THE MSSM AND WHY IT WORKS

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An introduction to the minimal supersymmetric standard model is presented. We emphasize phenomenological motivations for this model, along with examples of experimental tests. Particular attention is paid to the Higgs sector of the theory.

1 INTRODUCTION

The Standard Model of particle physics is in stupendous agreement with experimental measurements; in some cases it has been tested to a precision of greater than .1%. Why then expand our model? The reason, of course, is that the Standard Model contains a variety of nagging theoretical problems which cannot be solved without the introduction of some new physics. We have no understanding of masses or why there are three generations of quarks and leptons. The origin of electroweak symmetry breaking is a complete mystery. The source of CP violation is not known. The list goes on and on......

Supersymmetry is, at present, many theorists' favorite candidate for new physics beyond the Standard Model. Unfortunately, merely constructing a supersymmetric version of the Standard Model does not answer many of the open questions. String theories may answer some of these questions, although a phenomenologically viable string theory has yet to be constructed. In these lectures, we will try to be very explicit about the benefits and drawbacks of various SUSY models.

The most important aspect of the Standard Model which has not yet been verified experimentally is the Higgs sector. The Standard Model without the Higgs boson is incomplete since it predicts massless fermions and gauge bosons. Furthermore, the electroweak radiative corrections to observables such as the W and Z boson masses would be infinite if there were no Higgs boson. The simplest means of curing these defects is to introduce a single $SU(2)_L$ doublet of Higgs bosons. When the neutral component of the Higgs boson gets a vacuum expectation value, (VEV), the $SU(2)_L \times U(1)_Y$ gauge symmetry is broken, giving the W and Z gauge bosons their masses. The chiral symmetry forbidding fermion masses is broken at the same time, allowing the fermions to become massive. The coupling of the Higgs boson to gauge bosons is just that required to cancel the infinities in electroweak radiative corrections.

The electroweak symmetry breaking of the Standard Model has the special feature that we know the energy scale at which it must occur. The argument follows from the scattering of longitudinally polarized gauge bosons. At high energy, $\sqrt{s} >> M_W$, the amplitude for this process is,[1]

$$\mathcal{A}(W_L^+W_L^- \rightarrow W_L^+W_L^-) = -\frac{G_F M_h^2}{8\sqrt{2}\pi}\left\{2+\frac{M_h^2}{s-M_h^2}-\frac{M_h^2}{s}\log\left(1+\frac{s}{M_h^2}\right)\right\} \quad . \quad (1)$$

For a light Higgs boson, we have the limit,

$$\mathcal{A}(W_L^+W_L^- \rightarrow W_L^+W_L^-) \longrightarrow_{s>>M_h^2} -\frac{G_F M_h^2}{4\pi\sqrt{2}} \quad . \quad (2)$$

Applying the unitarity condition to the $I=J=0$ partial wave for this process, $\mid a_0^0 \mid < \frac{1}{2}$, gives the restriction,

$$M_h < 860 \ GeV \quad . \quad (3)$$

We thus are reasonably confident that a weakly interacting Higgs boson, if it exists, will appear below the TeV scale.

Given the nice features of the Standard Model with a single Higgs boson, what then is the problem with this simple and economical picture? The argument against the simplest version of the Standard Model is purely theoretical and arises when radiative corrections to the Higgs boson mass are computed. At one loop, the quartic self- interactions of the Higgs boson generate a quadratically divergent contribution to the Higgs boson mass which must be cancelled by a mass counterterm. This counterterm must be fine tuned at each order in perturbation theory. The quadratic growth of the Higgs boson mass beyond tree level in perturbation theory is one of the driving motivations behind the introduction of supersymmetry, which we will see cures this problem. The cancellation of quadratic divergences will be discussed extensively in the next section.

In these lectures, we discuss the theoretical motivation for supersymmetric theories and introduce the minimal low energy effective supersymmetric theory, (MSSM). We consider the MSSM and its simplest grand unified extension, along with models where the supersymmetry is broken by the gauge interactions, (GMSB). The particles and their interactions are examined with particular attention paid to the Higgs sector of SUSY models. Finally, we discuss indirect limits on the SUSY partners of ordinary matter coming from precision measurements at LEP and direct production searches at the Tevatron and LEPII. Search strategies for SUSY at both future e^+e^- and hadron colliders are briefly touched on. There exist numerous excellent reviews of both

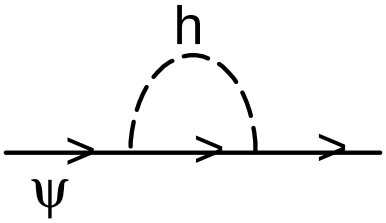

Figure 1: Fermion mass renormalization from a Higgs loop.

the more formal aspects of supersymmetric model building [2,3,4] and the phenomenology of these models [4,5,6] and the reader is referred to these for more details.

2 Quadratic Divergences

The vanishing of quadratic divergences in a supersymmetric theory is typically advertised as one of the primary motivations for introducing supersymmetry. As such, it is important to examine the question of quadratic divergences in detail. A nice discussion is given in the lecture notes by Drees.[7] We begin by considering a theory with a single fermion, ψ, coupled to a massive Higgs scalar,

$$\mathcal{L}_\phi = \overline{\psi}(i\gamma^\mu\partial_\mu)\psi + \mid \partial_\mu\phi \mid^2 - m_S^2 \mid \phi \mid^2 - \left(\frac{\lambda_F}{2}\overline{\psi}\psi\phi + \text{h.c.}\right) \quad . \tag{4}$$

We will assume that this Lagrangian leads to spontaneous symmetry breaking and so take $\phi = (h+v)/\sqrt{2}$, with h the physical Higgs boson. (The $\sqrt{2}$ is arbitrary at this point and is put in only because it is conventional.) After spontaneous symmetry breaking, the fermion acquires a mass, $m_F = \lambda_F v/\sqrt{2}$. First, let us consider the fermion self-energy arising from the scalar loop corresponding to Fig. 1.

$$-i\Sigma_F(p) = \left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2 \int \frac{d^4k}{(2\pi)^4} \frac{(k + m_F)}{[k^2 - m_F^2][(k-p)^2 - m_S^2]} \quad . \tag{5}$$

The renormalized fermion mass is $m_F^r = m_F + \delta m_F$, with

$$\begin{aligned} \delta m_F &= \Sigma_F(p) \mid_{p=m_F} \\ &= i\frac{\lambda_F^2}{32\pi^4}\int_0^1 dx \int d^4k' \frac{m_F(1+x)}{[k'^2 - m_F^2 x^2 - m_S^2(1-x)]^2} \quad . \end{aligned} \tag{6}$$

Since many of you probably only know how to calculate loop diagrams using dimensional regularization, we will take a brief aside to discuss the calculation of Eq. 6 using a momentum space cutoff. (This discussion directly parallels that of the renormalization of the electron self-energy in Bjorken and Drell.[8]) The integral can be performed in Euclidean space, which amounts to making the following transformations,

$$\begin{aligned} k_0 &\rightarrow ik_4 \\ d^4k' &\rightarrow id^4k_E \\ k'^2 &\rightarrow -k_E^2 \quad . \end{aligned} \tag{7}$$

Since the integral of Eq. 6 depends only on k_E^2, it can easily be performed using the result (valid for symmetric integrands),

$$\int d^4k_E f(k_E^2) = \pi^2 \int_0^{\Lambda^2} ydyf(y) \quad . \tag{8}$$

In Eq. 8, Λ is a high energy cut-off, presumably of the order of the Planck scale or a GUT scale. The renormalization of the fermion mass is then,

$$\begin{aligned} \delta m_F &= -\frac{\lambda_F^2 m_F}{32\pi^2}\int_0^1 dx(1+x)\int_0^{\Lambda^2}\frac{ydy}{[y+m_F^2x^2+m_S^2(1-x)]^2} \\ &= -\frac{3\lambda_F^2 m_F}{64\pi^2}\log\left(\frac{\Lambda^2}{m_F^2}\right)+.... \end{aligned} \tag{9}$$

where the .... indicates terms independent of the cutoff or which vanish when $\Lambda \rightarrow \infty$. This correction clearly corresponds to a well-defined expansion for m_F. Fermion masses are said to be *natural.* In the limit in which the fermion mass vanishes, Eq. 4 is invariant under the chiral transformations,

$$\begin{aligned} \psi_L &\rightarrow e^{i\theta_L}\psi_L \\ \psi_R &\rightarrow e^{i\theta_R}\psi_R, \end{aligned} \tag{10}$$

and so we see that setting the fermion mass to zero increases the symmetry of the theory. Since the Yukawa coupling (proportional to the fermion mass term) breaks this symmetry, the corrections to the mass must be proportional to m_F.

The situation is quite different, however, when we consider the renormalization of the scalar mass from a fermion loop (Fig. 2) using the same Lagrangian (Eq. 4),

$$-i\Sigma_S(p^2) = \left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2(-1)\int\frac{d^4k}{(2\pi)^4}\frac{Tr[(k+m_F)((k-p)+m_F)]}{(k^2-m_F^2)[(k-p)^2-m_F^2]} \quad . \tag{11}$$

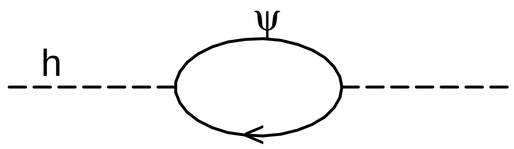

Figure 2: Higgs mass renormalization from a fermion loop.

The minus sign is the consequence of Fermi statistics and will be quite important later. Integrating with a momentum space cutoff as above we find the contribution to the Higgs mass, $(\delta M_h^2)_a \equiv \Sigma_S(m_S^2)$,

$$\begin{aligned}(\delta M_h^2)_a &= -\frac{\lambda_F^2}{8\pi^2}\Biggl[\Lambda^2 + (m_S^2 - 6m_F^2)\log\left(\frac{\Lambda}{m_F}\right) \\ &+ (2m_F^2 - \frac{m_S^2}{2})\left(1 + I_1\left(\frac{m_S^2}{m_F^2}\right)\right)\Biggr] + \mathcal{O}\left(\frac{1}{\Lambda^2}\right), \qquad (12)\end{aligned}$$

where $I_1(a) \equiv \int_0^1 dx \log(1 - ax(1-x))$. The Higgs boson mass diverges *quadratically*! The Higgs boson thus does not obey the decoupling theorem and this quadratic divergence appears independent of the mass of the Higgs boson. Note that the correction is *not* proportional to M_h. This is because setting the Higgs mass equal to zero does not increase the symmetry of the Lagrangian. There is nothing that protects the Higgs mass from these large corrections and, in fact, the Higgs mass wants to be close to the largest mass scale in the theory.

Since we know that in the Standard Model, the physical Higgs boson mass, M_h, must be less than around $1\ TeV$ (in order to keep the WW scattering cross section from violating unitarity), we have the unpleasant result,

$$M_h^2 = M_{h,0}^2 + \delta M_h^2 + \text{counterterm}, \qquad (13)$$

where the counterterm must be adjusted to a precision of roughly 1 part in 10^{15} in order to cancel the quadratically divergent contributions to δM_h^2. This adjustment must be made at each order in perturbation theory. This is known as the "*hierarchy problem*".

Of course, the quadratic divergence can be renormalized away in exactly the same manner as is done for logarithmic divergences by adjusting the cutoff. There is nothing formally wrong with this fine tuning. Most theorists,

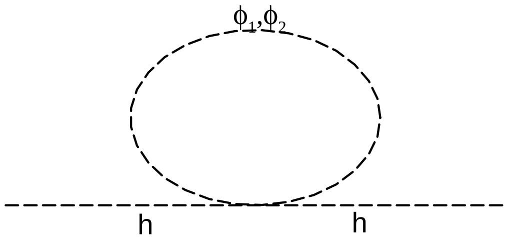

Figure 3: Higgs mass renormalization from scalar loops.

however, regard this solution as unattractive. On the other hand, if the calculation is performed in dimensional regularization, one obtains only $1/\epsilon$ singularities which are absorbed into the definitions of the counterterms. Hence, the problem of quadratic divergences does not become apparent when using dimensional regularization. It arises only when one attempts to import a physical significance to the cut-off Λ.

The effect of scalar particles on the Higgs mass renormalization is quite different from that of fermions. We introduce two complex scalar fields, ϕ_1 and ϕ_2, interacting with the Standard Model Higgs boson, h, (the reason for introducing 2 scalars is that with foresight we know that a supersymmetric theory associates 2 complex scalars with each fermion – we could just as easily make the argument given below with one additional scalar and slightly different couplings),

$$\begin{aligned}\mathcal{L} &= \mid \partial_\mu\phi_1 \mid^2 + \mid \partial_\mu\phi_2 \mid^2 - m_{s_1}^2 \mid \phi_1 \mid^2 - m_{s_2}^2 \mid \phi_2 \mid^2 \\ &\quad + \lambda_S \mid \phi \mid^2 \biggl(\mid \phi_1 \mid^2 + \mid \phi_1 \mid^2 \biggr) + \mathcal{L}_\phi \quad . \end{aligned} \tag{14}$$

From the diagram of Fig. 3, we find the contribution to the Higgs mass renormalization,

$$\begin{aligned}(\delta M_h^2)_b &= -\lambda_S \int \frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2-m_{s_1}^2}+\frac{i}{k^2-m_{s_2}^2}\right] \\ &= \frac{\lambda_S}{16\pi^2}\biggl\{2\Lambda^2 - 2m_{s_1}^2\log\left(\frac{\Lambda}{m_{s_1}}\right) - 2m_{s_2}^2\log\left(\frac{\Lambda}{m_{s_2}}\right)\biggr\} \\ &\quad +\mathcal{O}\left(\frac{1}{\Lambda^2}\right). \end{aligned} \tag{15}$$

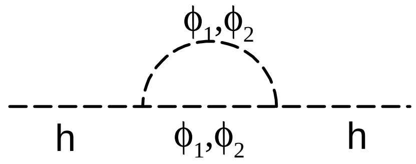

Figure 4: Higgs mass renormalization from scalar loops.

From Eqs. 12 and 15, we see that if

$$\lambda_S = \lambda_F^2, \tag{16}$$

the quadratic divergences coming from these two terms cancel each other. Notice that the cancellation occurs independent of the masses, m_F and m_{s_i}, and of the magnitude of the couplings λ_S and λ_F.

In the Standard Model, one could attempt to cancel the quadratic divergences in the Higgs boson mass by balancing the contribution from the Standard Model Higgs quartic coupling with that from the top quark loop in exactly the same manner as above. This approach would give a prediction for the Higgs boson mass in terms of the top quark mass. However, since there is no symmetry to enforce this relationship, this attempt to cancel quadratic divergences must fail at 2− loops.

After the spontaneous symmetry breaking, Eq. 14 also leads to a cubic interaction shown in Fig. 4. These also graphs give a contribution to the Higgs mass renormalization, although they are not quadratically divergent.

$$(\delta M_h^2)_c = \frac{\lambda_S^2 v^2}{16\pi^2}\left\{-1 + 2\log\left(\frac{\Lambda}{m_{s_1}}\right) - I_1\left(\frac{M_h^2}{m_{s_1}^2}\right)\right\} + (m_{s_1} \to m_{s_2}) + \mathcal{O}\left(\frac{1}{\Lambda^2}\right). \tag{17}$$

Combining the three contributions to the Higgs mass and assuming $\Lambda_S = \Lambda_F^2$ and $m_{s_1} = m_{s_2}$, we find no quadratic divergences in the Higgs mass renormalization,

$$(\delta M_h^2)_{tot} = \frac{\lambda_F^2}{4\pi^2}\left\{m_{s_1}^2 \log\left(\frac{\Lambda}{m_{s_1}}\right) - m_F^2 \log\left(\frac{\Lambda}{m_F}\right)\right\} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right). \tag{18}$$

If the mass splitting between the fermion and scalar is small, $\delta m^2 \equiv m_F^2 - m_{s_1}^2$, then we have the approximate result,

$$(\delta M_h^2)_{tot} = \frac{\lambda_F^2}{4\pi^2}\delta^2 \quad . \tag{19}$$

Therefore, as long as the mass splitting between scalars and fermions is "small", no unnatural cancellations will be required and the theory can be considered "natural". In this manner, a theory with nearly degenerate fermions and scalars and carefully adjusted couplings solves the hierarchy problem.

3 WHAT IS SUPERSYMMETRY?

In the previous section we saw that if the fermion and scalar couplings of a theory are carefully adjusted, it is possible to cancel the quadratically divergent contributions to the Higgs boson mass. Of course, in order for this cancellation to persist to all orders in perturbation theory it must be the result of a symmetry. This symmetry is **supersymmetry**.

Supersymmetry is a symmetry which relates the masses and couplings of particles of differing spin, (in the above example, fermions and scalars). The particles are combined into a *superfield*, which contains fields differing by one-half unit of spin.[9,10] The simplest example, the scalar (or chiral) superfield, contains a complex scalar, S, and a two- component Majorana fermion, ζ. (A Majorana fermion, ζ, is one which is equal to its charge conjugate, $\zeta^c = \zeta$. A familiar example is a Majorana neutrino.) The supersymmetry completely specifies the allowed interactions. In this simple case, the Lagrangian is

$$\begin{aligned} \mathcal{L} &= -\partial_\mu S^* \partial^\mu S - i\overline{\zeta}\overline{\sigma}^\mu \partial_\mu \zeta - \frac{1}{2} m(\zeta\zeta + \overline{\zeta\zeta}) \\ &\quad -cS\zeta\zeta - c^* S^* \overline{\zeta\zeta} - \mid mS + cS^2 \mid^2, \end{aligned} \tag{20}$$

(where $\overline{\sigma}^\mu \equiv (1, -\vec{\sigma})$, $\vec{\sigma}$ are the Pauli matrices, and c is an arbitrary coupling constant.) This Lagrangian is invariant (up to a total derivative) under supersymmetry transformations which take the scalar into the fermion and *vice versa*. The scalar self interactions of Eq. 20,

$$V(S, S^*) = \mid mS + cS^2 \mid^2 \tag{21}$$

clearly yield a potential which is positive definite, which is a general feature of an unbroken supersymmetric theory. An unbroken supersymmetric theory has its minimum at $\langle V \rangle = 0$.

Since the scalar and fermion interactions have the same coupling, the cancellation of quadratic divergences occurs automatically, as in Eq. 16. One thing that is immediately obvious is that this Lagrangian contains both a scalar and a fermion *of equal mass*. Supersymmetry connects particles of differing spin, but with all other characteristics the same. That is, they have the same quantum numbers and the same mass.

- Particles in a superfield have the same masses and quantum numbers and differ by 1/2 unit of spin in a theory with unbroken supersymmetry.

It is clear, then, that **supersymmetry must be a broken symmetry** if it is to be a theory of low energy interactions. There is no scalar particle, for example, with the mass and quantum numbers of the electron. In fact, there are no candidate supersymmetric scalar partners for any of the fermions in the experimentally observed spectrum. We will take a non-zero mass splitting between the particles of a superfield as a signal for supersymmetry breaking.

Supersymmetric theories are easily constructed according to the rules of supersymmetry. I present here a cookbook approach to constructing the minimal supersymmetric version of the Standard Model. The first step is to pick the particles in superfields.[11] There are two types of superfields relevant for our purposes:

1. *Chiral (Scalar) Superfields*: These consist of a complex scalar field, S, and a 2-component Majorana fermion field, ζ.

2. *Massless Vector Superfields*: These consist of a massless gauge field with field strength $F^A_{\mu\nu}$ and a 2-component Majorana fermion field, λ_A, termed a *gaugino*. The index A is the gauge index.

A chiral superfield, Φ, can be written in terms of anti-commuting Grassman variables, θ as,

$$\Phi(x) = S(x) + \sqrt{2}\theta\zeta(x) + \theta\theta F(x) \tag{22}$$

and all of the interactions can be written in terms of Φ. This form makes it clear that all the components of a superfield must have the same mass and gauge interactions. In the chiral superfield we see that the number of scalar degrees of freedom is equal to the number of fermion degrees of freedom. Similarly, a massless vector boson and a Majorana gaugino also have 2 degrees of freedom. The superfields also contain "auxiliary fields", F, which are fields with no kinetic energy terms in the Lagrangian.[9] These fields are not important for our purposes, although they are important for constructing the interactions.

3.1 The Particles of the MSSM

The MSSM respects the same $SU(3) \times SU(2)_L \times U(1)_Y$ gauge symmetries as does the Standard Model. The particles necessary to construct the minimal supersymmetric version of the Standard Model are shown in Tables 1 and 2 in terms of the superfields, (which are denoted by the superscript "hat"). Since there are no candidates for supersymmetric partners of the observed particles, we must double the entire spectrum, placing the observed particles in superfields with new postulated superpartners. There are, of course, quark and lepton superfields for all 3 generations and we have listed in Table 1 only the members of the first generation to simplify the notation. The superfield $\hat{Q}$ thus consists of an $SU(2)_L$ doublet of quarks:

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \tag{23}$$

and their scalar partners which are also in an $SU(2)_L$ doublet,

$$\tilde{Q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix} \quad . \tag{24}$$

Similarly, the superfield $\hat{U}^c$ ($\hat{D}^c$) contains the right-handed up (down) anti-quark, $\overline{u}_R$ ($\overline{d}_R$), and its scalar partner, $\tilde{u}_R^*$ ($\tilde{d}_R^*$). The scalar partners of the quarks are called squarks. We see that each quark has 2 scalar partners, one corresponding to each quark chirality. The leptons are contained in the $SU(2)_L$ doublet superfield $\hat{L}$ which contains the left-handed fermions,

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \tag{25}$$

and their scalar partners,

$$\tilde{L} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix} \quad . \tag{26}$$

Finally, the right-handed anti-electron, $\overline{e}_R$, is contained in the superfield $\hat{E}^c$ and has a scalar partner $\tilde{e}_R^*$. The scalar partners of the leptons are termed sleptons.

The $SU(3) \times SU(2)_L \times U(1)_Y$ gauge fields all obtain Majorana fermion partners in a SUSY model. The $\hat{G}^a$ superfield contains the gluons, $G^{a\mu}$, and their partners the gluinos, $\tilde{g}^a$; $\hat{W}_i$ contains the $SU(2)_L$ gauge bosons, W_i^μ and their fermion partners, $\tilde{\omega}_i$ (winos); and $\hat{B}$ contains the $U(1)_Y$ gauge field, B^μ, and its fermion partner, $\tilde{b}$ (bino). The usual notation is to denote the supersymmetric partner of a fermion or gauge field with the same letter, but with a tilde over it.

One feature of Table 1 requires explanation. The Standard Model contains a single $SU(2)_L$ doublet of scalar particles, dubbed the "Higgs doublet". In the supersymmetric extension of the Standard Model, this scalar doublet acquires a SUSY partner which is an $SU(2)_L$ doublet of Majorana fermion fields, $\tilde{h}_1$ (the Higgsinos), which contribute to the triangle $SU(2)_L$ and $U(1)_Y$ gauge anomalies. Since the fermions of the Standard Model have exactly the right

Table 1: Chiral Superfields of the MSSM

Superfield	SU(3)	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{Q}$	3	2	$\frac{1}{6}$	(u_L, d_L), $(\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	$\overline{3}$	1	$-\frac{2}{3}$	$\overline{u}_R$, $\tilde{u}^*_R$
$\hat{D}^c$	$\overline{3}$	1	$\frac{1}{3}$	$\overline{d}_R$, $\tilde{d}^*_R$
$\hat{L}$	1	2	$-\frac{1}{2}$	(ν_L, e_L), $(\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^c$	1	1	1	$\overline{e}_R$, $\tilde{e}^*_R$
$\hat{H}_1$	1	2	$-\frac{1}{2}$	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	$\frac{1}{2}$	$(H_2, \tilde{h}_2)$

Table 2: Vector Superfields of the MSSM

Superfield	SU(3)	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{G}^a$	8	1	0	G^μ, $\tilde{g}$
$\hat{W}^i$	1	3	0	W_i^μ, $\tilde{\omega}_i$
$\hat{B}$	1	1	0	B^μ, $\tilde{b}$

quantum numbers to cancel these anomalies, it follows that the contribution from the fermionic partner of the Higgs doublet remains uncancelled.[12] Since gauge theories cannot have anomalies, these contributions must be cancelled somehow if the SUSY theory is to be sensible. The simplest way is to add a second Higgs doublet with precisely the opposite $U(1)_Y$ quantum numbers as the first Higgs doublet. In a SUSY model, this second Higgs doublet will also have fermionic partners, $\tilde{h}_2$, and the contributions of the fermion partners of the two Higgs doublets to gauge anomalies will precisely cancel each other, leaving an anomaly free theory. It is easy to check that the fermions of Table 1 satisfy the conditions for anomaly cancellation:

$$Tr(Y^3) = Tr(T_{3L}^2 Y) = 0 \quad . \tag{27}$$

We will see later that 2 Higgs doublets are also required in order to give both the up and down quarks masses in a SUSY theory. The requirement that there be at least 2 $SU(2)_L$ Higgs doublets is a feature of all models with weak scale supersymmetry.

- In general, supersymmetric extensions of the Standard Model have extended Higgs sectors leading to a rich phenomenology of Higgs scalars.

It is instructive to consider the cancellation of quadratic divergences in the Higgs boson mass renormalization from the complete set of particles contained in the MSSM.[13] Now gauge bosons, gauginos, Higgs self-interactions, Higgsinos, fermions, and sfermions all contribute. The cancellation of quadratic divergences in this case uses in a fundamental manner the fact that the trace of the hypercharge generator over the particle spectrum vanishes.[7] In order not to have large contributions of the form of Eq. 19, the argument is usually made that the SUSY particles must have masses below around 1 TeV.

3.2 Aside on 2− Component Notation

Supersymmetry is most naturally formulated using 2− component Majorana spinors. However, for practical purposes, it is usually more convenient to obtain Feynman rules for the fermion interactions in terms of 4− component spinors. It is necessary, then, to develop techniques for going back- and- forth between the two formulations.[9]

A 4− component fermion, ψ, can be written in terms of 2− component spinors, ζ and $\overline{\eta}$, as,

$$\psi = \begin{pmatrix} \zeta \\ \overline{\eta} \end{pmatrix} . \tag{28}$$

For a Majorana spinor, ψ_M, we have $\psi_M = \psi_M^c$, where ψ_M^c is the charge conjugated spinor. This requires that a Majorana fermion have the form,

$$\psi_M = \begin{pmatrix} \zeta \\ \overline{\zeta} \end{pmatrix} . \tag{29}$$

It is most straightforward to work with fermions of definite helicity, $\psi_{R,L} = P_\pm \psi$ with $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$,

$$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} . \tag{30}$$

We then have the following useful results for translating between 2− and 4− component notation,

$$\begin{aligned} \overline{\psi}_a P_- \psi_b &= \eta_a \zeta_b \\ \overline{\psi}_a P_+ \psi_b &= \overline{\eta}_b \overline{\zeta}_a \\ \overline{\psi}_a \gamma^\mu P_- \psi_b &= \overline{\zeta}_a \overline{\sigma}^\mu \zeta_b \\ \overline{\psi}_a \gamma^\mu P_+ \psi_b &= -\overline{\eta}_b \overline{\sigma}^\mu \eta_a \quad . \end{aligned} \tag{31}$$

Many more results of this type can be found in Refs. 2 and 9. The results of Eq. 31 contain suppressed ϵ tensors,

$$\zeta\eta \equiv \zeta^\alpha \eta_\alpha = \zeta^\alpha \epsilon_{\alpha\beta} \eta^\beta = -\eta^\beta \epsilon_{\alpha\beta} \zeta_\alpha = \eta\zeta \tag{32}$$

with $\epsilon_{12} = -\epsilon^{21} = \epsilon^{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = \epsilon^{11} = \epsilon^{22} = 0$. We also have,

$$\overline{\zeta}\overline{\sigma}^\mu \eta = -\eta \sigma^\mu \overline{\zeta} \quad . \tag{33}$$

The minus sign of Eq. 33 is vital for deriving the correct Feynman rules for Majorana particles.

As an example, consider the Lagrangian for a $4-$ component Dirac fermion,

$$\mathcal{L} = \overline{\psi}\Big(i\partial_\mu \gamma^\mu - m\Big)\psi, \tag{34}$$

where we work in the basis,

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \quad . \tag{35}$$

Using Eq. 31, the Dirac Lagrangian therefore becomes in $2-$ component notation,

$$\mathcal{L} = -i\overline{\zeta}\overline{\sigma}^\mu \partial_\mu \zeta - i\overline{\eta}\overline{\sigma}^\mu \partial_\mu \eta - m(\eta\zeta + \overline{\eta}\overline{\zeta}) \quad . \tag{36}$$

Another interesting application of Eq. 31 is to consider the coupling of two gluinos to a gluon. From gauge invariance, the coupling must have the form

$$\mathcal{L} = f_{ijk}\overline{\tilde{g}}^i \gamma^\mu (a + b\gamma_5)\tilde{g}^j G^k_\mu, \tag{37}$$

where $\tilde{g}^i$, $i = 1,.,8$, is the color octet Majorana gluino (in 4-component notation),

$$\tilde{g}^i = \begin{pmatrix} \zeta^i \\ \overline{\zeta}^i \end{pmatrix}. \tag{38}$$

and f_{ijk} is the anti-symmetric $SU(3)$ tensor. Consider the axial vector piece of Eq. 37:

$$\begin{aligned} f_{ijk}\overline{\tilde{g}}^i \gamma^\mu \gamma_5 \tilde{g}^j G^k_\mu &= f_{ijk}\Big[\overline{\tilde{g}}^i \gamma^\mu P_+ \tilde{g}^j - \tilde{g}^i \gamma^\mu P_- \tilde{g}^j G^k_\mu\Big] \\ &= f_{ijk}\Big[-\overline{\zeta}^j \overline{\sigma}^\mu \zeta^i - \overline{\zeta}^i \overline{\sigma}^\mu \zeta^j\Big] G^k_\mu \\ &= 0, \end{aligned} \tag{39}$$

where the last line follows from the anti-symmetry of f_{ijk}. Hence the fact that the gluino is a Majorana particle has a physical consequence: it can only have a vector coupling to the gluon.

3.3 The Interactions of the MSSM

Having specified the superfields of the theory, the next step is to construct the supersymmetric Lagrangian.[14] There is very little freedom in the allowed interactions between the ordinary particles and their supersymmetric partners. It is this feature of a SUSY model which gives it predictive power (and makes it attractive to theorists!). It is important to note here, however, that there is nothing to stop us from adding more superfields to those shown in Tables 1 and 2 as long as we are careful to add them in such a way that any additional contributions to gauge anomalies cancel among themselves. The MSSM which we concentrate on, however, contains only those fields given in the tables.

The supersymmetry associates each 2-component Majorana fermion with a complex scalar. The massive fermions of the Standard Model are, however, Dirac fermions and we will use the more familiar 4- component notation when writing the fermion interactions.[2] The fields of the MSSM all have canonical kinetic energies:

$$\begin{aligned}\mathcal{L}_{KE} &= \sum_i \Big\{ (D_\mu S_i^*)(D^\mu S_i) + i\overline{\psi}_i D\psi_i \Big\} \\ &\quad + \sum_A \Big\{ -\frac{1}{4} F^A_{\mu\nu} F^{\mu\nu A} + \frac{i}{2}\overline{\lambda}_A D\lambda_A \Big\}, \qquad (40)\end{aligned}$$

where D is the $SU(3)\times SU(2)_L \times U(1)_Y$ gauge invariant derivative. The $\sum_i$ is over all the fermion fields of the Standard Model, ψ_i, and their scalar partners, S_i, and also over the 2 Higgs doublets with their fermion partners as given in Table 1. The $\sum_A$ is over the $SU(3)$, $SU(2)_L$ and $U(1)_Y$ gauge fields with their fermion partners, the gauginos, λ_A.

The interactions between the chiral superfields of Table 1 and the gauginos and the gauge fields of Table 2 are completely specified by the gauge symmetries and by the supersymmetry, as are the quartic interactions of the scalars,

$$\mathcal{L}_{int} = -\sqrt{2}\sum_{i,A} g_A \Big[S_i^* T^A \overline{\psi}_{iL}\lambda_A + \text{h.c.}\Big] - \frac{1}{2}\sum_A \Big(\sum_i g_A S_i^* T^A S_i \Big)^2 \quad , \qquad (41)$$

where g_A is the relevant gauge coupling constant. We see that the interaction strengths are given in terms of the gauge couplings. **There are no adjustable parameters.** For example, the interaction between a quark, its scalar partner, the squark, and the gluino is governed by the strong coupling constant, g_s. A complete set of Feynman rules for the minimal SUSY model described here is given in the review by Haber and Kane.[2] A good rule of thumb is to take an interaction involving Standard Model particles and replace two of the particles by their SUSY partners to get an approximate strength for the interaction.

The only freedom in constructing the supersymmetric Lagrangian (once the superfields and the gauge symmetries are chosen) is contained in a function called the **superpotential**, W. The superpotential is a function of the chiral superfields of Table 1 only (it is not allowed to contain their complex conjugates) and it contains terms with 2 and 3 chiral superfields. Terms in the superpotential with more than 3 chiral superfields would yield non-renormalizable interactions in the Lagrangian. The superpotential is an analytic function of the superfields and thus is not allowed to contain derivative interactions. From the superpotential can be found both the scalar potential and the Yukawa interactions of the fermions with the scalars:

$$\mathcal{L}_W = -\sum_i \mid \frac{\partial W}{\partial z_i} \mid^2 - \frac{1}{2}\sum_{ij}\left[\overline{\psi}_{iL}\frac{\partial^2 W}{\partial z_i \partial z_j}\psi_j + \text{h.c.}\right], \tag{42}$$

where z_i is a chiral superfield. To obtain the interactions, we take the derivatives of W with respect to the superfields, z_i, and then evaluate the result in terms of the scalar components of z_i, ϕ_i. This form of the Lagrangian is dictated by the supersymmetry and by the requirement that it be renormalizable. An explicit derivation of Eq. 42 can be found in Ref. 9.

The usual approach is to write the most general $SU(3)\times SU(2)_L \times U(1)_Y$ invariant superpotential with arbitrary coefficients for the interactions,

$$\begin{aligned} W &= -\epsilon_{ij}\mu \hat{H}_1^i \hat{H}_2^j + \epsilon_{ij}\left[\lambda_L \hat{H}_1^i \hat{L}^{cj}\hat{E}^c + \lambda_D \hat{H}_1^i \hat{Q}^j \hat{D}^c + \lambda_U \hat{H}_2^j \hat{Q}^i \hat{U}^c\right] \\ &\quad +\epsilon_{ij}\left[\lambda_1 \hat{L}^i \hat{L}^j \hat{E}^c + \lambda_2 \hat{L}^i \hat{Q}^j \hat{D}^c\right] + \lambda_3 \hat{U}^c \hat{D}^c \hat{D}^c, \end{aligned} \tag{43}$$

(where i,j are $SU(2)$ indices which are typically not written explicitely). In principle, a bi-linear term $\epsilon_{ij}\hat{L}^i\hat{H}_2^j$ could also be included in the superpotential since $\hat{L}$ and $\hat{H}_1$ have the same gauge and Lorentz quantum numbers. It is possible, however, to rotate the lepton field, $\hat{L}$, such that this term vanishes. It can be reintroduced through renormalization group effects if the rotation is performed at the GUT scale, but these effects are small except for neutrino masses and so we will ignore them.[15] We have written the superpotential in terms of the fields of the first generation. In principle, the λ_i could all be matrices which mix the interactions of the 3 generations.

The $\mu \hat{H}_1 \hat{H}_2$ term in the superpotential gives mass terms for the Higgs bosons when we apply $\mid \partial W/\partial z \mid^2$ and so μ is often called the Higgs mass parameter. We shall see later that the physics is very sensitive to the sign of μ. The terms in the square brackets proportional to λ_L, λ_D, and λ_U give the usual Yukawa interactions of the fermions with the Higgs bosons from the term $\overline{\psi}_i(\partial^2 W/\partial z_i \partial z_j)\psi_j$. Hence these coefficients are determined in terms of the fermion masses and the vacuum expectation values of the neutral members of the scalar components of the Higgs doublets and are not free parameters at all.

The Lagrangians of Eqs. 40, 41 and 42 cannot, however, be the whole story as all the particles (fermions, scalars, gauge fields) are massless at this point.

3.4 R Parity

The terms in the second line of Eq. 43 (proportional to λ_1, λ_2 and λ_3) are a problem. They contribute to lepton and baryon number violating interactions and can mediate proton decay at tree level through the exchange of the scalar partner of the down quark. If the SUSY partners of the Standard Model particles have masses on the TeV scale, then these interactions are severely restricted by experimental measurements.[16] We write the R-parity violating contributions to the superpotential as

$$W_{RP} = \lambda_1^{\alpha\beta\gamma} \hat{L}^\alpha \hat{L}^\beta \hat{E}^{c\gamma} + \lambda_2^{\alpha\beta\gamma} \hat{L}^\alpha \hat{Q}^\beta \hat{D}^{c\gamma} + \lambda_3^{\alpha\beta\gamma} \hat{U}^{c\alpha} \hat{D}^{c\beta} \hat{D}^{c\gamma}, \tag{44}$$

where the indices α, β, γ label the 3 generations of quarks and leptons.

There are several possible approaches to the problem of the lepton and baryon number violating interactions. The first is simply to make the coefficients, λ_1, λ_2, and λ_3 small enough to avoid experimental limits.[17] This artificial tuning of parameters is regarded as unacceptable by many theorists, but is certainly allowed experimentally. Another tactic is to make either the lepton number violating interactions, λ_1 and λ_2, or the baryon number violating interaction, λ_3, zero, (while allowing the others to be non-zero) which would forbid proton decay. The experimental limit on proton decay involves the couplings to the first generation,[18]

$$\mid \lambda_2^{11\alpha} \lambda_3^{11\alpha} \mid < 10^{-27} \left(\frac{M_{\tilde{d}_\alpha}}{100\ GeV} \right)^2 . \tag{45}$$

Many other possible interactions involving the $\lambda_2^{\alpha\beta\gamma}$ are forbidden by low energy interactions. For example, ν-less double beta decay requires,[19]

$$\mid \lambda_2^{111} \mid < 7 \times 10^{-3} \left(\frac{M_{\tilde{q}}}{200\ GeV} \right) \left(\frac{M_{\tilde{g}}}{1\ TeV} \right)^{1/2} . \tag{46}$$

Other processes, such as $\mu \to e\gamma$ restrict different combinations of R parity violating operators. A review is given in Ref. 19. One could simply fine tune these couplings to be small. There is, however, not much theoretical support for this approach since one of the motivations for introducing supersymmetry is to avoid the fine tuning of couplings.

The usual strategy is to require that all of these undesirable lepton and baryon number violating terms be forbidden by a symmetry. (If they are forbidden by a symmetry, they will not re-appear at higher orders of perturbation theory.) The symmetry which does the job is called **R parity**.[15,20] R parity can be defined as a discrete Z_2 symmetry under which the anti-commuting variable $\theta \to -\theta$. Eq. 22 makes it clear that a scalar and its fermionic partner transform oppositely under R parity. If we define the superfields to transform under R parity such that

$$\begin{aligned}(\hat{Q}, \hat{U}^c, \hat{D}^c, \hat{L}, \hat{E}^c) &\longrightarrow -(\hat{Q}, \hat{U}^c, \hat{D}^c, \hat{L}, \hat{E}^c) \\ (\hat{H}_1, \hat{H}_2) &\longrightarrow (\hat{H}_1, \hat{H}_2)\end{aligned} \tag{47}$$

then it is clear that the terms of Eq. 44 are forbidden. R parity is thus a multiplicative quantum number such that all particles of the Standard Model have R parity +1, while their SUSY partners have R parity -1. R parity can also be defined as,

$$R \equiv (-1)^{3(B-L)+s} \quad , \tag{48}$$

for a particle of spin s. It is worth noting that in the Standard Model, this problem of baryon and lepton number violating interactions does not arise, since such interactions are forbidden by the gauge symmetries to contribute to dimension- 4 operators and first arise in dimension- 6 operators which are suppressed by factors of some heavy mass scale.

The assumption of R parity conservation has profound experimental consequences which go beyond the details of a specific model. Because R parity is a multiplicative quantum number, it implies that the number of SUSY partners in a given interaction is always conserved modulo 2.

- SUSY partners can only be pair produced from Standard Model particles.

Furthermore, a SUSY particle will decay in a chain until the lightest SUSY particle is produced (such a decay is called a *cascade decay*). This lightest SUSY particle, called the LSP, must be absolutely stable when R parity is conserved.

- A theory with R parity conservation will have a lightest SUSY particle (LSP) which is stable.

The LSP must be neutral since there are stringent cosmological bounds on light charged or colored particles which are stable.[21] If R parity is violated then it is possible for some other particle (such as the gluino) to be the LSP.[22] Hence the LSP is stable and neutral and is not seen in a detector (much like a neutrino) since it interacts only by the exchange of a heavy virtual SUSY particle.

- The LSP will interact very weakly with ordinary matter.
- A generic signal for R parity conserving SUSY theories is missing transverse energy from the non-observed LSP.

In theories without R parity conservation, there will not be a stable LSP, and the lightest SUSY particle will decay into ordinary particles (possibly within the detector). Missing transverse energy will no longer be a robust signature for SUSY particle production.[23,24]

The assumption of R-parity conservation in SUSY models has been under attack because of some recent experimental results. The HERA experiments see an excess of events at large Q^2, ($Q^2 > 1.5\times 10^4 GeV^2$), in e^+p deep inelastic scattering events, although the statistics are poor.[25] This excess is difficult to explain within the context of the Standard Model. One possibility is that these events are a signal for SUSY models with R-parity violating interactions. An R-parity violating superpotential could introduce interactions of the form,

$$e^+ d \rightarrow \tilde{u}_L, \tilde{c}_L, \tilde{t}_L \quad , \tag{49}$$

along with interactions involving sea quarks. The excess events at HERA could be interpreted as resonant squark production through the λ_2 interaction of Eq. 44. The kinematics and number of events would then give restrictions of the mass of the exchanged squark and the relevant $\lambda_2^{\alpha\beta\gamma}$ operator.[19]

To summarize, R parity violating theories have a number of features which are different from those of the Standard Model:

- Baryon and/or lepton number is violated in some interactions.
- The SUSY partners of ordinary particles can be singly produced.
- The LSP is not stable. Hence the missing E_T signature for supersymmetry is degraded.
- The LSP need not be the neutralino, since the LSP is no longer stable.

3.5 Supersymmetry Breaking

The mechanism for supersymmetry breaking is not well understood. At this point we have constructed a SUSY theory containing all of the Standard Model particles, but the supersymmetry remains unbroken and the particles and their SUSY partners are massless. This is clearly unacceptable. Suppose we try to break the supersymmetry spontaneously, by giving some set of scalars VEVs. The difficulty in doing this arises from a fundamental problem. Just as in our simple example of Eq. 20, the scalar potential of the MSSM is positive definite,

$$V(\phi_i, \phi_i^*) = \sum_i \mid \frac{\partial W}{\partial z_i} \mid^2 + \frac{1}{2} \sum_{A,i,j} g_A^2 \mid \phi_i^* T_{ij}^A \phi_j \mid^2 \geq 0 \quad . \tag{50}$$

Clearly the second term, $\frac{1}{2} g_A^2 \mid \phi_i^* T_{ij}^A \phi_j \mid^2$ is minimized if $\langle \phi_i \rangle = 0$. In order to spontaneously break the supersymmetry, we need a non- trivial contribution from the first term. Such a contribution will contribute to the mass matrices and a spontaneouly broken SUSY theory satisfies a mass-squared sum rule:[2]

$$STrM^2 \equiv 3TrM_V^2 - 2TrM_F^2 + TrM_S^2 = 0. \tag{51}$$

This sum rule holds independent of the values of the scalar fields. Satisfying it is problematic, since we want *all* SUSY particles to be heavier than their Standard Model partners. This sum rule has caused most theorists to give up on models with spontaneous supersymmetry breaking.

At the moment the usual approach is to assume that the MSSM, which is the theory at the electroweak scale, is an effective low energy theory.[26] It is typically assumed that the SUSY breaking occurs at a high scale, say M_{pl}, and perhaps results from some complete theory encompassing gravity. The supersymmetry breaking is implemented by including explicit "soft" mass terms for the scalar members of the chiral multiplets and for the gaugino members of the vector supermultiplets in the Lagrangian. These interactions are termed soft because they do not re-introduce the quadratic divergences which motivated the introduction of the supersymmetry in the first place. The dimension of soft operators in the Lagrangian must be 3 or less, which means that the possible soft operators are mass terms, bi-linear mixing terms ("B" terms), and tri-linear scalar mixing terms (" A terms"). The origin of these supersymmetry breaking terms is left unspecified. The complete set of soft SUSY breaking terms (which respect R parity and the $SU(3) \times SU(2)_L \times U(1)_Y$ gauge symmetry) for the first generation is given by the Lagrangian:[14,27]

$$-\mathcal{L}_{soft} = m_1^2 \mid H_1 \mid^2 + m_2^2 \mid H_2 \mid^2 - B\mu\epsilon_{ij}(H_1^i H_2^j + \text{h.c.}) + M_{\tilde{Q}}^2(\tilde{u}_L^* \tilde{u}_L + \tilde{d}_L^* \tilde{d}_L)$$

$$
\begin{aligned}
&+M_{\tilde{u}}^2 \tilde{u}_R^* \tilde{u}_R + M_{\tilde{d}}^2 \tilde{d}_R^* \tilde{d}_R + M_{\tilde{L}}^2 (\tilde{e}_L^* \tilde{e}_L + \tilde{\nu}_L^* \tilde{\nu}_L) + M_{\tilde{e}}^2 \tilde{e}_R^* \tilde{e}_R \\
&+\frac{1}{2}\left[M_3 \overline{\tilde{g}} \tilde{g} + M_2 \overline{\tilde{\omega}_i} \tilde{\omega}_i + M_1 \overline{\tilde{b}} \tilde{b} \right] + \frac{g}{\sqrt{2} M_W} \epsilon_{ij} \left[\frac{M_d}{\cos\beta} A_d H_1^i \tilde{Q}^j \tilde{d}_R^* \right. \\
&\left. + \frac{M_u}{\sin\beta} A_u H_2^j \tilde{Q}^i \tilde{u}_R^* + \frac{M_e}{\cos\beta} A_e H_1^i \tilde{L}^j \tilde{e}_R^* + \text{h.c.} \right] \quad . \qquad (52)
\end{aligned}
$$

This Lagrangian has arbitrary masses for the scalars and gauginos and also arbitrary tri-linear and bi-linear mixing terms. The scalar and gaugino mass terms have the desired effect of breaking the degeneracy between the particles and their SUSY partners. The tri-linear A-terms have been defined with an explicit factor of mass and we will see later that they affect primarily the particles of the third generation.[a] When the A_i terms are non-zero, the scalar partners of the left- and right-handed fermions can mix when the Higgs bosons get vacuum expectation values and so they are no longer mass eigenstates. The B term mixes the scalar components of the 2 Higgs doublets.

The philosophy is to add all of the mass and mixing terms which are allowed by the gauge symmetries. To further complicate matters, all of the mass and interaction terms of Eq. 52 may be matrices involving all three generations. $\mathcal{L}_{soft}$ has clearly broken the supersymmetry since the SUSY partners of the ordinary particles have been given arbitrary masses. This has come at the tremendous expense, however, of introducing a large number of unknown parameters (Haber [28] has called this model the MSSM-124 to emphasize the number of free parameters!). It is one of the wonderful features of supersymmetry that even with all these new parameters, the theory is still able to make some definitive predictions. This is, of course, because the gauge interactions of the SUSY particles are completely fixed.

We have now constructed the Lagrangian describing a softly broken supersymmetric theory which is assumed to be the effective theory at the weak scale. In the next section we will examine how the electroweak symmetry is broken in this model and study the mass spectrum and interactions of the new particles.

[a]We have also included an angle β in the normalization of the A terms. The factor β is related to the vacuum expectation values of the neutral components of the Higgs fields and is defined in the next section. The normalization is, of course, arbitrary and the normalization we have chosen reflects theoretical prejudices.

The Higgs Potential

The Higgs sector of the MSSM is very similar to that of a general 2 Higgs doublet model and can be constructed from Eq. 50. The first contribution to the Higgs potential is called the "D" term, V_D,

$$\begin{aligned} V_D = &\quad \textstyle\sum_A \frac{1}{2} D^A D^A \\ D^A \equiv &\quad -g_A \phi_i^* T_{ij}^A \phi_j \quad , \end{aligned} \tag{53}$$

is called the "D"-term. For $H_1(H_2)$, we have $Y = -\frac{1}{2}(\frac{1}{2})$ and so the D-terms are given by,

$$\begin{aligned} U(1)_Y: \quad D^1 = &\quad -\tfrac{g'}{2}\Big(\mid H_2 \mid^2 - \mid H_1 \mid^2 \Big) \\ SU(2)_L: \quad D^a = &\quad -\tfrac{g}{2}\Big(H_1^{i*} \tau_{ij}^a H_1^j + H_2^{i*} \tau_{ij}^a H_2^j \Big), \end{aligned} \tag{54}$$

(where $T^a = \frac{\tau^a}{2}$). The D terms then contribute to the scalar potential:

$$V_D = \frac{g'^2}{8}\Big(\mid H_2 \mid^2 - \mid H_1 \mid^2 \Big)^2 + \frac{g^2}{8}\Big(H_1^{i*} \tau_{ij}^a H_1^j + H_2^{i*} \tau_{ij}^a H_2^j \Big)^2 \quad . \tag{55}$$

Using the $SU(2)$ identity,

$$\tau_{ij}^a \tau_{kl}^a = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} \tag{56}$$

we find,

$$\begin{aligned} V_D \quad = \quad & \frac{g^2}{8}\Big\{ 4 \mid H_1^* \cdot H_2 \mid^2 - 2(H_1^* \cdot H_2)(H_2^* \cdot H_2) + \Big(\mid H_1 \mid^2 + \mid H_2 \mid^2 \Big)^2 \Big\} \\ & + \frac{g'^2}{8}\Big(\mid H_2 \mid^2 - \mid H_1 \mid^2 \Big)^2 . \end{aligned} \tag{57}$$

Adding the remaining contribution to the potential, the "F"-term,

$$V_F = \sum_i \mid \frac{\partial W}{\partial z_i} \mid^2, \tag{58}$$

we find the supersymmetric potential,

$$V = \mid \mu \mid^2 \Big(\mid H_1 \mid^2 + \mid H_2 \mid^2 \Big) + \frac{g^2 + g'^2}{8}\Big(\mid H_2 \mid^2 - \mid H_1 \mid^2 \Big)^2 + \frac{g^2}{2} \mid H_1^* \cdot H_2 \mid^2 . \tag{59}$$

This potential has its minimum at $\langle H_1^0 \rangle = \langle H_2^0 \rangle = 0$, giving $\langle V \rangle = 0$ and so represents a model with no electroweak symmetry breaking and no SUSY breaking. Adding all possible soft SUSY breaking terms as in Eq. 52, we find the scalar potential involving the Higgs bosons,

$$
\begin{aligned}
V_H &= \left(| \mu |^2 + m_1^2 \right) | H_1 |^2 + \left(| \mu |^2 + m_2^2 \right) | H_2 |^2 - \mu B \epsilon_{ij} \left(H_1^i H_2^j + \text{h.c.} \right) \\
&\quad + \frac{g^2 + g'^2}{8} \left(| H_1 |^2 - | H_2 |^2 \right)^2 + \frac{1}{2} g^2 \, | H_1^* H_2 |^2 \quad . \qquad (60)
\end{aligned}
$$

The Higgs potential of the SUSY model can be seen to depend on 3 independent combinations of parameters,

$$
\begin{aligned}
&| \mu |^2 + m_1^2, \\
&| \mu |^2 + m_2^2, \\
&\mu B \quad , \qquad (61)
\end{aligned}
$$

where B is a new mass parameter. This is in contrast to the general 2 Higgs doublet model where there are 6 arbitrary coupling constants (and a phase) in the potential. From Eq. 41, it is clear that the quartic couplings are fixed in terms of the gauge couplings and so they are not free parameters. This leaves only the mass terms of Eq. 60 unspecified. Note that V_H automatically conserves CP since any complex phase in μB can be absorbed into the definitions of the Higgs fields.

Clearly, if $\mu B = 0$ then all the terms in the potential are positive and the minimum of the potential occurs with $V = 0$ and $\langle H_1^0 \rangle = \langle H_2^0 \rangle = 0$. Hence all 3 parameters of Eq. 61 must be non-zero in order for the electroweak symmetry to be broken. [b]

In order for the electroweak symmetry to be broken and for the potential to be stable at large values of the fields, the parameters must satisfy the relations,

$$
\begin{aligned}
(\mu B)^2 &> \left(| \mu |^2 + m_1^2 \right) \left(| \mu |^2 + m_2^2 \right) \\
| \mu |^2 + \frac{m_1^2 + m_2^2}{2} &> | \mu B | \quad . \qquad (62)
\end{aligned}
$$

[b] We assume that the parameters are arranged in such a way that the scalar partners of the quarks and leptons do not obtain vacuum expectation values. Such vacuum expectation values would spontaneously break the $SU(3)$ color gauge symmetry or lepton number. This requirement that the proper vacuum be chosen gives a restriction on $A_i/\tilde{M}$, where $\tilde{M}$ is a generic squark or slepton mass.

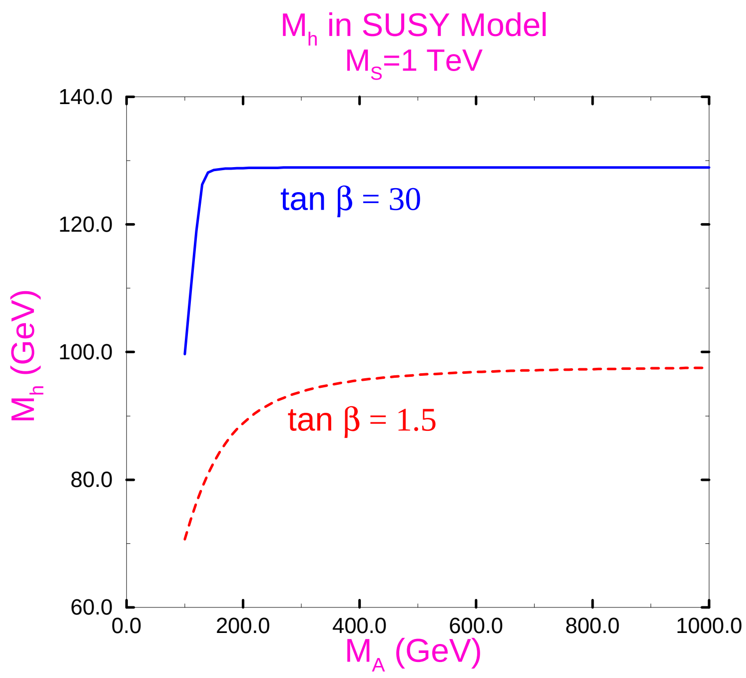

Figure 5: Mass of the lightest neutral Higgs boson as a function of the pseudoscalar mass, M_A, and $\tan\beta$. This figure includes one-loop radiative corrections to the Higgs mass as in Eq. 70, assumes a common scalar mass of 1 TeV, and neglects mixing effects, $(A_i = \mu = 0)$.

We will assume that these conditions are met. The symmetry is broken when the neutral components of the Higgs doublets get vacuum expectation values,

$$\begin{aligned} \langle H_1^0 \rangle &\equiv v_1 \\ \langle H_2^0 \rangle &\equiv v_2 \quad . \end{aligned} \tag{63}$$

By redefining the Higgs fields, we can always choose v_1 and v_2 positive.

In the MSSM, the Higgs mechanism works in the same manner as in the Standard Model. When the electroweak symmetry is broken, the W gauge boson gets a mass which is fixed by v_1 and v_2,

$$M_W^2 = \frac{g^2}{2}(v_1^2 + v_2^2) \quad . \tag{64}$$

Before the symmetry was broken, the 2 complex $SU(2)_L$ Higgs doublets had 8 degrees of freedom. Three of these were absorbed to give the W and Z gauge bosons their masses, leaving 5 physical degrees of freedom. A charged Higgs boson, $H^\pm$, a CP -odd neutral Higgs boson, A, and 2 CP-even neutral Higgs bosons, h and H remain in the spectrum. After fixing $v_1^2 + v_2^2$ such that the W boson gets the correct mass, the Higgs sector is then described by 2 additional parameters which can be chosen however you like. The usual choice is

$$\tan\beta \equiv \frac{v_2}{v_1} \tag{65}$$

and M_A, the mass of the pseudoscalar Higgs boson. Once these two parameters are given, then the masses of the remaining Higgs bosons can be calculated in terms of M_A and $\tan\beta$. Note that we can chose $0 \leq \beta \leq \frac{\pi}{2}$ since we have chosen $v_1, v_2 > 0$.

Higgs Boson Masses

It is straightforward to find the physical Higgs bosons and their masses in terms of the parameters of Eq. 60. Details can be found in Ref. 29. The neutral Higgs masses are found by diagonalizing the 2×2 Higgs mass matrix and by convention, h is taken to be the lighter of the neutral Higgs bosons. At tree level, the masses of the neutral Higgs bosons are given by,

$$M_{h,H}^2 = \frac{1}{2}\left\{ M_A^2 + M_Z^2 \mp \left((M_A^2 + M_Z^2)^2 - 4 M_Z^2 M_A^2 \cos^2 2\beta \right)^{1/2} \right\}. \tag{66}$$

The pseudoscalar mass is given by,

$$M_A^2 = \frac{2 \mid \mu B \mid}{\sin 2\beta}, \tag{67}$$

and the charged scalar mass is,

$$M_{H^\pm}^2 = M_W^2 + M_A^2 \quad . \tag{68}$$

We see that at tree level [29], Eq. 60 gives important predictions about the relative masses of the Higgs bosons,

$$M_{H+} \quad > \quad M_W$$

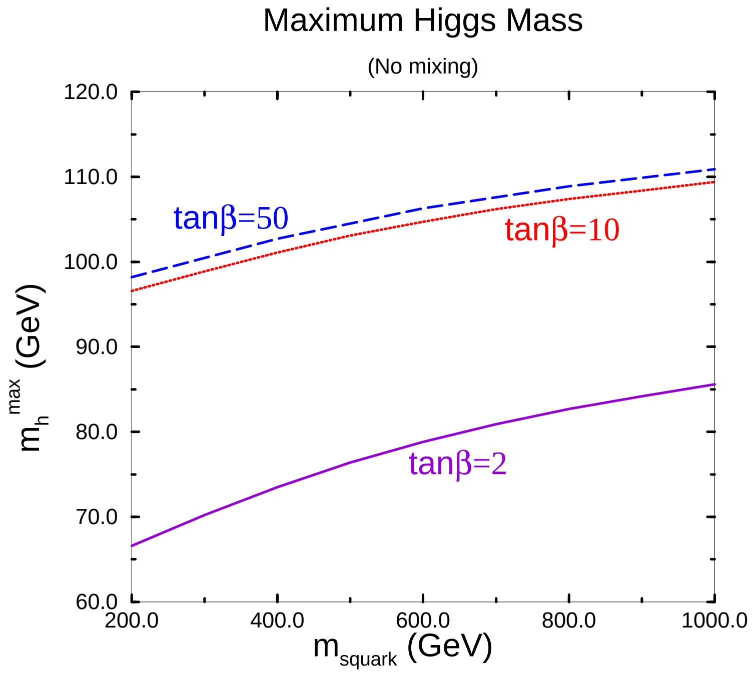

Figure 6: Maximum value of the lightest Higgs boson mass as a function of the squark mass. This figure includes 2-loop radiative corrections and renormalization group improvements. (We have assumed degenerate squarks and set the mixing parameters $A_i = \mu = 0$.)

$$\begin{aligned} M_H &> M_Z \\ M_h &< M_A \\ M_h &< M_Z \mid \cos 2\beta \mid \quad . \end{aligned} \tag{69}$$

These relations yield the desirable prediction that the lightest neutral Higgs boson is lighter than the Z boson and so must be observable at LEPII. However, it was realized several years ago that loop corrections to the relations of Eq. 69 are large. In fact the corrections to M_h^2 grow like $G_F M_T^4$ and receive contributions from loops with both top quarks and squarks. In a model with unbroken supersymmetry, these contributions would cancel. Since the supersymmetry has been broken by splitting the masses of the fermions and their scalar partners, the neutral Higgs boson masses become at one- loop,[30]

$$\begin{aligned} M_{h,H}^2 &= \frac{1}{2}\biggl\{ M_A^2 + M_Z^2 + \frac{\epsilon_h}{\sin^2\beta} \pm \biggl[\biggl((M_A^2 - M_Z^2)\cos 2\beta + \frac{\epsilon_h}{\sin^2\beta} \biggr)^2 \\ &\quad + \biggl(M_A^2 + M_Z^2 \biggr)^2 \sin^2 2\beta \biggr]^{1/2} \biggr\} \qquad (70) \end{aligned}$$

where ϵ_h is the contribution of the one-loop corrections,

$$\epsilon_h \equiv \frac{3G_F}{\sqrt{2}\pi^2} M_T^4 \log\left(\frac{\tilde{M}^2}{M_T^2}\right) \quad . \qquad (71)$$

We have assumed that all of the squarks have equal masses, $\tilde{M}$, and have neglected the smaller effects from the mixing parameters, A_i and μ. In Fig. 5, we show the lightest Higgs boson mass as a function of the pseudoscalar mass, M_A, and for two values of $\tan\beta$. For $\tan\beta > 1$, the mass eigenvalues increase monotonically with increasing M_A and give an upper bound to the mass of the lightest Higgs boson,

$$M_h^2 < M_Z^2 \cos^2 2\beta + \epsilon_h \quad . \qquad (72)$$

The corrections from ϵ_h are always positive and increase the mass of the lightest neutral Higgs boson with increasing top quark mass. From Fig. 5, we see that M_h always obtains its maximal value for rather modest values of the pseudoscalar mass, $M_A > 300\ GeV$. The radiative corrections to the charged Higgs mass-squared are proportional to M_T^2 and so are much smaller than the corrections to the neutral masses.

There are many analyses [30] which include a variety of two-loop effects, renormalization group effects, the effects of large mixing in the squark sector, etc., but the important point is that for given values of $\tan\beta$ and the squark masses, there is an upper bound on the lightest neutral Higgs boson mass. The maximum value of the lightest Higgs mass is shown in Fig. 6 including 2-loop and renormalization group effects and we see that there is still a light Higgs boson even when radiative corrections are included. From Figures 5 and 6, we see that including 2-loop effects, SUSY particle threshold effects, and renormalization group group improvements lowers the upper bound on the neutral Higgs boson mass. For large values of $\tan\beta$ the limit is relatively insensitive to the value of $\tan\beta$ and with a squark mass less than about $1\ TeV$, the upper limit on the Higgs mass is about $110\ GeV$ if mixing in the top squark sector is negligible ($A_T \sim 0$). For large mixing, this limit is raised to around $130\ GeV$.

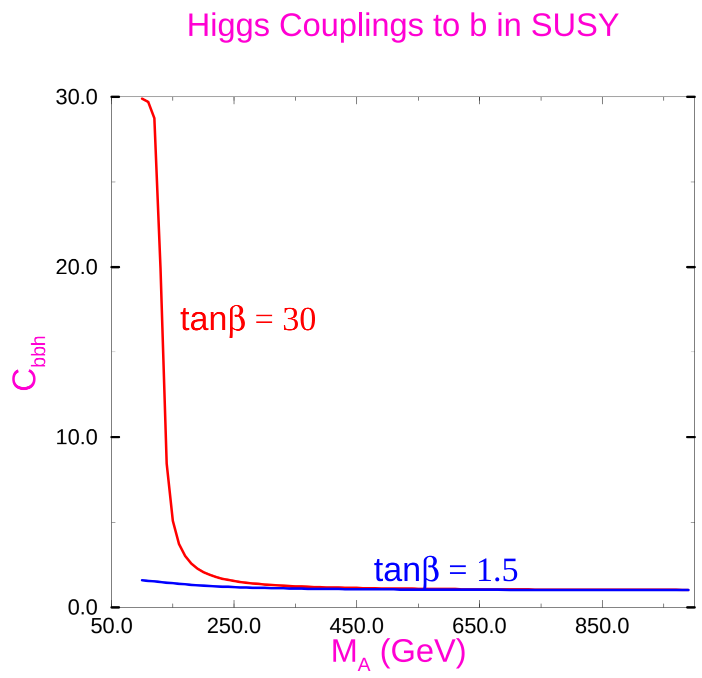

Figure 7: Coupling of the lightest Higgs boson to charge $-1/3$ quarks including radiative corrections [34] in terms of the couplings defined in Eq. 77. The value $C_{bbh} = 1$ corresponds to the Standard Model coupling of the Higgs boson to charge $-1/3$ quarks.

- The minimal SUSY model predicts a neutral Higgs boson with a mass less than around 130 GeV.

Such a mass scale will be accessible at LEPII or the LHC and provides a definitive test of the MSSM.

In a more complicated SUSY model with a richer Higgs structure, this bound will, of course, be changed. However, the requirement that the Higgs self coupling remain perturbative up to the Planck scale gives an upper bound on the lightest SUSY Higgs boson of around 150 GeV in all models with only singlets and doublets of Higgs bosons.[31] This is a very strong statement. It implies that either there is a relatively light Higgs boson (which would be accessible experimentally at LEPII or the LHC) or else there is some new physics between the weak scale and the Planck scale which causes the Higgs couplings to become non-perturbative.

As an example of the effects of adding additional fields to the MSSM, consider including a gauge singlet superfield, $\hat{N}$, in the superpotential,

$$W = ... + \lambda_N \hat{H}_1 \hat{H}_2 \hat{N} + \frac{\kappa}{3}\hat{N}^3. \tag{73}$$

There will now be 3 neutral Higgs boson described by a 3×3 mass matrix. A limit on the lightest Higgs boson mass can still be obtained, however, using the fact that the smallest value of a real symmetric $n\times n$ matrix is smaller than the smallest eigenvalue of the upper 2×2 matrix. Eq. 72 then becomes,[32]

$$M_h^2 < M_Z^2 \cos^2 2\beta + \epsilon_h + \frac{2M_Z^2\lambda_N^2}{g^2+g'^2}\sin^2 2\beta. \tag{74}$$

The bound on M_h therefore grows with λ_N and it is only by making further assumptions that a numerical bound can be obtained. If we require that λ_N be perturbative up to the Planck scale, then we have roughly the same bound as in the MSSM. However, if we relax this assumption and require only that λ_N be perturbative up to say 10^5 GeV, then the bound increases to up to around 150 GeV, depending on $\tan\beta$.

Higgs Boson Couplings to Fermions

The Higgs boson couplings to fermions are dictated by the gauge invariance of the superpotential and at lowest order are completely specified in terms of the two parameters, M_A and $\tan\beta$. From Eq. 43, we see that the charge $2/3$ quarks get their masses entirely from v_2, while the charge $-1/3$ quarks receive their masses from v_1. This is a consequence of the $U(1)_Y$ hypercharge assignments for H_1 and H_2 given in Table 1. In the Standard Model, it is possible to give both the up and down quarks mass using a single Higgs doublet. This is because in the Standard Model the up quarks can get their masses from the charge conjugate of the Higgs doublet. Terms involving the charge conjugates of the superfields are not allowed in SUSY models, however, and so a second Higgs doublet with opposite $U(1)_Y$ hypercharge from the first Higgs doublet is necessary in order to give the up quarks mass. Requiring that the fermions have their observed masses fixes the couplings in the superpotential of Eq. 43,[33]

$$\lambda_D = \frac{gM_d}{\sqrt{2}M_W\cos\beta}$$

Table 3: Higgs Boson Couplings to fermions

f	C_{ffh}	C_{ffH}	C_{ffA}
u	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\cot\beta$
d	$-\frac{\sin\alpha}{\cos\beta}$	$\frac{\cos\alpha}{\cos\beta}$	$\tan\beta$

$$\begin{aligned} \lambda_U &= \frac{g M_u}{\sqrt{2} M_W \sin\beta} \\ \lambda_L &= \frac{g M_l}{\sqrt{2} M_W \cos\beta} \quad , \end{aligned} \tag{75}$$

where g is the $SU(2)_L$ gauge coupling, $g^2 = 4\sqrt{2} G_F M_W^2$. We see that the only free parameter in the superpotential now is the Higgs mass parameter, μ, (along with the angle β in the λ_i couplings).

In the MSSM, the μ parameter is a source of concern. It cannot be set to zero (see Eq. 60) because then there would be no symmetry breaking. The Z-boson mass can be written in terms of the radiatively corrected neutral Higgs boson masses and μ:

$$M_Z^2 = 2\left[\frac{M_h^2 - M_H^2 \tan^2\beta}{\tan^2\beta - 1}\right] - 2\mu^2. \tag{76}$$

In order to get the observed value of M_Z, a delicate cancellation between the Higgs masses and μ is required. This is sometimes called the μ problem.[35]

It is convenient to write the couplings for the neutral Higgs bosons to the fermions in terms of the Standard Model Higgs couplings,

$$\mathcal{L} = -\frac{g m_i}{2 M_W}\left[C_{ffh}\overline{f}_i f_i h + C_{ffH}\overline{f}_i f_i H + C_{ffA}\overline{f}_i \gamma_5 f_i A\right], \tag{77}$$

where C_{ffh} is 1 for a Standard Model Higgs boson. The C_{ffi} are given in Table 3 and plotted in Figs. 7 and 8 as a function of M_A. We see that for small M_A and large $\tan\beta$, the couplings of the neutral Higgs boson to fermions can be significantly different from the Standard Model couplings; the b-quark coupling becomes enhanced, while the t-quark coupling is suppressed. It is obvious from Figs. 7 and 8 that when M_A becomes large the Higgs-fermion couplings approach their standard model values, $C_{ffh} \to 1$. In fact even for $M_A \sim 300\ GeV$, the Higgs-fermion couplings are very close to their Standard Model values.

The Higgs boson couplings to gauge bosons are fixed by the $SU(2)_L \times U(1)_Y$ gauge invariance. Some of the phenomenologically important couplings

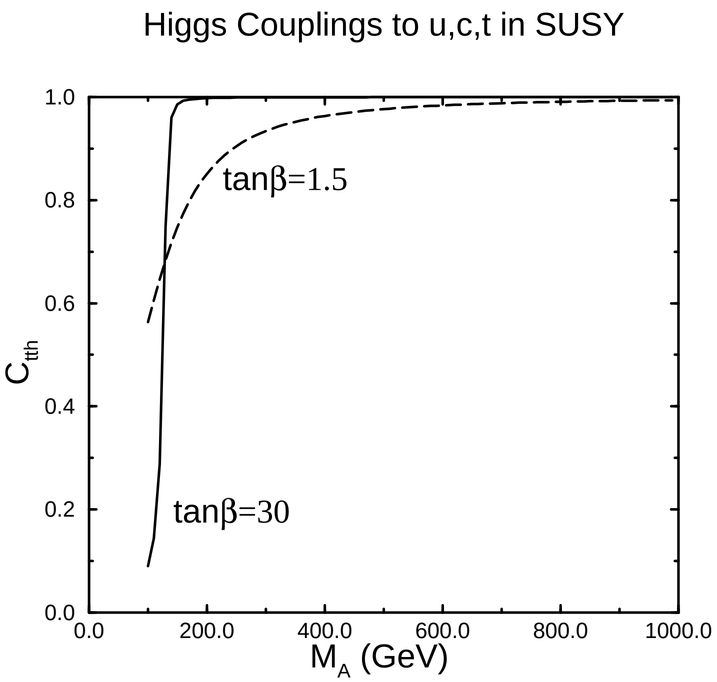

Figure 8: Coupling of the lightest Higgs boson to charge 2/3 quarks including radiative corrections [34] in terms of the couplings defined in Eq. 77. The value $C_{tth} = 1$ yields the Standard Model coupling of the Higgs boson to charge 2/3 quarks.

are:

$$\begin{aligned}
Z^\mu Z^\nu h : &\qquad \frac{igM_Z}{\cos\theta_W}\sin(\beta-\alpha)g^{\mu\nu} \\
Z^\mu Z^\nu H : &\qquad \frac{igM_Z}{\cos\theta_W}\cos(\beta-\alpha)g^{\mu\nu} \\
W^\mu W^\nu h : &\qquad igM_W\sin(\beta-\alpha)g^{\mu\nu} \\
W^\mu W^\nu H : &\qquad igM_W\cos(\beta-\alpha)g^{\mu\nu}
\end{aligned}$$

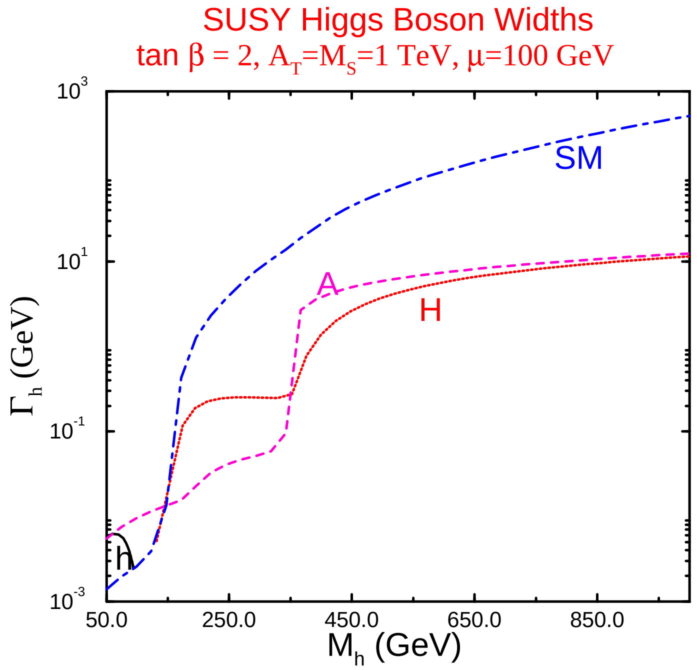

Figure 9: Total SUSY Higgs boson decay widths including two-loop radiative corrections as a function of the Higgs masses. The width for the Standard Model Higgs boson is shown for comparison. The curve for the lightest Higgs boson is cut off at the maximum M_h. The program HDECAY [34] was used to obtain this plot.

$$
\begin{aligned}
Z^\mu h(p)A(p') : &\qquad \frac{g\cos(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu \\
Z^\mu H(p)A(p') : &\qquad -\frac{g\sin(\beta-\alpha)}{2\cos\theta_W}(p+p')^\mu \quad .
\end{aligned}
\tag{78}
$$

We see that the couplings of the Higgs bosons to the gauge bosons all depend on the same angular factor, $\beta-\alpha$. The pseudoscalar, A, has no tree level coupling to pairs of gauge bosons. The angle β is a free parameter while the neutral Higgs mixing angle, α, which enters into many of the couplings, can be found in terms of the M_A and β masses:

$$\tan 2\alpha = \frac{(M_A^2 + M_Z^2)\sin 2\beta}{(M_A^2 - M_Z^2)\cos 2\beta + \epsilon_h / \sin^2\beta} \quad . \tag{79}$$

With our conventions, $-\frac{\pi}{2} \leq \alpha \leq 0$. It is clear from Eq. 78 that the couplings of the SUSY Higgs bosons to gauge bosons are always suppressed relative to those of the Standard Model.

A complete set of couplings for the Higgs bosons (including the charged and pseudoscalar Higgs) at tree level can be found in Ref. 29. These couplings completely determine the decay modes of the SUSY Higgs bosons and their experimental signatures. The important point is that (at lowest order) all of the couplings are completely determined in terms of M_A and $\tan\beta$. When radiative corrections are included there is a dependence on the squark masses and the mixing parameters. This dependence is explored in detail in Ref. 36.

It is an important feature of the MSSM that for large M_A, the Higgs sector looks exactly like that of the Standard Model. As $M_A \rightarrow \infty$, the masses of the charged Higgs bosons, $H^\pm$, and the heavier neutral Higgs, H, become large leaving only the lighter Higgs boson, h, in the spectrum. In this limit, the couplings of the lighter Higgs boson, h, to fermions and gauge bosons take on their Standard Model values. We have,

$$\begin{aligned} \sin(\beta-\alpha) &\qquad \rightarrow 1 \text{ for } M_A \rightarrow \infty \\ \cos(\beta-\alpha) &\qquad \rightarrow 0 \quad . \end{aligned} \tag{80}$$

From Eq. 78, we see that the heavier Higgs boson, H, decouples from the gauge bosons in the heavy M_A limit, while the lighter Higgs boson, h, has Standard Model couplings. Figs. 7 and 8 demonstrate that the Standard Model limit is rapidly approached in the fermion-Higgs couplings for $M_A > 300\ GeV$. In the limit of large M_A, it will thus be exceedingly difficult to differentiate a SUSY Higgs sector from the Standard Model Higgs boson.

- The SUSY Higgs sector with large M_A looks like the Standard Model Higgs sector.

In this case, it will be difficult to discover SUSY through the Higgs sector. Instead, it will be necessary to find some of the other SUSY partners of the observed particles.

The total width of the Higgs boson depends sensitively on $\tan\beta$ and is illustrated in Fig. 9 for $\tan\beta = 2$.[34] We see that the lightest Higgs boson has a width $\Gamma_h \sim 10 - 100\ MeV$, while the heavier Higgs boson has a width $\Gamma_H \sim .1 - 1\ GeV$, which is considerably narrower than the width of the Standard

Model Higgs boson with the same mass. (The curve for the lighter Higgs boson is cut off at the kinematic upper limit.) The pseudoscalar, A, is also narrower than a Standard Model Higgs boson with the same mass.

3.7 The Squark and Slepton Sector

We turn now to a discussion of the scalar partners of the quarks and leptons. The left-handed $SU(2)_L$ quark doublet has scalar partners,

$$\tilde{Q} = \left(\begin{array}{c} \tilde{u}_L \\ \tilde{d}_L \end{array} \right) \quad . \tag{81}$$

The right-handed quarks also have scalar partners, $\tilde{u}_R$ and $\tilde{d}_R$. The L and R subscripts denote which helicity quark the scalars are partners of– **they are for identification purposes only. These are ordinary complex scalars**. Before SUSY is broken the fermions and scalars have the same masses and this mass degeneracy is split by the soft mass terms of Eq. 52. The tri-linear A terms allow the scalar partners of the left- and right-handed fermions to mix to form the mass eigenstates. In the top squark sector, the mixing between the scalar partners of the left- and right handed top (the stops), $\tilde{t}_L$ and $\tilde{t}_R$, is given by

$$M_{\tilde{t}}^2 = \left(\begin{array}{cc} M_{\tilde{Q}}^2 + M_T^2 & M_T(A_T - \mu \cot\beta) \\ \quad + M_Z^2(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\cos 2\beta & \\ M_T(A_T - \mu\cot\beta) & M_{\tilde{u}}^2 + M_T^2 \\ & \quad + \frac{2}{3}M_Z^2 \sin^2\theta_W \cos 2\beta \end{array} \right) , \tag{82}$$

while in the $b-$ squark system the mass-squared matrix is

$$M_{\tilde{b}}^2 = \left(\begin{array}{cc} M_{\tilde{Q}}^2 + M_b^2 & M_b(A_b - \mu \tan\beta) \\ \quad - M_Z^2(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W)\cos 2\beta & \\ M_b(A_b - \mu\tan\beta) & M_{\tilde{d}}^2 + M_b^2 \\ & \quad - \frac{1}{3}M_Z^2 \sin^2\theta_W \cos 2\beta \end{array} \right) . \tag{83}$$

For the scalars associated with the lighter quarks, the mixing effects will be negligible if we assume that all of the A_i are of similar size (as often happens in GUT models), since the mixing is assumed to be proportional to the quark mass. If $\tan\beta >> 1$, then $\tilde{b}_L - \tilde{b}_R$ mixing could be large and so be phenomenologically relevant. In principle, these mixing matrices could involve all three generations of squarks and so could be 6×6 matrices in the $(\tilde{q}_{iL}, \tilde{q}_{iL})$ basis, $(i = 1, 2, 3)$.

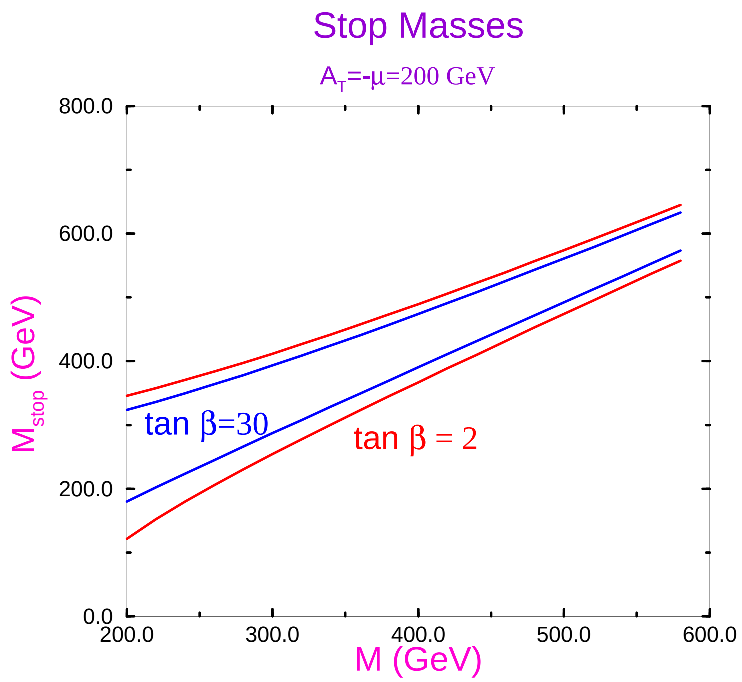

Figure 10: Stop squark masses for large mixing parameters, $A_T = \mu = 200\ GeV$, and for $\tan\beta = 2$ and $\tan\beta = 30$. $M \equiv M_{\tilde{Q}} = M_{\tilde{u}}$ are the squark mass parameters of Eq. 52.

From Eqs. 82 and 83, we see that there are two important cases to consider. If the soft breaking occurs at a large scale with all the soft masses roughly equal and much greater than M_Z, M_T, and A_T, then all the soft masses will be approximately equal, and we will have 12 degenerate squarks with masses $\tilde{M} \sim M_{\tilde{Q}} \sim M_{\tilde{u}} \sim M_{\tilde{d}}$. On the other hand, if the soft masses and the tri-linear mixing term, A_T, are on the order of the electroweak scale, then mixing effects become important.

If mixing effects are large, then one of the stop squarks will become the lightest squark, since the mixing effects are proportional to the relevant quark masses and hence will be largest in this sector. The case where the lightest squark is the stop is particularly interesting phenomenologically. In Fig. 10, we show the stop squark masses for $M_{\tilde{Q}} = M_{\tilde{u}} \equiv \tilde{M}$ and for several values of $\tan\beta$. Of course the mixing effects cannot be too large, or the stop squark mass-squared will be driven negative, leading to a breaking of the color $SU(3)$ gauge symmetry. Typically, the requirement that the correct vacuum be chosen leads to a restriction on the mixing parameter on the order of $\mid A_T \mid < \tilde{M}$.

The couplings of the squarks to gauge bosons are completely fixed by gauge invariance, with no free parameters. A few examples of the couplings are:

$$\begin{aligned}
\gamma^\mu \; \tilde{q}_{L,R}(p) \; \tilde{q}^*_{L,R}(p') : &\qquad -ieQ_q(p+p')^\mu \\
W^{\mu -} \; \tilde{u}_L(p) \; \tilde{d}^*_L(p') : &\qquad -\frac{ig}{\sqrt{2}}(p+p')^\mu \\
Z^\mu \; \tilde{q}_{L,R}(p) \; \tilde{q}^*_{L,R}(p') : &\qquad -\frac{ig}{\cos\theta_W}\left[T_3 - Q_q \sin^2\theta_W\right](p+p')^\mu \quad ,
\end{aligned} \tag{84}$$

where T_3 and Q_q are the quantum numbers of the corresponding quark. The strength of the interactions are clearly given by the relevant gauge coupling constants. A complete set of Feynman rules can be found in Ref. 2.

The mixing in the slepton sector is analogous to that in the squark sector and we will not pursue it further. From Table 1, we see that the scalar partner of the ν_L, $\tilde{\nu}_L$, has the same gauge quantum numbers as the H_2^0 Higgs boson. It is possible to give $\tilde{\nu}_L$ a vacuum expectation value and use it to break the electroweak symmetry. Such a vacuum expectation value would break lepton number (and R parity) thereby giving the neutrinos a mass and so its magnitude is severely restricted. [20]

3.8 The Chargino Sector

There are four charge- 1, spin- $\frac{1}{2}$ Majorana fermions; $\tilde{\omega}^\pm$, the fermionic partners of the $W^\pm$ bosons, (winos), and $\tilde{h}^\pm$, the charged fermion partners of the Higgs bosons, termed the Higgsinos. Since they have the same quantum numbers, these fermions can mix to form the 4$-$ component Dirac spinors,

$$\begin{aligned}
\psi^+ &= (-i\tilde{\omega}^+, \tilde{h}_2^+) \\
\psi^- &= (-i\tilde{\omega}^-, \tilde{h}_1^-) \quad .
\end{aligned}$$

Note that ψ^+ and ψ^- are two independent Dirac spinors and so it will take two different mixing matrices to find the mass eigenstates. The physical mass states, $\chi^\pm_{1,2}$, are linear combinations formed by diagonalizing the mass matrix and are usually called charginos. We define these 2$-$ component states in terms of the mixing matrices,

$$\begin{aligned} \chi_i^+ &\equiv V_{ij}\psi_j^+ \\ \chi_i^- &\equiv U_{ij}\psi_j^-, \qquad i=1,2 \end{aligned} \tag{85}$$

(with $\psi_1^+ \equiv -i\tilde{\omega}^+, \;\; \psi_2^+ \equiv \tilde{h}^+$, etc.). The 4− component Majorana chargino mass eigenstates can be formed as in Eq. 29,

$$\tilde{\chi}_i^+ = \begin{pmatrix} \chi_i^+ \\ \overline{\chi}_i^+ \end{pmatrix} \quad . \tag{86}$$

The mass matrix for the charginos can be found from Eqs. 42 and 43,[c]

$$\mathcal{L} = -\frac{1}{2}(\psi^+,\psi^-)\begin{pmatrix} 0 & M^T \\ M & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c.} \tag{87}$$

where

$$M = \begin{pmatrix} M_2 & \sqrt{2}M_W\cos\beta \\ \sqrt{2}M_W\sin\beta & \mu \end{pmatrix} \quad . \tag{88}$$

It is clear that $\tilde{\omega}^\pm$ and $\tilde{h}^\pm$ are not mass eigenstates, although for $M_2, \mu >> M_W$, the mass of the lightest chargino is approximately $min(M_2,\mu)$. For $M_2 >>\mid \mu \mid$, the chargino is termed "Higgsino-like", while for $M_2 <<\mid \mu \mid$ it is called "gaugino-like". The properties of the chargino in these two regimes are significantly different.

The diagonal chargino mass matrix, M_{χ^+}, can be found by diagonalizing the mass matrix using the 2 unitary matrices, U and V,

$$M_{\chi^+} = U^* M V^{-1} \tag{89}$$

with,

$$V_{ij} = \begin{pmatrix} \cos\phi_+ & \sin\phi_+ \\ \sin\phi_+ & -\cos\phi_+ \end{pmatrix}, \qquad U_{ij} = \begin{pmatrix} \cos\phi_- & \sin\phi_- \\ -\sin\phi_- & \cos\phi_- \end{pmatrix} \quad . \tag{90}$$

It is straightforward to find analytic expressions for the mixing angles :

$$\begin{aligned} \tan 2\phi_+ &= \frac{2\sqrt{2}M_W(\mu\sin\beta + M_2\cos\beta)}{M_2^2-\mu^2-2M_W^2\cos 2\beta} \\ \tan 2\phi_- &= \frac{2\sqrt{2}M_W(\mu\cos\beta + M_2\sin\beta)}{M_2^2-\mu^2+2M_W^2\cos 2\beta}. \end{aligned} \tag{91}$$

[c] Note that some older references define $\tan\beta = v_1/v_2$. In comparing with the literature, it is also important to check the definition of the sign(μ).

Useful expressions for the mixing angles in terms of the chargino mass eigenstates are given in Refs. 37 and 38. One of the mass eigenstates (say $M_{\tilde{\chi}_2^+}$) will be negative if

$$Det(M) = M_2\mu - M_W^2 \sin(2\beta) > 0 \quad . \tag{92}$$

The easiest way to deal with a negative mass eigenstate is to define a sign factor, η_i, which is 1 for positive mass and -1 for negative mass. If we consistently replace V_{2i} by $\eta_i V_{2i}$ in all couplings, then the correct Feynman rules are obtained. U and V are always assumed to be chosen such that the mass eigenstates are real and positive.

The mass eigenstates are given by,

$$\begin{aligned} M^2_{\tilde{\chi}^\pm_{1,2}} &= \frac{1}{2}\Bigg\{ M_2^2 + 2M_W^2 + \mu^2 \mp \Bigg[(M_2^2-\mu^2)^2 + 4M_W^4 \cos^2 2\beta \\ &\qquad + 4M_W^2(M_2^2+\mu^2+2M_2\mu\sin^2\beta)\Bigg]^{1/2}\Bigg\}. \end{aligned} \tag{93}$$

By convention $M_{\tilde{\chi}_1^\pm}$ is the lighter chargino. In Fig 11, we show the mass of the lightest chargino as a function of μ for several values of M_2 and $\tan\beta$. From Eq. 93, it is clear that for $\mu \to 0$, there will be an almost massless chargino, which is clearly seen in Fig 11. This possibility is excluded by experiment.

Combining our results, we find the interaction Lagrangian of the $4-$ component charginos with the gauge bosons,

$$\begin{aligned} \mathcal{L} &= e\overline{\tilde{\chi}}_i^+ \gamma^\mu \tilde{\chi}_j^+ A_\mu \delta_{ij} + \left(\frac{g}{\cos\theta_W}\right) \overline{\tilde{\chi}}_i^+ \gamma^\mu \left[C_{ij}^+ P_+ + C_{ij}^- P_- \right] \tilde{\chi}_j^+ Z_\mu \\ &\quad - g\left[\overline{e}_L P_+ \tilde{\chi}_i^{+c} \tilde{\nu}_L V_{i1} + \overline{\nu}_L P_+ \tilde{\chi}_i^+ \tilde{e}_L U_{i1} + h.c. \right] \quad , \end{aligned} \tag{94}$$

where $e < 0$, $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ and

$$\begin{aligned} C_{ij}^+ &= -V_{i1}V_{j1}^* - \frac{1}{2} V_{i2}V_{j2}^* + \delta_{ij}\sin^2\theta_W \\ C_{ij}^- &= -U_{i1}^*U_{j1} - \frac{1}{2} U_{i2}^*U_{j2}^* + \delta_{ij}\sin^2\theta_W \quad . \end{aligned}$$

The Feynman rules derived from Eq. 94 can be used to compute processes of physical interest and can be found in Ref. 2. As an example, in the Appendix we compute $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, where there is an interesting interplay between sneutrino exchange and the $s-$ channel γ- and Z -exchange.

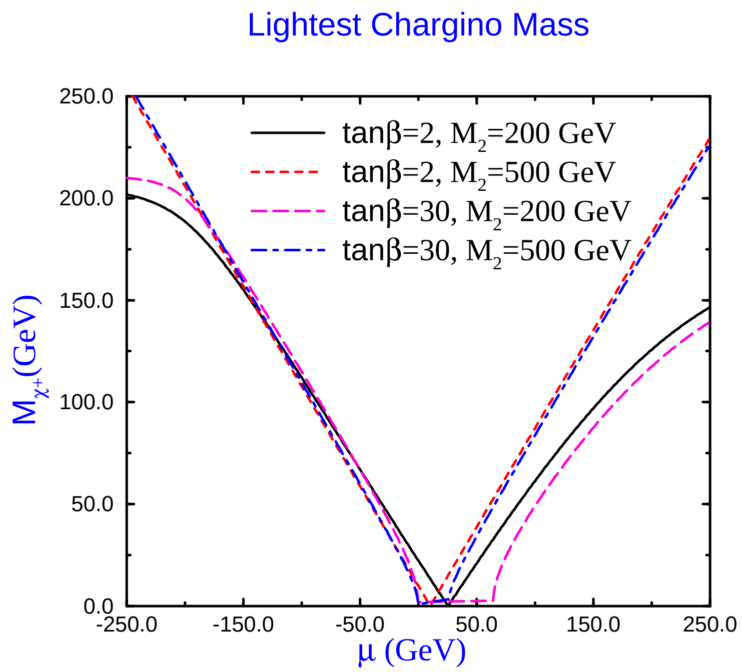

Figure 11: Lightest chargino mass for fixed $\tan\beta$ and M_2.

3.9 The Neutralino Sector

In the neutral fermion sector, the neutral fermion partners of the B and W^3 gauge bosons, $\tilde{b}$ and $\tilde{\omega}^3$, can mix with the neutral fermion partners of the Higgs bosons, $\tilde{h}_1^0, \tilde{h}_2^0$ to form the mass eigenstates. Hence the physical states, $\tilde{\chi}_i^0$, termed neutralinos, are found by diagonalizing the 4×4 mass matrix,

$$M_{\tilde{\chi}_i^0} = \begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin\theta_W & M_Z\sin\beta\sin\theta_W \\ 0 & M_2 & M_Z\cos\beta\cos\theta_W & -M_Z\sin\beta\cos\theta_W \\ -M_Z\cos\beta\sin\theta_W & M_Z\cos\beta\sin\theta_W & 0 & -\mu \\ M_Z\sin\beta\sin\theta_W & -M_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix} \tag{95}$$

where θ_W is the electroweak mixing angle and we work in the $\tilde{b}, \tilde{\omega}^3, \tilde{h}_1^0, \tilde{h}_2^0$ basis. The physical masses can be defined to be positive and by convention,

$M_{\tilde{\chi}_1^0} < M_{\tilde{\chi}_2^0} < M_{\tilde{\chi}_3^0} < M_{\tilde{\chi}_4^0}$. In general, the mass eigenstates do not correspond to a photino, (a fermion partner of the photon), or a zino, (a fermion partner of the Z), but are complicated mixtures of the states. The photino is only a mass eigenstate if $M_1 = M_2$. Physics involving the neutralinos therefore depends on M_1, M_2, μ, and $\tan\beta$. The lightest neutralino, $\tilde{\chi}_1^0$, is usually assumed to be the LSP.

4 WHY DO WE NEED SUSY?

Having introduced the MSSM as an effective theory at the electroweak scale and briefly discussed the various new particles and interactions of the model, we turn now to a consideration of the reasons for constructing a SUSY theory in the first place. We have already considered the cancellation of the quadratic divergences, which is automatic in a supersymmetric model. There are, however, many other reasons why theorists are excited about supersymmetry.

4.1 Coupling constants run!

In a gauge theory, coupling constants scale with energy according to the relevant β-function. Hence, having measured a coupling constant at one energy scale, its value at any other energy can be predicted. At one loop,

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(\mu)} + \frac{b_i}{2\pi}\log\left(\frac{\mu}{Q}\right) \quad . \tag{96}$$

In the Standard (non-supersymmetric) Model, the coefficients b_i are given by,

$$\begin{aligned} b_1 &= \frac{4}{3}N_g + \frac{N_H}{10} \\ b_2 &= -\frac{22}{3} + \frac{4}{3}N_g + \frac{N_H}{6} \\ b_3 &= -11 + \frac{4}{3}N_g \quad , \end{aligned} \tag{97}$$

where $N_g = 3$ is the number of generations and $N_H = 1$ is the number of Higgs doublets. The evolution of the coupling constants is seen to be sensitive to the particle content of the theory. We can take $\mu = M_Z$ in Eq. 96, input the measured values of the coupling constants at the Z-pole and evolve the couplings to high energy. The result is shown in Fig. 12. There is obviously no meeting of the coupling constants at high energy.

Suppose we assume that the unifying gauge group is $SU(5)$. This requires that the $SU(3)$, $SU(2)_L$, and $U(1)_Y$ generators, T_i, be normalized in the same manner. Each generation of fermions is contained in a $\overline{5}$ and 10 of $SU(5)$ with the $\overline{5}$ given by,

$$\overline{5} = (\overline{d}, \overline{d}, \overline{d}, e^-, \nu). \tag{98}$$

The $SU(3)$ and $SU(2)_L$ generators both satisfy $Tr(T_i)^2 = \frac{1}{2}$ for the $\overline{5}$. The $U(1)_Y$ generator for the $\overline{5}$ must have the same normalization,

$$Y_{GUT} = \sqrt{\frac{3}{5}} diag(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}). \tag{99}$$

By comparing with Table 1, we see that $Y_{GUT} = \sqrt{\frac{3}{5}} Y$ and so we must have

$$g' = g^* \sqrt{\frac{3}{5}}, \tag{100}$$

where g^* is the GUT coupling constant, in order to obtain the correct couplings of the fermions to the gauge bosons.

If the theory is supersymmetric, then the spectrum is different and the new particles contribute to the evolution of the coupling constants. In this case we have at one loop,[39]

$$\begin{aligned} b_1 &= 2N_g + \frac{3}{10} N_H \\ b_2 &= -6 + 2N_g + \frac{N_H}{2} \\ b_3 &= -9 + 2N_g \quad . \end{aligned} \tag{101}$$

Because a SUSY model of necessity contains two Higgs doublets, we have $N_H = 2$. If we assume that the mass of all the SUSY particles is around $1\ TeV$, then the coupling constants scale as shown in Fig.13. We see that the coupling constants meet at a scale around 10^{16} GeV.[14,40,41] This meeting of the coupling constants is a necessary feature of a Grand Unified Theory (GUT).

- SUSY theories can be naturally incorporated into Grand Unified Theories.

It is interesting to use the requirement of unification to predict $\alpha_s(M_Z)$. This requirement gives the prediction:

$$\frac{b_1 - b_2}{\alpha_3(\mu)} + \frac{b_2 - b_3}{\alpha_1^*(\mu)} + \frac{b_3 - b_1}{\alpha_2(\mu)} = 0, \tag{102}$$

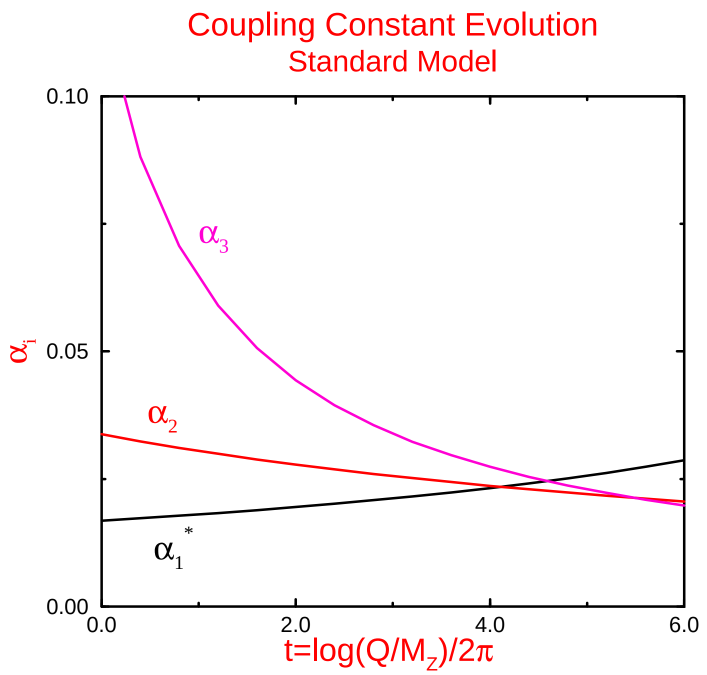

Figure 12: Evolution of the gauge coupling constants in the Standard Model from the experimentally measured values at the Z-pole. $\alpha_1^* \equiv 5/3\alpha_1$, since this is the relevant coupling in SU(5)-like Grand Unified Theories.

valid at any scale μ between M_X and M_Z. If we input the SUSY β-functions, $\alpha_1(M_Z) = 1/128$, and $\sin^2\theta_W(M_Z) = .2315$, we obtain a prediction from the MSSM at 1$-$loop:

$$\alpha_s(M_Z) = .116, \tag{103}$$

in reasonable agreement with the measured value at LEP, $\alpha_s(M_Z) = .118 \pm .003$.

Unfortunately, the picture is not so rosy when we attempt to take the MSSM seriously and include two loop beta functions, effects from passing through SUSY particle thresholds, etc.[41] Typically, the prediction for $\alpha_s(M_Z)$

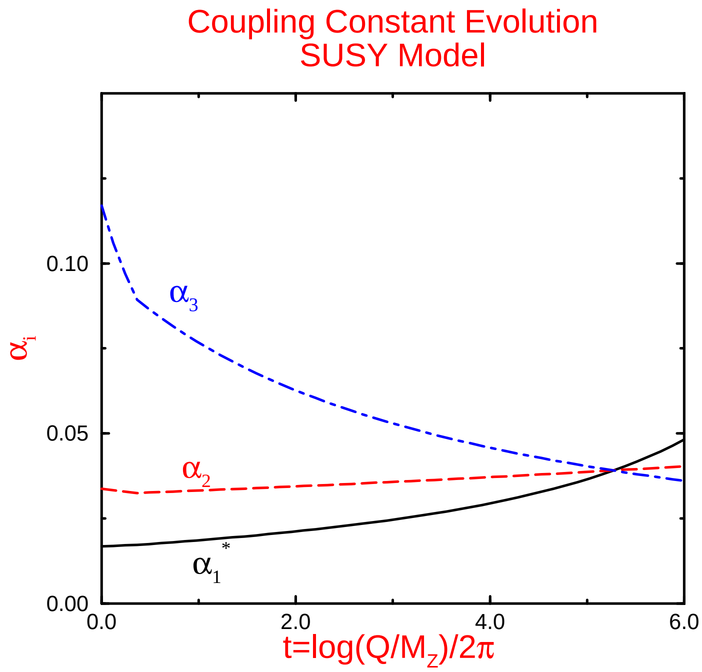

Figure 13: Evolution of the coupling constants in a low energy SUSY model from the experimentally measured values at the Z-pole. The SUSY thresholds are taken to be at 1 TeV. $\alpha_1^* \equiv 5/3\alpha_1$, since this is the relevant coupling in Grand Unified Theories.

becomes significantly larger that the experimental value. The goal is to learn something about the underlying GUT theory by computing the threshold corrections at the GUT scale and seeing which models are consistent with the data.[43]

4.2 SUSY GUTS

The observation that the measured coupling constants tend to meet at a point when evolved to high energy assuming the β-function of a low energy SUSY model has led to widespread acceptance of a standard SUSY GUT model. We assume that the $SU(3)\times SU(2)_L\times U(1)_Y$ gauge coupling constants are unified at a high scale $M_X\sim 10^{16}\ GeV$:

$$\sqrt{\frac{5}{3}}g_1(M_X)=g_2(M_X)=g_s(M_X)\equiv g^* \quad . \tag{104}$$

4.3 CMSSM

We will describe two types of possible GUT models which differ in the source of the soft SUSY breaking terms. The first model is sometimes called the constrained MSSM (CMSSM) or "super-gravity inspired" (SUGRA) MSSM.

Along with the coupling constants, the gaugino masses, M_i, are assumed to unify,

$$M_i(M_X)\equiv m_{1/2} \quad . \tag{105}$$

At lowest order, the gaugino masses then scale in the same way as the corresponding coupling constants,

$$M_i(M_W)=m_{1/2}\frac{g_i^2(M_W)}{g^{*2}} \tag{106}$$

yielding

$$\begin{aligned} M_2(M_W) &= \left(\frac{\alpha(M_W)}{\sin^2\theta_W(M_W)}\right)\left(\frac{1}{\alpha_s(M_W)}\right)M_3(M_W)\\ M_1(M_W) &= \frac{5}{3}\tan^2\theta_W(M_W)M_2(M_W) \quad . \end{aligned} \tag{107}$$

The gluino mass is therefore always the heaviest of the gaugino masses. This relationship between the gaugino masses is a fairly robust prediction of SUSY GUTS and as we will see in the next section persists in models where the supersymmetry is broken through the gauge interactions.

Typical SUSY GUTS of this type also assume that there is a common scalar mass at M_X,

$$\begin{aligned} m_1^2(M_X)= &\quad m_2^2(M_X)\equiv m_0^2\\ M_{\tilde{Q}}^2(M_X)=M_{\tilde{d}}^2(M_X)= M_{\tilde{u}}^2(M_X)= &\quad M_{\tilde{L}}^2(M_X)=M_{\tilde{e}}^2(M_X)\equiv m_0^2 \end{aligned} \tag{108}$$

The neutral Higgs boson masses at M_X are then $M_{h,H}^2=m_0^2+\mu^2$.

It is instructive to study the scalar masses within this scenario. The evolution of the sleptons between M_X and M_W is small and we have the approximate result for the slepton masses,[5,42]

$$M_{\tilde{L}}^2(M_W)\sim M_{\tilde{e}}^2(M_W)\sim m_0^2, \tag{109}$$

while the squark masses are roughly

$$M^2_{\tilde{q}}(M_W) \sim m_0^2 + 4m^2_{1/2} \quad . \tag{110}$$

Since the squarks have strong interactions, (which drives the masses upwards), their masses at the weak scale tend to be larger than the sleptons. Once all the particle masses have been computed in this scheme, then their production cross sections and decay rates at any given accelerator can be computed unambiguously.

As a final simplifying assumption, a common A parameter is assumed,

$$A_T(M_X) = A_b(M_X) = ... \equiv A_0 \quad . \tag{111}$$

With these assumptions, the SUSY sector is completely described by 5 input parameters at the GUT scale,[44]

1. A common scalar mass, m_0.
2. A common gaugino mass, $m_{1/2}$.
3. A common trilinear coupling, A_0.
4. A Higgs mass parameter, μ.
5. A Higgs mixing parameter, B.

The picture is that there is a hidden sector of the theory containing fields which do not transform under the $SU(3) \times SU(2)_L \times U(1)_Y$ gauge group. We assume that the supersymmetry is broken in this hidden sector and communicated to the fields of the MSSM by gravitational interactions.[26] When the supersymmetry is broken at the scale M_{SUSY}, the gravitino will obtain a mass,

$$M_{3/2} \sim \frac{M^2_{SUSY}}{M_{pl}} \quad . \tag{112}$$

The soft mass terms of Eq. 52 will then be,

$$m_{soft} \sim M_{3/2} \sim \frac{M^2_{SUSY}}{M_{pl}} \quad . \tag{113}$$

To obtain $M_{soft} \sim 1~TeV$ (which we argued was necessary in order that supersymmetry solve the hierarchy problem), we need to break supersymmetry at a scale,

$$M_{SUSY} \sim 10^{11}~GeV \quad . \tag{114}$$

If the fields of the hidden sector have canonical kinetic energy terms, then the scalar masses will satisfy the relationships of Eq. 108. Although this framework is somewhat *ad hoc*, it does provide guidance to reduce the immense parameter space of a SUSY model.

The strategy is now to input the 5 parameters given above at M_X and to use the renormalization group equations to evolve the parameters to M_W. In fact, the requirement that the Z boson obtain its measured value when the parameters are evaluated at low energy can be used to restrict $\mid \mu B \mid$, leaving the $sign(\mu)$ as a free parameter. We can also trade the parameter B for $\tan\beta$. In this way the parameters of the model become

$$m_0, m_{1/2}, A_0, \tan\beta, \mathrm{sign}(\mu) \quad . \tag{115}$$

This form of a SUSY theory is extremely predictive, as the entire low energy spectrum is predicted in terms of a few input parameters. Also, all phenomenological limits can be expressed as limits on these parameters. Within this scenario, contours for the various SUSY particle masses can be found as a function of m_0 and $m_{1/2}$ for given values of $\tan\beta$, A_0 and $\mathrm{sign}(\mu)$.[42,44]

Changing the input parameters at M_X (for example, assuming non-universal scalar masses) of course changes the phenomenology at the weak scale. A preliminary investigation of the sensitivity of the low energy predictions to these assumptions has been made in Ref. 24. For now, we will consider the Grand Unified Model described above as a starting point for phenomenological investigations into SUSY.

4.4 GMSB

An alternative picture of the SUSY breaking is gauge mediated SUSY breaking.[45,46] In this type of model, the SUSY breaking again occurs in a hidden sector. The hidden sector is assumed to contain new chiral supermultiplets, called messenger fields, which transform under the $SU(3) \times SU(2)_L \times U(1)_Y$ gauge group. When supersymmetry is broken, the messengers obtain a mass, Λ.

The SUSY breaking is communicated to the MSSM particles through the gauge interactions. The gauginos obtain mass at $1-$ loop,

$$M_i \sim \frac{\alpha_i}{4\pi}\Lambda \quad . \tag{116}$$

We see that the gauginos satisfy Eq. 106. Similarly, the scalars of the MSSM obtain masses at two loops from diagrams involving the gauge fields and the messenger fields. The scalar masses are,

$$\tilde{M}^2 \sim \left(\frac{\Lambda}{4\pi}\right)^2 \left\{ \alpha_s^2 C_3 + \alpha_2^2 C_2 + \alpha_1^2 C_1 \right\}, \tag{117}$$

where C_i are the quadratic Casimir operators for the $SU(3) \times SU(2)_L \times U(1)_Y$ gauge groups. The squarks and sleptons with the same gauge quantum numbers will automatically have the same masses. For example,

$$M_{\tilde{e}} = M_{\tilde{\mu}} = M_{\tilde{\tau}}, \tag{118}$$

etc. In order to obtain soft masses for the gauginos and scalars around 1 TeV, we need

$$\Lambda \sim 10^4 - 10^5 \ GeV. \tag{119}$$

Note that this scale is much smaller than the intermediate scale found in the CMSSM.

The most important difference between the CMSSM and the GMSB models is that here the LSP is the gravitino, $\tilde{G}_{3/2}$. In this case the gravitino mass is,

$$m_{3/2} \sim \frac{\Lambda^2}{M_{pl}} \sim 10^{-10} \ GeV \quad . \tag{120}$$

This leads to strikingly different phenomenology from the CMSSM since this model allows

$$\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}_{3/2}, \tag{121}$$

giving a signal for SUSY of missing E_T plus photons. A review of GMSB models can be found in Ref. 46.

4.5 Electroweak Symmetry Breaking

The CMSSM model has the appealing feature that it explains the mechanism of electroweak symmetry breaking. In the Standard Model (non-supersymmetric) with a single Higgs doublet, ϕ, the scalar potential is given by:

$$V(\phi) = \mu^2 \mid \phi \mid^2 + \lambda(\mid \phi \mid^2)^2 \quad . \tag{122}$$

By convention, $\lambda > 0$. If $\mu^2 > 0$, then $V(\phi) > 0$ for all ϕ not equal to 0 and there is no electroweak symmetry breaking. If, however, $\mu^2 < 0$, then the minimum of the potential is not at $\phi = 0$ and the potential has the familiar Mexican hat shape. When the Lagrangian is expressed in terms of the physical field, $\phi' \equiv (\phi - v)/\sqrt{2}$, which has zero vacuum expectation value, then the electroweak symmetry is broken and the W and Z gauge bosons acquire non-zero masses. We saw in the previous sections that this same mechanism gives the W and Z gauge bosons their masses in the MSSM. This simple picture leaves one looming question:

$$\text{Why is } \mu^2 < 0? \tag{123}$$

It is this question which the SUSY GUT models can answer.

In the minimal CMSSM model, the neutral Higgs bosons both have masses, $M_{h,H}^2 = m_0^2 + \mu^2$, at M_X, while the squarks and sleptons have mass m_0 at M_X. Clearly, at M_X, the electroweak symmetry is not broken since the Higgs bosons have positive mass-squared. The masses scale with energy according to the renormalization group equations.[47] If we neglect gauge couplings and consider only the scaling of the third generation scalars we have,[48]

$$\begin{aligned} \frac{d}{d\log(Q)} \begin{pmatrix} M_h^2 \\ M_{\tilde{t}_R}^2 \\ M_{\tilde{Q}_L^3}^2 \end{pmatrix} &= -\frac{8\alpha_s}{3\pi} M_3^2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \\ &+ \frac{\lambda_T^2}{8\pi^2} \left(M_{\tilde{Q}_L^3}^2 + M_{\tilde{t}_R}^2 + M_h^2 + A_T^2 \right) \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \end{aligned} \tag{124}$$

where $\tilde{Q}_L^3$ is the $SU(2)_L$ doublet containing $\tilde{t}_L$ and $\tilde{b}_L$, h is the lightest neutral Higgs boson, λ_T is the top quark Yukawa coupling constant given in Eq. 75, and Q is the effective scale at which the masses are measured. The signs are such that the Yukawa interactions (proportional to M_T) decrease the masses, while the gaugino interactions increase the masses. Because of the $3-2-1$ structure of the last term in Eq. 124, the Higgs mass decreases faster than the squark masses and it is possible to drive $M_h^2 < 0$ at low energy, while keeping $M_{\tilde{Q}_L^3}^2$ and $M_{\tilde{t}_R}^2$ positive. A generic set of scalar masses in a typical SUSY GUT model is shown in Fig. 14. We can clearly see that the lightest Higgs boson mass becomes negative around the electroweak scale.[49]

For large λ_T, we have the approximate solution,

$$M_h^2(Q) = M_h^2(M_X) - \frac{3}{8\pi^2}\lambda_T^2 (M_{\tilde{Q}_L^3}^2 + M_{\tilde{t}_R}^2 + M_h^2 + A_T^2) \log\left(\frac{M_X}{Q}\right) \quad . \tag{125}$$

Hence the larger M_T is, the faster M_h^2 goes negative. This of course generates electroweak symmetry breaking. If M_T were light, M_h^2 would remain positive.[48] This observation was made fifteen years ago when we thought the top quark was light, ($\sim 40\ GeV$). At that time it was ignored as not being phenomenologically relevant. In fact, this mechanism only works for $M_T \sim 175\ GeV$!

- SUSY GUTS can explain electroweak symmetry breaking. The lightest Higgs boson mass is negative, $M_h^2 < 0$, because M_T is large.

The requirement that the electroweak symmetry breaking occur through the renormalization group scaling of the Higgs boson mass also restricts the allowed

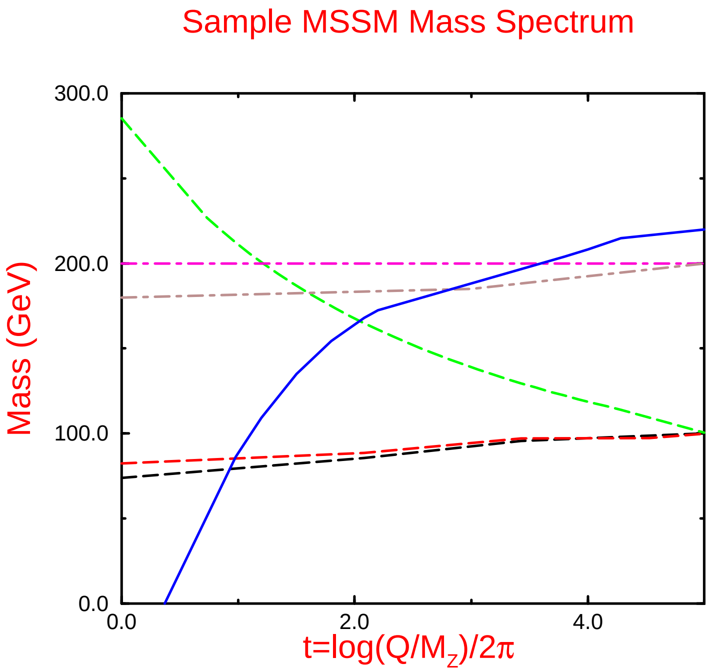

Figure 14: Sample masses of SUSY particles in a SUSY GUT. At the GUT scale M_X, we have taken $m_0 = 200\ GeV, m_{1/2} = 100\ GeV, \mu = 100\ GeV$ and $A_i = 0$. The solid line is the lightest neutral Higgs boson mass. The dashed lines are the gaugino masses (the largest is the gluino) and the dot-dashed lines are typical squark masses.

values of $\tan\beta$ to $\tan\beta > 1$. (Remember that λ_T depends on β through Eq. 75.)

4.6 Fixed Point Interactions

In the previous subsection we saw that a large top quark mass could generate electroweak symmetry breaking in a SUSY GUT model. The top quark mass is determined in terms of its Yukawa coupling and scales with energy, Q,[50]

$$\lambda_T(Q) = \frac{M_T(Q)}{M_W} \frac{g}{\sqrt{2}\sin\beta} \quad . \tag{126}$$

Including both the gauge couplings and the Yukawa couplings to the t- and b-quarks, the scaling is:

$$\frac{d\lambda_T}{d\log(Q)} = \frac{\lambda_T}{16\pi^2}\left\{-\frac{13}{9}g'^2 - 3g^2 - \frac{16}{3}g_s^2 + 6\lambda_T^2 + \lambda_B^2\right\} \quad . \tag{127}$$

To a good approximation, we can consider only the contributions from the strong coupling constant, g_s, and the top quark Yukawa coupling, λ_T. If we begin our scaling at M_X and evolve λ_T to lower energy, we will come to a point where the evolution of the Yukawa coupling stops,

$$\frac{d\lambda_T}{d\log(Q)} = 0 \quad . \tag{128}$$

At this point we have roughly,

$$-\frac{16}{3}g_s^2 + 6\lambda_T^2 = 0 \tag{129}$$

which gives,

$$\lambda_T \sim \frac{4}{3}\sqrt{2\pi\alpha_s} \sim 1, \tag{130}$$

or

$$M_T \sim (200\ GeV)\sin\beta \quad . \tag{131}$$

This point where the top quark mass stops evolving is called a *fixed point*. What this means is that no matter what the initial condition for λ_T is at M_X, it will always evolve to give the same value at low energy. For $\tan\beta \sim 2$, the fixed point value for the top quark mass is close to the experimental value. More sophisticated analyses do not change this picture substantially.

- SUSY GUTS can naturally accommodate a large top quark mass for $\tan\beta \sim 1-3$.

4.7 $b-\tau$ Unification

The unification of the b- and τ- Yukawa coupling constants, λ_B and λ_τ, at the GUT scale is a concept much beloved by theorists since

$$\lambda_B(M_X) = \lambda_\tau(M_X) \tag{132}$$

occurs naturally in many GUT models (such as the $SU(5)$ GUT). Imposing the boundary condition of Eq. 132 and requiring that the b quark have its experimental value at low energy leads to a prediction for the top quark mass in terms of $\tan\beta$. There are two solutions which yield $M_T = 175\ GeV$,[50]

$$\begin{aligned} & \tan\beta \sim 1 \\ \text{or} \quad & \tan\beta \sim \frac{M_T}{M_b} \quad . \end{aligned} \tag{133}$$

The first solution roughly corresponds to the fixed point solution of the previous subsection. The second solution with $\tan\beta \sim 35$ has interesting phenomenological consequences, since for large $\tan\beta$ the coupling of the lightest Higgs boson to b quarks is enhanced relative to the Standard Model. (See Fig. 7). The values in the $\tan\beta - M_T$ plane allowed by $b - \tau$ unification depend sensitively on the exact value of the strong coupling constant, α_s, used in the evolution and so there is a significant uncertainly in the prediction.[51]

- SUSY GUTs allow for the unification of the $b - \tau$ Yukawa coupling constants at the GUT scale along with the experimentally observed value for the top quark mass.

Similar relationships to Eq. 132 involving the first two generations do not work.

4.8 Comments

We see that SUSY plus grand unification has many desirable features which are not sensitive to the exact mechanism of SUSY breaking or to the details of the underlying GUT:

1. There are no troubling quadratic divergences requiring disagreeable cancellations.

2. M_T is large because λ_T evolves from the GUT scale to its fixed point.

3. Electroweak symmetry is broken, $M_h^2 < 0$, because M_T is large.

4. $b - \tau$ unification can be incorporated, leading to the experimentally observed value for the top quark mass.

5 SEARCHING FOR SUSY

5.1 Indirect Hints for SUSY

One might hope that the precision measurements at the Z-pole could be used to garner information on the SUSY particle spectrum. Since the precision electroweak measurements are overwhelmingly in excellent agreement with the predictions of the Standard Model, it would appear that stringent limits could be placed on the existence of SUSY particles at the weak scale. There are two reasons why this is not the case.

The first is that SUSY is a *decoupling theory*. With the exception of the Higgs particles, the effects of SUSY particles at the weak scale are suppressed by powers of M_W^2/M_{SUSY}^2, where M_{SUSY} is the relevant SUSY mass scale, and so for M_{SUSY} larger than a few hundred GeV, the SUSY particles give negligible contributions to electroweak processes. The second reason why there are not stringent limits from precision results at LEP has to do with the Higgs sector. The Higgs bosons are the only particles in the spectrum which do not decouple from the low energy physics when they are very massive. The fits to electroweak data tend to prefer a Higgs boson in the 100 GeV mass range.[21] Since the MSSM requires a light Higgs boson with a mass in this region anyways, the electroweak data is completely consistent with a SUSY model with a light Higgs boson and all other SUSY particles significantly heavier.

Attempts have been made to perform global fits to the electroweak data and to fix the SUSY spectrum this way.[52,53] It is possible to obtain a fit where the χ^2/degree of freedom is roughly the same as in the Standard Model fit. Although the fits do not yield stringent limits on the SUSY particle masses, they do exhibit several interesting features. They tend to prefer either small $\tan\beta$, $\tan\beta \sim 2$, or relatively large values, $\tan\beta \sim 30$. In addition, the fitted values for the strong coupling constant at M_Z, $\alpha_s(M_Z)$, are slightly smaller in SUSY models than in the Standard Model. (For $\tan\beta = 1.6$, Ref. 52 finds $\alpha_s(M_Z) = .116 \pm .005$ and for $\tan\beta = 34$, they find $\alpha_s(M_Z) = .119 \pm .005$.) It is clear that all precision electroweak measurements can be accommodated within a SUSY model, but the data show no clear preference for these models.

5.2 Limits from the ρ Parameter

One of the most precisely measured electroweak quantities is the ρ parameter,

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \quad . \tag{134}$$

A large mass splitting between the stop and sbottom squarks can give a significant contribution to the ρ parameter, just as does the $t-b$ mass splitting. If we define θ_t to be the mixing angle associated with the stop mass matrix, Eq. 82, and neglect the mixing in the $\tilde{b}_L - \tilde{b}_R$ sector, there is a contribution to the ρ parameter from the squark sector of[54]

$$\begin{aligned}\delta\rho^{SUSY} &= \frac{3G_F}{8\sqrt{2}\pi^2}\biggl\{-\frac{1}{4}\sin^2 2\theta_t f(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}) \\ &\qquad +\cos^2\theta_t f(M^2_{\tilde{t}_1}, M^2_{\tilde{b}_L}) + \sin^2\theta_t f(M^2_{\tilde{t}_2}, M^2_{\tilde{b}_L})\biggr\}, \end{aligned} \qquad (135)$$

where t_1 and t_1 are the stop mass eigenstates and

$$f(m_1^2, m_2^2) = m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2}\log\left(\frac{m_1^2}{m_2^2}\right) \quad . \qquad (136)$$

The function,$f(m_1^2, m_2^2)$, has the property that it vanishes for degenerate squark masses,

$$f(m^2, m^2) = 0 \quad . \qquad (137)$$

If one of the masses is much heavier than the other, then we have

$$f(m^2, 0) \rightarrow m^2 \quad . \qquad (138)$$

Hence the contribution of a squark doublet can be very large if the mass splitting is large. In the limit in which there is no mixing in the stop sector and $M_{\tilde{t}} >> M_{\tilde{b}}$, the ρ parameter becomes,

$$\delta\rho^{SUSY} \rightarrow \frac{3G_F}{8\sqrt{2}\pi} M^2_{\tilde{t}}. \qquad (139)$$

For squark masses in the 200 GeV region, the contribution to the ρ parameter is typically in the range of 10^{-3}, depending on assumptions about $\tan\beta$ and the mixing parameters. This can be used to limit the allowed values of the squark masses and the squark mass splittings.[54]

5.3 Flavor Changing Neutral Currents

From the squark mass matrices of Eq. 82, it is apparent that the unitary matrices, $\tilde{U}$, which diagonalize the squark mass matrices are not, in general, the same as the CKM matrix, V, which diagonalizes the quark mass matrix.

The physical mass eigenstates are given by,

$$\begin{aligned} q_i^p &= \sum_j V_{ij} q_j \\ \tilde{q}_i^p &= \sum_j \tilde{U}_{ij} \tilde{q}_j \quad . \end{aligned} \tag{140}$$

These matrices work their way into the various squark-couplings and introduce flavor off-diagonal interactions, which are severely restricted by limits on rare decays.

One of the most restrictive limits on flavor off-diagonal interactions is from the CLEO measurement of the inclusive decay $B \to X_s \gamma$,[55]

$$BR(B \to X_s \gamma) = (2.32 \pm .67) \times 10^{-4}, \qquad \text{CLEO} \tag{141}$$

which is sensitive to loops containing the new particles of a SUSY model. The contribution from $tH^{\pm}$ loops always adds constructively to the Standard Model result. There are additional contributions from squark-chargino loops, squark-neutralino loops, and squark-gluino loops. The contributions from the squark-neutralino and squark-gluino loops are small and are typically neglected. The dominant contribution from the squark-chargino loops is proportional to $A_T \mu$ and thus can have either sign relative to the Standard Model and charged Higgs loop contributions. There will therefore be regions of SUSY parameter space which are excluded depending upon whether there is constructive or destructive interference between the Standard Model/ charged Higgs contributions and the squark-chargino contribution.[56] At small $\tan\beta$, the $b \to s\gamma$ branching ratio is close to the Standard Model value for most of the parameter space. For large $\tan\beta$, the squark-chargino contribution is completely dominant and the limit which can be obtained is very sensitive to the sign$(A_T \mu)$. Neglecting QCD corrections (which are significant) we have,[57]

$$\frac{BR(b \to s\gamma)}{BR(b \to ce\overline{\nu})} \sim \frac{\mid V_{ts}^* V_{tb} \mid^2}{\mid V_{cb} \mid^2} \frac{6\alpha}{\pi} \left\{ C + \frac{M_T^2 A_T \mu}{M_{\tilde{t}}^4} \tan\beta \right\}^2 \quad , \tag{142}$$

where C (positive) is the contribution from the Standard Model and charged Higgs loops and $M_{\tilde{t}}$ is the stop mass. For $A_T \mu$ positive, this leads to a larger branching ratio, $BR(b \to s\gamma)$, than in the Standard Model. Since the Standard Model prediction is already somewhat above the measured value, we require $A_T \mu < 0$ to avoid conflict with the experimental measurement if $M_{\tilde{t}}$ is at the electroweak scale and $\tan\beta$ is large. Detailed plots of the allowed regions for various assumptions about $\tan\beta$, μ, and A_T are given in Refs. 52 and 58.

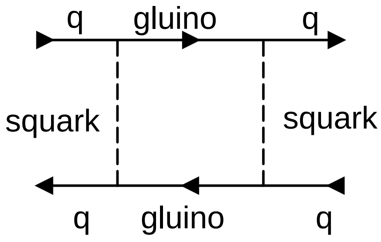

Figure 15: Flavor changing neutral current effects in $K^0 - \overline{K}^0$ mixing from off- diagonal $q\tilde{q}\tilde{g}$ couplings.

Depending on $\tan\beta$ and the sign of $A_T\mu$, this process probes stop masses in the $100 - 300\ GeV$ region.

Another class of important indirect limits on SUSY models comes from flavor changing neutral current (FCNC) processes such as $K^0 - \overline{K}^0$ mixing. Consider the squark-squark-gluino coupling resulting from Eq. 140,

$$\begin{aligned} \mathcal{L} &= g_s\tilde{g}\sum_{i=1}^{3} q_i\tilde{q}_i^* + \text{h.c.} \\ &\sim g_s\tilde{g}\sum_{l,j}\left(\tilde{U}V^\dagger\right)_{lj} q_j^p\tilde{q}_l^{p*} + \text{h.c.} \quad , \end{aligned} \tag{143}$$

where i is a flavor label.

Diagrams such as Fig. 15, can then introduce large contributions. The amplitude from Fig. 15 will be schematically,

$$\mathcal{A} \sim \alpha_s^2\sum_{\alpha,\beta}\left(V\tilde{U}^\dagger\right)_{i,\alpha}\left(V\tilde{U}^\dagger\right)^*_{j,\alpha}\left(V\tilde{U}^\dagger\right)_{i,\beta}\left(V\tilde{U}^\dagger\right)^*_{j,\beta} F(M_{\tilde{\alpha}}^2, M_{\tilde{\beta}}^2), \tag{144}$$

where $F(M_{\tilde{\alpha}}^2, M_{\tilde{\beta}}^2)$ is a complicated function of the squark masses in general. If the squarks are degenerate, however, then F is independent of α and β and the sum of Eq. 144 can be performed since,

$$\sum_\alpha\left(V\tilde{U}^\dagger\right)_{i,\alpha}\left(V\tilde{U}^\dagger\right)^*_{j,\alpha} = 0 \qquad \text{for } i \neq j \tag{145}$$

The contributions from the off-diagonal squark-quark-gluino couplings vanish if the squarks have degenerate masses and so the limits are of the form:

$$\frac{\Delta\tilde{M}^2}{\tilde{M}^2} < \mathcal{O}(10^{-3}) \qquad , \tag{146}$$

where $\Delta\tilde{M}^2$ is the mass-squared splitting between the different squarks and $\tilde{M}$ is the average squark mass. A detailed discussion of FCNCs in SUSY models and references to the literature is given in Refs. 59 and 60. As a practical matter, the assumption is often made that there are 10 degenerate squarks, corresponding to the scalar partners of the u, d, c, s, and b quarks, while the stop squarks are allowed to have different masses from the others. This avoids phenomenological problems with FCNCs. In both GUT models which we have considered, the CMSSM and the GMSB, the squarks are degenerate at the GUT scale and so flavor changing effects are introduced only through renormalization group effects and are therefore small.

6 Experimental Limits and Search Strategies

We turn now to a discussion of some of the existing experimental limits on the various SUSY particles and also to the search strategies applicable at present and future accelerators. We focus primarily on the Higgs sector. Detailed discussions are given in the contributions of Refs. 61 and 70.

6.1 Observing SUSY Higgs Bosons

The goal in the Higgs sector is to observe the 5 physical Higgs particles, $h, H, A, H^\pm$, and to measure as many couplings as possible to verify that the couplings are those of a SUSY model. The lightest neutral Higgs boson in the minimal SUSY model is unique in the SUSY spectrum because there is an upper bound to its mass,

$$M_h < 130\ GeV. \tag{147}$$

All other SUSY particles in the model can be made arbitrarily heavy by adjusting the soft SUSY breaking parameters in the model and so can be just out of reach of today's or tomorrow's accelerators (although if they are heavier than around 1 TeV, much of the motivation for low energy SUSY disappears). The lightest SUSY Higgs boson, however, cannot be much outside the range of LEPII and can almost certainly be observed at the LHC . Hence an extraordinary theoretical effort has gone into the study of the reach of various accelerators in the SUSY Higgs parameter space since in this sector it will be possible to experimentally exclude the MSSM if a light Higgs boson is not observed.

If we find a light neutral Higgs boson, then we want to map out the parameter space to see if we can distinguish it from a Standard Model Higgs boson. The only way to do this is to measure a variety of production and decay modes and attempt to extract the various couplings of the Higgs bosons to fermions and gauge bosons. Since as $M_A \rightarrow \infty$, the h couplings approach those of the Standard Model, there will clearly be a region where the SUSY Higgs boson and the Standard Model Higgs boson are indistinguishable. This is obvious from Figs. 7 and 8.

The search strategies for the SUSY Higgs boson depend sensitively on the Higgs boson branching ratios, which in turn depend on $\tan\beta$. In Figs. 16 and 17, we show the branching ratios for the lightest SUSY Higgs boson, h, into some interesting decay modes assuming that there are no SUSY particles light enough for the h to decay into. (These figures include radiative corrections to the branching ratios, which can be important.[34]) For a Higgs boson below the WW threshold, the decay into $b\overline{b}$ is completely dominant. Unfortunately, there are large QCD backgrounds to this decay mode and so it is often necessary to look at rare decay modes. The branching ratios to $b\overline{b}$, $\tau^+\tau^-$, and $\mu^+\mu^-$ are relatively insensitive to $\tan\beta$, but the WW^*, ZZ^*, and $\gamma\gamma$ rates have strong dependences on $\tan\beta$ as we can see from Figs. 16 and 17.

6.2 Higgs Bosons at LEP and LEPII

Direct limits on SUSY Higgs production have been obtained at LEP and LEPII by searching for the complementary processes,[36]

$$
\begin{aligned}
e^+e^- &\rightarrow Zh \\
e^+e^- &\rightarrow Ah \quad .
\end{aligned}
\tag{148}
$$

From the couplings of Eq. 78, we see that the process $e^+e^- \rightarrow Zh$ is suppressed by $\sin^2(\beta-\alpha)$ relative to the Standard Model Higgs boson production process, while $e^+e^- \rightarrow Ah$ is proportional to $\cos^2(\beta-\alpha)$. The moral is that it is impossible to suppress both processes simultaneously if both the h and the A are kinematically accessible! Because the Higgs sector (at lowest order) can be described by the two parameters, M_h and $\tan\beta$, searches exclude a region in this plane. At LEPII, the cross section for either Zh (small $\tan\beta$) or Ah (large $\tan\beta$) is roughly .5 pb. With a luminosity of $150/pb/yr$, this leads to 75 events/yr before the inclusion of branching ratios. Using preliminary data at $\sqrt{s}=183~GeV$, ALEPH and DELPHI exclude the region at 95% confidence

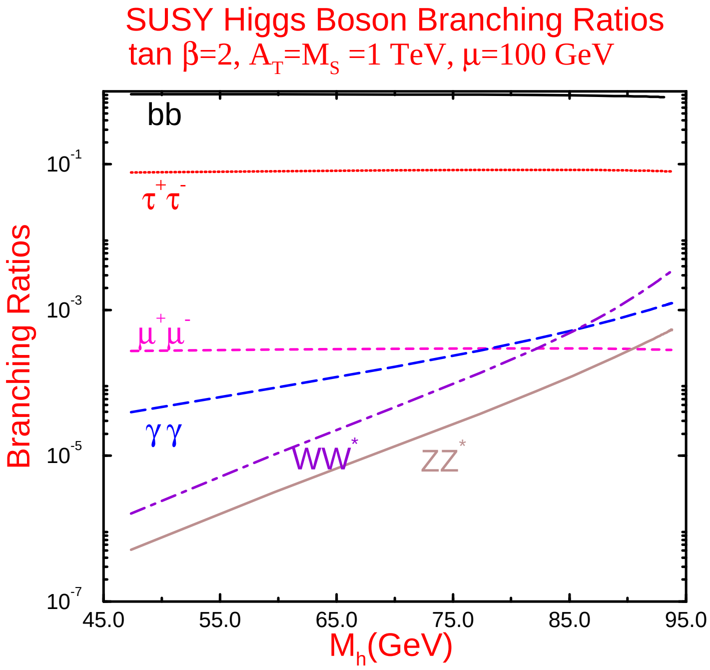

Figure 16: Branching ratios of the lightest Higgs boson assuming decays into other SUSY particles are kinematically forbidden. WW^* and ZZ^* denote decays with one off-shell gauge boson and M_S is a typical squark mass.

level,[62]

$$M_h > 73\ GeV, \text{ for any } \tan\beta \quad . \tag{149}$$

For a given value of $\tan\beta$, there may be a stronger bound. It is important to note that the LEP and LEPII searches do not leave any window for a very light (on the order of a few GeV) Higgs boson. The limit on a SUSY Higgs boson is weaker than the corresponding limit on the Standard Model Higgs boson,[62] $M_h^{SM} > 77.5\ GeV$, due to the suppression in the couplings of the Higgs boson to vector bosons.

The limits on the Higgs boson mass could be substantially altered if there is

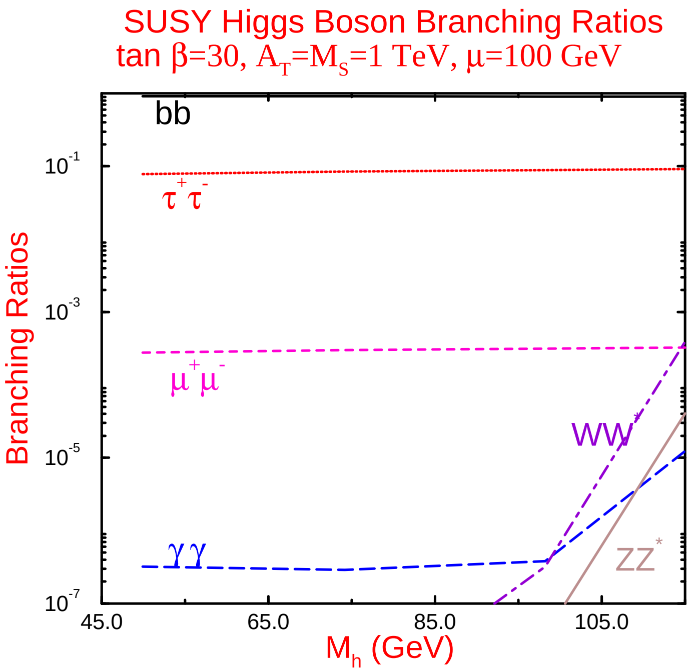

Figure 17: Branching ratios of the lightest Higgs boson assuming decays into other SUSY particles are kinematically forbidden.

a significant branching rate into invisible decay modes, such as the neutralinos,

$$h, A \to \tilde{\chi}_1^0 \tilde{\chi}_1^0 \quad . \tag{150}$$

These branching ratios could be as high as 80%, but are extremely model dependent since they depend sensitively on the parameters of the neutralino mixing matrix. In Fig. 18, we show the branching ratio of the lightest Higgs boson to $\tilde{\chi}_1^0 \tilde{\chi}_1^0$ for several choices of parameters. For $\tan\beta = 2$, with the set of parameters which we have chosen, the branching ratio is always greater than 40%. If the invisible decay modes are significant, a different search strategy for the Higgs boson must be utilized and LEPII can put a limit on the product

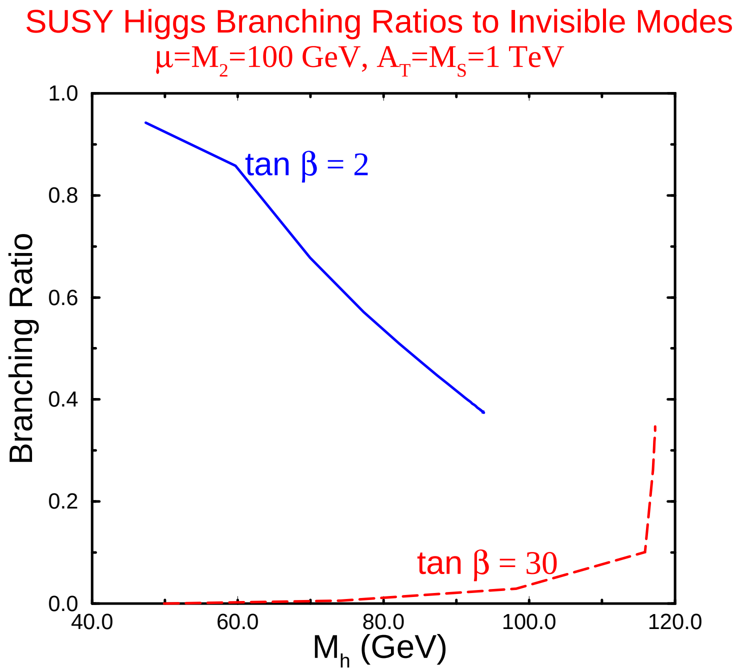

Figure 18: Branching ratio of the lightest Higgs boson to $\tilde{\chi}_1^0\tilde{\chi}_1^0$. The curve with $\tan\beta = 30$ has $M_{\chi_1^0} = 33\ GeV$, while that with $\tan\beta = 2$ has $M_{\chi_1^0} = 7\ GeV$.[34]

of the Higgs boson mixing angles, $\beta - \alpha$, and the branching ratio to invisible modes:

$$
\begin{aligned}
R_1^2 &\equiv \sin^2(\beta-\alpha) BR(h \to \text{visible}) \\
R_2^2 &\equiv \sin^2(\beta-\alpha) BR(h \to \text{invisible}) \quad .
\end{aligned}
\tag{151}
$$

Studies of the expected limits on R_1 and R_2 at various LEPII energies can be found in Ref. 36.

6.3 *Higgs Bosons at $\mu^+\mu^-$ Colliders*

A $\mu^+\mu^-$ collider could in principle obtain stringent bounds on a SUSY Higgs boson through its s-channel couplings to the Higgs.[63] Since these couplings are proportional to the lepton mass, the s-channel Higgs couplings will be much larger at a $\mu^+\mu^-$ collider than at an e^+e^- collider. For large $\tan\beta$, the lighter SUSY Higgs boson could be found in the process $e^+e^- \rightarrow Zh$ at LEPII or at an NLC.[36,64] However, for large $\tan\beta$, the coupling of the heavier Higgs boson to gauge boson pairs is highly suppressed, (see Eq. 78), so the H can not be discovered through $e^+e^- \rightarrow ZH$. Instead the H can be found through $\mu^+\mu^- \rightarrow H \rightarrow b\bar{b}$, which is enhanced by the factor $\tan^2\beta$ relative to $\mu^+\mu^- \rightarrow h_{SM} \rightarrow b\bar{b}$.

A muon collider could also be very useful for obtaining precision measurements of the lighter Higgs boson mass and couplings. The idea is that the h has been discovered through either the process $e^+e^- \rightarrow Zh$ or $\mu^+\mu^- \rightarrow Zh$ and so we have a rough idea of the Higgs boson mass. A muon collider could be tuned to sit right on the resonance, $\mu^+\mu^- \rightarrow h$. By doing an energy scan around the region of the resonance, a precise value of the mass could be obtained due in large part to the narrowness of the muon beam as compared to the beam in an electron collider. (The narrowness of the beam is due to the suppression of synchrotron radiation in a muon collider.)

The narrowness of the $\mu^+\mu^-$ beam is parameterized in terms of the beam energy resolution, R, as

$$\delta_E = (7\ MeV)\left(\frac{R}{.01\%}\right)\left(\frac{\sqrt{s}}{100\ GeV}\right). \tag{152}$$

If we compare the beam energy spread with the width of a SUSY Higgs boson, we see that for $M_h \sim 100\ GeV$ and $\delta_E < .01$ the energy spread is less than the Higgs width, $\delta_E < \Gamma_h$. In this limit the effective cross section is given by,

$$\sigma_{eff} = \pi\frac{\sqrt{2\pi}\Gamma(h \rightarrow \mu^+\mu^-)BR(h \rightarrow X)}{M_h^2\delta_E}, \tag{153}$$

making it clear that the smallest possible R gives the best measurement of the Higgs mass. For $M_h = 100\ GeV$ and $\mathcal{L} = 100 fb^{-1}$, Ref. 63 finds that a 1 σ measurement of 60 MeV will be possible for the Higgs mass. In Fig. 19, we show the cross section for $\mu^+\mu^- \rightarrow h$ for several values of R. This process requires that the beam energy be adjusted to within δ_E of M_h. Both the lighter Higgs boson, h, and the heavier neutral Higgs bosons, H, and A, can be studied at a muon collider through their s-channel production essentially up to the kinematic limit over much of the parameter space.

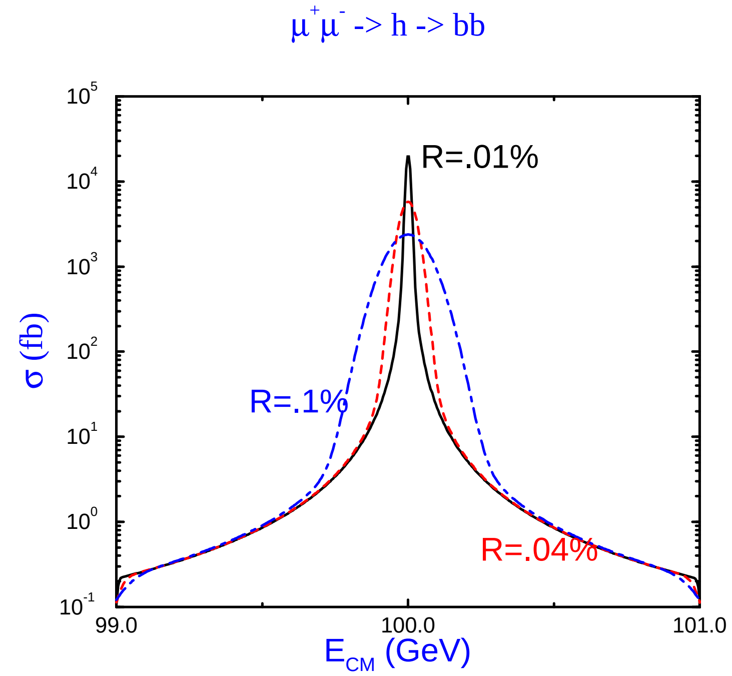

Figure 19: Cross section for $\mu^+\mu^- \to h$ for several values of R. The cross section must be multiplied by the model dependent couplings $(C_{\mu\mu h}C_{bbh})^2$.

6.4 Higgs Bosons at the LHC

At the LHC, for most Higgs masses the dominant production mechanism is gluon fusion, $gg \to h, H$ or A. These processes proceed through triangle diagrams with internal b and t quarks and also through squark loops. In the limit in which the top quark is much heavier than the Higgs boson, the top quark contribution is a constant, while the b quark contribution is suppressed by $(M_b/v)^2 \log(M_h/M_b)$ and so only the top quark contribution is numerically important. For large $\tan\beta$, however, the dominance of the top quark loop is overtaken by the large $\overline{b}bh$ coupling and the bottom quark contribution becomes important, (as seen in Fig. 7). The production rate is therefore extremely sensitive to $\tan\beta$. Both QCD corrections and squark loops can also

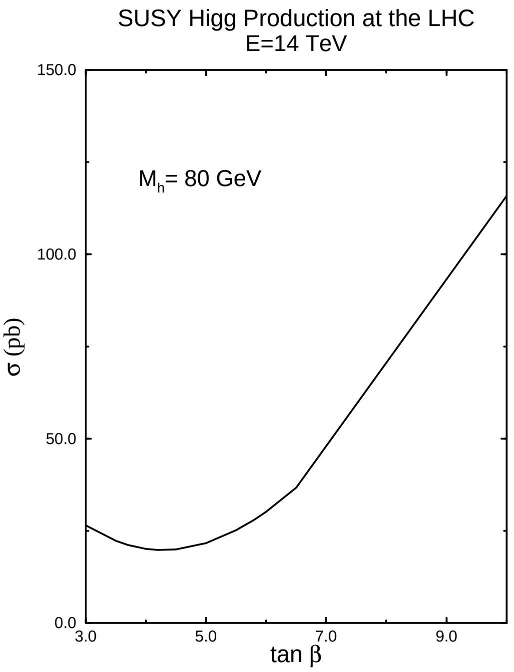

Figure 20: Cross section for production of the lightest SUSY Higgs boson at the LHC as a function of $\tan\beta$.

be numerically important.[65] In fact, the QCD corrections increase the rate by a factor between 1.5 and 2. The rate for $pp \rightarrow h$ at the LHC is shown in Fig. 20 as a function of $\tan\beta$ for $M_h = 80\ GeV$. We see that there are a relatively large number of events. For example, for $M_h \sim 80\ GeV$, the LHC cross section is roughly 100 pb. With a luminosity of $10^{33}/cm^2/sec$, this yields 10^6 events/year.

Unfortunately, there are large backgrounds to the dominant decay modes, ($b\overline{b}, \mu^+\mu^-$, and $\tau^+\tau^-$), for a Higgs boson in the 100 GeV region.[66] The decay $h \rightarrow ZZ^*$ will be useful, but its branching ratio decreases rapidly with decreasing Higgs mass. In order to cover the region around $M_h \sim 80 - 100\ GeV$, it will be necessary to look for the Higgs decay to $\gamma\gamma$,

$$gg \rightarrow h, H \rightarrow \gamma\gamma \quad . \tag{154}$$

(From Figs. 16 and 17, we see that the $BR(h \to \gamma\gamma)$ is typically $< 10^{-3}$–10^{-5}.) This process will be extremely difficult to observe at the LHC due to the small rate and the desire to observe the $h \to \gamma\gamma$ decay has been one of the driving forces behind the design of both LHC detectors.[67] For large M_A, the rate is roughly independent of $\tan\beta$ for $\tan\beta > 3$ and can be used to exclude $M_A > 150\ GeV$ with the full design luminosity of $3 \times 10^5/pb$. (With a smaller luminosity of 3×10^4/pb, the $h \to \gamma\gamma$ process is sensitive to roughly $M_A > 270\ GeV$. See Fig. 21 for the exact region.)

In order to exclude the region with smaller $\tan\beta$, the process $pp \to Wh \to l\nu\bar{b}b$ can be used.[68] This process can exclude a region with $M_A > 100\ GeV$ and $\tan\beta < 4$ (see Fig. 15) and demonstrates the crucial need for b-tagging at the LHC in order to cover all regions of SUSY parameter space. In Fig. 21, we see the excluded region formed by combining potential LHC and LEP limits.[69] A variety of Higgs production and decay channels can be utilized in order to probe the entire $\tan\beta - M_A$ plane. The most striking feature of Fig. 21 is the region around $M_A \sim 100\ GeV$ for $\tan\beta > 5$ where the lightest Higgs boson cannot be observed. In the region with $M_A \sim 100 - 200\ GeV$, both the $h\bar{t}t$ coupling and the $h \to \gamma\gamma$ branching ratios are suppressed relative to the Standard Model rates. Furthermore, the dominant decays, $h \to b\bar{b}$ and $h \to \tau^+\tau^-$, have large backgrounds from Z decays. It will be necessary to look for the decays of the heavier neutral Higgs boson, H, or the pseudoscalar, A, to $\tau^+\tau^-$ pairs in order to probe this region,

$$H, A \to \tau^+\tau^- \to l\nu\bar{q}q \quad . \tag{155}$$

Detector studies by the ATLAS and CMS collaborations suggest that these decay modes may be accessible.

7 Finding the Zoo of SUSY Particles

In addition to the multiple Higgs particles associated with SUSY models, there is a whole zoo of other new particles. There are the squarks and gluinos which are produced through the strong interactions and the sleptons, charginos, and neutralinos which are produced weakly.

We begin by discussing some generic signals for supersymmetry. All SUSY particles in a theory with R parity conservation eventually decay to the LSP, which is typically taken to be the lightest neutralino, $\tilde{\chi}_1^0$, although in GMSB models it is the gravitino.[26] The LSP's interactions with matter are extremely weak and so it escapes detection leading to missing energy.

- A basic SUSY signature is missing energy, E_T^{miss}, from the undetected LSP.

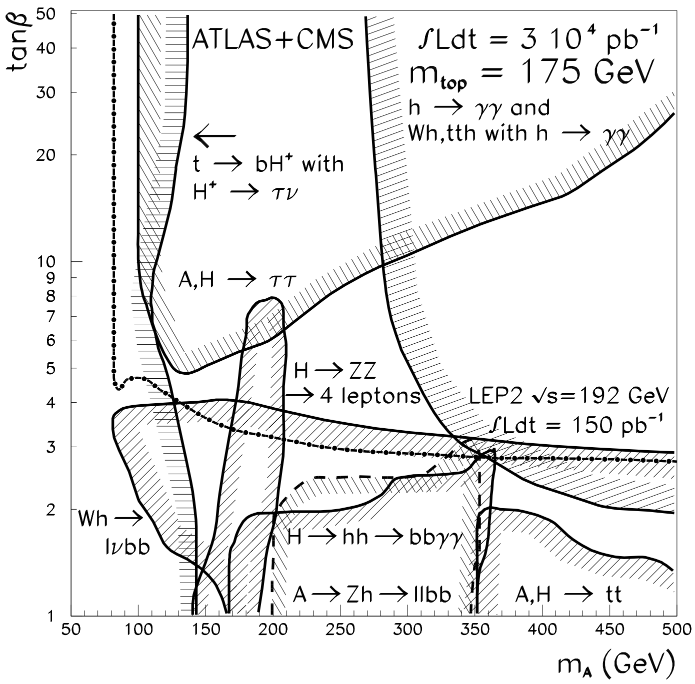

Figure 21: LHC (with low luminosity) and LEPII discovery limits for SUSY Higgs bosons. Figure from Ref. [69].

A SUSY interaction typically produces a cascade of decays, until the final state consists of only the LSP plus jets and leptons. Hence typical final states are:

- $l^{\pm} + \text{jets} + E_T^{miss}$
- $l^{\pm}l^{\pm} + \text{jets} + E_T^{miss}$
- $l^{\pm}l^{\mp} + \text{jets} + E_T^{miss}$.

Because of the presence of the LSP in the final state of an R parity convserving theory, it is not possible to completely reconstruct the masses of the SUSY particles, although a significant amount of information about the masses can be obtained from the event structure.

- A combination of characteristic signatures may determine the SUSY model.

Because the gluinos are Majorana particles, they have some special characteristics which may be useful for their experimental detection. They have the property:

$$\Gamma(\tilde{g} \to l^+ X) = \Gamma(\tilde{g} \to l^- X) \quad . \tag{156}$$

Hence gluino pair production can lead to final states with same sign $l^\pm l^\pm$ pairs.[44] The standard model background for this type of signature is rather small.

- Same sign di-lepton pairs are a useful signature for gluino pair production.

Another generic signature for SUSY particles is tri-lepton production.[70] If we consider the process of chargino-neutralino production,then it is possible to have the process:

$$\tilde{\chi}_1^\pm \tilde{\chi}_2^0 \to l\nu\tilde{\chi}_1^0 + \overline{l}' l' \tilde{\chi}_1^0 \quad . \tag{157}$$

Again this is a signature with a small standard model background.

How these signatures can be observed at the LHC is the subject of F. Paige's lectures at this school.[70]

7.1 Chargino and Neutralino Production

As an example of SUSY particle searches, we consider the search for chargino pair production at an electron-positron collider,

$$e^+ e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^- \quad , \tag{158}$$

(where $\tilde{\chi}_1^\pm$ are the lightest charginos.) The chargino mass matrix has a contribution from both the fermionic partner of the $W^\pm$, $\tilde{\omega}^\pm$, and from the fermionic partner of the charged Higgs, $\tilde{h}^\pm$, and so depends on the two unknown parameters in the mass matrix, μ and M_2. (See Eq. 87). The calculation of the cross section is presented in the Appendix.

The search proceeds by looking for the decay $\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 \overline{f}' f$. The assumption is made that the $\tilde{\chi}_1^0$ is stable and escapes the detector unseen. Using this technique, ALEPH obtains a limit,[71]

$$\begin{aligned} M_{\tilde{\chi}^\pm} &> 85.5\ GeV \quad \text{for } \mu = -500\ GeV, \quad \mid \mu \mid >> M_2 \\ M_{\tilde{\chi}^\pm} &> 85.0\ GeV \quad \text{for } M_2 = 500\ GeV, \quad M_2 >> \mid \mu \mid \end{aligned} \tag{159}$$

based on a total of 21.5 pb^{-1} of data at energies between $\sqrt{s} = 161.3\ GeV$ and 172.3 GeV and assuming $M_{\tilde{\nu}} > 200\ GeV$. This limit is not very sensitive to $\tan\beta$. In the gaugino region, $\mid \mu \mid >> M_2$, there is a strong sensitivity to the mass of the sneutrino as the sneutrino mass is lowered.

It is interesting to compare the search for charginos and neutralinos at LEP with the search at the Tevatron and the LHC. At the LHC one clear signature will be,[72]

$$pp \rightarrow \tilde{\chi}_1^{\pm} \tilde{\chi}_2^0 \tag{160}$$

with,

$$\begin{aligned} \tilde{\chi}_1^{\pm} &\rightarrow l'^{\pm} \nu \tilde{\chi}_1^0 \\ \tilde{\chi}_2^0 &\rightarrow l \bar{l} \tilde{\chi}_1^0 \quad . \end{aligned} \tag{161}$$

The cross section for this process is $\sigma \sim 1 - 100\ pb$ for masses below 1 TeV. This gives a "tri-lepton signature" with three hard, isolated leptons, significant E_T and little jet activity. The dominant Standard Model backgrounds are from $t\bar{t}$ production (which can be eliminated by requiring that the 2 fastest leptons have the same sign) and $W^{\pm}Z$ production (which is eliminated by requiring that $M_{ll} \neq M_Z$).

At the LHC the largest background to chargino and neutralino production is from other SUSY particles, such as squark and gluino production, which also give events with leptons, multi-jets, and missing E_T.

- The biggest background to SUSY is SUSY itself.

Since the squarks and gluinos are strongly interacting, they will generate more jets and a harder missing E_T spectrum than the charginos and neutralinos. This can be used to separate squark and gluino production from the chargino and neutralino production process of interest.

- The tri-lepton signal offers the possibility of untangling the $\tilde{\chi}^+ \tilde{\chi}^0$ signal from the gluino and squark background.

CDF has searched for this decay chain and obtains a limit which is sensitive to $\tan\beta$ and μ. For $\tan\beta = 4$ and $\mu = -200\ GeV$, they obtain a limit $M_{\tilde{\chi}^+} > 70\ GeV$.[73] This is somewhat weaker than the limit found at LEPII.

Aside from observing the process and verifying the existence of charginos and neutralinos, we would also like to obtain a handle on the masses of the SUSY particles. The kinematics are such that,

$$0 < M_{ll} < M_{\tilde{\chi}_2^0} - M_{\tilde{\chi}_1^0} \quad , \tag{162}$$

and hence the distribution $d\sigma/dM_{ll}$ has a sharp cut-off at the kinematic boundary which can be used to obtain information on the masses. Recently, significant progress has been made in our understanding of the capabilities of a hadron collider for extracting values of the SUSY particle masses from different event distributions.[70]

7.2 Squarks, Gluinos, and Sleptons

Squarks and sleptons,$(\tilde{f}_i)$, can be produced at both e^+e^- and hadron colliders. At LEP and LEPII, they would be pair produced via

$$e^+e^- \rightarrow \gamma, Z \rightarrow \tilde{f}_i \tilde{f}_i^* . \tag{163}$$

If there were a scalar with mass less than half the Z mass, it would increase the total width of the Z, Γ_Z. Since Γ_Z agrees quite precisely with the Standard Model prediction, the measurement of the Z lineshape gives

$$M_{\tilde{q}} > 35 - 40 \; GeV \tag{164}$$

for squarks and sleptons.[21] The limit from the Z width is particularly important because it is independent of the squark or slepton decay mode and so applies for any model with low energy supersymmetry. (Remember that the coupling of the sfermions to γ, Z is fixed by gauge invariance.)

There are also limits on the direct production of squarks and gluinos from the Tevatron. The rates for squark and gluino production at both the Tevatron and the LHC are shown in Figs. 17 and 18 and analytic expressions for the Born cross sections can be found in Ref. 75. The QCD radiative corrections to these process are large and increase the cross sections by up to a factor of two.[76] We neglect the mixing effects in the squark mass matrix and assume that there are 10 degenerate squarks associated with the light quarks. (The top squarks are assumed to be different since here mixing effects can be relevant.) The cross sections are significant, around 1 *pb* for squarks and gluinos in the few hundred GeV range at the Tevatron.

The cleanest signatures for squark and gluino production are jets plus missing E_T from the undetected LSP, assumed to be $\tilde{\chi}_1^0$, and jets plus multileptons plus missing E_T.[77] It will clearly be exceedingly difficult to separate the effects of squarks and gluino production, since they both contribute to the same experimental signature. Limits from the Tevatron require (at the 95% confidence level) that [78]

$$\begin{aligned} M_{\tilde{g}} &> 175 \; GeV \\ M_{\tilde{q}} &> 175 \; GeV \quad \text{for } M_{\tilde{g}} < 300 \; GeV \quad . \end{aligned} \tag{165}$$

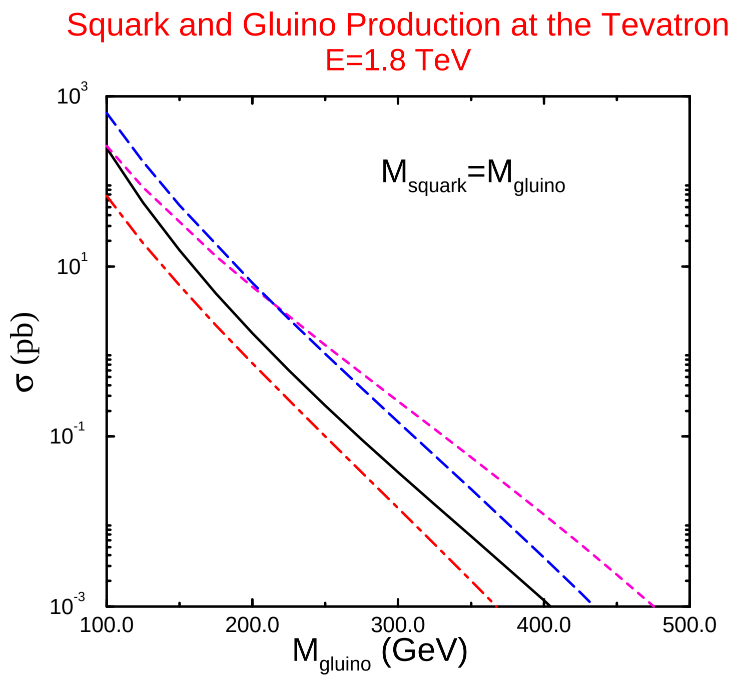

Figure 22: Squark and gluino production at the Tevatron assuming $M_{\tilde{q}} = M_{\tilde{g}}$. The solid line is $p\overline{p} \rightarrow \tilde{g}\tilde{g}$, the dot-dashed $\tilde{q}\tilde{q}$, the dotted $\tilde{q}\tilde{q}^*$, and the dashed is $\tilde{q}\tilde{g}$. This figure includes only the Born result and assumes 10 degenerate squarks.

Details about squark and gluino searches at the Tevatron can be found in the lectures of Lammel at this school, Ref. 61, and about searches at the LHC in Ref. 70.

Limits on the stop squark are particularly interesting since in many models the stop is the lightest squark. There are two types of stop squark decays which are relevant. The first is,

$$\tilde{t} \rightarrow b\tilde{\chi}_1^+ \rightarrow bf\overline{f}'\tilde{\chi}_i^0 \quad . \tag{166}$$

The signal for this decay channel is jets plus missing energy. This signal shares many features with the dominant top quark decay, $t \rightarrow bW^+$. Another possible decay chain for the stop squark is

$$\tilde{t} \rightarrow c\tilde{\chi}_1^0, \tag{167}$$

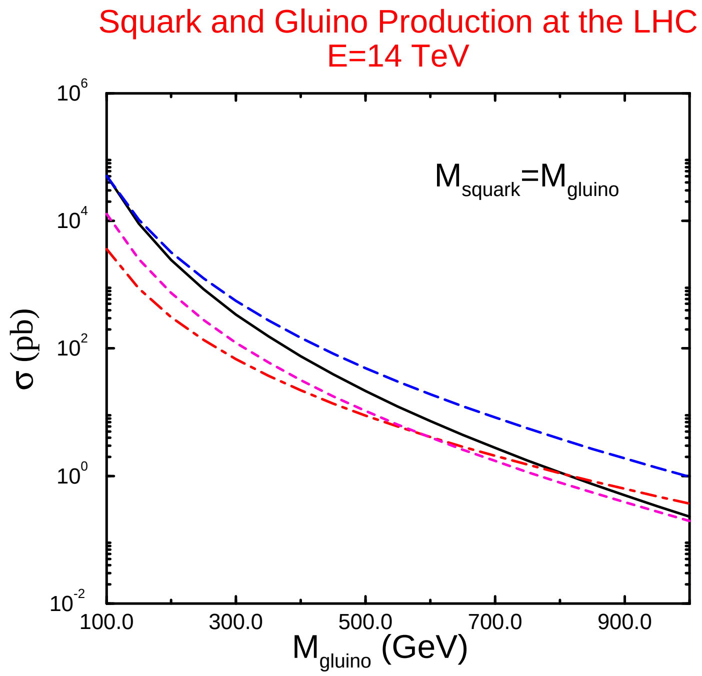

Figure 23: Squark and gluino production at the LHC assuming $M_{\tilde{q}} = M_{\tilde{g}}$. The solid line is $pp \rightarrow \tilde{g}\tilde{g}$, the dot-dashed $\tilde{q}\tilde{q}$, the dotted $\tilde{q}\tilde{q}^*$, and the dashed is $\tilde{q}\tilde{g}$. This figure includes only the Born result and assumes 10 degenerate squarks.

which also leads to jets plus missing energy. The two cases must be analyzed separately. Limits from LEPII require that the lightest stop squark mass be greater than 67 GeV. This limit is independent of the mixing in the stop mass sector, but is sensitive to the lightest neutralino mass.[79] A slightly higher bound is found at the Tevatron,

$$M_{\tilde{t}} > 93 \; GeV, \tag{168}$$

again depending on the mass of the lightest neutralino.[80]

From the examples we have given, it is clear that searching for SUSY at

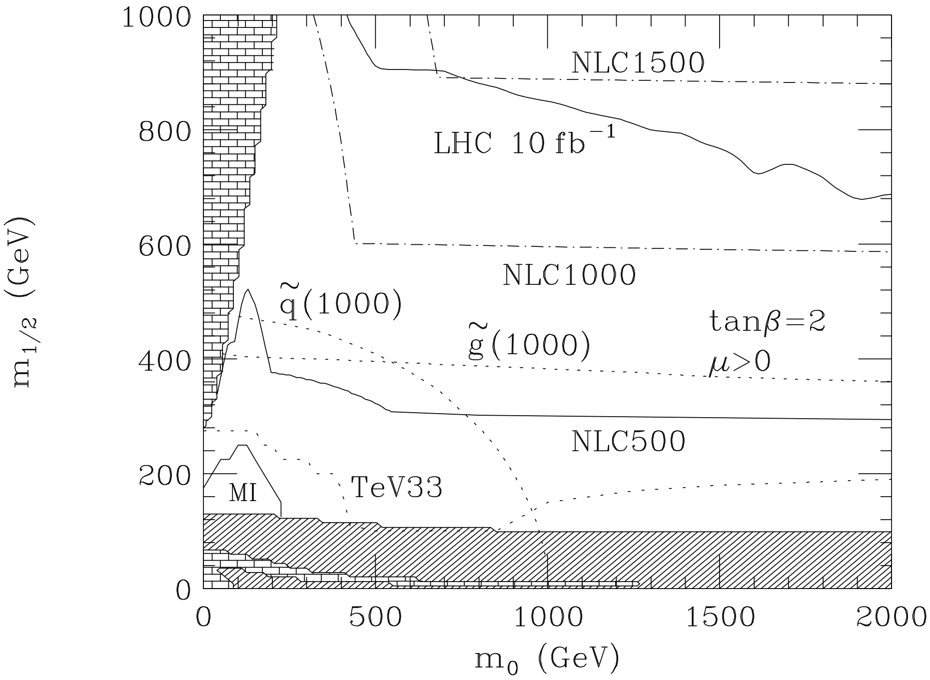

Figure 24: Discovery reach in the CMSSM at various possible future accelerators.

a hadron collider is particularly challenging since there will typically be many SUSY particles which are kinematically accessible. Hadron colliders thus have a large discovery potential, but it is difficult to separate the various processes. To a large extent, one must trust the generic signatures of supersymmetry: $E_T^{\rm miss}$, plus multi-jet and multi-lepton signatures. One will need to observe a signal in many channels in order to verify the consistency of the model.

8 CONCLUSION

A preliminary investigation of the differing capabilities for observing supersymmetry at various colliders was made at the Snowmass 1996 meeting. This study considered the CMSSM and mapped out the region in $m_0 - m_{1/2}$ space which would be accessible at the different machines. For each machine, a number of different discovery channels were considered. By combining the various channels, the curves of Fig. 24, were obtained. The study came to the rough conclusion that a high energy e^+e^- collider with $\sqrt{s} \sim 1.2-1.5\,TeV$ had a similar SUSY discovery reach to that of the LHC with 10 fb^{-1}, as can be seen in Fig. 24.[74] Such a strong conclusion is only possible because the CMSSM relates the masses of the various particles to each other. The Snowmass study only required the discovery of the existence of supersymmetry in a single channel and did not consider how to elucidate the characteristics of a SUSY model.

Weak scale supersymmetry is a theory in need of experimental confirmation. The theoretical framework has evolved to a point where predictions for cross sections, branching ratios, and decay signatures can be reliably made. In many cases, calculations exist beyond the leading order in perturbation theory. However, without experimental observation of a SUSY particle or a precision measurement which disagrees with the Standard Model (which could be explained by SUSY particles in loops) there is no way of choosing between the many possible manifestations of low energy SUSY and thereby fixing the parameters in the soft SUSY breaking Lagrangian. With the coming of LEPII, the Fermilab Main Injector, and the LHC, large regions of SUSY parameter space will be explored and we can only hope that some evidence for supersymmetry will be uncovered.

Appendix: $e^+e^- \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-$

In this appendix we compute the cross section for $e^+e^- \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-$ using the Lagrangian of Eq.94. This result can be found in many standard references [37,38] and our derivation follows that of Ref. 38. There are 3 diagrams which contribute to this process; s-channel γ and Z exchange and $t-$ channel sneutrino exchange. The amplitudes in terms of $4-$ component Dirac spinors are:

$$\begin{aligned}
\mathcal{A}_\gamma &= \frac{e^2}{s}\overline{v}(e^+)\gamma^\mu u(e)\overline{u}(\tilde{\chi}^+)\gamma_\mu u(\tilde{\chi}^-) \\
\mathcal{A}_Z &= \frac{g^2}{2\cos^2\theta_W}\frac{1}{s-M_Z^2}\overline{v}(e^+)\gamma^\mu\left(R_eP_+ + L_eP_-\right) \\
&\quad \cdot u(e)\overline{u}(\tilde{\chi}^+)\gamma_\mu\left(C_{ij}^+P_+ + C_{ij}^-P_-\right)u(\tilde{\chi}^-) \\
\mathcal{A}_{\tilde{\nu}} &= -\frac{g^2}{t-\tilde{m}_\nu^2}V_{i1}V_{j1}^*\overline{u}(\tilde{\chi}^-)P_-u(e^-)\overline{v}(e^+)P_+u(\tilde{\chi}^+)\ , \qquad (169)
\end{aligned}$$

with $P_\pm = \frac{1}{2}(1\pm\gamma_5)$, $R_e = 2\sin^2\theta_W$, $L_e = 2\sin^2\theta_W$, and $C_{ij}^\pm$ defined in Eq. 95. Since the charginos are Majorana particles, there is no distinction between spinor and anti-spinor for them. (In obtaining $\mathcal{A}_Z$ from Eq. 94, we have used

Eq. 33.) We can use the Fiertz rearrangement,

$$(\overline{\psi}_1 P_- \psi_2)(\overline{\psi}_3 P_+ \psi_4) = \frac{1}{2}(\overline{\psi}_3 \gamma^\mu P_- \psi_2)(\overline{\psi}_1 \gamma_\mu P_+ \psi_4) \tag{170}$$

to rewrite the sneutrino contribution so that it has the same form as the γ and Z contributions,

$$\mathcal{A}_{\tilde{\nu}} = \frac{1}{2}\frac{g^2}{(t - \tilde{m}_\nu^2)} V_{i1} V_{j1}^* \overline{v}(e^+)\gamma^\mu P_- u(e^-)\overline{u}(\tilde{\chi}^+)\gamma_\mu P_- u(\tilde{\chi}^-) \quad . \tag{171}$$

In terms of helicity eigenstates the total amplitude is then

$$\mathcal{A}(e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^-) = \frac{e^2}{s} \sum_{m,n=\pm} X_{mn} \overline{v}(e^+)\gamma^\mu P_m u(e^-)\overline{u}(\tilde{\chi}^+)\gamma_\mu P_n u(\tilde{\chi}^-) \tag{172}$$

where

$$\begin{aligned}
X_{++} &= 1 + \frac{L_e C_{ij}^-}{2\sin^2\theta_W \cos^2\theta_W}\frac{1}{1 - M_Z^2/s} + \frac{V_{i1}V_{j1}^*}{2\sin^2\theta_W}\frac{s}{t - \tilde{m}_\nu^2} \\
X_{--} &= 1 + \frac{R_e C_{ij}^+}{2\sin^2\theta_W \cos^2\theta_W}\frac{1}{1 - M_Z^2/s} \\
X_{+-} &= 1 + \frac{R_e C_{ij}^-}{2\sin^2\theta_W \cos^2\theta_W}\frac{1}{1 - M_Z^2/s} \\
X_{-+} &= 1 + \frac{L_e C_{ij}^+}{2\sin^2\theta_W \cos^2\theta_W}\frac{1}{1 - M_Z^2/s} \quad .
\end{aligned} \tag{173}$$

This result makes clear the importance of the relative sign between the sneutrino contribution and the $s-$ channel diagrams. The sneutrino exchange interferes destructively with the γ and Z diagrams for small sneutrino mass. In Fig. 20, we show the cross section for pair production of the lightest chargino. In the Higgsino-like region, $\mid \mu \mid << M_2$, the cross section is relatively insensitive to the sneutrino mass, while in the gaugino-like region, $\mid \mu \mid >> M_2$, there is a strong suppression of the cross section for $\tilde{m}_\nu < 200\ GeV$.

It is straightforward to find the differential cross section,

$$\begin{aligned}
\frac{d\sigma}{d\cos\theta} &= \frac{\alpha^2\beta\pi}{8s}\Big\{(X_{++}^2 + X_{--}^2)(1 - \beta\cos\theta)^2 \\
&\quad + (X_{+-}^2 + X_{-+}^2)(1 + \beta\cos\theta)^2 \\
&\quad + \frac{8M_{\tilde{\chi}^+}^2}{s}(X_{++}X_{-+} + X_{--}X_{+-})\Big\} \quad ,
\end{aligned} \tag{174}$$

where $\beta^2 = 1 - 4M_{\tilde{\chi}^+}^2/s$.

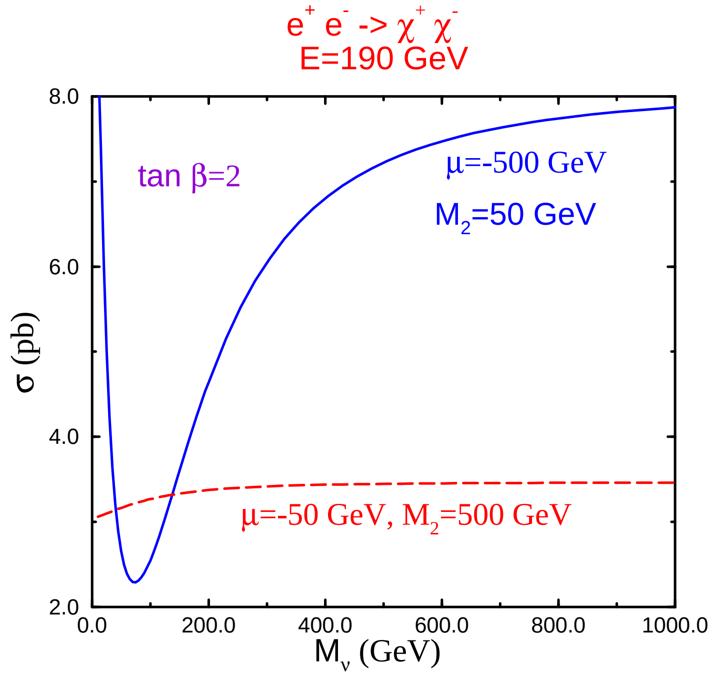

Figure 25: Cross section for pair production of the lightest chargino as a function of the sneutrino mass, $M_{\tilde{\nu}}$.

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