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FUNDAMENTAL STUDY IN LOW-DENSITY GAS DYNAMICS

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ABSTRACT

This report, the second of a series, outlines the progress on a theoretical and experimental investigation of rarefied, internal gas dynamics. This research is being conducted under Research Grant 43-001-023 sponsored by The National Aeronautics and Space Administration.

An experimental facility has been constructed to permit the experimental study of rarefied gas dynamics. This report presents the experimental results which have been obtained for rarefied gas flow through annuli. Data for only one configuration are reported.

A theoretical model has been developed for predicting the flow through annuli ranging from the continuum to the free molecule regimes. Some difficulty has been encountered in the numerical integration scheme on part of the theoretical model but it is anticipated that these difficulties will be resolved shortly.

I. INTRODUCTION

An investigation of rarefied, internal gas dynamics with emphasis on shaft sealing applications was initiated on May 1, 1966, at the University of Tennessee in the Department of Mechanical and Aerospace Engineering. The investigation is being conducted for the National Aeronautics and Space Administration under Research Grant 43-001-023, and this report presents the progress of the investigation for the period November 1, 1966 through May 1, 1967.

A critical review of all available literature led to the identification of a number of basic problem areas requiring study in order to advance rarefied gas dynamics technology to the state required by today's applications (1). Other reports describe the results of a long tube investigation (2) and a nozzle investigation (3) which were completed before the present study was initiated. The first report in this series (4) describes the results of a short tube study.

II. OBJECTIVES

The research effort during this period has followed the program outlined in the original proposal. The objective for this period was to study rarefied gas flow through annuli. A theoretical model was to be developed and an experimental investigation was to be conducted to support and verify the theoretical study.

III. ACTIVITIES

Facility Construction

The experimental apparatus was completed early in this reporting period and a "shake down" test series was conducted. During this test series the experimental apparatus performed essentially as planned. However, some modifications were made in the test equipment, instrumentation, and experimental procedures to improve the precision and efficiency of the experimental effort and to correct some problems which were revealed during the initial test. All shop work has been completed except for maintenance and repair.

Experimental Study

During this reporting period the test facility has been operated for 295 hours with 114 test runs being completed on Test Configuration 1. Argon and Freon -12 were used as the test fluids.

Initially the data reduction and computation was done by hand. During this time the experimental technique and computation procedures were refined and standardized. A computer program was prepared for reduction and computation of subsequent test data and the initial data have been computer processed. A detailed discussion of the experimental procedures and results are presented in Section VI of this report.

Theoretical Study

The continuum equation of motion has been solved with slip boundary conditions and a model has been developed for the additional flow component due to diffusion. Due to the complexity of the self-diffusion model it was necessary to use a numerical integration scheme. Some difficulty has been encountered in the computer program developed to perform this integration and as of this date no reliable results have been obtained from this part of the theoretical study. It is anticipated that the difficulties in the self-diffusion term will be resolved shortly. A detailed discussion of the theoretical study is presented in Section V of this report.

IV. PROPOSED SCHEDULE

During the period May 1, 1967 - November 1, 1967 the following efforts are scheduled:

1. Complete experimental tests on annuli. Complete by June 1, 1967.
2. Complete the development of theoretical model for annuli and correlate with experimental results. Complete by July 1, 1967.
3. Modify test facility to provide the capability of rotating the inner boundary of an annulus space. Provisions will be made to allow a complete study of grooved shaft and/or housing geometries.

4. Start development of a theoretical model to allow the visco-seal concept to be extended to rarefied gas seal applications.

V. THEORETICAL ANALYSIS

Previous investigations (2,3,4,5,6) have indicated that a single equation for gas flow through tubes, applicable to the flow regimes extending from molecular to viscous flow, can be derived by formulating corrections to the continuum equation for viscous flow. It is necessary to account for the slip velocity at the wall and to account for the diffusion which can occur in each cylindrical flow lamella due to a concentration gradient. An attempt to develop such an equation for annulus flow has been made and the development is presented below.

Continuum with Slip

The analysis will be carried out utilizing a flow model consisting of the continuum forms of the equations of momentum and mass conservation together with a slip boundary condition. For fully developed laminar flow, the continuum momentum equation for an annular duct is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{1}{\eta}\frac{dp}{dz} \tag{1}$$

and the slip boundary conditions may be expressed as

$$u_1 = A\lambda \left.\frac{du}{dr}\right|_{r=r_1}$$
$$u_2 = -A\lambda \left.\frac{du}{dr}\right|_{r=r_2} \tag{2}$$

Various investigators have developed expressions for the proportionally constant "A" ranging from 2/3 (7) to $\frac{2-f}{f}$ (8) where f is the fraction of their tangential momentum which molecules give up upon striking a solid boundary. Published values (6,9) for f range from 0.82 for air flowing over shellac (after two days of shellac drying) to 1.02 for CO_2 flowing over scratched brass. A value of f equal to one appears to be fairly realistic for the flow of argon over machined stainless steel. Thus the

value for the constant "A" in Equations 2 was assumed to be unity.

Integration of Equation 1 using the boundary conditions given in Equation 2 results in an expression for the velocity at any value of radius in the annular duct. The flow rate through the duct can be obtained by integrating this velocity over the flow area.

$$\dot{N} = n \int_0^{2\pi} \int_{r_1}^{r_2} u r \, dr \, d\theta$$

Evaluation of this integral results in an expression for the molecular flow rate through the duct.

$$\dot{N} = \frac{\pi r_2^4}{8\eta(kT)l} \bar{p} \Delta p \left[1 - \kappa^4 + 4N_k (1-\kappa^2)(\kappa^2 - \kappa + 1) + \frac{\kappa(1-\kappa^2)^2 (1+2N_k)^2}{\kappa \ell n \kappa - N_k (1-\kappa)} \right] \quad (3)$$

This equation does not agree with a similar expression derived by Sparrow (10) but upon checking this reference it was concluded that Equation 3 is the correct expression. The flow rate predicted by Equation 3 is compared with experimental data in Section VI of this report. If continuum no slip boundary conditions are applied to the integrated form of Equation 1 the following continuum equation results.

$$\dot{N} = \frac{\pi r_2^4}{8\eta(kT)l} \bar{p} \Delta p \left[1 - \kappa^4 + \frac{(1-\kappa^2)^2}{\ell n \kappa} \right] \quad (4)$$

Self-Diffusion

The molecular transfer due to concentration gradients was obtained by evaluating the net number of molecules crossing a plane in the annular region. The evaluation was done by considering separately the molecules crossing the plane which came from the outer wall and from the inner wall.

Flow Coming from Outer Wall

In order to develop the equation take as a physical model the annular region shown in Figure 1. For simplicity the annulus will be

assumed to be infinitely long so that end effects can be neglected. Let dS' be a small region on the surface of the outer flow tube. The number of molecules which leave dS' per unit time in the direction of the element dS is found to be (7)

$$-d^2\dot{N}'_w = n\bar{v}\, dS' \cos\Theta' \frac{d\omega}{4\pi} \tag{5}$$

where $d\omega$ is the elemental solid angle subtended by dS at the center of dS' and the minus sign indicates flow in the negative z-direction. Let r be the length of the line joining the centers of dS and dS', and θ' be the angle between r and the normal to dS'.

Of those molecules leaving dS' in the direction of dS, some will travel the entire distance without a collision. The probability of a molecule traveling a distance r without a collision is $e^{-r/\lambda}$ (7), where λ is the mean free path length. Thus the number of molecules which leave dS' per unit time in the direction of dS and actually arrive there is found to be

$$-d^2\dot{N}_w = n\bar{v}\, dS' \cos\Theta' \frac{d\omega}{4\pi} e^{-r/\lambda} \tag{6}$$

From geometry considerations it can be shown that

$$d\omega = \frac{dS\cos\Theta}{r^2}$$

where θ is the angle between r and the normal to the surface dS. Also it can be shown that

$$d\omega' = \frac{dS\cos\Theta'}{r^2}$$

But $dS'\cos\theta = (sd\phi)rd\theta = (r\sin\theta d\phi)rd\theta = r^2\sin\theta d\phi\, d\theta$

With these considerations Equation 6 becomes

$$-d^2\dot{N}_w = n\bar{v}\,\frac{dS'\cos\Theta'}{r^2}\,\frac{dS\cos\Theta}{4\pi}\,e^{-r/\lambda}$$

or

$$-d^2\dot{N}_w = \frac{n\bar{v}}{4\pi}\sin\Theta\cos\Theta\,e^{-r/\lambda}\,d\phi\,d\Theta \tag{7}$$

Next consider the number of molecules which pass through dS directly after a molecular collision somewhere along r. Let d^3N_g be the number of molecules per unit time from this source of flow through dS. If $d\tau$ is an elemental volume on r, $\frac{nv}{\lambda}\,d\tau$ is the collision frequency in $d\tau$ (7); of this number $\frac{d\omega_2}{4\pi}$ leave $d\tau$ in the direction of dS where $d\omega_2$ is the solid angle subtended by dS at the center of $d\tau$. Of the number leaving $d\tau$ in the direction of dS, $e^{-\rho/\lambda}$ will make it without a collision if ρ is the distance from $d\tau$ to dS.
Thus it is found that

$$d^3\dot{N}_g = -\frac{n\bar{v}}{\lambda}\,d\tau\,\frac{d\omega_2}{4\pi}\,e^{-\rho/\lambda} \tag{8}$$

However, from geometry considerations

$$d\omega_2 = \frac{dS\cos\Theta}{\rho^2}$$

and

$$d\tau = (d\rho)(\rho\,d\Theta)(\rho\sin\Theta\,d\phi) = \rho^2\sin\Theta\,d\rho\,d\phi\,d\Theta$$

so that

$$-d^3\dot{N}_g = \frac{n\bar{v}}{4\pi\lambda}\cos\Theta\sin\Theta\,e^{-\rho/\lambda}\,d\rho\,d\Theta\,d\phi \tag{9}$$

Now at any position, ρ, along r, the molecules passing through dS directly following a molecular collision at ρ can be written

$$-d^2N_g = \frac{\bar{v}}{4\pi\lambda} dS \cos\Theta \sin\Theta \, d\Theta \, d\Phi \int_0^r n e^{-\rho/\lambda} d\rho \qquad (10)$$

At this point something needs to be known about the molecular density, n, in order to perform the indicated integration in Equation 10. As a simple model, which has been shown to work well in self-diffusion through a tube, it will be assumed that

$$n = n_o + z\left(\frac{dn}{dz}\right)$$

where n_o and $\frac{dn}{dz}$ are constant.

Now at any distance ρ along r,

$$z = \rho \cos\Theta$$

and thus

$$n = n_o + \rho\cos\Theta\left(\frac{dn}{dz}\right)$$

With this definition of n it is possible to perform the integration in Equation 10 as follows

$$\int_0^r n e^{-\rho/\lambda} d\rho = \int_0^r \left(n_o + \rho\cos\Theta\frac{dn}{dz}\right) e^{-\rho/\lambda} d\rho$$

$$= n_o \int_0^r e^{-\rho/\lambda} d\rho + \cos\Theta \frac{dn}{dz} \int_0^r \rho e^{-\rho/\lambda} d\rho$$

$$= n_o\left(-\lambda e^{-r/\lambda} + \lambda\right) + \cos\Theta \frac{dn}{dz}\left(-\lambda r e^{-r/\lambda} - \lambda^2 e^{-r/\lambda} + \lambda^2\right)$$

$$= \lambda\left(1 - e^{-r/\lambda}\right)\left(n_o + \lambda\cos\Theta\frac{dn}{dz}\right) - \lambda\cos\Theta\frac{dn}{dz} r e^{-r/\lambda}$$

And now Equation 10 can be written

$$-d^2\dot{N}_g = \frac{\bar{v}}{4\pi} dS \cos\theta \sin\theta \left\{ (1-e^{-r/\lambda})\left(n_o + \lambda \cos\theta \frac{dn}{dz}\right) \right.$$

$$\left. - \cos\theta \frac{dn}{dz} r e^{-r/\lambda} \right\} d\theta\, d\phi \qquad (11)$$

Thus the total number of molecules per unit time passing through dS (from right to left) is given by

$$-d^2\dot{N} = -d^2\dot{N}_w - d^2\dot{N}_g \qquad (12)$$

Where d^2N_w and d^2N_g are given by Equations 7 and 11 respectively. Since Equation 11 will be the same for all elements dS at a radius of r', let $dS = 2\pi r' dr'$, and Equation 12 becomes

$$-d^2\dot{N} = \frac{n\bar{v}}{2} r' dr' \sin\theta \cos\theta\, e^{-r/\lambda}\, d\phi\, d\theta$$

$$+ \frac{\bar{v}}{2} r' dr' \sin\theta \cos\theta \left\{ (1-e^{-r/\lambda})\left(n_o + \lambda \cos\theta \frac{dn}{dz}\right) \right.$$

$$\left. - \cos\theta \frac{dn}{dz} e^{-r/\lambda} \right\} d\theta\, d\phi$$

which simplifies to

$$-d^2\dot{N} = \frac{\bar{v}}{2} r' \sin\theta \cos\theta \left\{ n_o + \lambda \cos\theta \frac{dn}{dz}\left(1-e^{-r/\lambda}\right) \right\} dr'\, d\phi\, d\theta \qquad (13)$$

(Note that r' is a function of r, ϕ and θ)

Now let N be the total molecular flux through dS from right to left from along all possible values of r. This can be expressed as

$$\dot{N} = \int_{-\phi_o}^{\phi_o} \int_{0}^{\pi/2} d^2\dot{N}$$

Utilizing the symmetry of the annular region this can be rewritten

$$\dot{N} = 2\int_0^{\phi_o}\int_0^{\pi/2} d^2\dot{N}$$

or using Equation 13

$$\dot{N} = -\bar{v}\int_0^{\phi_o}\int_0^{\pi/2} r' \sin\theta\cos\theta\left\{n_o + \lambda\cos\theta\frac{dn}{dz}\left(1-e^{-r/\lambda}\right)\right\}dr'd\phi\,d\theta \tag{14}$$

Next the flow through the annular plane can be written as

$$\dot{N}_{PLANE} = -\bar{v}\int_{r_1}^{r_2}\int_0^{\phi_o}\int_0^{\pi/2} r' \sin\theta\cos\theta\left\{n_o + \lambda\cos\theta\frac{dn}{dz}\left(1-e^{-r/\lambda}\right)\right\}dr'd\phi\,d\theta \tag{15}$$

(flow from right to left)

In order to consider the flux of molecules from left to right through the annular plane it is only necessary to redefine the molecular density as

$$n = n_o - z\frac{dn}{dz}$$

and to realize that now the flow is in the positive z-direction. With these two changes Equation 15 can be modified.

$$\dot{N}_{PLANE} = -\bar{v}\int_{r_1}^{r_2}\int_0^{\phi_o}\int_0^{\pi/2} r' \sin\theta\cos\theta\left\{n_o - \lambda\cos\theta\frac{dn}{dz}\left(1-e^{-r/\lambda}\right)\right\}dr'd\phi\,d\theta \tag{16}$$

(flow from left to right)

So that the net flow of molecules through the annular plane from the outer wall is obtained by adding Equations 15 and 16 so as to obtain

$$\dot{N}_{Net} = -\bar{v}\int_{r_1}^{r_2}\int_{0}^{\phi_0}\int_{0}^{\pi/2} r' \sin\theta\cos\theta\left(2\lambda\cos\theta\frac{dn}{dz}\right)\left(1-e^{-r/\lambda}\right)dr'd\phi d\theta$$

This simplifies to

$$\dot{N}_{Net} = -2\lambda\bar{v}\frac{dn}{dz}\int_{r_1}^{r_2}\int_{0}^{\phi_0}\int_{0}^{\pi/2} r' \sin\theta\cos^2\theta\left(1-e^{-r/\lambda}\right)dr'd\phi d\theta \qquad (17)$$

Before integration it is necessary to express r in terms of r', ϕ and θ. This can be found by geometry and the Pythagorean Theorem to be

$$r = \frac{-r'\cos\phi + \sqrt{r_1^2\cos^2\phi + (r_2^2 - r'^2)}}{\sin\theta} \qquad (18)$$

Flow Coming From Inner Wall

If the variables in Equation 17 are modified slightly to fit the model shown in Figure 2, the flow coming from the inner wall can be expressed as

$$\dot{N}_{Net} = -2\lambda\bar{v}\frac{dn}{dz}\int_{r_1}^{r_2}\int_{\phi_0}^{\pi}\int_{0}^{\pi/2} r' \sin\theta\cos^2\theta\left(1-e^{-r/\lambda}\right)dr'd\phi d\theta \qquad (19)$$

Again it is necessary to find r in terms of θ', ϕ and r' before integration of the equation. This time it is found that

$$r = \frac{-r'\cos\phi - \sqrt{r'^2\cos^2\phi - (r'^2 - r_1^2)}}{\sin\theta'} \qquad (20)$$

Solution To Governing Equations:

Equations 17 and 19 were solved separately by numerical integration on a digital computer after non-dimensionalization. This was accomplished by the following definitions;

$$R = \frac{r}{r_2 - r_1} \tag{21}$$

$$R' = \frac{r'}{r_2 - r_1} \tag{22}$$

$$N_k = \frac{\lambda}{r_2 - r_1} \tag{23}$$

With these definitions Equation 17 becomes

$$\dot{N}_{\substack{\text{Net} \\ \text{Outer} \\ \text{Wall}}} = -2\bar{v}\lambda \frac{dn}{dz}(r_2 - r_1)^2 \int_{R_1}^{R_2} \int_0^{\phi_0} \int_0^{\pi/2} R' \sin\Theta \cos^2\Theta \left(1 - e^{-R/N_k}\right) dR'\, d\phi\, d\Theta \tag{24}$$

and Equation 19 becomes

$$\dot{N}_{\substack{\text{Net} \\ \text{Inner} \\ \text{Wall}}} = -2\bar{v}\lambda \frac{dn}{dz}(r_2 - r_1)^2 \int_{R_1}^{R_2} \int_{\phi_0}^{\pi} \int_0^{\pi/2} R' \sin\Theta' \cos^2\Theta' \left(1 - e^{-R/N_k}\right) dR'\, d\phi\, d\Theta' \tag{25}$$

To date the numerical procedure being used to solve Equation 24 and 25 has not resulted in reliable results. It is anticipated that the difficulties will be resolved shortly. When these difficulties are corrected the flow rate predicted for the continuum-slip boundary condition (Equation 3) will be combined with that predicted for the self-diffusion to yield a single equation for use with annular ducts in the regimes from

continuum viscous flow to free molecular flow.

VI. EXPERIMENTAL STUDY

The experimental portion of the annuli study has proceeded to the point of obtaining experimental data to support the theoretical analysis. The investigation has included a flow study of argon in the lower continuum, slip and free molecular flow regimes for a single test configuration. A limited amount of data for Freon-12 were obtained in the continuum and slip flow regimes.

Experimental Procedure

The molecular flow rate through the annular test section was determined using a quasi-steady flow technique. A rigid tank upstream of the test section served as a stagnation reservoir. The volume of this tank was carefully established by measurement. The ideal gas equation of state was used to determine the number of molecules which left the stagnation tank during a measured time interval. This was accomplished by applying the ideal gas equation of state before and after the time interval of flow. Downstream of the test section another rigid tank served as a downstream sink for the flow. This downstream tank was connected through a large diameter, quick acting, flow control valve to a pumping system capable of maintaining the desired downstream pressures and flow rates. Figure 3 is a schematic diagram of the experimental facility.

The time increment for each data point was chosen such that the pressure in the stagnation reservoir decreased less than 20% during the measured time interval. Thus the average stagnation pressure $\overline{P}_T$ was approximately constant for any one time interval. All pressures were measured using McLeod gauges. Temperature instrumentation inside the stagnation tank during previous investigations (3) revealed that the expansion of the gas remaining in the tank was essentially isothermal during the flow intervals. Thus the molecular flow rate through the test section could be determined as

$$\dot{N} = -\left(\frac{V}{kT}\right)\dot{P}_T \tag{26}$$

By using Equation 26, it was possible to obtain molecular flow rates for a known pressure potential across the test section. By varying the stagnation pressure P_T and the downstream pressure P_b, it was possible to obtain data for different pressure ratios across the test section and various degrees of rarefication. In the analysis of the experimental data, suitable corrections were applied to Equation 26 to account for very small atmospheric leakage into the stagnation tank and for the molecular flow into the downstream tank following closure of the flow control valve.

The quasi-steady flow test technique was previously verified using a long tube as a test section (2). Weber (5), and others have shown very good agreement between experimental data and theoretically predicted values for the long tube. The experimental results (2) for flow through a tube with a radius of 0.342 cm and a length-to-radius ratio of 336 agreed very well with the theoretical values. This gave a high confidence level in the experimental procedure used to study the annulus.

Flow Configurations

All experimental data obtained during this reporting period were for a single geometry. Configuration 1 is an annulus having a length of 26.70 cm. The inner radius is 0.509 cm with an outer radius of 0.611 cm. This results in a radial clearance of 0.102 cm.

The manufacturing of a second annulus, Configuration 2, has been completed. This geometry will be obtained by replacing the Configuration 1 shaft with one having a radius of 0.412 cm. This will increase the radial clearance to 0.199 cm. As yet, no experimental tests have been made with Configuration 2. A schematic of the test configuration is shown in Figure 4.

Results

The experimental data obtained for all tests were reduced to a form giving the molecular flow rate per unit pressure potential across the test section. These specific molecular flow rate data were then correlated versus the degree of rarefication of the flow. Knudsen number which is the ratio mean free path of the gas to a characteristic system dimension was chosen as the index of rarefication. The radial clearance was chosen as the annulus characteristic dimension. Thus using the Chapman relationship for the mean free path, the Knudsen number was determined as

$$N_k = \frac{\lambda}{r_2 - r_1} = \frac{\eta\left(2\pi \frac{R_0}{M} T\right)}{2\bar{P}\,(r_2 - r_1)} \tag{27}$$

The mean free path was based on the average test section pressure

$$\bar{P} = \frac{\bar{R}_T + P_b}{2} \tag{28}$$

The flow in the annulus was treated as isothermal with the stagnation reservoir temperature being used in the mean free path calculation and to establish the viscosity value.

Figure 5 presents specific molecular flow rate data for argon versus inverse Knudsen number over a range of Knudsen numbers from 0.0023 to 42. This covers the lower continuum to free molecular flow regimes. The experimental data approaches the theoretical continuum (Equation 4) value at a Knudsen number of approximately 0.0025. This point where the character of the flow ceases to behave as a continuum compares to a previously reported (10) theoretical value for Knudsen number of 0.003.

The experimental data for argon indicates a rather flat minimum value of specific flow rate at a Knudsen number of approximately 1.7. The location of this $N/\Delta p$ minimum for the Configuration 1 annulus compares with a minimum located at a Knudsen number of 2.8 for a

long tube (2).

The influence of pressure ratio across the test section on the molecular flow rate was investigated using argon. Two sets of data corresponding to approximate pressure ratios of 0.06 and 0.34 respectively are shown in Figure 5. These data indicate that the specific flow rate is not greatly influenced by pressure ratio and that the molecular flow varies directly in proportion to the magnitude of the pressure potential across the test section.

A limited amount of annulus flow data for a gas with a molecular weight greater than argon was obtained using Freon-12. These data were taken to investigate a range of molecular weights and are presented in Figure 6. A reaction between Freon-12 and the sealing oil of the mechanical vacuum pump prevented the acquiring of data for Knudsen numbers greater than 0.14.

The theoretical continuum flow (Equation 4) and the theoretical continuum plus slip (Equation 3) are shown in Figures 5 and 6 for argon and Freon-12 respectively. These curves are based on a temperature of 294°K which is in very close agreement with all experimental data. Table I presents the ratio of the theoretical continuum plus slip flow (Equation 3) to the experimental data values as a function of Knudsen number. The range of Knudsen numbers over which data for the two gases were obtained is limited. However, the preliminary conclusion based on these data is that the present theoretical equation (Equation 3) does properly account for the molecular weight of the gas to the same degree for light or heavy gases. Present plans include experimental tests using xenon to more thoroughly investigate the flow of a heavy molecular weight gas. The flow of argon in the Configuration 2 annulus will also be evaluated.

TABLE I - RATIO OF SPECIFIC MOLECULAR FLOW RATES, THEORETICAL EQUATION 3/EXPERIMENTAL DATA

N_k	$1/N_k$	ARGON	F-12
0.0025	400	1.0	1.0
0.0100	100	0.85	0.81
0.025	40	0.75	0.75
0.100	10	0.71	0.71
0.25	4	0.70	----
1.0	1	0.66	----
2.5	0.4	0.60	----
10.0	0.10	0.54	----
25.0	0.04	0.49	----

NOMENCLATURE

h	radial clearance
k	Boltzmann constant
κ	ratio of annulus radii (r_1/r_2)
l	test section length
M	molecular weight
n	molecular density
$\dot{N}$	molecular flow rate
N_k	Knudsen number
$\dot{P}_T$	change in stagnation pressure per unit time
$\bar{P}$	mean test section pressure
P_b	downstream or back pressure
P_T	upstream or stagnation pressure
$\bar{P}_T$	mean upstream pressure
ΔP	pressure drop across test section
r	coordinate distance
r_1	inner radius of annulus
r_2	outer radius of annulus
R_o	universal gas constant
s	distance shown in Figure 1
T	absolute temperature
u	axial continuum velocity component
$\bar{v}$	mean molecular speed
V	volume of stagnation tank
z	axial coordinate
λ	mean free path of gas molecule
η	absolute viscosity

μHg	micron of mercury pressure
$\omega_2, \omega, \omega'$	solid angles
θ, θ', ϕ	angles defined in Figures 1 and 2
ρ	distance defined in Figure 1
τ	volume

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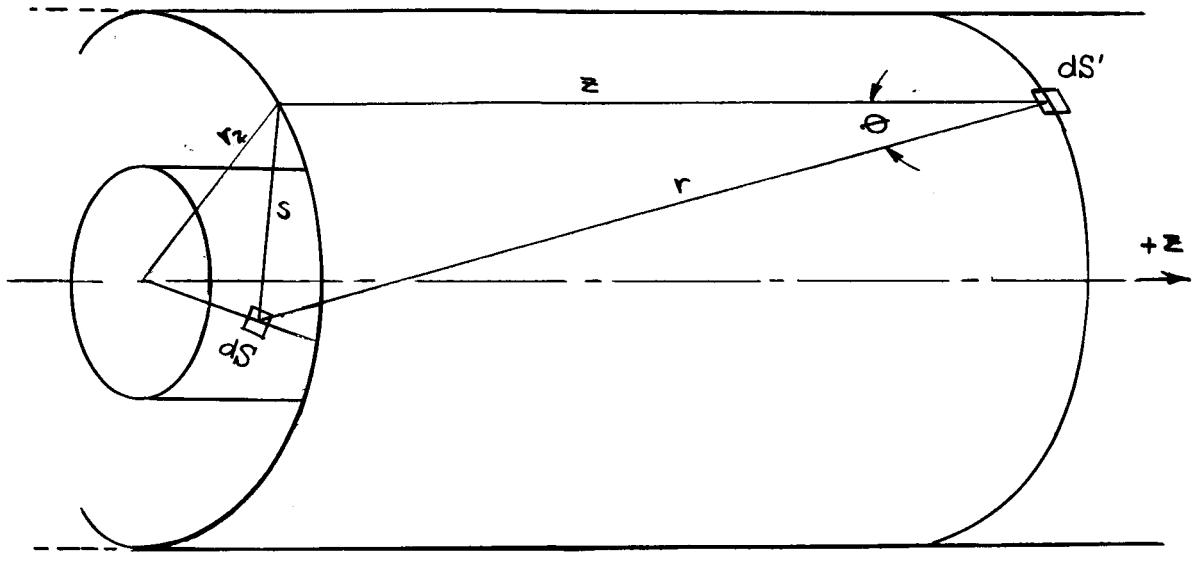
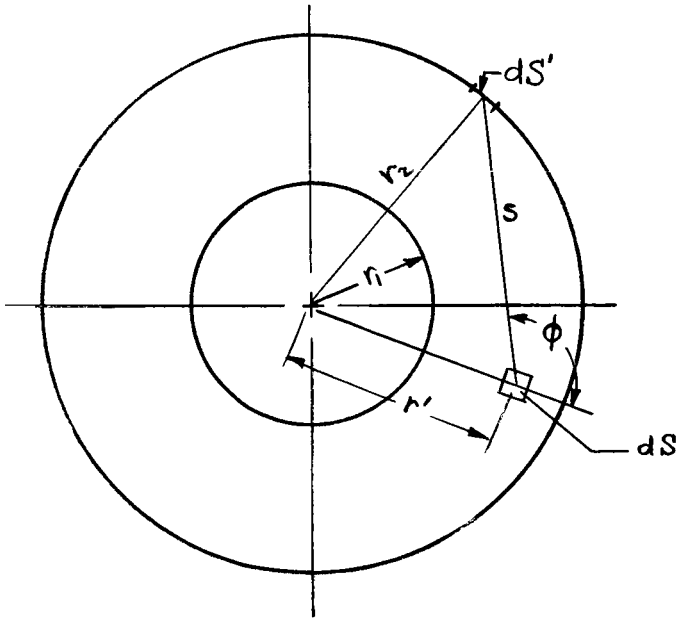
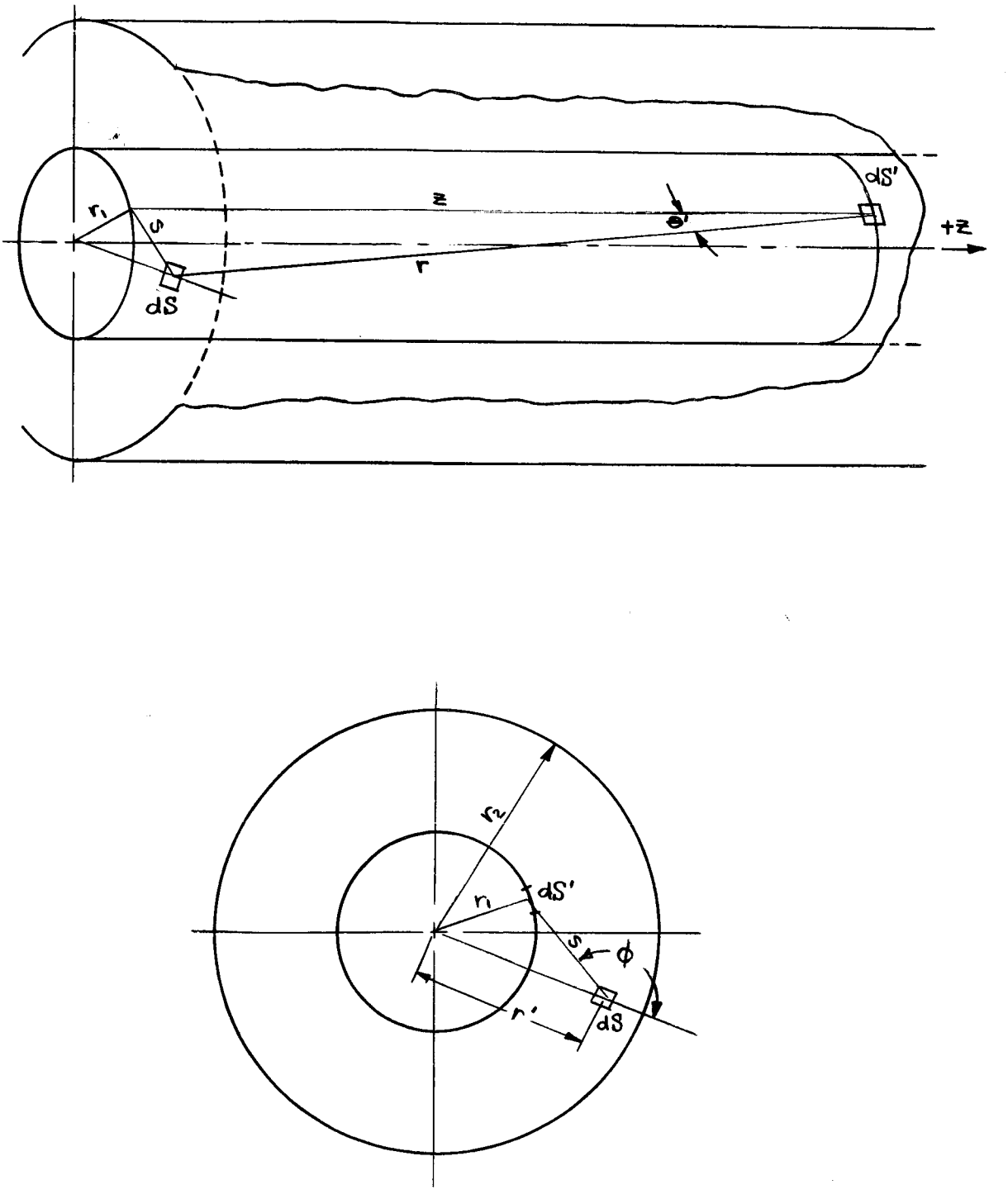




End View of Typical Section

Figure 1. Model for Flow Coming from Outer Wall



End View of a Typical Section

Figure 2. Model for Flow Coming from Inner Wall

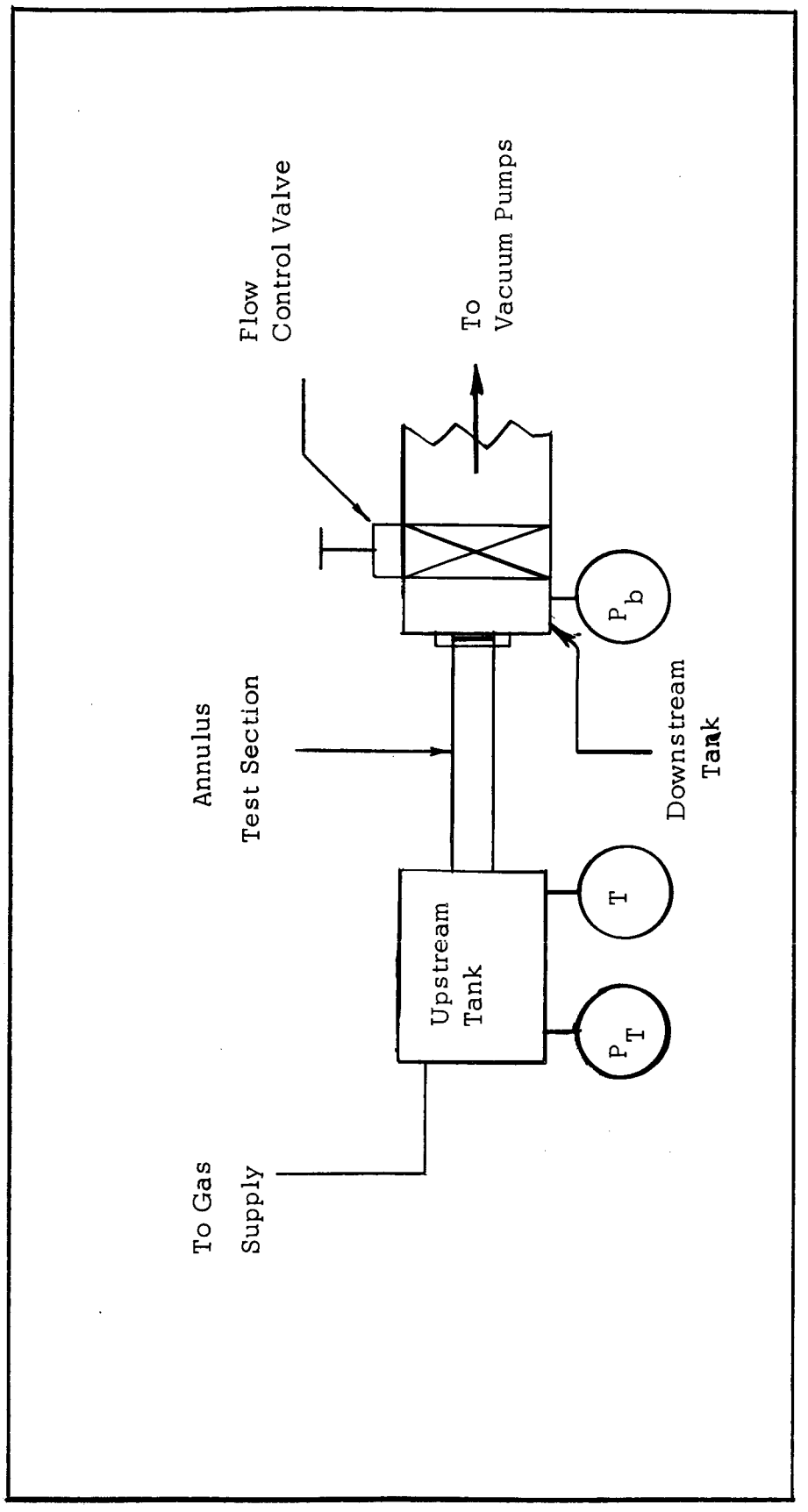


Figure 3 Low-Density Research System Schematic

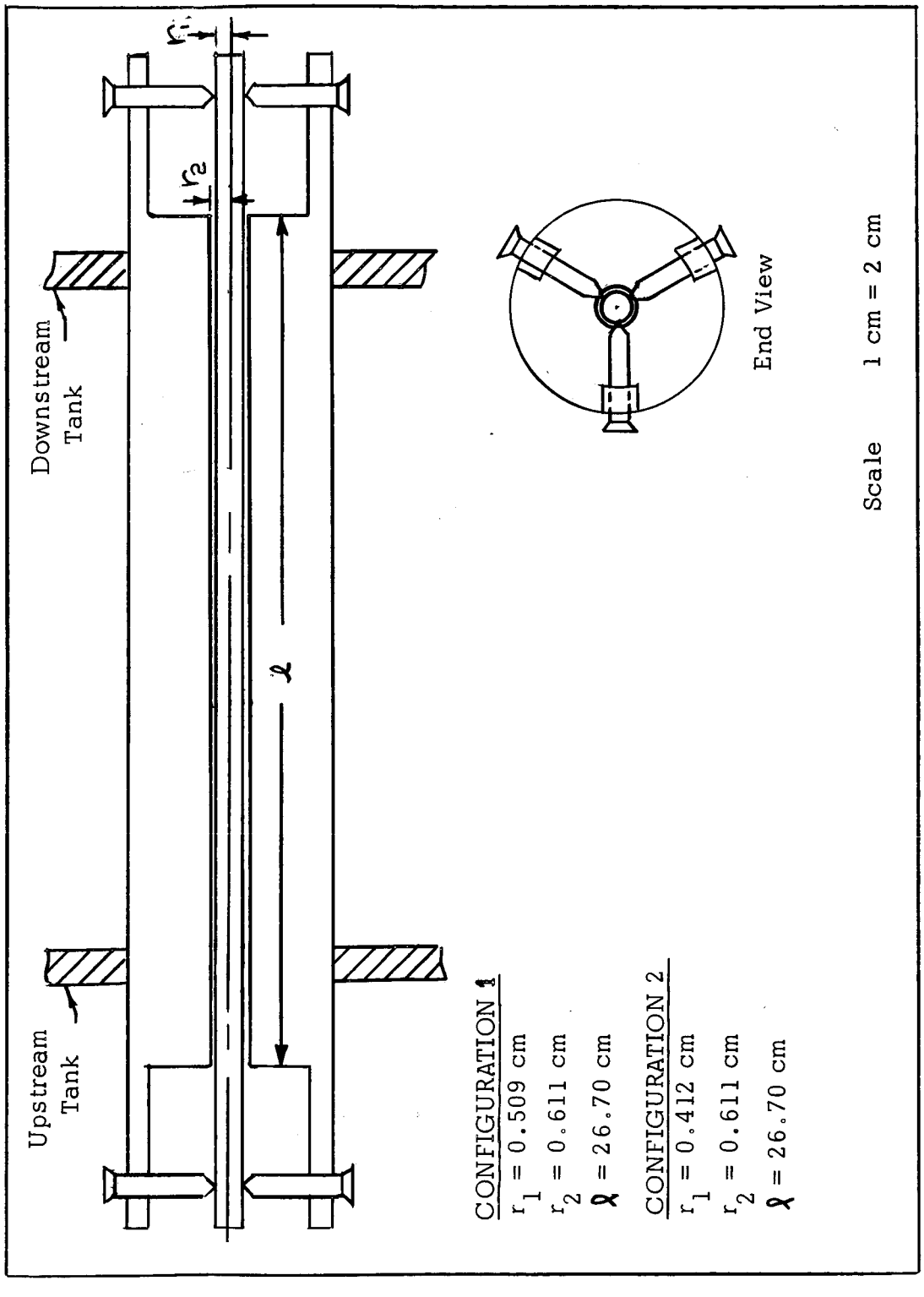


Figure 4 Annulus Test Section Schematic

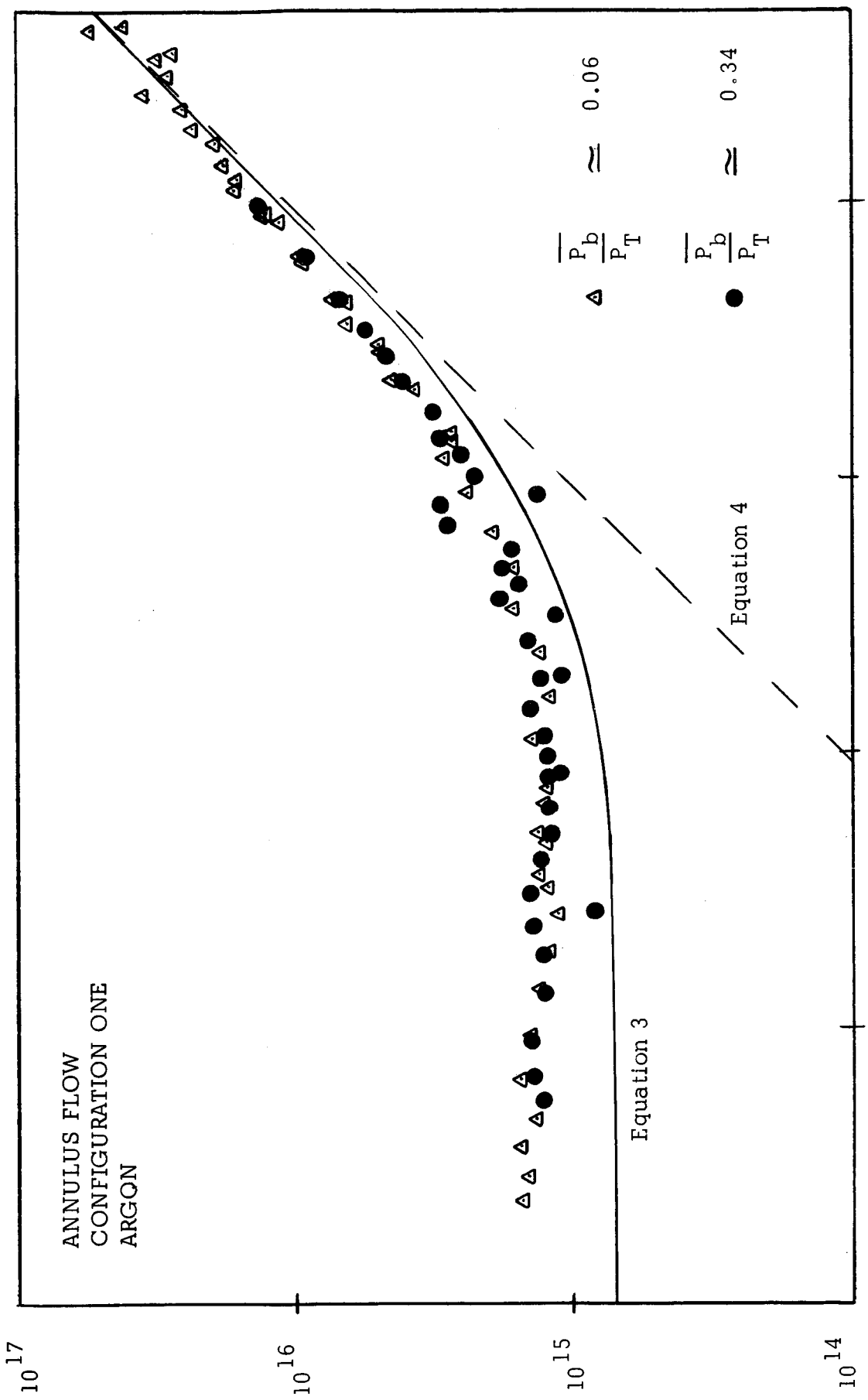


Figure 5 Argon Specific Flow Rate Versus Inverse Knudsen Number

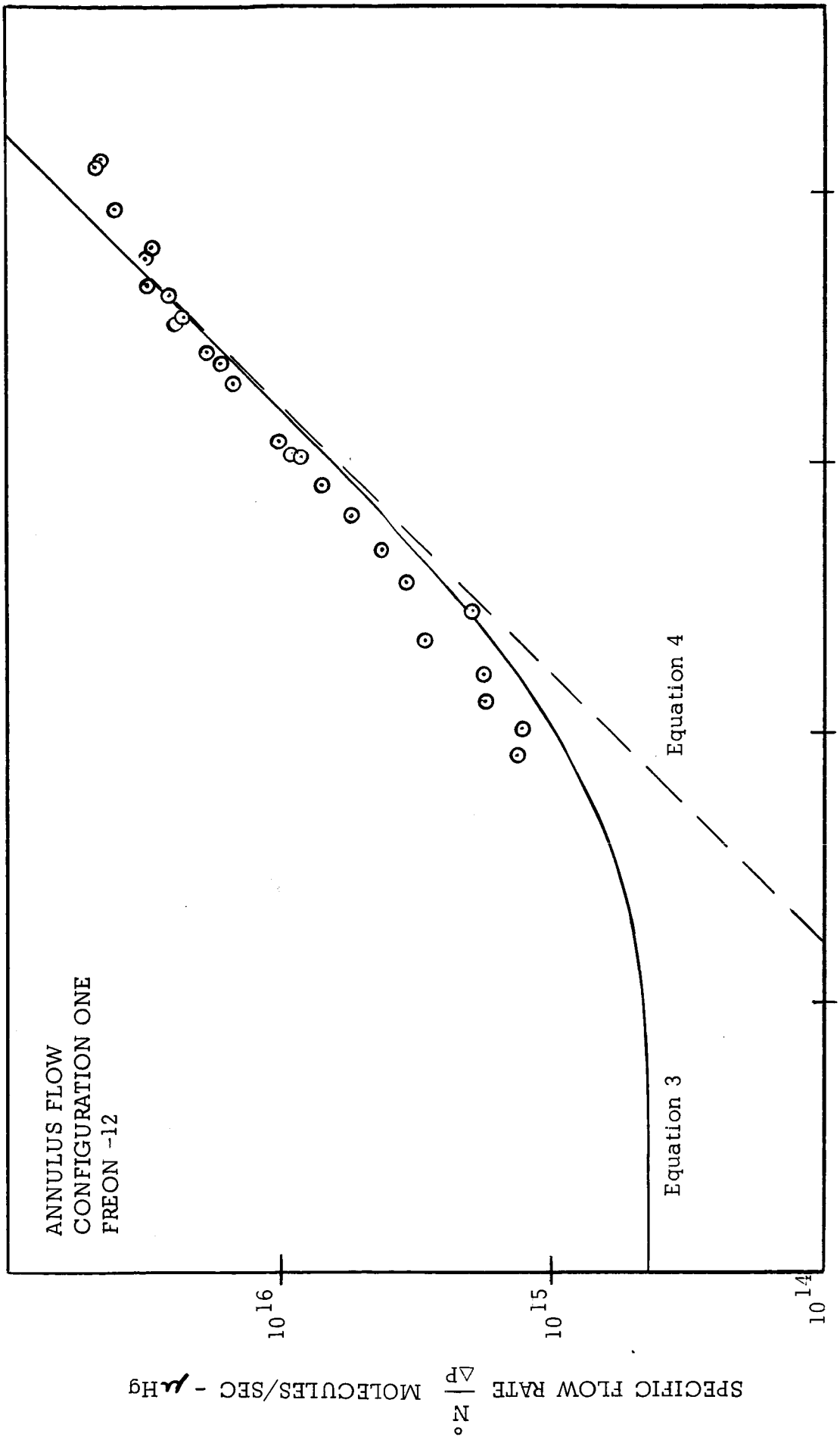


Figure 6 Freon-12 Specific Flow Rate Versus Inverse Knudsen Number