AN INTEGRABLE IF AMPLIFIER OF HIGH STABILITY

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The use of the voltage amplifier or VCVS in simple single loop ctive RC networks has been described in detail by Sallen & Key (Ref 1) Figure 1 shows two variations of the network which allow of ve gain and a negative gain amplifier respectively Figure 2 show e ci as a function of K for these two circuits The poles of the positiv n s reach the jω axis at K = 3 0 and cross the axis at right angles The ve system is absolutely stable and achieves high Q by moving the poles r om the origin as the magnitude of K increases The important characteris of se two networ are shown in Table I The sensitivity of Q to gain c e h and m s it very difficult to use the positive gain realization a a Q The n tive gain realization minimizes this sensitivity ires very n in and requires inordinately large capacitors The high g equired precludes the use of this method at either high Q or high frequency A pair of complex poles can be obtained with a differential voltage amplifier and a single feedback loop by using a feedback network having zeros of transmission (phantom zeros) on the jω axis as shown in Figure 3a (Ref 2) These phantom zeros terminate the complex pole loci at the jω axis and thereby greatly reduce the Q sensitivity to amplifier gain change The zeros prevent the poles from moving into the right half plane and thereby ensure an insensitive absolutely stable system The root loci for this network are shown in Figure 3b The reduction in Q sensitivity is evident since the pole motion is small for a large change in gain if the gain is high as it must be for high Q Actually, the Q sensitivity is both small and approximately constant ($S_K^Q \cong +1$) As can be seen the circuit complexity is increased (six passive elements are required in order to obtain two complex poles) and the amplifier gain required is relatively high (for Q = 50 K = 199), thereby severely limiting the high frequency capability of this circuit In addition the large amount of negative feedback well away from center frequenc can cause difficulty in amplifier stabilization due to incidental phase shifts caused by parasitic elements The configuration shown in Figure 3a is a modified form of the phantom zero network in which one zero is located at the origin and one at infinity so that a true second-order bandpass function is obtained This is done by applying the input signal to the normally grounded section of the twin-T

This work will describe the use of right-half-plane complex-phantom zeros to achieve a significant gain reduction for a given pole Q, thereby increasing the high frequency capability without greatly increasing the Q sensitivity Network realizations will be shown for both lumped and distributed RC networks Design equations are given for the lumped element case and design charts for the distributed case Plots of the gain required for a given Q of the Q sensitivity to gain change and the frequency of maximum response as a function of the real part of the right-half-plane phantom zero position σ complete the design data

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Right Half Plane Phantom Zeros Using Lumped Elements

For the twin-T shown in Figure 4, the transfer function is

$$\frac{E_{out}}{E_{in}} = T(p) = \frac{p^2 + \alpha p + 1}{p^2 + \beta p + 1} \tag{1}$$

where

$$\alpha = b + \frac{b}{k} - 1 \tag{2}$$

$$\beta = b + \frac{b}{k} + \frac{1}{k} + \frac{1}{b} \tag{3}$$

The definition of b and k is indicated in Figure 4. If we now combine this network with a negative gain amplifier as shown in Figure 5, we produce a true band-pass function (one zero at D. C. and one at infinity) as given below:

$$T(p) = -\left(\frac{K}{1 + K}\right) \frac{(\beta - \alpha)p}{p^2 + [(\beta + \alpha K)/(1 + K)]\, p + 1} \tag{4}$$

K will be used to indicate the magnitude of the amplifier gain in all of the following equations. The Q of the poles in equation (4) is

$$Q \cong \frac{1 + K}{\beta + \alpha K} \tag{5}$$

and therefore the gain required for a given Q is

$$K = \frac{\beta Q - 1}{1 - \alpha Q} \tag{6}$$

Note that negative values of α (that is right-half-plane phantom zeros) reduce K, and that small β is also advantageous. The system gain at center frequency, $\omega_o = 1$ rps, is

$$T(p)\,]_{p=j1} = \frac{-K(\beta - \alpha)}{\beta + \alpha K} \tag{7}$$

If we now determine the sensitivity of Q to changes in K we find that

$$S_K^Q \triangleq \frac{\partial Q/Q}{\partial K/K} = \frac{K}{Q}\frac{\partial Q}{\partial K} = \frac{K}{1 + K}\left(\frac{\beta - \alpha}{\beta + \alpha K}\right) \tag{8}$$

Since the root locus is circular, the frequency sensitivity to changes in K is negligibly small at high Q. We are now in a position to select values of α and β

(consistent with b, k) to achieve a particular K to Q relation, and then determine the sensitivity S_K^Q If this is not acceptable, a second iteration is usually all that is necessary to obtain an acceptable set of values. Note that $\alpha = 0$ gives $S_K^Q = \frac{K}{1 + K}$

If we take for example, b = 0.727, k = 10, ($\alpha = -0.2$, $\beta = 2.28$) in order to place the zeros in the right half plane and minimize β, we find from equation (1) that the transfer function of the twin-T is

$$T_1(p) = \frac{p^2 - 0.2p + 1}{p^2 + 2.28p + 1} \tag{9}$$

The complex zeros are in the right half plane at $p = \sigma + j\omega = 0.1 \pm j\,0.995$. Combining this network with a negative gain amplifier as in Figure 5, we obtain the following transfer function

$$T_2(p) = -\left(\frac{K}{1 + K}\right) \frac{2.48p}{p^2 + \dfrac{2.28 - 0.2K}{1 + K}\, p + 1} \tag{10}$$

If we now select Q = 50, we find from equation (6) that K = 10.3 is required, that the system gain at center frequency is [eq (7)], $|T(p)|_{p = j1} = 206$, and that the sensitivity to amplifier gain change is, [eq (8)], $S_K^Q = 10.3$ The reduction in gain and consequent increase in center frequency capability by a factor of 19.3 compared to the symmetrical twin-T (when the real part of the phantom zero position, $\sigma = -\alpha/2 = 0$, K = 199 for the symmetrical twin-T or K = 113 for the b, k of this example when $\alpha = 0$) is accompanied by a greatly increased freedom from parasitic oscillations. The Q sensitivity is increased, of course, but this is controllable by choice of σ (and therefore α), and an appropriate compromise can be made between the gain reduction and the sensitivity increase. The gain reduction illustrated above is about as much as is practical. The Q sensitivity to changes in the passive elements is quite complex, but if we look at the effect of changes in b or k, Figure 4, we find that the resonant frequency is independent of changes in b and k, that they affect α and β as given by equations (2) and (3), and that the Q sensitivity to changes in α, and β are

$$S_\alpha^Q = -\frac{\alpha\beta(1 + K)}{(\beta - \alpha)(\beta + \alpha K)} = -\frac{\alpha\beta Q}{\beta - \alpha} \tag{11}$$

$$S_\beta^Q = \frac{\alpha\beta(1 + K)}{(\beta - \alpha)(\beta + \alpha K)} = \frac{\alpha\beta Q}{\beta - \alpha} \tag{12}$$

Equations (11) and (12) show that the use of negative values of α increase the sensitivity to changes in either α or β and that these sensitivities increase with Q. This indicates that large negative values of α should be avoided, but of course, equation (8) also precludes the use of large negative values of α due to the intolerably large Q sensitivity to amplifier gain change that results. The fact that S_α^Q and S_β^Q have opposite signs can be used to produce some degree of cancellation of the two effects, but, of course, this must be interpreted in terms of expected changes in b, k The normalized resonant frequency [eq (10)] is $\omega_o = 1$ rps Appropriate impedance and frequency scaling can be used as desired The Q and the gain K are unaffected by this scaling A single-tuned bandpass filter amplifier was designed for use at a frequency of 4.78 MHz with a Q of 50. The values used were $\sigma = -\frac{\alpha}{2} = 0.1$, and k = 10. This results in b = 0.727 from equation (2), and $\beta = 2.28$, from equation (3), as in the previous example and therefore that K = 10.3 and $S_K^Q = 10.3$. The circuit is shown in Figure 6 The measured performance of this filter-amplifier is shown in Figure 7. The actual gain required was 12.7 and as shown in Figure 7, the center frequency was 4.55 MHz. Since the discrepancy between computed and experimental gain requirements was significant, tests were made at a lower center frequency (50 kHz) to reduce the effect of stray capacities and amplifier phase shift. Good agreement was found between calculated and measured amplifier gain and center frequency While the 4.5 MHz filter performance as measured shows the indicated deviations in gain and center frequency due to these effects, the difference is not sufficiently great to prevent use of this network at high frequency in most applications

Right Half Plane Phantom Zeros Using Distributed Elements

The distributed RC network suited to the production of right-half-plane complex-zeros (Ref. 3) is shown in Figure 8 The resistance and capacitance are assumed to be uniformly distributed. Computer analysis shows that this network has complex zeros as shown in Figure 9. Values of G_1 for specific complex-zero positions are shown along the root locus. Note that for $G_1 > 17.8$ mhos, the zeros are in the right half plane. As can be seen from the network of Figure 8, the element complexity is considerably reduced. If right half plane zeros are chosen by proper selection of G_1, the gain required is also considerably reduced as compared to $j\omega$ axis, or left half plane zero positions, thereby enhancing the high frequency capability. The value of R_1 (where $R_1 = 1/G_1$) for any given value of the real part of the zero position, σ, is approximately

$$R_1 \cong \frac{1}{17.80 + 5.40\sigma + 0.529\sigma^2} \tag{13}$$

accurate to within ±0.4% from $\sigma = 0$ to $\sigma = 5.0$. This is more than adequate for most purposes since the choice of R_1 is a compromise between the gain required and the resulting Q sensitivity, and is not at all critical. If we add a negative gain amplifier to the network of Figure 8, and apply the input to the normally grounded resistor, R_1, we obtain the circuit shown in Figure 10. We now have an overall transfer function with a zero at D.C. and a zero at infinity. A digital computer program was used to determine the Q, the frequency of maximum response, ω_{max}, and the Q sensitivity, S_K^Q, as functions of the gain, and the real part of the right-half-plane phantom-zero position, σ. Figure 11 shows the Q and the Q sensitivity, S_K^Q, as functions of σ and K. This allows an appropriate compromise between gain and sensitivity to be easily made. After an appropriate choice of σ from Figure 11, R_1 is determined by means of equation (13). The value of $\omega_{max} = \omega_o$ as a function of

K and σ is shown in Figure 12 and the overall system gain at ω_{max} as a function of K and σ is shown in Figure 13 Note that the amplifier gain, K, required to produce a given system gain is reduced as σ is increased (phantom zeros moved progressively further into the right half plane). This quality of the circuit is clearly shown in Figure 13 As a design example, we will assume that a bandpass filter-amplifier having Q = 50 and a Q sensitivity $S_K^Q \cong 10$ is required From Figure 11, we find that K = 42 and $\sigma \cong 0.8$ Solving equation (13), for R_1 gives a value of 0.045Ω. From Figure 12 we find that the frequency of maximum response is 12 rps, and from Figure 13, the system gain at ω_{max} is found to be approximately 51 dB. The resulting network is shown in Figure 14 Although the gain required is higher when the distributed RC line is used as compared to the lumped element of Figure 5 the Q sensitivity to changes in the passive elements is considerably reduced. Since only a single capacitor is used, changes in this capacitor cannot produce any change in the system Q, that is $S_C^Q = 0$. If the two resistors R_o and $1/G_1$ of Figure 8 are constructed in such a manner that changes in their value due to manufacturing tolerances or temperature will occur equally in both, then the Q sensitivity to changes in R_1 and R_o taken together is also <u>zero</u>. We need to be concerned, then, only with the Q sensitivity to active element change which is given in Figure 11 and was used as a design specification in the example.

Conclusions

The use of phantom zeros in the right-half-plane provides an additional parameter which allows a compromise between amplifier gain required and Q-sensitivity-to-amplifier-gain-change for a given Q. The lumped element network used was the twin-T with appropriately modified element values, and the signal input point changed in order to produce a true second order bandpass function Tapering the lumped element twin-T (k > 1) greatly reduced the gain required for a given Q, and the Q-sensitivity-to-gain-change. A set of design charts are provided which allow one to obtain similar performance with a uniformly distributed RC line. The use of a uniform RC line required considerably more amplifier gain for a given Q than the lumped element network. The distributed line could also be tapered, of course, and it would be expected that this would result in a considerable decrease in the gain required for a given Q. Design charts are not yet available for that case The primary advantages of the gain reduction obtained by the use of this synthesis method are the capability of operation at higher frequency and the reduced likelihood of parasitic oscillations.

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References

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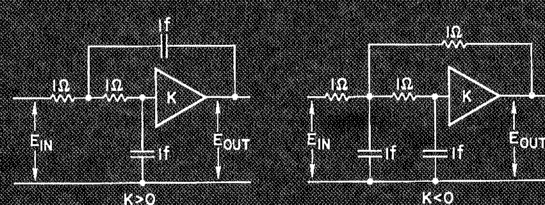
CHARACTERISTICS OF THE ACTIVE RC NETWORKS OF FIGURE 1

	K>0	K<0
POLE SENSITIVITY TO CHANGE IN C (Q>>1)	$-Q-j\,1/2$	$-1/4-j\,1/2$
POLE SENSITIVITY TO CHANGE IN K (Q>>1)	$-KQ+j\frac{K}{4Q}$	$0+j\,1/2$
TOTAL CAPACITANCE FOR ω_0 = 1rps	2f	8Qf
Q	$\frac{1}{3-K}$	$\frac{\sqrt{2-K}}{4}$
AMPLIFIER GAIN, K, FOR Q=5	2.80	-398

Table I

TYPICAL SINGLE LOOP VCVS ACTIVE RC NETWORKS



$$T(P)=\frac{E_{OUT}}{E_{IN}}=\frac{K}{P^2+(3-K)P+1}$$

$$T(P)=\frac{E_{OUT}}{E_{IN}}=\frac{K}{P^2+4P+2-K}$$

Fig. 1

ROOT LOCI OF THE NETWORK OF FIGURE 1

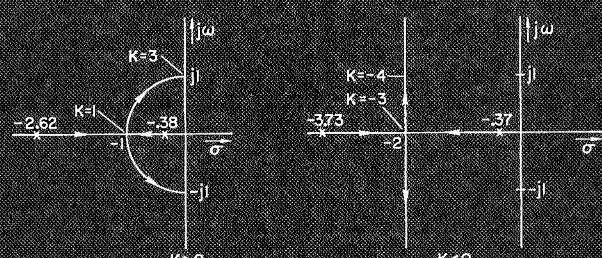


Fig. 2

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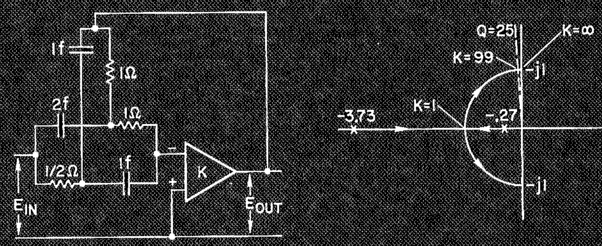
BAND PASS ACTIVE RC NETWORK WITH PHANTOM ZEROS ON THE $j\omega$ AXIS



ROOT LOCI

$$T(P) = -\left(\frac{K}{1+K}\right)\left(\frac{4P}{P^2+\frac{4}{1+K}P+1}\right)$$

Fig. 3

GENERAL TWIN-T

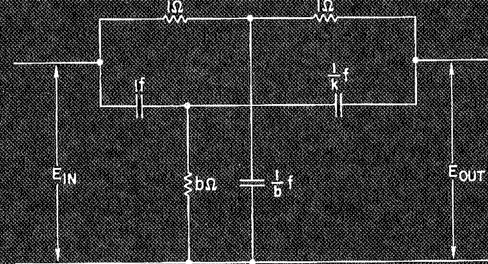
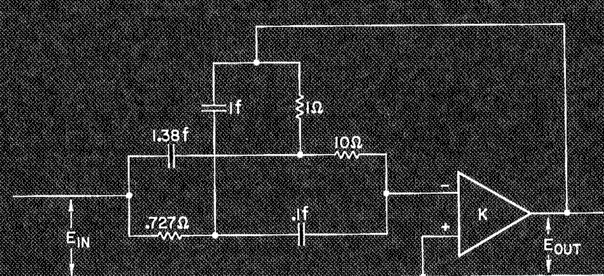


Fig. 4

BAND PASS ACTIVE RC NETWORK WITH PHANTOM ZEROS IN THE RIGHT HALF PLANE



$$T(P) = -\left(\frac{K}{1+K}\right)\left(\frac{2.48P}{P^2+\frac{2.28-.2K}{1+K}P+1}\right)$$

Fig. 5

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SINGLE TUNED BANDPASS FILTER AMPLIFIER

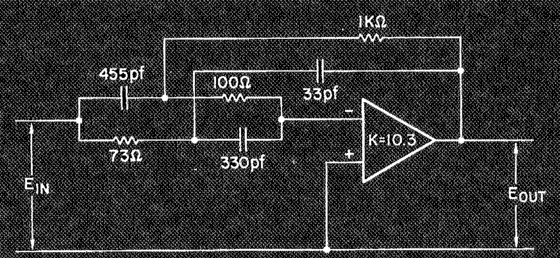

Fig. 6

MEASURED PERFORMANCE OF THE NETWORK OF FIGURE 6

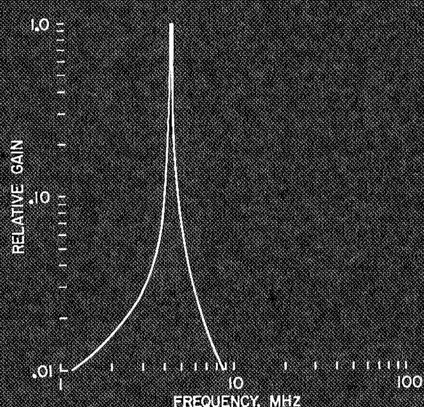

Fig. 7

GENERAL DISTRIBUTED RC NOTCH NETWORK

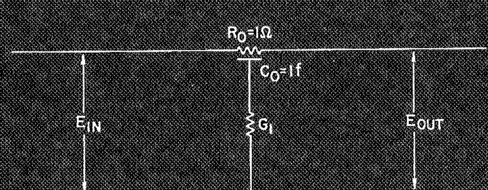

Fig. 8

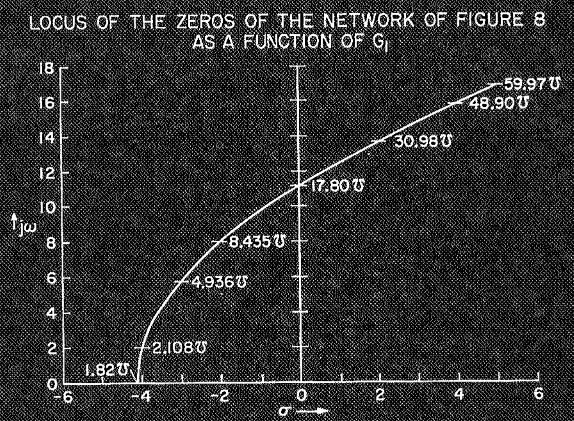

Fig. 9

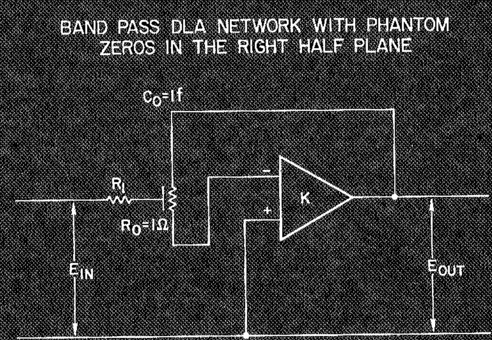

Fig. 10

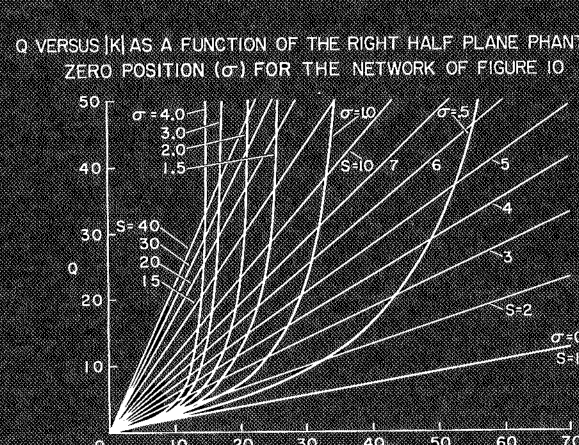

Fig. 11

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FREQUENCY OF MAXIMUM RESPONSE VERSUS AMPLIFIER GAIN AND σ

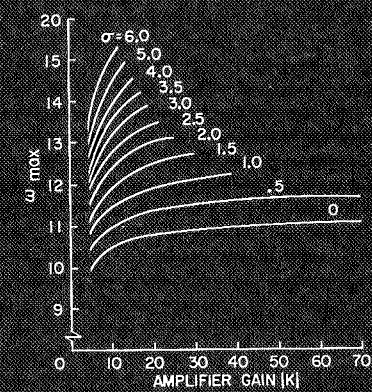

Fig. 12

SYSTEM GAIN VERSUS AMPLIFIER GAIN AND σ

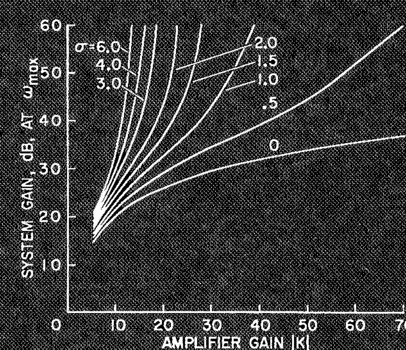

Fig. 13

DLA BAND PASS NETWORK FOR Q=50

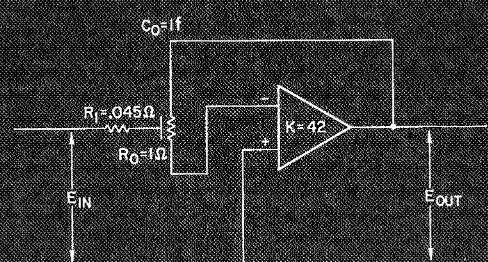

Fig. 14

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