Omega t+\mathrm{Im}(\sigma))(A^2-A'^2)\Big]\Big)\\ &= \frac{2}{\Lambda+1}e^{i\frac{\Omega\mathrm{Re}(\sigma)}{2\hbar\Lambda}\left(D\sin^2\Omega t+\mathrm{Im}(\sigma)\right)(A^2-A'^2)}\sum_{n=0}^{\infty}\Big(\frac{\Lambda-1}{\Lambda+1}\Big)^n\phi_n(A)\phi^*n(A')\ ,\end{aligned} \qquad (16)$$

where $\Lambda\equiv 1+D\sqrt{(\Omega t)^2-\sin^2\Omega t}$. The ϕ_n happen to be the energy eigenfunctions of a harmonic oscillator with natural frequency $\omega=\Omega\mathrm{Re}(\sigma)$:

$$-\hbar^2\frac{d^2}{dA^2}\phi_n(A)+[\Omega\mathrm{Re}(\sigma)]^2A^2\phi_n(A)=(2n+1)\hbar\Omega\mathrm{Re}(\sigma)\phi_n(A)\ . \qquad (17)$$

This precise form of $\rho(A,A';t)$ has the convenient property that

$$\begin{aligned}\langle A|\ln\hat\rho(t)|A'\rangle &= \exp\Big[i\frac{\Omega\mathrm{Re}(\sigma)}{2\hbar\Lambda}(D\sin^2\Omega t+\mathrm{Im}(\sigma))(A^2-A'^2)\Big]\\ &\qquad\times\Big(C_1-C_2\Big[\hbar^2\frac{d^2}{dA'^2}-(\Omega\mathrm{Re}(\sigma))^2A'^2\Big]\Big)\,\delta(A-A')\ ,\end{aligned} \qquad (18)$$

where C_1 and C_2 are constants that may readily be computed from Eqn. (16).

We can also determine from Eqn. (13) the operator valued functional $\frac{\delta\hat\rho(t)}{\delta\psi_i(A_i)}$, for any ψ_i. Even where the initial state is our special squeezed state $\psi(A_i,\tau)|_{\tau=t}$, carefully chosen with regard to the final time t, this operator variation is somewhat complicated. Its diagonal matrix elements, though, are quite simple:

$$\langle A|\frac{\delta\hat{\rho}(t)}{\delta\psi_i(A_i)}|A\rangle = \sqrt{\frac{\Omega}{2\pi\hbar\eta(t)}}\psi^*(A_i,t)\, e^{-\frac{\Omega}{2\hbar\eta(t)}[A-(\cos\Omega t-i\sigma^*(t)\sin\Omega t)A_i]^2}\ , \tag{19}$$

where $\eta(t)\equiv\sigma^*\sin^2\Omega t+\frac{D}{2}(\Omega t-\sin\Omega t\cos\Omega t)-i\sin\Omega t\cos\Omega t$. It is easy to see that the property (19) reflects conservation of $\mathrm{Tr}\hat{\rho}$.

The variation also has another property, much less trivial (and with a much more tedious derivation):

$$\begin{aligned}
&\Big[(\hbar^2\frac{d^2}{dA^2}-[\Omega\mathrm{Re}(\sigma)]^2A^2)e^{i\frac{\Omega\mathrm{Re}(\sigma)}{2\hbar\Lambda}\left(D\sin^2\Omega t+\mathrm{Im}(\sigma)\right)(A^2-A'^2)}\,\langle A|\frac{\delta\hat{\rho}(t)}{\delta\psi_i(A_i)}|A'\rangle\Big]_{A=A'}\\
&= \Big(C_3(t)\Big[A-A_i(\cos\Omega t-i\sigma^*\sin\Omega t)\Big]-C_4(t)A\Big)\\
&\qquad\times\Big[A-A_i(\cos\Omega t-i\sigma^*\sin\Omega t)\Big]\\
&\qquad\qquad\times\ \exp-\Big[\frac{\Omega}{2\hbar\eta(t)}[A-(\cos\Omega t-i\sigma^*(t)\sin\Omega t)A_i]^2\Big]\ ,
\end{aligned}\tag{20}$$

where $C_3(t)$ and $C_4(t)$ are functions whose exact form will be irrelevant to our discussion.

Combining Eqns. (18), (19), and (20), we find that

$$\begin{aligned}
&\int dA dA'\,\langle A'|[1+\ln\hat{\rho}(t)]|A\rangle\langle A|\frac{\delta\hat{\rho}(t)}{\delta\psi_i(A_i)}|A'\rangle\\
&= [1+C_1(t)-\frac{\hbar\eta(t)}{\Omega}C_2(t)C_3(t)]\psi^*(A_i,t)\ .
\end{aligned}\tag{21}$$

This is the constrained Euler-Lagrange equation (11); the initial state $\psi(A,t)$ of Eqn. (14) therefore minimizes the entropy of the reduced density matrix at time t. This is identical to the result obtained in Ref. [8] for the initial state which minimizes linear entropy at time t.

From Eqn. (13), it is apparent that the phase space translation

$$|\psi_i\rangle\rightarrow e^{\frac{i}{\hbar}a\hat{p}_A}e^{-\frac{i}{\hbar}b\hat{A}}|\psi_i\rangle\ , \tag{22}$$

where $\hat{p}_A$ is the canonical momentum operator conjugate to $\hat{A}$, leads to a unitary transformation of the density operator at time t:

$$\begin{aligned}
\hat{\rho}(t) &\rightarrow \hat{V}\hat{\rho}(t)\hat{V}^\dagger\\
\hat{V} &= e^{\frac{i}{\hbar}(\frac{p}{\Omega}\sin\Omega t+x\cos\Omega t)\hat{p}_A}e^{-\frac{i}{\hbar}(x\cos\Omega t-p\Omega\sin\Omega t)\hat{A}}\ .
\end{aligned}\tag{23}$$

The entropy of the state at time t is thus invariant under such phase space translations of the initial state. Therefore, the two-parameter set of initial wave functions $e^{-\frac{i}{\hbar}bA}\psi(A-a,t)$ also minimize $S(t)$. We conjecture that these are the only such minimizing states.

There is thus no initial pure state which will have minimum entropy at all times. However, after a few dynamical times, the squeezing and the imaginary part of $\sigma(t)$ become steadily less significant. Also, the state which instantaneously minimizes $S(t)$ oscillates back and forth, over time, around unsqueezed coherent states. It is therefore clear that the least mixing initial states, on average over a few dynamical times, are the coherent states. While it is only our special limit $\gamma \to 0$, $T \to \infty$ that has allowed us to implement the predictability sieve analytically, calculations in other models[15], as well as general arguments[8], support the conclusion that coherent states can be considered the natural pointer states for harmonic oscillators coupled linearly to an environment. From the more general analysis of our first section, it then follows that they are the natural pointer states of an electromagnetic field mode in a linear medium.

Since the field modes are decoupled, an initial direct product state of all modes will evolve into a final direct product of mixed states, for which the total entropy will be the sum of the individual entropies. It is therefore clear that the generalization of Equation (11) to all $8(K+1)^3$ decoupled field modes is solved by a direct product of such squeezed states, and that coherent states of all $8(K+1)^3$ oscillators are the optimum pointer states for the field in a homogeneous medium. Furthermore, it follows trivially from Equation (23) that the c-number parameters $x(t), p(t)$ labelling the pointer states obey the classical equations of motion. As long as environmental noise is not so strong that the Gaussian peak in (16) becomes too broad too fast, it is clear that classical mechanics provides a good effective description of the evolution of the pointer states. (Of course, the existence of classical histories follows so trivially from our instantaneous definition of pointer states only because the dynamics of our model is linear.)

While coherent states of single oscillators are typically interpreted as localized particles, a coherent state of a quantum field is a vacuum state displaced by an 'external' or 'background' field configuration. The localization associated with decoherence occurs not in the positions of particles, but in the amplitudes of field modes. In this way one can understand that the classical physics which emerges from quantum electrodynamics, in the presence of a linear environment, will naturally be a field theory and not a many-body particle theory.

We emphasize that this result is a significant addition to the observation that one can obtain field equations as classical limits of quantum dynamics. After all, the equation of motion for a quantum harmonic oscillator is exactly the same as it is in the classical case, but this does nothing towards providing a classical interpetation for a "Schrödinger's Cat" state. One must consider decoherence in order to establish the crucial additional point that the pointer states of the quantum system, in the presence of an environment, behave in a sufficiently classical manner. In the present problem, we have done this, and observed that the emergent pointer states are classical field configurations — a fact which is empirically familiar, but does not follow at all from the free quantum field theory.

We have therefore made contact, via the predictability sieve, between decoherence in quantum Brownian motion and the standard field theoretic notion of a classical background field. We have confirmed that such background fields really do behave classically, in that quantum interference between distinct background field configurations is rapidly eliminated by a dissipative medium, and that the coherent quantum states labelled by these background fields are themselves the states least affected by decoherence. We now turn to the other side of the field theoretic coin, and consider how photons excited above a background field may be affected by the environment.

4 Quantum halos

The first point to be made is that our model for the environment is not intended to describe a sensitive detector. It is a very poor model for an ultra-high-gain amplifier, such as is required to detect single quanta. So while our discussion concerns the emergence of classical electrodynamics, we do not really address quantum measurement itself. Leaving aside the issue of actually detecting photons, however, we still have a point to address. Before a photon reaches such a special environment as a film plate, we know from several classic experiments that it maintains quantum coherence, despite propagating through air or other media that are described by our model. A naive application of one-particle results to the case of a photon might make this seem problematic, but in fact the explanation is very simple.

We have found that coherent states are decohered least, on average over several dynamical times, of all initial pure states. For a single harmonic oscillator evolving under (13), it is well known that a "Schrödinger's Cat state"

$$|\psi\rangle = c_1 e^{-\frac{i}{\hbar}p_1\hat{A}} e^{\frac{i}{\hbar}x_1\hat{p}_A}|0\rangle + c_2 e^{-\frac{i}{\hbar}p_2\hat{A}} e^{\frac{i}{\hbar}x_2\hat{p}_A}|0\rangle \tag{24}$$

formed by superposing two coherent states, *well separated in* (a,b)*-space*, will decohere thoroughly and rapidly. At a time $t = \frac{2n\pi}{\Omega}$, the reduced density matrix that has evolved from this initial state will be

$$\begin{aligned}
\rho(Q,Q';\frac{2n\pi}{\Omega}) &= \Big[\frac{M\Omega}{\hbar\pi(1+2n\pi D)}\Big]\sum_{i,j=1}^{2}\exp\Big(-\frac{M\Omega}{4\hbar(1+2n\pi D)}\mathcal{R}_{ij}\Big)\\
\mathcal{R}_{ij} &= \Big[Q+Q'-x_i-x_j\Big]^2 + (1+2n\pi D)^2\Big[Q-Q'-\frac{x_i-x_j}{1+2n\pi D}\Big]^2\\
&\quad -\frac{4i}{M\Omega}\Big[Qp_i - Q'p_j + n\pi D(Q-Q')(p_i+p_j) - \frac{x_ip_i - x_jp_j}{2}\Big]\\
&\quad +2n\pi D\Big[(x_i-x_j)^2 + (\frac{1}{M\Omega})^2(p_i-p_j)^2\Big]\ .
\end{aligned} \tag{25}$$

From the last line of (25), we can infer that the timescale for decoherence of the two pointer states states is

$$t_D = \Big[\Omega D(\Delta^2 - 1)\Big]^{-1}\ , \tag{26}$$

where

$$\Delta^2 \equiv \frac{M\Omega}{2\hbar}\Big[(x_1-x_2)^2 + (\frac{p_1-p_2}{M\Omega})^2\Big]\ . \tag{27}$$

It is obvious that equation (26) makes sense only when $\Delta^2 > 1$. This has been pointed out, in somewhat different language, in Ref. [9]. We will further elaborate on this point below, when we consider what happens to a superposition of two orthogonal states whose wave functions are concentrated within a phase-space distance of order $\Delta^2 = 1$.

It is also obvious that during processes that occur over timescales shorter than some t_{max}, quantum coherence between two superposed states will *not* decay significantly, if the Wigner functions for the two states are concentrated within a phase space disc of radius $\sim 2\sqrt{t_{max}/\Omega D}$. This rather elementary fact is of considerable conceptual importance, as it clearly exhibits the limitations of environmental decoherence. From it, we can deduce a succinct refinement of our formulation of environmental-induced superselection, introducing a new term that complements the notion of a 'pointer state': *Every pointer state is surrounded, in Hilbert space, by a 'quantum halo' of states which are not effectively distinguished from it by the environment.*

The size of the quantum halo of a pointer state is in general a function both of the strength of environmental noise, and of the maximum timescale over which it is allowed to act. However, there is an upper bound to this timescale, past which the whole notion of environmentally-induced superselection breaks down anyway, and neither pointer states nor quantum halos are particularly meaningful. We can deduce from Eqn. (16) that the entropy for an initially coherent state after n periods of motion is

$$S(\frac{n\pi}{\Omega}) = (1 + \frac{n\pi D}{2}) \ln(1 + \frac{n\pi D}{2}) - \frac{n\pi D}{2} \ln(\frac{n\pi D}{2}) \ . \tag{28}$$

When $nD = \frac{2}{\pi}$, the entropy even for a pointer state is equal to that of a statistical mixture of four equally probable pure states. It is clear that decoherence this severe does not produce superselection, but merely swamps the system with environmental noise. For our pointer states to be meaningful, therefore, we must have $D\Omega t << 1$.

This means that, as long as decoherence is mild enough to be achieving superselection instead of mere randomization, the quantum halo surrounding a coherent state is bound to extend to at least a radius $\Delta \sim 1$. Even this minimal halo supports a two dimensional subspace of states: the first excited energy eigenstate of the oscillator resides within the quantum halo of the ground state (see Figure 1), and a similar halo state may easily be found for any coherent state. Higher excited states (excited above the classical background represented by the pointer state) may be accommodated within larger halos, and from the fact that the wave function $\psi_n(Q)$ of the nth state decays rapidly beyond the classical turning points $M\Omega Q^2 = (n + \frac{1}{2})\hbar$, we can estimate that a halo of radius Δ_h is a subspace spanned by linear combinations of all excited states up to $n \sim \Delta_h$.

Generalizing straightforwardly from the single oscillator to an electromagnetic field in a homogeneous medium, we can conclude that every background field configuration is surrounded by a quantum halo of photons. This explains why a dielectric medium does little to eliminate quantum interference in a double slit experiment, and why propagation through an environment will not necessarily destroy the long-range entanglement of an EPR pair. The quantum halo around a pointer state of a system consisting of N field modes can most simply be described when the Hilbert space is represented in the complex N-space of coherent states. Each point $\vec{\alpha} = (\alpha_1, ..., \alpha_N)$ in this space labels a pointer state, and the associated quantum halo is the ellipsoidal ball

$$\sum_{k=1}^{N} \frac{|z_k - \alpha_k|^2}{\Omega_k} < \frac{\hbar}{\gamma t_{max} k_B T} . \tag{29}$$

Arguing as for the single oscillator, this can be taken to imply that the halo for a field pointer state can accommodate on the order of

$$N_k \simeq \Big(\frac{\hbar\Omega_k}{k_B T}\Big) \times \Big(\frac{t_{diss}}{t_{max}}\Big) \tag{30}$$

photons per mode k, where the dissipation time $t_{diss} \equiv \gamma^{-1}$.

5 Conclusion

The pointer states of the quantum electromagnetic field, propagating in a homogeneous linear dielectric medium, are coherent states. When decoherence is not so strong that it merely swamps the field with noise, coherent states evolve almost freely. The pointer states therefore behave as classical field configurations, evolving under the classical equations of motion. We have therefore provided an insight into the emergence of classical electromagnetism from quantum electrodynamics.

Each pointer state of a quantum field is surrounded, in Hilbert space, by a quantum halo — a set of states which are negligibly decohered from the pointer state over whatever time period is of interest. When the environmental noise is weak enough that it does not significantly degrade the pointer states themselves, this quantum halo is large enough to contain at least a few particles, excited above the background classical field configuration represented by the pointer state. We have thus recovered the familiar field-theoretic dichotomy between background classical fields and N-particle excitations. The relative immunity of the particle excitations to decoherence, in comparison with the strong decoherence of superpositions of distinct pointer states, explains the co-existence of effective classical electrodynamics and coherent propagation of photons.

The n-particle excitations are not localized by our homogeneous environment. All localization occurs in the space of coherent state amplitudes, and not in position space. This result is consistent with the "indications" arrived at, in the perturbative study by Kübler and Zeh[10], by considering a first order "de-separation" parameter instead of entropy. It is also consistent with the semi-classical results of Kiefer[11], in which superpositions of well-separated coherent states of a scalar electromagnetic field are shown to decohere in the presence of a charged scalar source field. But it is in strong contrast with what one might expect decoherence to do, based on a naive translation of the particles studied in quantum Brownian motion into field quanta. Although a linear dielectric medium does not have the avalanche instability of a cloud chamber, one would still look to the concept of decoherence for a general explanation of why electrons, for example, should behave in the classical limit as localized particles. Indeed, we do expect that this is the case: the linearly coupled field we have analyzed in this paper differs in an essential way from the environmental coupling of a typical matter field.

For matter fields, the interaction Hamiltonian with an environment tends to be bilinear, rather than linear, in creation and annihilation operators. The crude rule of thumb, that pointer states should be eigenstates of operators that approximately commute with the interaction Hamiltonian[1], suggests then that the pointer states for matter fields should be n-particle states rather than coherent states. And while a photon can only impart information to a localized environmental degree of freedom by being absorbed by it, material particles can scatter, surviving the information transfer without having to rely on a rare recurrence event to re-emit them. Since even a charged scalar field would therefore be expected to have particle, rather than field, pointer states, the usual reference to the different statistics of fermions and bosons is clearly not the main reason for the distinction between material particles and fields of force, which is rather due to different forms of environmentally induced super-selection.[2]

6 Appendix

The reduced density matrix $\rho(Q,Q';t)$ which evolves under the propagator (13), from an initial squeezed state

$$\langle Q|\psi_I\rangle = \Big(\frac{M\Omega \mathrm{Re}(C)}{\pi\hbar}\Big)^{\frac{1}{4}} e^{-\frac{M\Omega}{2\hbar}CQ^2} \tag{31}$$

with complex C, is given by

$$\rho(Q,Q';t) = \sqrt{\frac{M\Omega\mathrm{Re}(C)}{\pi\hbar}}\sqrt{\alpha(t)}e^{-\frac{M\Omega}{4\hbar}\alpha(t)[(Q+Q')^2+\beta(t)(Q-Q')^2-2i\lambda(t)(Q^2-Q'^2)]}\ . \tag{32}$$

The dimensionless functions α, β, and λ are defined as

$$\begin{aligned}
[\alpha(t)]^{-1} &\equiv [\mathrm{Re}(C)]^2\sin^2\Omega t + D\mathrm{Re}(C)(\Omega t - \sin\Omega t\cos\Omega t) + [\mathrm{Im}(C)\sin\Omega t - \cos\Omega t]^2\ ;\\
\beta(t) &\equiv \mathrm{Re}(C)[1 + D^2(\Omega^2 t^2 - \sin^2\Omega t)] + D|C|^2(\Omega t - \sin\Omega t\cos\Omega t)\\
&\qquad + D(\Omega t + \sin\Omega t\cos\Omega t) - 2D\mathrm{Im}(C)\sin^2\Omega t\ ;\\
\lambda(t) &\equiv [|C|^2 - 1]\sin\Omega t\cos\Omega t - \mathrm{Im}(C)\cos 2\Omega t + D\mathrm{Re}(C)\sin^2\Omega t\ .
\end{aligned} \tag{33}$$

Final states evolving from other initial squeezed states may be obtained trivially from the result we exhibit, by applying translation operators as discussed in Section III.

[2]We point out that the field-like nature of a Bose condensate of atoms is not necessarily a counter-example to our assertion that field-like behavior is associated with linear coupling to an environment. Bose condensation only occurs in the presence of a chemical potential, which represents the exchange of particles between the system and an unobserved environment. The simplest such exchange is described by a rotating-wave-like coupling, through which a system particle can be annihilated as an environmental particle is created, or *vice versa*. Such a coupling is linear in the system field operator.

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Figure Captions.

Figure 1

Plots of the reduced density matrix $\rho(Q, Q'; t)$ at $t = 0$ (figs. 1a and 1a') and at $t = 2n\pi\Omega^{-1}$, with D chosen so that $2n\pi D = 0.05$ (figs. 1b and 1b'). The horizontal axes measure Q and Q', in units of $\frac{\hbar}{M\Omega}$. Figures 1a and 1b show the evolution of a superposition of two coherent states $\sqrt{2}\cos(\hat{p}/\sqrt{\hbar M\Omega})|0\rangle$. It is clear that decoherence is much advanced in Fig. 1b, but that the two diagonal peaks are essentially intact, as they represent pointer states. In contrast, Figures 1a' and 1b' show the evolution of an initial superposition of energy eignenstates, $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. The first excited state lies within the quantum halo of the ground state, and the superposition has not suffered any discernible loss of coherence.