Matter Effects on Long Baseline Neutrino Oscillation Experiments

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Abstract.
We calculate matter effects on neutrino oscillations relevant for long baseline neutrino oscillation experiments. In particular, we compare the results obtained with simplifying approximations for the density profile in the Earth versus results obtained with actual density profiles. We study the dependence of the oscillation signals on both $E/\Delta m^2_{atm.}$ and on the angles in the leptonic mixing matrix. The results show quantitatively how matter effects can cause significant changes in the oscillation signals, relative to vacuum oscillations and can be useful in amplifying these signals and helping one to obtain measurements of mixing parameters and the magnitude and sign of Δm^2.

INTRODUCTION

In a modern theoretical context, one generally expects nonzero neutrino masses and associated lepton mixing. Experimentally, there has been accumulating evidence for such masses and mixing. All solar neutrino experiments (Homestake, Kamiokande, SuperKamiokande, SAGE, and GALLEX) show a significant deficit in the neutrino fluxes coming from the Sun (1). This deficit can be explained by oscillations of the ν_e's into other weak eigenstate(s), with Δm^2_{sol} of the order 10^{-5} eV^2 for MSW solutions (2) or of the order of 10^{-10} eV^2 for vacuum oscillations. Accounting for the data with vacuum oscillations requires almost maximal mixing. The MSW solutions include one for small mixing angle (SMA) and one with essentially maximal mixing (LMA).

Another piece of evidence for neutrino oscillations is the atmospheric neutrino anomaly, observed by Kamiokande, SuperKamiokande, IMB, MACRO, and Soudan-2 (3). Of these, the Superkamiokande data has especially high statistics - roughly 52 kton-years worth of data at present. This data can be well fit by the inference of $\nu_\mu \to \nu_x$ oscillations with $\Delta m^2_{atm} \sim 3.5 \times 10^{-3}$ eV^2 (3) and maximal mixing $\sin^2 2\theta_{atm} = 1$, where $\nu_x = \nu_\tau$ is favored. The possibility $\nu_x = \nu_{sterile}$ is disfavored at the 2σ level (5). (The possibility that ν_x is predominantly ν_e is ruled out by both the Superkamiokande data and the CHOOZ experiment (4)).

In addition, the LSND experiment has reported observing $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ oscillations with $\Delta m^2_{LSND} \sim 0.1 - 1$ eV^2 and moderately small mixing angle. This result is not confirmed by a similar experiment, KARMEN (6).

There are currently strong efforts to confirm and extend the evidence for neutrino oscillations in all of the various sectors – solar, atmospheric, and accelerator. Some of these are currently running: the Sudbury Neutrino Observatory, SNO, the K2K pioneering long baseline experiment between KEK and Kamioka. Others are in development and testing phases, such as Borexino, KamLAND, MINOS, mini-BOONE, and the CERN-Gran Sasso program. Among the long baseline neutrino oscillation (LBLNO) experiments, the distances are $L \simeq 250$ km for K2K, 730 km for both MINOS, from Fermilab to Soudan and the proposed CERN-Gran Sasso experiments. The sensitivity of these experiments should reach the region $\Delta m^2 \sim$ few $\times 10^{-3}$eV^2. Another generation of experiments, with even higher sensitivity will be required for precision measurements of oscillation parameters. One of the physics capabilities of the Next generation Nucleon decay and Neutrino detector discussed at this NNN99 workshop would be as part of a LBLNO experiment. An interesting possibility that is being studied intensively is a muon collider or storage ring that would serve as a source of quite high intensity, flavor-pure ($\nu_\mu + \bar{\nu}_e$ beams from μ^- and $\bar{\nu}_\mu + \nu_e$ beam from μ^+) (anti)neutrino beams. Using these, one could perform LBLNO experiments with an existing deep underground detector, e.g., at Soudan, Gran Sasso, or Kamioka, the NNN detector, and/or a surface detector. Studies have shown that one can get hundreds of events per kiloton-year at distances of 7000-9000 km (7), (8). It is thus appropriate to begin planning for this next generation of very long baseline neutrino oscillation experiments.

An important effect that must be taken into account in such experiments is the matter-induced oscillations which neutrinos undergo along their flight path through the Earth from the source to the detector. In a hypothetical world in which there were only two neutrinos, ν_μ and ν_τ, the $\nu_\mu \to \nu_\tau$ oscillations in matter would be the same as in vacuum, since both have the same forward scattering amplitude, via Z exchange, with matter. However, in the realistic case of three generations, because of the indirect involvement of ν_e due to a nonzero U_{13}, and because of the fact that ν_e has a different forward scattering amplitude off of electrons, involving both Z and W exchange, there will be a matter-induced oscillation effect on $\nu_\mu \to \nu_\tau$ (as well as other channels). An early study of matter effects in the earth is (9); several recent studies are (8)-(15).

Here we shall report on a study that we have carried out (16) of matter effects relevant to LBLNO experiments. We consider the usual three flavors of active neutrinos, with no light sterile (= electroweak-singlet) neutrinos. This is sufficient to describe the more established evidence from the solar and atmospheric neutrino deficit (if one tried to fit also the LSND experiment with a neutrino oscillation scenario, one would be led to include light sterile neutrinos). As suggested by the solar and atmospheric data, we consider that there is only one mass scale relevant for long baseline and atmospheric neutrino oscillations, $\Delta m^2_{atm} \sim$ few $\times 10^{-3}$ eV^2 and we work with the hierarchy

$$\Delta m^2_{21} = \Delta m^2_{sol} \ll \Delta m^2_{31} \approx \Delta m^2_{32} = \Delta m^2_{atm} \qquad (1)$$

In our work we take into account the actual profile of the Earth, as given by geophysical seismic data (17) and compare the results with those calculated using the approximation of average density along the path of the neutrino. Further, we present the oscillation probabilities as functions of $E/\Delta m^2$ so one can determine which energies are best suited for precise measurements of Δm^2 in a given region. We study how these oscillation probabilities vary with the different input parameters and discuss the influence of the matter effects on the sensitivity to each of these parameters.

MATTER EFFECTS

The evolution of the flavor eigenstates is given by

$$i\frac{d}{dx}\nu = \left(\frac{1}{2E}UM^2U^\dagger + V\right)\nu \qquad (2)$$

where the flavor neutrino wavefunction is

$$\nu = U\nu_m \qquad (3)$$

in terms of the mass eigenstates

$$\nu_m = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad (4)$$

and

$$M^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, V = \begin{pmatrix} \sqrt{2}G_F N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (5)$$

where N_e is the electron number density and $\sqrt{2}G_F N_e[\text{eV}] = 7.6 \times 10^{-14} Y_e \rho$ [g/cm^3].

The leptonic mixing matrix U can be written as

$$U = R_{23}KR_{13}K^*R_{12}K' \qquad (6)$$

which is the standard CKM-type parametrization, with R_{ij} being the rotation matrix in the ij subspace, $c_{12} = \cos\theta_{12}$, $s_{12} = \sin\theta_{12}$, etc., $K = diag(e^{-i\delta}, 1, 1)$, and $K' = diag(e^{i\alpha_1}, e^{i\alpha_2}, 1)$ (the latter phases originate from the general presence of Majorana mass terms but will not be important here).

The atmospheric neutrino data suggests almost maximal mixing in the $2-3$ sector. However, a small but non-zero s_{13} is still allowed, and this produces the matter effect in the traversal of neutrinos through the Earth. We use $\sin^2(2\theta_{13}) \le 0.1$, consistent with the limits from the atmospheric neutrino data (3) and the CHOOZ experiment (4). We also assume the small mixing angle (SMA) MSW solution to the solar neutrino data. This assumption, together with the hierarchy of eq. (1), implies that, for the relevant energies $E \gtrsim 1$ GeV and pathlengths $L \sim 10^3 - 10^4$ km, only one squared mass scale, Δm^2_{atm}, is important for the oscillations, and the three-species neutrino oscillations can be described in terms of this quantity, Δm^2_{atm}, and the mixing parameters $\sin^2(2\theta_{23})$, and $\sin^2(2\theta_{13})$, with negligible dependence on $\sin^2(2\theta_{12})$ and δ; hence also, CP violation effects would be negligibly small here, and $P(\nu_a \to \nu_b) = P(\nu_b \to \nu_a)$, $P(\bar{\nu}_a \to \bar{\nu}_b) = P(\bar{\nu}_b \to \bar{\nu}_a)$. Although, *a priori*, CP violation would lead to $P(\nu_a \to \nu_b) \neq P(\bar{\nu}_a \to \bar{\nu}_b)$ in vacuum, this inequality is true in matter even in the absence of CP violation.

For our purposes, we recall that the Earth is composed of crust, mantle, liquid outer core, and solid inner core, together with additional sublayers in the mantle, with particularly strong changes in density between the lower mantle and outer core. The density profile of the Earth is shown in fig. 1 from (17). The core has average density $\rho_{core} = 11.83$ g/cm^3 and electron fraction $Y_{e,core} = 0.466$, while the mantle has average density $\rho_{mantle} = 4.66$ g/cm^3

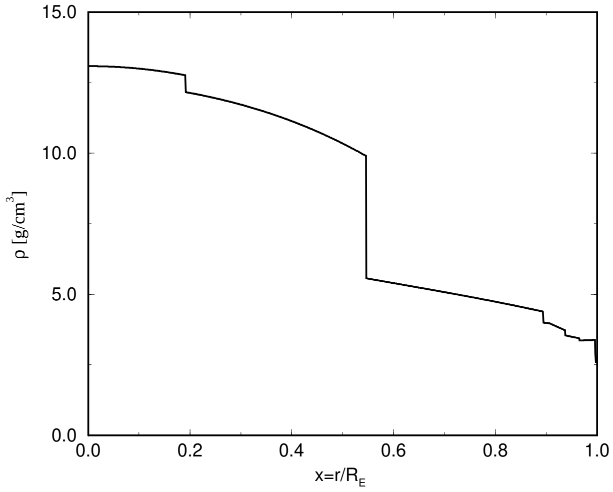


FIGURE 1. Density profile of the Earth

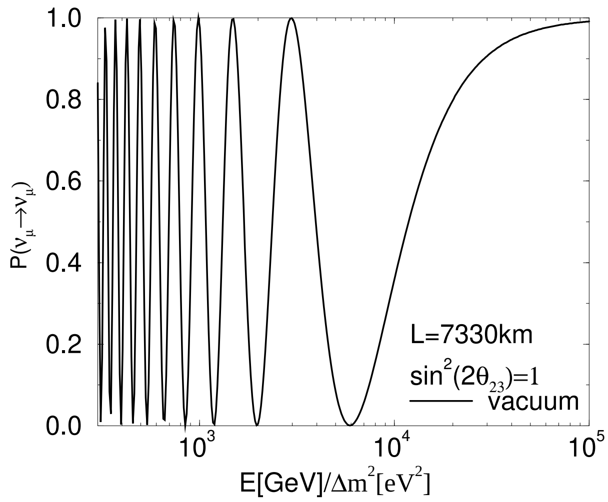


FIGURE 2.

and $Y_{e,mantle} = 0.494$. If one approximates the density as a constant along the neutrino flight path, the evolution equation can easily be solved, with well-known results. However, when one takes account of the actual variable-density situation in the earth, it is necessary to perform a numerical integration of the evolution equation, which we have done.

RESULTS AND DISCUSSION

For long baseline experiments like K2K, Fermilab to Minos, and CERN to Gran-Sasso, the neutrino flight path only goes through the upper mantle. The density in this region is practically constant, and the oscillation probabilities can easily be calculated. The matter effects are small, but possibly detectable for the longer baselines. However, there are several motivations for very long baseline experiments, since, with sufficiently high-intensity sources, these can be sensitive to quite small values of Δm^2 and since the matter effects, being larger, can amplify certain oscillations and can, in principle, be used to get information on the sign of Δm^2_{atm}. Hence we concentrate here on these very long baseline experiments; for these, the neutrino flight path goes through several layers of the Earth with different densities, including the lower mantle. We show results for $L \simeq 7330$ km, the distance from Fermilab to Gran Sasso. We have also performed calculations for the Fermilab to SuperKamiokande and Fermilab to SLAC path lengths, $\sim$ 9120 and 2880 km, respectively. We calculate the probabilities of oscillation in long baseline experiments as a function of $E/\Delta m^2$, rather than using a particular value for Δm^2 or the energy. The relevant ranges are $\Delta m^2 \sim$ few $\times 10^{-3}$ eV^2 and energies E of the order of tens of GeV. This way of presenting the results can be useful in studying the optimization of the beam energy. In our work (16) we calcualte the oscillation probabilities for different values of the mixing angles θ_{13} and θ_{23} allowed by the atmospheric neutrino data and the CHOOZ experiment; for this workshop report we only show results for $\sin^2(2\theta_{23}) = 1$. We consider both neutrinos and antineutrinos. The matter effects change sign in these two cases; for antineutrinos, V in (5) is replaced with $(-V)$. This implies that if Δm^2 is positive (as considered here), one can get a resonant enhancement of the oscillations for neutrinos, while for antineutrinos the matter effects would suppress the oscillations. The situation would be reversed if Δm^2 were negative.

We first study the survival probability of ν_μ. If the beam went through vacuum, the oscillation probability would look like the curve in fig. 2 for practically any value of $\sin^2(2\theta_{13})$. In matter, this probability becomes sensitive to all oscillation parameters, as can be seen from fig. 3 and fig. 4.

We also want to compare the solution in vacuum (fig. 2) with the solution in matter for neutrinos (fig. 3) and antineutrinos (fig. 5). In the legends for the figures with antineutrinos, "anti $\nu_a \to \nu_b$ means $\bar{\nu}_a \to \bar{\nu}_b$. One can see the opposite effects of matter on neutrinos and antineutrinos. The difference in the results for different mixing angles makes it possible in principle to use this probability for relatively precise measurements of the oscillation parameters. Measuring separately the probability for ν and $\bar{\nu}$ can be very useful in detecting the matter effects and using these to constrain the relevant mixings and squared mass difference. Clearly, if one could use two path lengths, as may be possible with a neutrino factory, this would provide more information and constraints.

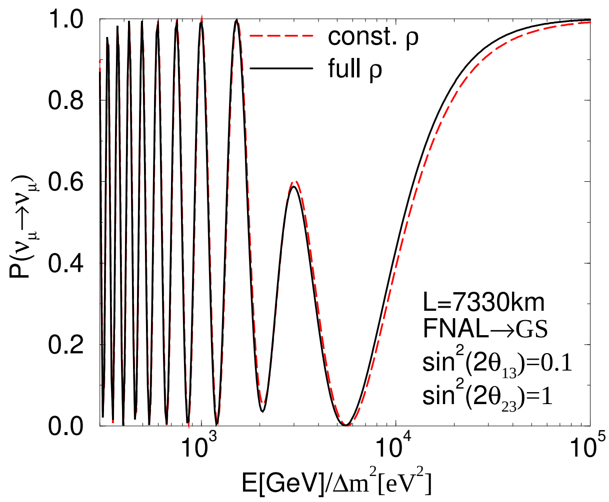
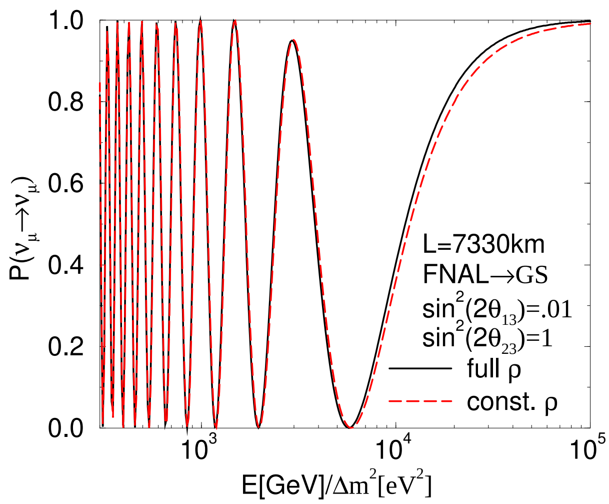


FIGURE 3.



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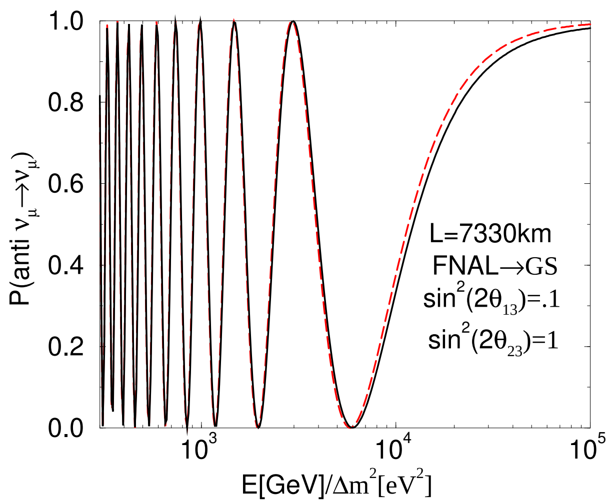

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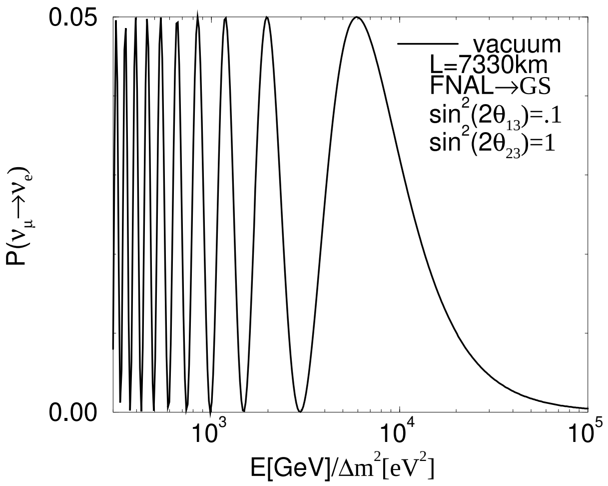
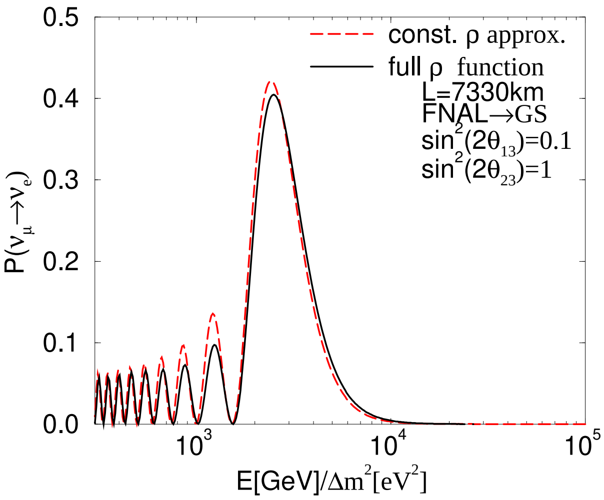


FIGURE 6.

The relative effects of matter can be especially dramatic in the oscillation probability $P(\nu_\mu \to \nu_e)$, since these directly involve ν_e. If the beam were to go through the vacuum, $P(\nu_\mu \to \nu_e)$ would be probably too small to detect (fig. 6). Because of the matter effect however, this probability can be strongly enhanced, as is evident in fig. 7. The enhancement is largest for $E/\Delta m^2$ around 3000 GeV/eV^2. This is essentially equal to the ratio that one would get using a beam energy of ~ 10 GeV, given the indication from the data that $\Delta m_{atm} = 3.5 \times 10^{-3}$ eV^2. Hence the matter effect can amplify $P(\nu_\mu \to \nu_e)$ and enable this transition to be measured with reasonable accuracy, thereby yielding important information on the oscillation parameters. This probability is very sensitive to the value of θ_{13} (figs. 7, 8), so one could use it for a good determination of this angle. The sensitivity to Δm^2 is also quite strong, due to the pronounced peak given by the matter effect in the relevant region. Note that for antineutrinos, the oscillation is suppressed (fig. 9), so an independent measurement of the two channels ($\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$) would be very valuable.

The atmospheric neutrino data tells us that the dominant oscillation channel is actually $\nu_\mu \to \nu_\tau$. Consequently, it would be very useful to measure $P(\nu_\mu \to \nu_\tau)$; this would provide further confirmation of this oscillation and could also provide accurate determinations of Δm^2 and θ_{23}. Fig. 10 shows $P(\nu_\mu \to \nu_\tau)$. Fig. 11 shows $P(\bar{\nu}_\mu \to \bar{\nu}_\tau)$.

Since with a muon collider or muon storage ring, ν_e ($\bar{\nu}_e$) beams would also be available, it would be interesting to study oscillation probabilities with these beams. We already have the results for $P(\nu_e \to \nu_\mu)$ since, as mentioned above, with our parameters, this is the same as $P(\nu_\mu \to \nu_e)$. We present here $P(\nu_e \to \nu_\tau)$ in fig. 12 and



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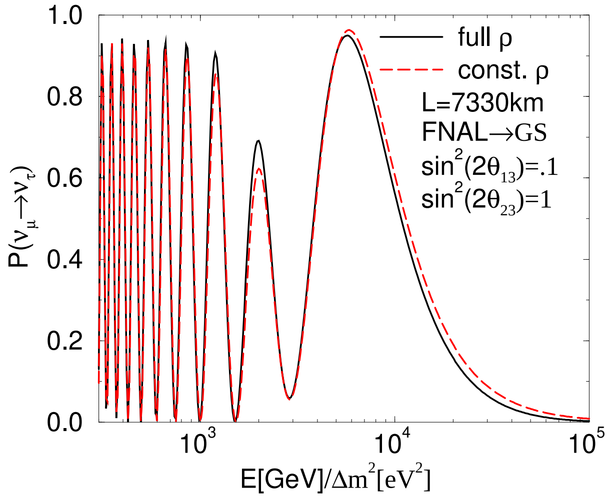
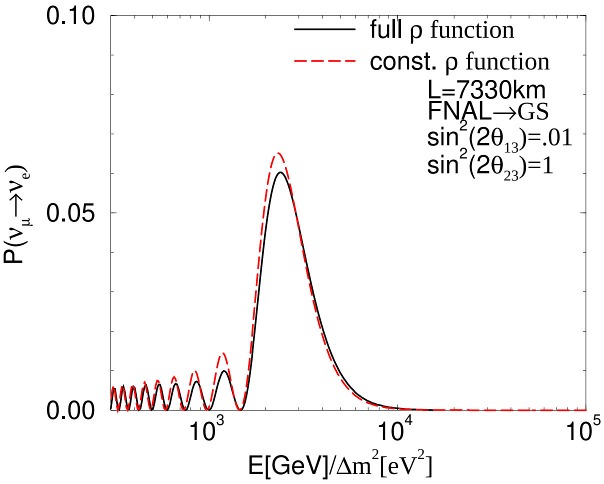


FIGURE 10.



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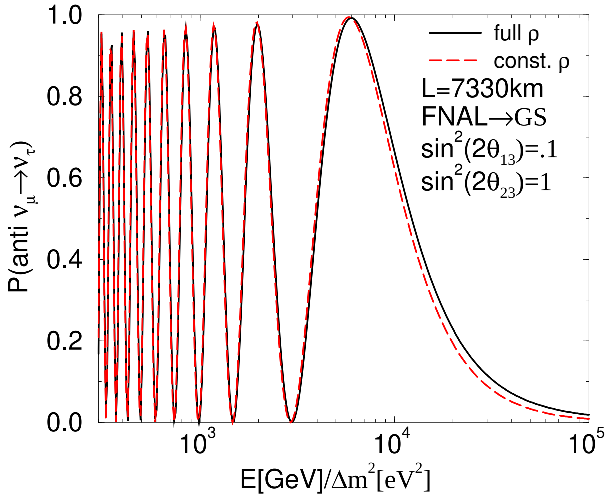

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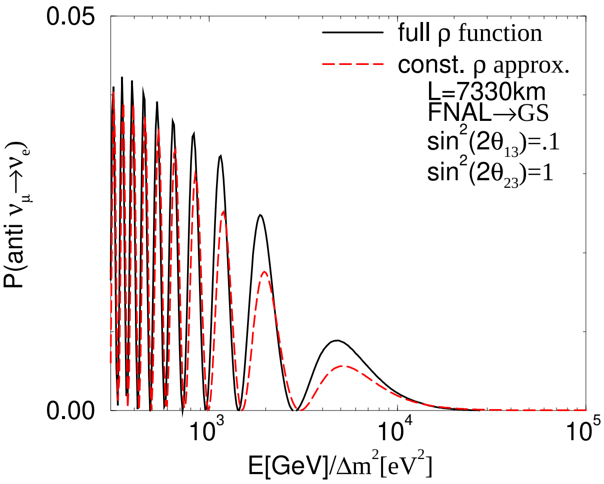

FIGURE 9.

$P(\bar{\nu}_e \to \bar{\nu}_\tau)$ in Fig. 13. These calculations show that matter effects are important and enhance oscillations of the neutrinos and suppress oscillations of antineutrinos in the relevant region of parameters.

To summarize, in planning for very long baseline neutrino oscillation experiments, it is important to take into account matter effects. We have performed a careful study of these, including realistic density profiles in the earth. Matter effects can be useful in amplifying neutrino oscillation signals and helping one to obtain measurements of mixing parameters and the magnitude and sign of Δm^2.

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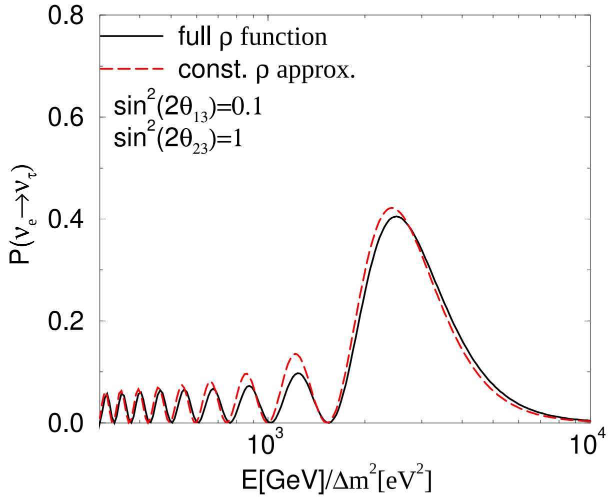

FIGURE 12.

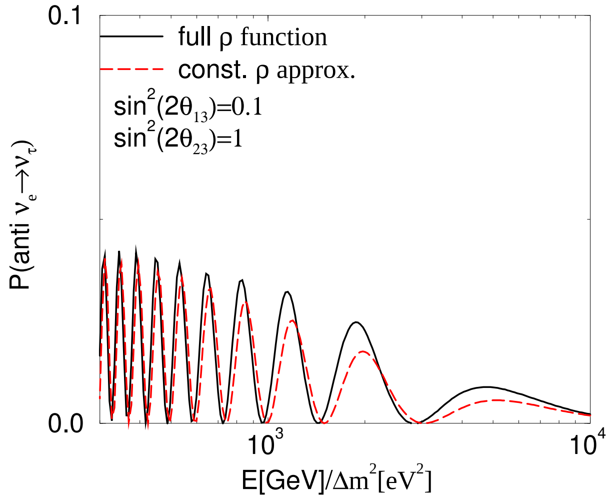

FIGURE 13.

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REFERENCES

1. Fits and references to the data include J. Bahcall, P. Krastev, A. Smirnov, Phys. Rev. **D58**, 096016 (1998); J. Bahcall and P. Krastev, Phys. Lett. **B436**, 243 (1998). Recent Superkamiokande data is reported in SuperKamiokande Collab., Y.Fukuda et al., Phys. Rev. Lett. **82**, 1810, 2430 (1999).
2. L. Wolfenstein, Phys. Rev. **D17**, 2369 (1978); S. P. Mikheyev and A. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov.J. Nucl. Phys. **42**, 913 (1986)], Nuovo Cim., **C9**, 17 (1986).
3. Kamiokande Coll., Y.Fukuda et al., Phys. Rev. Lett. **B335**, 237 (1994).

   IMB Coll., R.Becker-Szendy et al., Nucl. Phys.**B38**(Proc. Suppl.), 331 (1995).

   Soudan Coll., W.W.M. Allison et al, Phys. Lett. **B391**, 491 (1997).

   MACRO Coll., M. Ambrosio et al., Phys. Lett. **B434**, 451 (1998).

   Super-Kamiokande Coll., Y. Fukuda et al., Phys. Rev. Lett. **81**,1562 (1998); *ibid.*, **82**, 2644 (1999); hep-ex/9908049.
4. M. Apollonio et al., Phys. Lett. **B420**, 397 (1998); hep-ex/9907037.
5. J. Learned, this workshop.
6. LSND Coll., C. Athanassopoulous et al., Phys. Rev. Lett. **77**, 3082 (1996), LSND Coll., C. Athanassopoulous et al., Phys. Rev. Lett. **81**, 1774 (1998).

   KARMEN Coll., Nucl. Phys. Proc. Suppl. **70**, 210 (1999).
7. S. Geer, Phys.Rev.**D57**, 6989 (1998), erratum-ibid. **D59**, 039903 (1999).
8. V. Barger, S. Geer, K. Whisnant, hep-ph/9906487.
9. A. J. Baltz, J. Weneser, Phys. Rev. **D37**, 3364 (1988).
10. G. Fogli, E. Lisi, D. Montanino, Phys.Rev. D49, 3626 (1994).
11. E. Akhmedov, A. Dighe, P. Lipari, A. Smirnov, Nucl. Phys. **B542**, 3 (1999).
12. E. Akhmedov, Nucl.Phys. **B538**, 25 (1999).
13. P. Krastev, S. Petcov, Phys. Lett. **B205**, 8 (1988).

    E. Lisi, D. Montanino, Phys. Rev. **D56**, 1792 (1997).

    M. Chiznov, M. Maris, S. Petcov, hep-ph/9810501.

    M. Chiznov, S. Petcov, hep-ph/9903424.
14. P. Lipari, hep-ph/9903481.
15. M. Campanelli, A. Bueno, A. Rubbia, hep-ph/9905240.

    D. Dooling, C. Giunti, K. Kang, C. W. Kim, hep-ph/9908513.
16. I. Mocioiu. R. Shrock, YITP-SB-99-13.
17. A.Dziewonski, Earth Structure, in: ”The Encyclopedia of Solid Earth Geophysics”, D.E.James (Ed.), (Van Nostrand Reinhold, New York, 1989), p.331.