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DEPARTMENT OF MECHANICAL
AND AEROSPACE ENGINEERING

FUNDAMENTAL STUDY IN LOW-DENSITY GAS DYNAMICS

PROGRESS REPORT

November 1, 1968 - June 30, 1969

by

M. W. Milligan
H. J. Wilkerson

Investigation Conducted for the
National Aeronautics and Space Administration
under
Research Grant 43-001-023

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ABSTRACT

This report, the fifth of a series, outlines the progress on a theoretical and experimental investigation of rarefied, internal gas dynamics. This research is being conducted under Research Grant 43-001-023 sponsored by the National Aeronautics and Space Administration.

The activities reported herein are concerned with an investigation of rarefied-gas viscoseals. Section A presents a theoretical analysis which has been developed by applying non-continuum boundary conditions to the Reynold's Lubrication equations. Comparisons between the performance predicted by this model and the experimental data of several investigators are presented. Section B is a progress report of the experimental efforts to investigate rarefied-gas viscoseal performance.

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INTRODUCTION

An investigation of rarefied, internal gas dynamics with emphasis on shaft sealing applications was initiated on May 1, 1966, at the University of Tennessee in the Department of Mechanical and Aerospace Engineering. The investigation is being conducted for the National Aeronautics and Space Administration under Research Grant 43-001-023, and this report presents the progress of the investigation for the period November 1, 1968 through June 30, 1969.

A critical review of all available literature led to the identification of a number of basic problem areas requiring study in order to advance rarefied gas dynamics technology to the state required by today's application. Other reports describe the results of a long tube investigation (1)* and a nozzle investigation (2)* which were completed before the present study was initiated. The first report in this series (3)* describes the results of a short tube study. The second report in this series (4)* was an interim report on the annuli investigation which was reported in full in reference (5)*. The fourth report (6)* contains three different topics of rarefied gas dynamics including a progress report primarily concerned with experimental efforts to investigate flow through annuli with a rotating inner boundary, a flow analysis

*Section B References, Page 24

for an eccentric annulus using slip boundary conditions, and an experimental investigation of flow through ultra fine filtering media.

This report presents interim results of an investigation of rarefied-gas viscoseals. Section A contains an analysis developed by applying non-continuum boundary conditions to the Reynold's Lubrication equations. The presentation in Section A is a paper entitled "Theoretical Performance of Rarefied-Gas Viscoseals," which is being considered for the ASLE-ASME Joint Conference, October 1969, in Houston, Texas. Section B is a progress report of the experimental investigation.

OBJECTIVES

The research effort during this period followed the program generally outlined in the proposal with the exception that overall progress has not been as rapid as originally estimated. A primary factor in this delay was the three month late delivery of the spiral grooved shaft by the supplier.

PROPOSED SCHEDULE

During the period July 1, 1969 - December 31, 1969 the following efforts are scheduled:

1. Experimental data acquisition on at least one grooved shaft configuration.
2. Continued development of theoretical models to be used to predict viscoseal performance in the gas flow regime between laminar-continuum and free molecule.

3. An attempt to correlate experimental data with theoretical results for the visco-seal geometry.

SECTION A

THEORETICAL PERFORMANCE

OF

RAREFIED-GAS VISCOSEALS

THEORETICAL PERFORMANCE OF RAREFIED-GAS VISCOSEALS

Mancil W. Milligan[1] and Harvey J. Wilkerson[2]

ABSTRACT

A theoretical model has been developed to predict the performance of spiral groove pumping seals when the sealant is a rarefied gas. The model has been developed by applying non-continuum boundary conditions to the Reynold's lubrication equations. Comparisons between the performance predicted by this model and the experimental data of several investigators are presented. Very limited experimental data are available in the rarefied-gas regime but qualitative comparisons indicate that the proposed model is valid for predicting the onset of non-continuum effects and the consequent degradation of seal performance.

NOMENCLATURE

A	slip coefficient
C_1, C_2, C_3, C_4	constants of integration
D	seal diameter
L	seal length
Q	sealant flow
U	surface velocity

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a	axial land width
b	axial groove width
c	radial clearance
h	groove depth
h	film thickness (h_g,h_r)
l	length of seal thread
N_K	Knudsen number
p	pressure
t	tangent of helix angle α
u,v,w	velocity components in ξ , η , z coordinates
x,y,z	coordinates
ξ, η	coordinates
α	helix angle
β	$\frac{h + c}{c}$
γ	$\frac{b}{a + b}$
Λ	sealing coefficient
λ	mean free path of gas molecule
μ	absolute viscosity

Subscripts

r	refers to land
g	refers to groove
ξr	refers to ξ direction in region of land
ηr	refers to η direction in region of land
ξg	refers to ξ direction in region of groove
ηg	refers to η direction in region of groove
y	refers to axial component

INTRODUCTION

The spiral groove pumping seal is a pressure generation device which is characterized by two surfaces moving relative to each other, with very small film thickness and having one or both surfaces grooved. The cylindrical form, viscoseal, has been the subject of many recent investigations. Most of these studies have been concerned with liquids as the sealant with emphasis on the continuum regimes both laminar and turbulent. The requirement of restricting the flow of fluids to space has necessitated the development of seals which operate in the rarefied gas flow regime.

Little experimental investigation has been devoted to rarefied-gas sealing. Baron (1) performed experiments using air and hydrogen as the sealants in viscoseals but his data are well within the continuum regime. Hodgson (2) performed experiments with mercury vapor as the sealant but great difficulty was experienced in this investigation. King (3) performed experiments using air, argon, helium, and sulfur hexaflouride gas and a portion of these data are in the non-continuum regime. Hodgson and Milligan (4) obtained data for air but again these were in the continuum regime.

The theoretical analysis of viscoseals having rarefied gas as the sealant are also very limited. King (3) attempted a theoretical solution using a non-continuum boundary condition and a simplified model. Hodgson (2) attempted a molecular analysis and a continuum solution but included no investigation of the transition regime between continuum and molecular flows.

In the work presented here the laminar continuum model of Boon and Tal (5) has been modified by using non-continuum boundary conditions and the resulting solution has been used to predict the onset of non-continuum effects and the performance of viscoseals in rarefied near-continuum operation.

THEORETICAL ANALYSIS

Consider a screw formed on a shaft located concentrically within a cylindrical housing with a radial clearance c. The annular space is filled with a gas and the shaft is moving relative to the housing with an angular velocity, ω . Figure 1 shows a developed view of the viscoseal geometry. The (x,y) axes are along and normal to the direction of relative motion and the (ξ , η) axes are parallel and normal to the grooves. The (x,y) and (ξ , η) coordinates systems are related by:

$$\xi = x\cos\alpha + y\sin\alpha$$

$$\eta = y\cos\alpha - x\sin\alpha$$

Most investigators (5, 6) have reduced the describing partial differential equations to the Reynold's lubrication equations. For gases this appears to be even more realistic than for liquids. The describing mathematical model is taken as:

$$\frac{d^2u}{dz^2} = \frac{1}{\mu}\frac{\partial p}{\partial \xi} \qquad [1]$$

$$\frac{d^2 v}{dz^2} = \frac{1}{\mu}\frac{\partial p}{\partial \eta} \qquad [2]$$

Integration of [1] and [2] gives:

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial \xi} z^2 + C_1 z + C_2 \qquad [3]$$

$$v = \frac{1}{2\mu}\frac{\partial p}{\partial \eta} z^2 + C_3 z + C_4 \qquad [4]$$

The integration constants are determined by the boundary conditions. To account for the non-continuum effects slip boundary conditions are introduced. The boundary conditions are taken as:

Along the lands:

$$u_r\big|_{z=0} = U\cos\alpha + A\lambda \frac{du_r}{dz}\bigg|_{z=0} \equiv u_1$$

$$u_r\big|_{z=h_r} = -A\lambda \frac{du_r}{dz}\bigg|_{z=h_r} \equiv u_2$$

Along the groove:

$$u_g\big|_{z=0} = -U\cos\alpha + A\lambda \frac{du_g}{dz}\bigg|_{z=0} \equiv u_3$$

$$u_g\big|_{z=h_g} = -A\lambda \frac{du_g}{dz}\bigg|_{z=h_g} \equiv u_4$$

Across the lands:

$$V_r\Big|_{z=0} = -U\sin\alpha + A\lambda\frac{dv_r}{dz}\Big|_{z=0} \equiv V_1$$

$$V_r\Big|_{z=h_r} = -A\lambda\frac{dv_r}{dz}\Big|_{z=h_r} \equiv V_2$$

Across the groove:

$$V_g\Big|_{z=0} = -U\sin\alpha + A\lambda\frac{dv_g}{dz}\Big|_{z=0} \equiv V_3$$

$$V_g\Big|_{z=h_g} = -A\lambda\frac{dv_g}{dz}\Big|_{z=h_g} \equiv V_4$$

Using the above boundary conditions the following velocity components are determined:

Along the lands:

$$u_r = \frac{1}{2\mu}\frac{\partial p}{\partial \xi}\left(z^2 - h_r z\right) + \left(\frac{u_2 - u_1}{h_r}\right)z + u_1 \qquad [5]$$

Along the grooves:

$$u_g = \frac{1}{2\mu}\frac{\partial p}{\partial \xi}\left(z^2 - h_g z\right) + \left(\frac{u_4 - u_3}{h_g}\right)z + u_3 \qquad [6]$$

Across the lands:

$$V_r = \frac{1}{2\mu}\frac{\partial p}{\partial \eta}\left(z^2 - h_r z\right) + \left(\frac{V_2 - V_1}{h_r}\right) z + V_1 \qquad [7]$$

Across the grooves:

$$V_g = \frac{1}{2\mu}\frac{\partial p}{\partial \eta}\left(z^2 - h_g z\right) + \left(\frac{V_4 - V_3}{h_g}\right) z + V_3 \qquad [8]$$

The slip velocities at the walls can now be solved for and the following velocity distribution equations obtained:

$$u_r = \frac{1}{2\mu}\frac{\partial p}{\partial \xi}\left(z^2 - h_r z\right) + \left(\frac{-U\cos\alpha}{1 + 2A\lambda/h_r}\right)\frac{z}{h_r}$$

$$+ U\cos\alpha\left(\frac{1 + A\lambda/h_r}{1 + 2A\lambda/h_r}\right) - \frac{1}{2\mu}\frac{\partial p}{\partial \xi} h_r^2 \frac{A\lambda}{h_r} \qquad [9]$$

$$u_g = \frac{1}{2\mu}\frac{\partial p}{\partial \xi}\left(z^2 - h_g z\right) + \left(\frac{-U\cos\alpha}{1 + 2A\lambda/h_g}\right)\frac{z}{h_g}$$

$$+ U\cos\alpha\left(\frac{1 + A\lambda/h_g}{1 + 2A\lambda/h_g}\right) - \frac{1}{2\mu}\frac{\partial p}{\partial \xi} h_g^2 \frac{A\lambda}{h_g} \qquad [10]$$

$$v_r = \frac{1}{2\mu}\frac{\partial p}{\partial \eta}(z^2 - h_r z) + \left(\frac{U\sin\alpha}{1 + 2A\lambda/h_r}\right)\frac{z}{h_r}$$

$$- U\sin\alpha\left(\frac{1 + A\lambda/h_r}{1 + 2A\lambda/h_r}\right) - \frac{1}{2\mu}\frac{\partial p}{\partial \eta} h_r^2 \frac{A\lambda}{h_r} \qquad [11]$$

$$v_g = \frac{1}{2\mu}\frac{\partial p}{\partial \eta}(z^2 - h_g z) + \left(\frac{U\sin\alpha}{1 + 2A\lambda/h_g}\right)\frac{z}{h_g}$$

$$- U\sin\alpha\left(\frac{1 + A\lambda/h_g}{1 + 2A\lambda/h_g}\right) - \frac{1}{2\mu}\frac{\partial p}{\partial \eta} h_g^2 \frac{A\lambda}{h_g} \qquad [12]$$

Noting that the axial velocity components are

$$u_y = u\sin\alpha \qquad [13]$$

$$v_y = v\cos\alpha \qquad [14]$$

the axial flow rate components $Q_{\xi r}$, $Q_{\xi g}$, $Q_{\eta r}$ and $Q_{\eta g}$ may be determined. The width of the flow path for the ξ land flow component is $(1-\gamma)\pi D$ and the path width for the ξ groove flow is $\gamma\pi D$. The ratio of the groove width to the groove plus land width is defined as γ. The axial component of the ξ coordinate land flow is:

$$Q_{\xi r} = (1-\gamma)\pi D \int_0^{h_r} u_{ry}\, dz = (1-\gamma)\pi D \int_0^{h_r} u_r \sin\alpha\, dz \qquad [15]$$

Substituting Eq. [9] and integrating gives,

$$Q_{\xi r} = (1-\gamma)\pi D \sin\alpha \left[\frac{1}{2\mu}\left(\frac{\partial p}{\partial \xi}\right)_r h_r^3\left(-\frac{A\lambda}{h_r} - \frac{1}{6}\right) + U\cos\alpha\,\frac{h_r}{2}\right] \qquad [16]$$

In a similar manner

$$Q_{\xi g} = \gamma\pi D \sin\alpha \left[\frac{1}{2\mu}\left(\frac{\partial p}{\partial \xi}\right)_g h_g^3\left(-\frac{A\lambda}{h_g} - \frac{1}{6}\right) + U\cos\alpha\,\frac{h_g}{2}\right] \qquad [17]$$

$$Q_{\eta r} = \pi D \cos\alpha \left[\frac{1}{2\mu}\left(\frac{\partial p}{\partial \eta}\right)_r h_r^3\left(-\frac{A\lambda}{h_r} - \frac{1}{6}\right) - U\sin\alpha\,\frac{h_r}{2}\right] \qquad [18]$$

$$Q_{\eta g} = \pi D \cos\alpha \left[\frac{1}{2\mu}\left(\frac{\partial p}{\partial \eta}\right)_g h_g^3\left(-\frac{A\lambda}{h_g} - \frac{1}{6}\right) - U\sin\alpha\,\frac{h_g}{2}\right] \qquad [19]$$

The pressure gradients in Eqs. [16] through [19] may be replaced by the more convenient axial gradients by noting that:

$$\frac{\partial p}{\partial \xi} = \frac{\partial p}{\partial y} \sin \alpha \qquad [20]$$

and

$$(1-\gamma)\left(\frac{\partial p}{\partial \eta}\right)_r + \gamma\left(\frac{\partial p}{\partial \eta}\right)_g = \frac{\partial p}{\partial y} \cos\alpha \qquad [21]$$

From the continuity of mass:

$$Q_{\eta r} = Q_{\eta g} \qquad [22]$$

Thus the total flow is given by:

$$Q = Q_{\xi r} + Q_{\xi g} + Q_{\eta r} = Q_{\xi r} + Q_{\xi g} + Q_{\eta g} \qquad [23]$$

Using Eqs. [16] through [23] the following expression for the axial flow is obtained.

$$Q = -\frac{1}{12\mu}\frac{\partial p}{\partial y} c^3 \pi D \frac{1}{(1+t^2)}\left[t^2(1-\gamma)\left(1+\frac{6A\lambda}{c}\right) + \left(1+\frac{6A\lambda}{\beta c}\right)\left(\beta^3\right)\left(\gamma t^2 + \frac{1}{\psi}\right)\right]$$

$$+ \frac{U c \pi D}{2}\,\frac{t(1-\gamma)(\beta-1)}{1+t^2}\left[\frac{\beta^3\left(1+\frac{6A\lambda}{\beta c}\right)}{\psi\left(1+\frac{6A\lambda}{c}\right)} - 1\right] \qquad [24]$$

where

$$\beta = \frac{h+c}{c}$$

$$\gamma = \frac{b}{a+b}$$

$$\psi = (1-\gamma)\beta^3 \left(\frac{1+\frac{6A\lambda}{\beta c}}{1+\frac{6A\lambda}{c}} \right) + \gamma$$

$$t = \tan\alpha$$

The most useful situation for the viscoseal as a sealing device would be to have Q equal to zero. Taking Q = 0 and $\frac{\partial p}{\partial y} = \frac{\Delta p}{L}$ the sealing coefficient may be obtained as:

$$\Lambda \equiv \frac{6\mu U L}{\Delta p c^2} =$$

$$\frac{t^2(1-\gamma)(1+6AN_k) + \left(1+\frac{6AN_k}{\beta}\right)(\beta^3)\left(\gamma t^2 + \frac{1}{\psi}\right)}{t(1-\gamma)(\beta-1)\left[\frac{\beta^3\left(1+6\frac{AN_k}{\beta}\right)}{\psi(1+6AN_k)} - 1\right]} \quad [25]$$

where the Knudsen Number is defined as

$$N_K \equiv \frac{\lambda}{c} \quad [26]$$

The slip coefficient, A, has a value near unity for surfaces of practical interest and is taken to be unity in all of the results presented here. The Knudsen Number indicates the degree of rarefication since it becomes large as the mean free path, λ , becomes large.

It should be noted that all experimental Knudsen Numbers reported here have been determined using the average pressure in the seal as detailed in Ref. (7).

RESULTS AND CONCLUSIONS

Experimental data obtained by other investigators are presented in Figs. 2, 3, and 4. Also presented on these figures are the results obtained from the theoretical model (Eq. 25) which has been developed. In Figure 2 the data of Hodgson and Milligan (4) are compared with the results from Eq. [25]. These data were obtained at a $N_K = 0.00147$. The experimental results indicate a slightly larger sealing coefficient than that predicted by the theory but agree within the uncertainties associated with the seal dimensions. In Figure 3 the data of Baron (1) are compared with theoretical predictions. Baron's sealing parameter is related to the sealing coefficient in the manner shown on the ordinate in Fig. 3. The theory certainly predicts the trends of the data although there is a significant deviation between theory and experiment. As in the case of Ref. (4) these data are all well within the continuum regime. In Figure 4 the data of King (3) are compared with theoretical predictions. As shown in the figure the theory indicates that non-continuum effects will occur for Knudsen Numbers greater than 0.001 in the form of a decrease in sealing performance. The data also indicate this same trend although there is a significant deviation between theory and experiment.

In Figs. 5, 6, and 7 the sealing coefficient is presented as a function of geometry for different values of Knudsen Number. These theoretical predictions indicate that there is a four-fold decrease in sealing performance as the Knudsen Number goes from continuum ($N_K = 0.01$) to rarefied ($N_K = 1.0$). Also these results indicate that the optimum value of β increases as the degree of rarefication increases. In addition there is less sensitivity of the sealing coefficient to a precise value of β as indicated by the flatness of the minimums.

It is concluded that the theoretical model presented here can be used to predict the performance of viscoseals in the rarefied non-continuum regime. As the degree of rarefication increases it is necessary to correct this model to account for self-diffusion flow and thus the utility of this model is limited to the near continuum regime. It is anticipated that in the near future a complete theoretical model will be available to predict performance up to and including free molecular conditions. It should be noted that the small Knudsen Number limit for this model is identical to the continuum (no-slip boundary conditions) solution previously presented in References (5) and (6).

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of this research by the National Aeronautics and Space Administration, Lewis Center, under Grant NGR-43-001-023.

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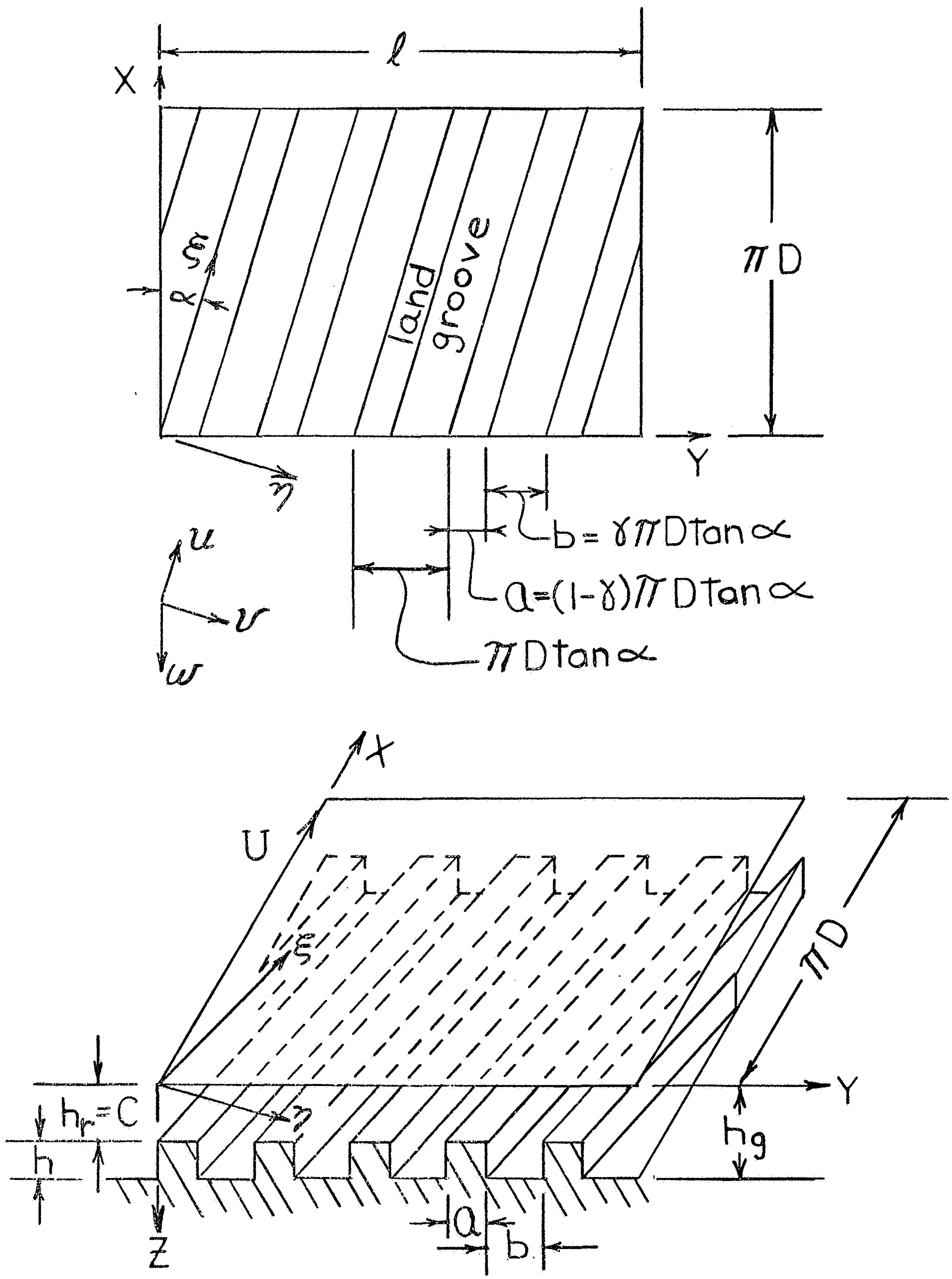

Figure 1. Developed Viscoseal Geometry

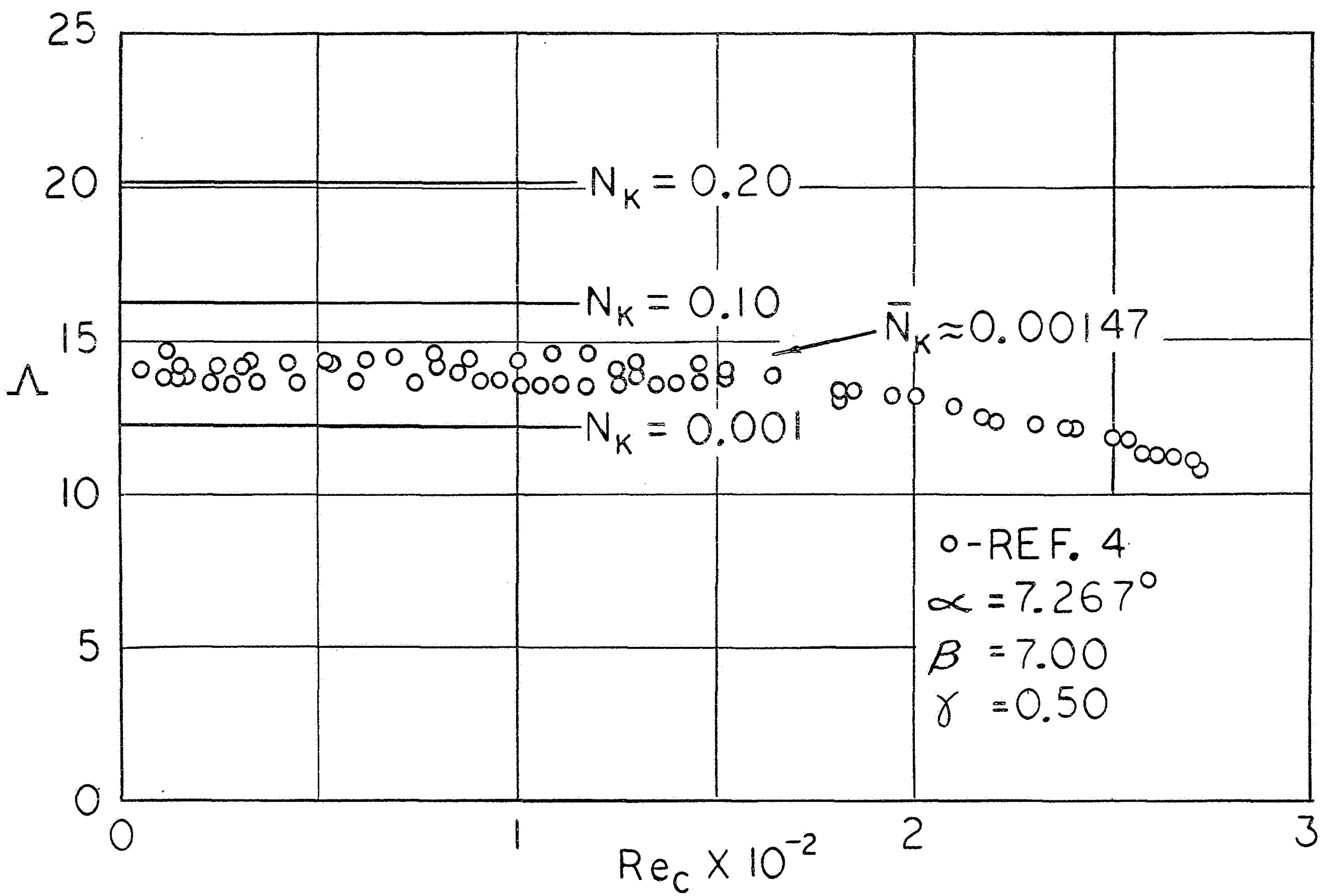

Figure 2. Sealing Coefficient versus Clearance Reynold Number

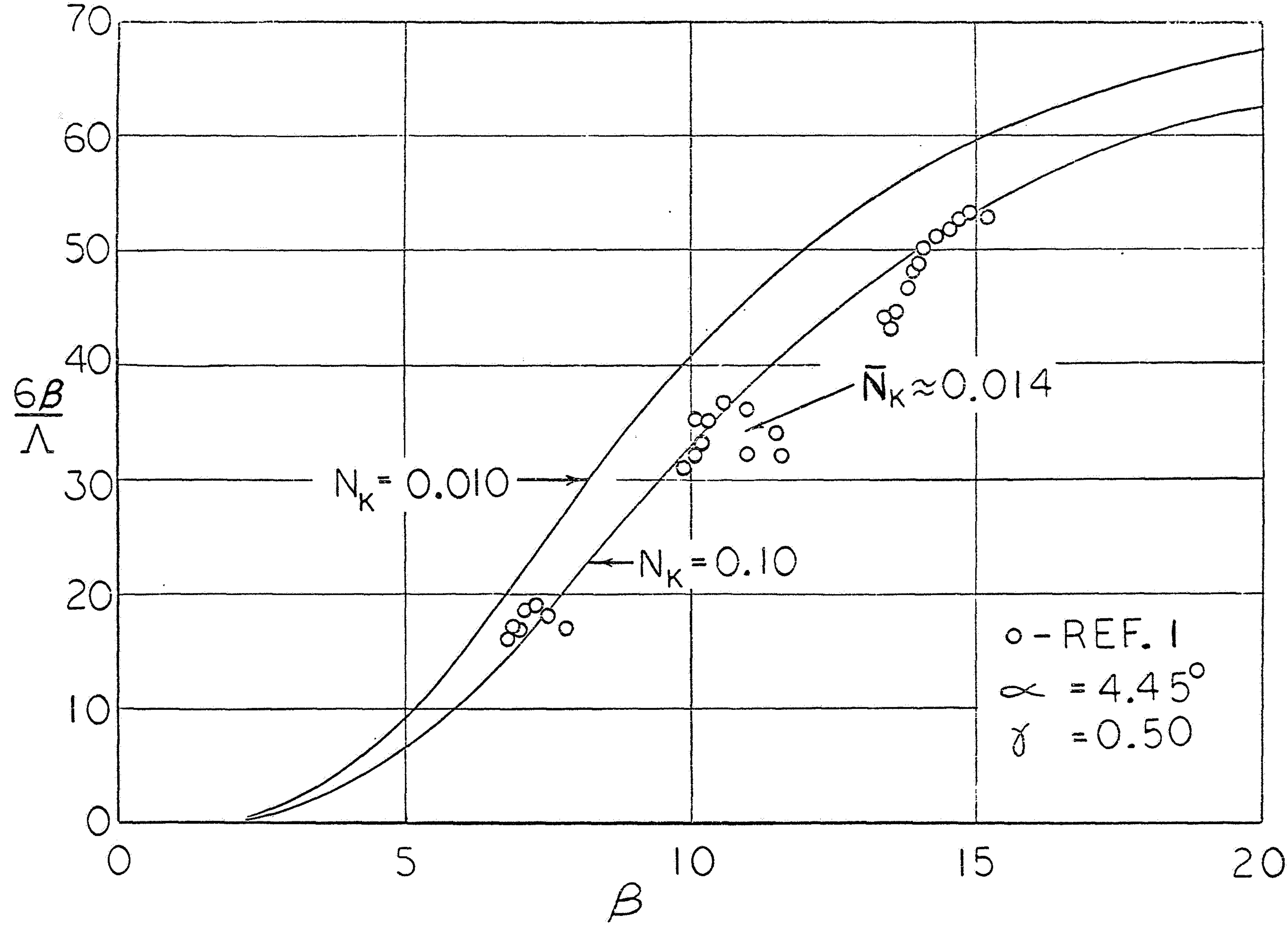

Figure 3. Baron's Sealing Parameter versus β

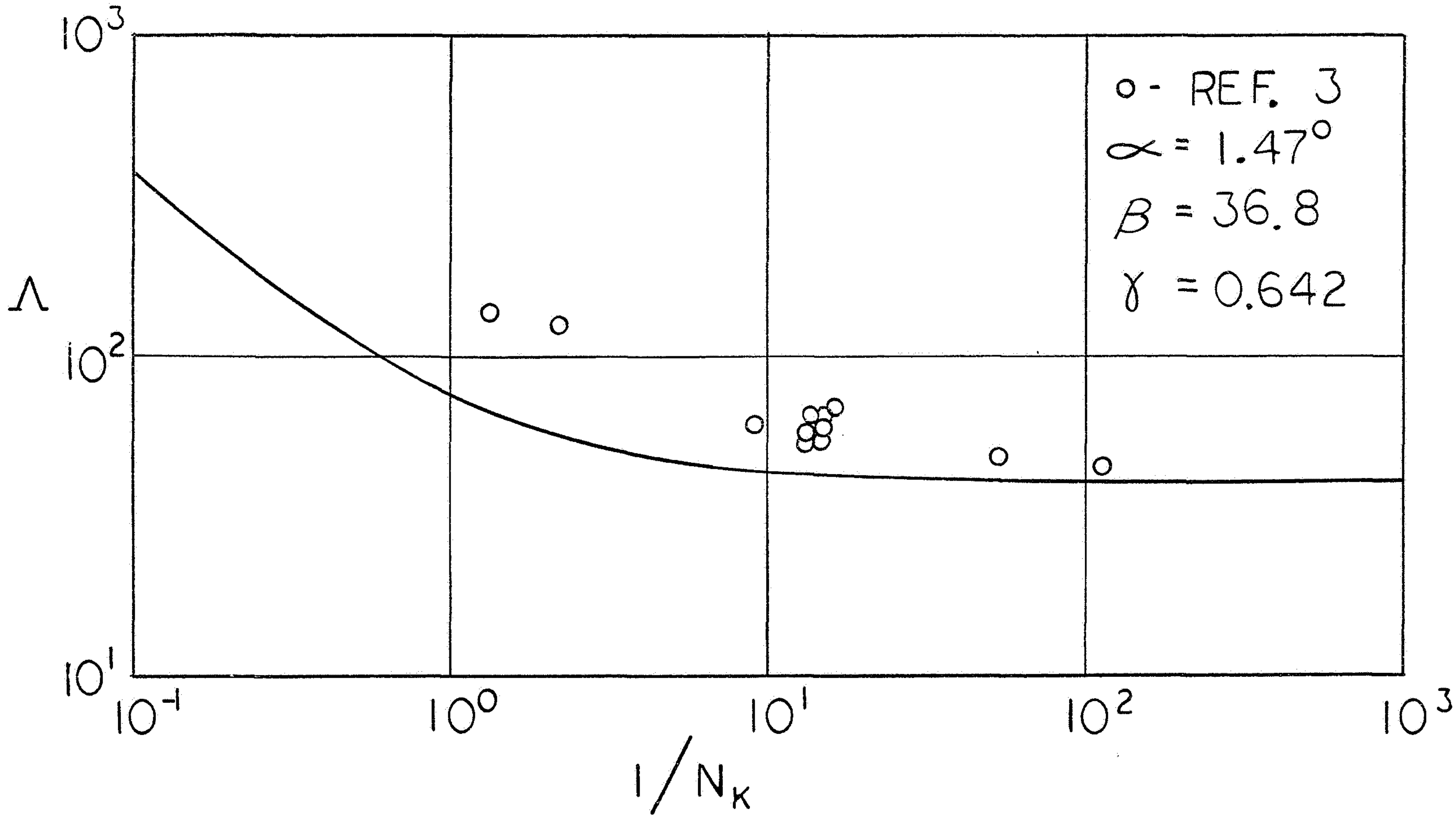

Figure 4. Sealing Coefficient versus Inverse Knudsen Number

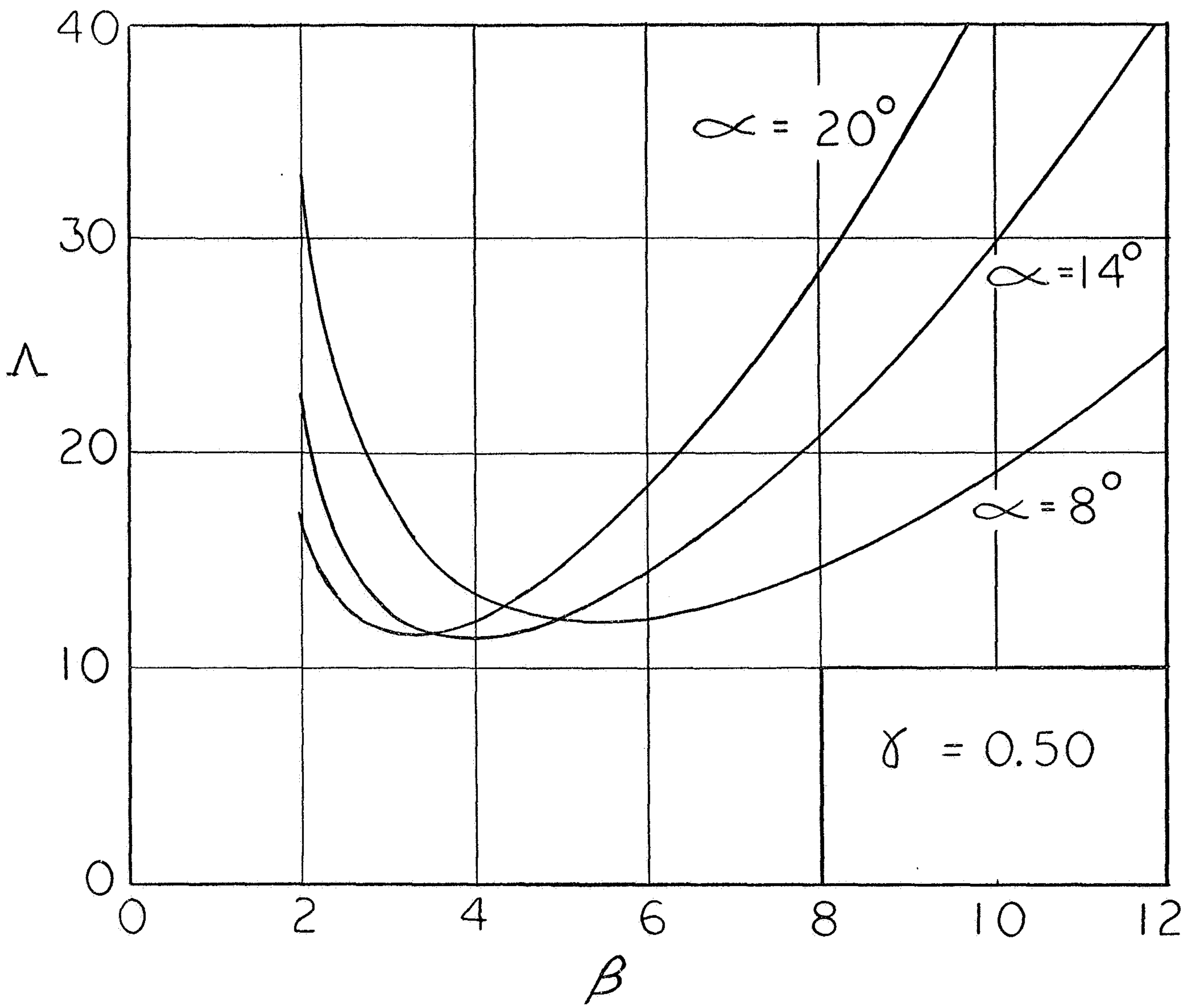

Figure 5. Sealing Coefficient versus β for $N_K = 0.01$

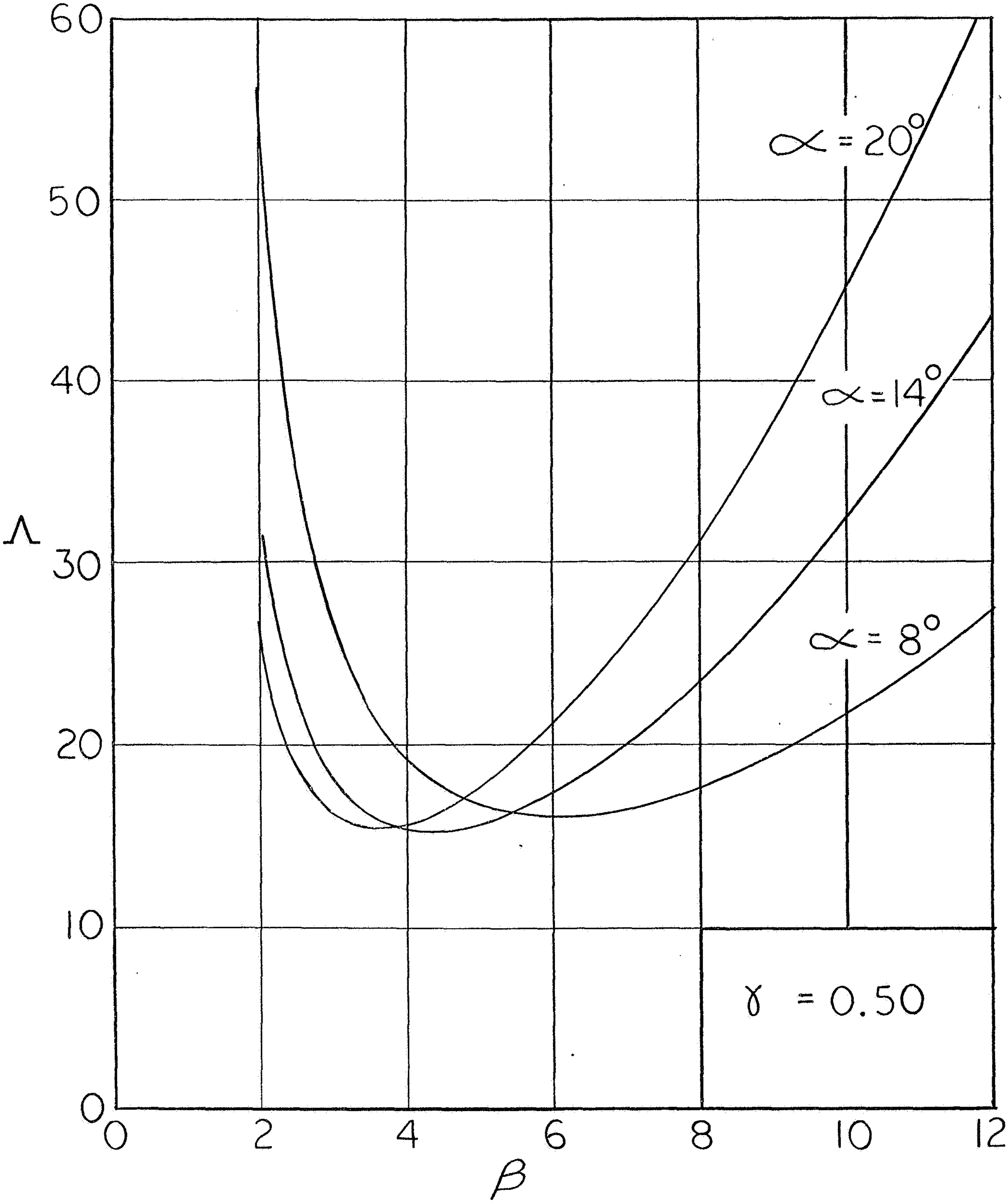

Figure 6. Sealing Coefficient versus β for N_K = 0.1

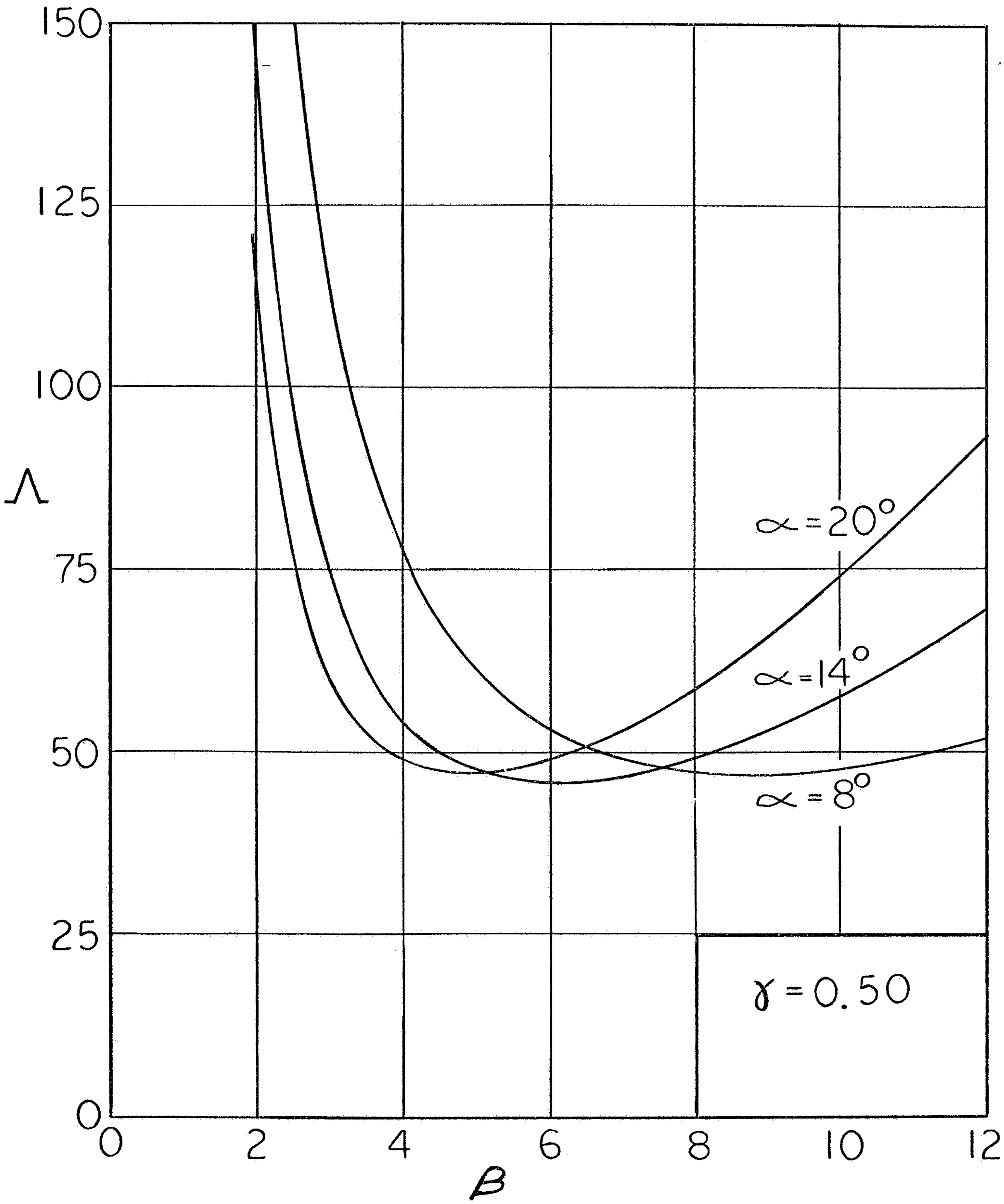

Figure 7. Sealing Coefficient versus β for N_K = 1.0

<u>SECTION B</u>

PROGRESS REPORT

EXPERIMENTAL INVESTIGATION

OF

RAREFIED-GAS VISCOSEALS

INTRODUCTION

The experimental apparatus is designed to investigate viscoseal performance in the gas flow regime between continuum and free molecule. The experimental setup, procedure, and preliminary results obtained for gas flow through a smooth-walled annulus with a rotating inner boundary were described in reference (6).

ACTIVITIES

Experimental efforts during this period did not progress as rapidly as originally estimated due to the three month late delivery of the spiral grooved shaft by the supplier. The proximity detectors which are used for shaft-housing alignment are presently being calibrated for the grooved shaft surface. These and other preliminary efforts such as instrument calibrations and minor revisions to the setup are being conducted in preparation for sealing performance tests on the grooved shaft. It is anticipated that the majority of experimental data will be acquired during the next three months. Current plans call for the viscoseal to be run over a wide speed range up to 30,000 rpm and over a density range from continuum to free-molecular.

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