Nuclei from QCD : Strategy, Challenges and Status

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Abstract. I describe progress that is being made toward calculating the properties and interactions of nuclei from QCD.

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INTRODUCTION

Perhaps the greatest challenge facing those of us working in the area of strong interaction physics is to be able to rigorously compute the properties and interactions of nuclei. The many decades of theoretical and experimental investigations in nuclear physics have, in many instances, provided a very precise phenomenology of the strong interactions in the non-perturbative regime. However, at this point in time we have little understanding of much of this phenomenology in terms of the underlying theory of the strong interactions, Quantum Chromo Dynamics (QCD). I wish to discuss a strategy for making a connection between QCD and nuclear physics, which ultimately will allow for the calculation of nuclear properties and processes in terms of the light quark masses, the scale of the strong interactions, and the electroweak couplings.

QCD TO NUCLEI: STRATEGY

The ultimate goal is to be able to rigorously compute the properties and interactions of nuclei from QCD. This includes determining how the structure of nuclei depend upon the fundamental constants of nature. Perhaps as important, we would then be in the position to reliably compute quantities that cannot be accessed, either directly or indirectly, by experiment.

The only way to rigorously compute strong-interaction quantities in the nonperturbative regime is with lattice QCD. One starts with the QCD Lagrange density and performs a Monte-Carlo evaluation of Euclidean space Green functions directly from the path integral. To perform such an evaluation, space-time is latticized and computations are performed in a finite volume, at finite lattice spacing, and at this point in time, with quark masses that are larger than the physical quark masses. To compute any given quantity, contractions are performed in which the valence quarks that propagate on any given gauge-field configuration are “tied together”. For simple processes such as nucleon-nucleon scattering, such contractions do not require significant computer time compared with lattice or propagator generation. However, as one explores processes involving more hadrons, the number of contractions grows rapidly (for a nucleus with atomic number A and charge Z, the number of contractions is $(A+Z)!(2A-Z)!$), and a direct lattice QCD calculation of the properties of a large nucleus is quite impractical simply due to the computational time required.

The way to proceed is to establish a small number of effective theories, each of which have well-defined expansion parameters and can be shown to be the most general form consistent with the symmetries of QCD. Each theory must provide a complete description of nuclei over some range of atomic number. Calculations in two "adjacent" theories are performed for a range of atomic numbers for which both theories converge. One then matches coefficients in one EFT to the calculations in the other EFT or to the lattice, and thereby one can make an indirect, but rigorous connection between QCD and nuclei. It appears that four different matchings are required:

1. **Lattice QCD**. Lattice QCD calculations of the properties of the very lightest nuclei will be possible at some point in the not so distant future [1]. Calculations for $A \leq 4$ as a function of the light-quark masses, would uniquely define the interactions between nucleons up to and including the four-body operators. Depending on the desired precision, one could possibly imagine calculations up to $A \sim 8$.
2. **Exact Many-Body Methods**. During the past decade one has seen remarkable progress in the calculation of nuclear properties using Green Function Monte-Carlo (GFMC) with the AV_{18}-potential (e.g. Ref. [2]) and also the No-Core Shell Model (NCSM) (e.g. Ref. [3]) using chiral potentials. Starting with the chiral potentials, which are the most general interactions between nucleons consistent with QCD, one would calculate the properties of nuclei as a function of all the parameters in the chiral potentials with GFMC or the NCSM out to some given order in the chiral expansion. A comparison between such calculations and lattice QCD calculations will determine these parameters to some level of precision. These parameters can then be used in the calculation of nuclear properties up to atomic numbers $A \sim 20-30$. The computer time for these many-body theories suffers from the same $\sim (A!)^2$ blow-up that lattice QCD does, and for a sufficiently large nucleus, such calculations become impractical.
   Another recent development that shows exceptional promise is the latticization of the chiral effective field theories [4, 5, 6, 7, 8]. This should provide a model-independent calculation of nuclear processes once matched to lattice QCD calculations.
3. **Coupled Cluster Calculations**. In order to move to larger nuclei, $A \lesssim 100$ a technique that has shown promise is to implement a coupled-clusters expansion (e.g. Ref. [9]). One uses the same chiral potential that will have been matched to lattice QCD calculations, and then performs a diagonalization of the nuclear Hamiltonian, after truncating the cluster expansion, which itself contains arbitrary coefficients. The results of these calculations will be matched to those of the NCSM or GFMC for $A \sim 20-30$ to determine the arbitrary coefficients. This method is unlikely to be practical for very large atomic numbers.
4. **Density Functional Theory (??) and Very Large Nuclei** To complete the periodic table one needs to have an effective theory that is valid for very large nuclei and

nuclear matter. A candidate that has received recent attention is Density Function Theory (DFT) (e.g. Refs. [10, 11]). It remains to be seen if this is in fact a viable candidate. There is reason to hope that this will be useful because there is clearly a density expansion in large nuclei with a power-counting that is consistent with the Naive Dimensional Analysis (NDA) of Georgi and Manohar [12]. The application of DFT to large nuclei is presently the least rigorously developed component of this program.
The latticized chiral theory mentioned previously can also be applied to the infinite nuclear matter problem. This work is still in the very earliest stages of exploration, but this looks promising [6].

QCD TO NUCLEI: ONE OF THE CHALLENGES

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QCD TO NUCLEI: STATUS

During the past few years there has been substantial progress toward being able to compute nuclear properties from QCD using the strategy already outlined.

Lattice QCD

Lattice QCD has entered an era in which reliable calculations of strong interaction quantities can be performed with fully-dynamical QCD calculations at small lattice spacings and in large volumes (large and small are defined relative to the scale of chiral symmetry breaking). The lattice actions have good chiral symmetry through the invention of Domain-Wall fermions [30, 31] and Overlap fermions [32]. Further, there has been substantial progress in chiral EFT's that, in addition to describing the light-quark mass dependence and allowing for rigorous chiral extrapolations, facilitate the removal of finite-lattice spacing [33] and finite-volume effects inherent in the lattice QCD calculations, e.g. Refs. [34, 35].

There has been much effort over the years to precisely determine strong interaction matrix elements required to extract parameters of the electroweak theory, such as V_{bc}. A subset of these were described in Chris Sachrajda's talk [36], and I will not discuss them here. A recent calculation involving the light mesons of interest to nuclear physicists is the calculation of $I = 2$ $\pi\pi$ scattering in fully dynamical QCD by the NPLQCD collaboration [37], as shown in fig. 1. One finds good agreement with the predictions of chiral perturbation theory, and the calculations are at small enough pion masses where the perturbative expansion is reliable (see also Ref. [38]).

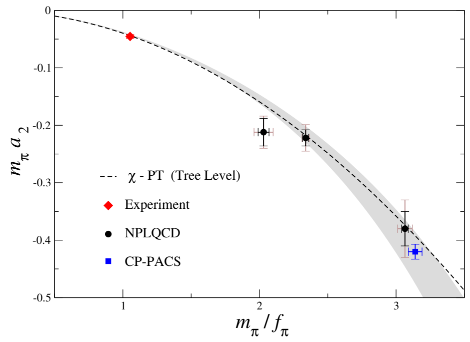

FIGURE 1. $I = 2$ $\pi\pi$ scattering from fully-dynamical lattice QCD [37]. (This figure is taken from Ref. [37].)

Compared to the meson sector, there has been somewhat less emphasis on the baryon sector. However, this is changing through the significant investment in lattice QCD at the Jefferson Laboratory by nuclear physics DOE and SciDAC. There are several hundred processors available for lattice calculations, and more importantly is the work of Robert Edwards and his team to develop and make available the lattice software suite *Chroma* [39, 40].

There has been very impressive recent work by LHPC [41] computing the matrix element of the light-quark axial current in the nucleon at low quark masses and large volumes, as shown in fig. 2. Further, there are some preliminary results from the NPLQCD

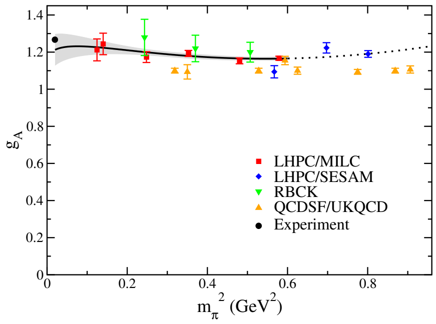

FIGURE 2. The light-quark isovector axial current matrix element in the nucleon computed in fully dynamical lattice QCD [41], and its chiral extrapolation. (This figure is taken from Ref. [41].)

collaboration for the nucleon-nucleon scattering lengths in fully-dynamical lattice QCD, as shown in fig. 3.

Light Nuclei, Chiral Symmetry and the Renormalization Group

There has been impressive progress in the calculation of properties of nuclei when the NN, 3N and 4N interactions are specified. Calculations using GFMC have developed to the stage where, in addition to the ground states, the excited states of the light nuclei

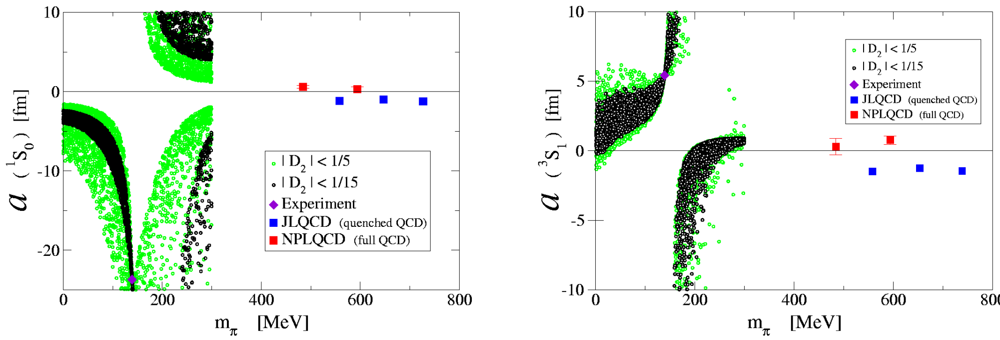

FIGURE 3. The nucleon-nucleon scattering lengths in the $^1S_0$ channel (left panel) and the $^3S_1 - ^3D_1$ coupled channels (right panel) as a function of the pion mass. The light (green) and dark (black) sets of points denote present theoretical estimates of the quark-mass dependence of the scattering lengths based upon EFT arguments [42]. The QCD data points at $m_\pi \sim 500$ MeV and ~ 600 MeV are the **preliminary** results of the NPLQCD exploratory investigation, while the other data points are the results of a quenched calculation [43].

can be extracted [2]. The agreement between the calculated energy-levels and those

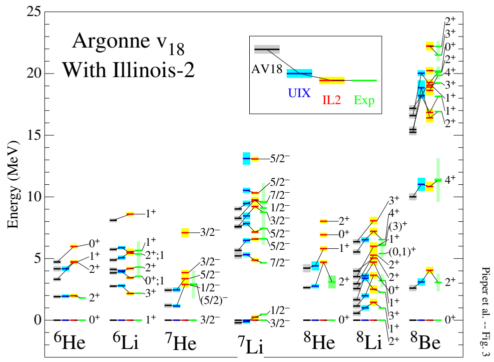

FIGURE 4. The spectra of the $A = 6,7,8$ nuclei computed with a GFMC from the AV_{18} and IL2 interactions [2]. (This figure is taken from Ref. [2].)

observed is truly impressive, and clearly demonstrates the strength of this technique. An example of this agreement can be seen in fig. 4. One would like to see these calculations performed with chiral potentials, so that they could be matched to lattice QCD calculations of the future.

Classifying and computing the interactions between nucleons based upon the approximate chiral symmetry of QCD [21, 22, 23, 24, 25, 26] is now at an advanced stage of development, e.g. Refs. [44, 45, 46, 47]. Initiated by the pioneering papers of Weinberg in the early 1990's, the field is currently at the stage of having determined the small expansion parameter and to have essentially determined (in terms of *apriori* unknown counterterms) the interactions between two, three and four nucleons out to four orders in the expansion [48], see fig. 5. The importance of this effort cannot be overstated. In order to make rigorous, model-independent predictions and calculations in nuclear physics, the most general form of the interactions consistent with QCD must be known. The established power-counting finds that contributions from operators involving four or more nucleons are parametrically suppressed. In addition to establishing a rigorous

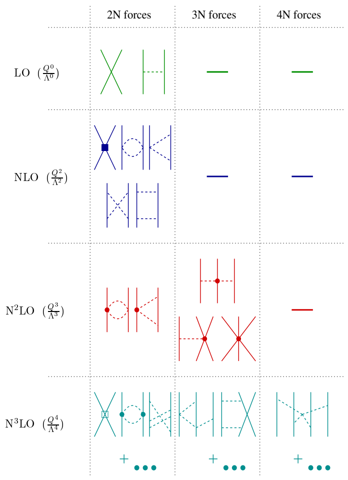

FIGURE 5. Interactions between nucleons as classified in Weinbergs' power-counting scheme [21, 22]. The solid lines denote nucleons, while the dashed lines denote pions. (This figure is taken from Ref. [44].)

framework, the light-quark mass dependence of nuclear interactions is provided by these same interactions.

The RG is a valuable tool for studying quantum systems and has been employed by particle physicists for decades. In the course of developing the EFT's for nuclear physics, it was shown that the RG is also a powerful tool for nuclear physics [24, 25, 26, 27, 28]. Important nuclear physics phenomenology has arisen from applying the RG framework to the modern phenomenological NN interactions, such as the AV_{18} potential, CD-Bonn potential and Idaho A potential. It was shown that by evolving these potential down to a sufficient low scale, $\Lambda \sim 600$ MeV, they all coincide in momentum-space [50, 51] to what is now referred to as $V_{\mathrm{low}\,k}$, (for a nice overview see Ref. [52]), as shown for the ${}^1S_0$ channel in fig. 6. The success of the chiral EFT program for these interactions meant that

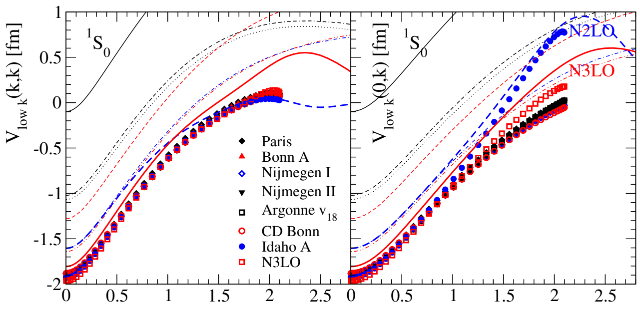

FIGURE 6. Diagonal (left) and off-diagonal (right) momentum-space matrix element for $V_{\mathrm{low}\,k}$ (symbols) in the ${}^1S_0$ channel versus relative momentum derived from different modern potential models for $\Lambda = 2.1\,\mathrm{fm}^{-1}$ [52]. The bare interactions are shown as lines, while the thick sold and thick dashed lines denote the N2LO (Idaho A) or N3LO interactions, resp. (This figure is taken from Ref. [52].)

this result had to be true. As the renormalization scale of $V_{\mathrm{low}\,k}$ is lower than the typical scale of the hard-core interaction in the "bare"-potentials, the interactions are softer, and as a result the many-body calculations in nuclei are significantly more convergent than with the bare potentials.

The No-Core Shell Model

A significant step toward the rigorous calculation of nuclear properties is the development of the NCSM. The entire goal of this program is to implement effective interaction theory, and “take the model out of the shell model”. Historically, the shell model implies that there is a small number of “active” nucleons in shell model orbits outside an inert core of nucleons. The Hamiltonian of the active space is diagonalized to yield the energy eigenstates and energies. The NCSM treats all nucleons as active particles. Some arbitrary but complete basis is chosen, conventionally that of a harmonic oscillator (HO), and the Hamiltonian is constructed in this basis for realistic NN, $3N$ and recently $4N$ interactions. If an infinite number of HO states were included in the calculation, the eigenstates and energies would be independent of the HO parameter. This is impractical, but as the model space in enlarged the eigenenergies and states become less dependent upon the scale of the HO. A sufficient number of HO levels can be included to obtain the desired precision (for a discussion see Ref. [53]). Recent calculations with the softer potentials that result from the chiral EFT’s or $V_{\mathrm{low}\,k}$ are very encouraging. In particular, calculations have been done for $A = 10$ with the NN interaction out to NNNLO, the 3-body interaction out to NNLO and fit to the properties of $A = 3$ and $A = 4$ nuclei. Fig. 7 shows the energy levels of ^{10}B computed in the NCSM with the CD-Bonn potential, and with the chiral potential [1]. Clearly, the convergence is greatly improved when the chiral potential is used.

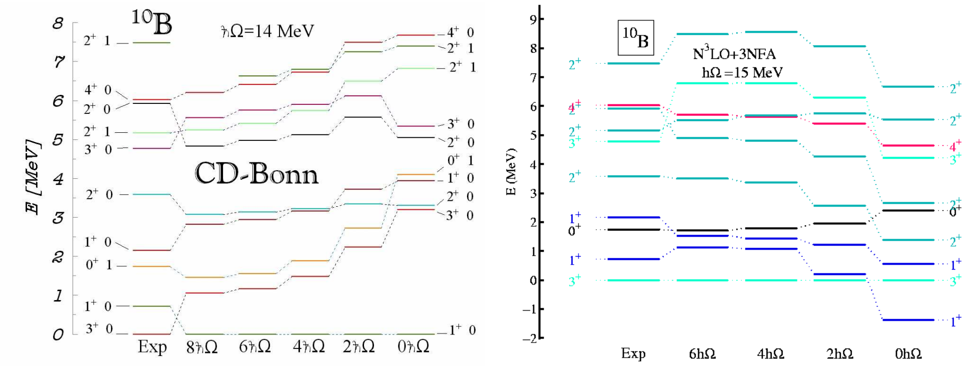

FIGURE 7. The spectrum of ^{10}B, determined in the NCSM with the CD-Bonn potential (left panel) and the chiral potential (right panel) outlined in the text, as a function of the size of the basis.

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[1] I would like to thank Erich Ormand and his collaborators for allowing me to show these figures.

The Braaten-Hammer Conjecture

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SUMMARY AND OUTLOOK

Important progress has been made in the areas necessary for calculation of the properties and interactions of nuclei from QCD. It appears that we are entering an era in which lattice QCD calculations in the $A = 2,3,4$ systems will be matched onto the few-nucleon chiral interactions. These interactions will then be used to compute the properties of nuclei via GFMC, the NCSM, or potentially the latticized chiral theory.

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