Systematics in TagMix and $\sin 2\beta$

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$\sin 2\beta$ Meeting

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Outline

- Statistical Issues in $\sin 2\beta$
- General Systematic Issues
  - Effect of $\sigma_{CP} \neq \sigma_{TM}$
  - Consequences of Systematic Shifts in Dilutions
  - Possible Δz dependence of dilution
- Charged Bs and related problems
  - Problem is Generic
  - Can We Ignore $\tau = 0$ (continuum) Background?
  - Can We Ignore Oscillating Background?
  - Simplified Picture
  - Proposed confirming analysis
- Comments

Purely statistical issues in $\sin 2\beta$

For a single tag category,

$$\sigma(A)^{-2} = ND^2 J(DA, \sigma, \ldots)$$

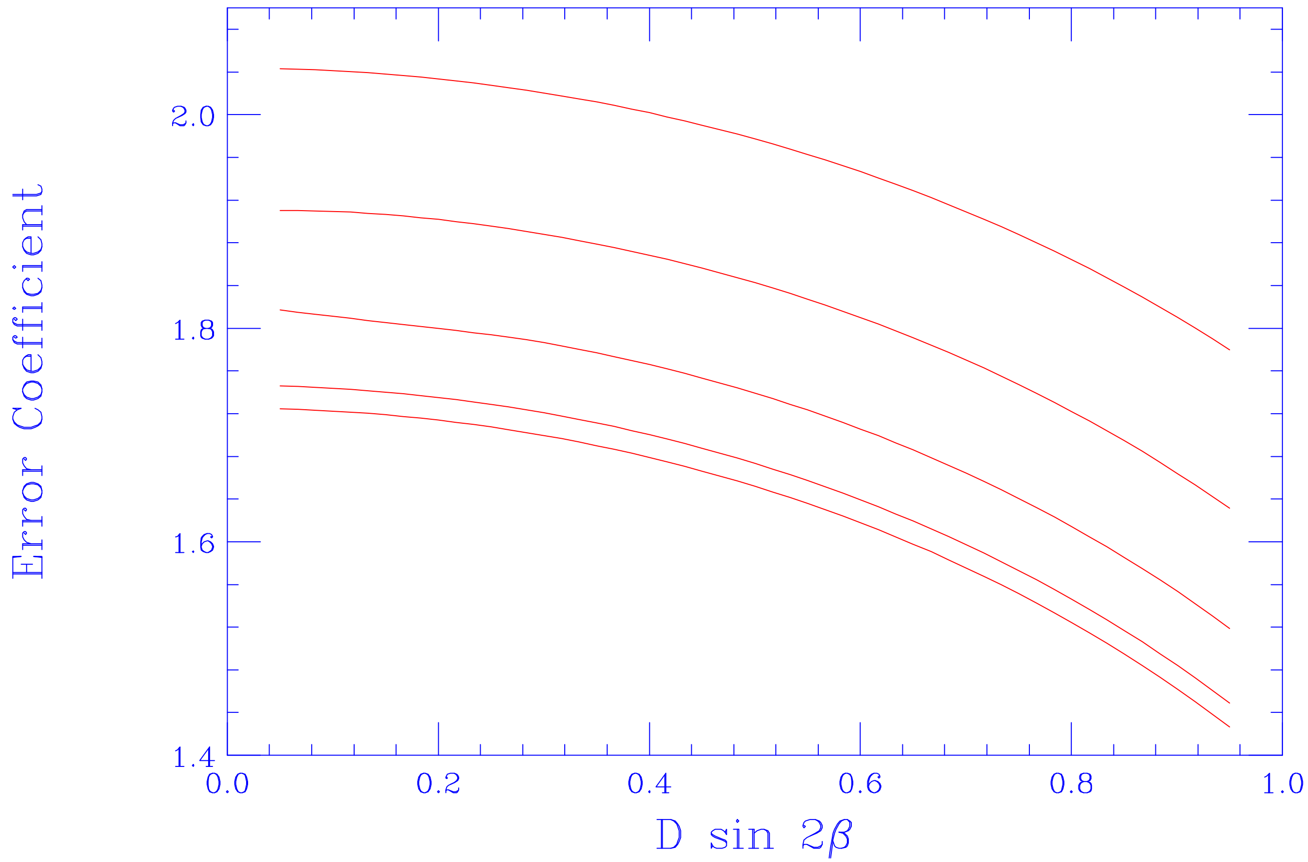
where for perfect resolution ($\sigma = 0$) and small DA,

$$J \to \frac{2x^2}{1+4x^2} \approx \frac{1}{3}$$

With several tagging categories α

$$\sigma(A)^{-2} = N \sum_{\alpha} \epsilon_\alpha D_\alpha^2 J(D_\alpha A, \sigma)$$

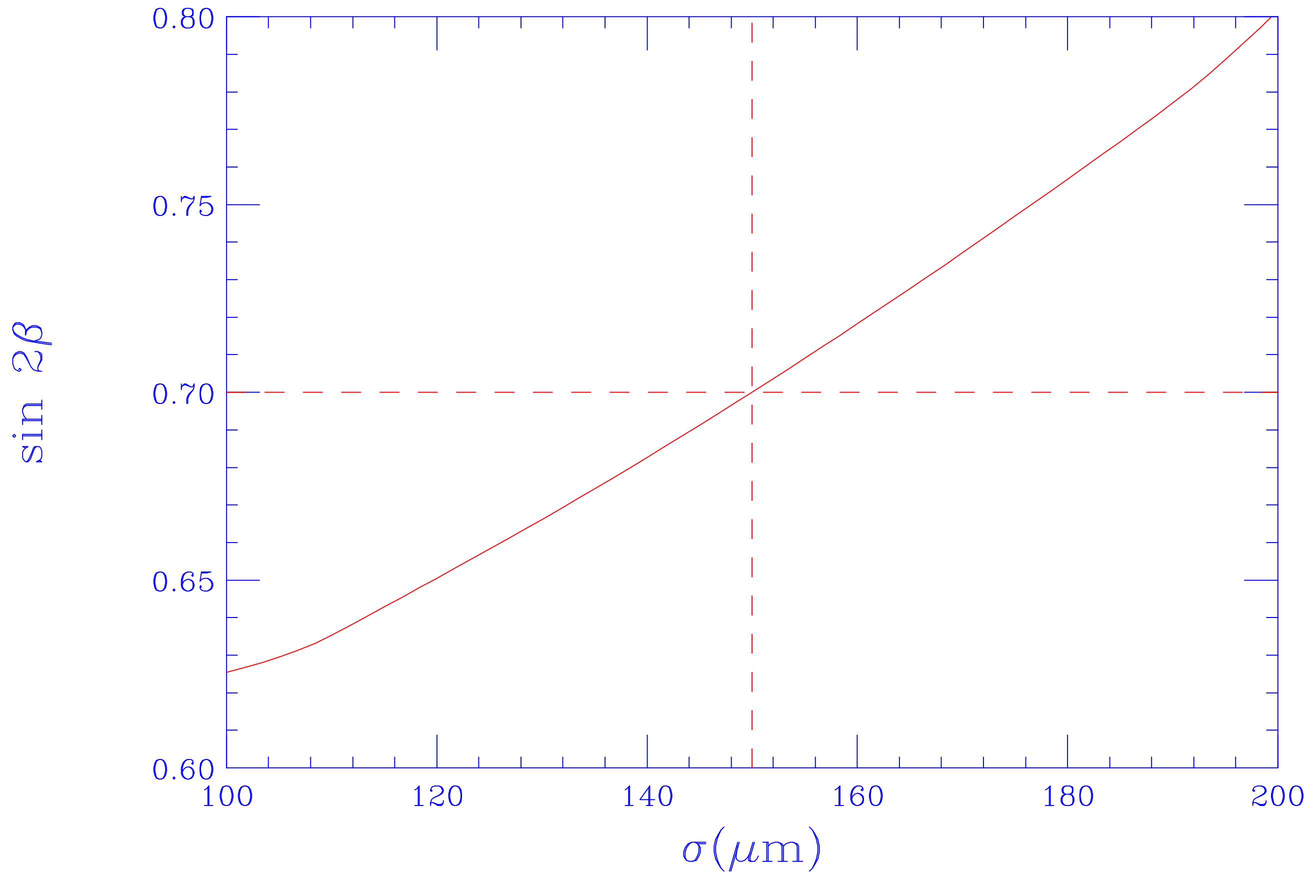
$$\sigma(A = \sin 2\beta) = \mathrm{coef}/\sqrt{D^2 N}$$
$$\sigma(\Delta z) = 10, 50, 100, 150, 200\ \mu\text{m}$$



If $\sigma(\Delta z)_{CP} \neq \sigma(\Delta z)_{TM}$

True values: $\sin 2\beta = 0.7, \sigma = 150\mu$m

$\sin 2\beta$ deduced vs. σ_{CP} used



What to set as systematic error on $|\sigma_{TM} - \sigma_{CP}|$, $|\sigma_{MC\ \psi K_S} - \sigma_{B\ reco}|$?

Consequence of Systematic Shifts in Dilutions

Maximum of log-likelihood shifts

$$\delta D_\alpha = D_\alpha - \overline{D}_\alpha$$

$$\mathcal{L} = \mathcal{L}_0 + \tfrac{1}{2}\delta A^2 \frac{\partial^2 \mathcal{L}}{\partial A^2} + \delta A \delta D_\alpha \frac{\partial^2 \mathcal{L}}{\partial D_\alpha \partial A} + \ldots$$

$$\delta A = -\sum_\alpha \frac{\partial^2 \mathcal{L}}{\partial D_\alpha \partial A}\delta D_\alpha / \frac{\partial^2 \mathcal{L}}{\partial A^2} = -\frac{\sum_\alpha \epsilon_\alpha D_\alpha A \delta D_\alpha J(D_\alpha A)}{\sum_\alpha \epsilon_\alpha D_\alpha^2 J(D_\alpha A)}$$

If we ignore variation in $J(DA)$ due to different dilutions,

$$\frac{\delta A}{A} = -\frac{\sum_\alpha \epsilon_\alpha D_\alpha \delta D_\alpha}{\sum_\alpha \epsilon_\alpha D_\alpha^2} = -\frac{\delta\sqrt{\epsilon D^2}}{\sqrt{\epsilon D^2}}$$

McMahon, McMahon, and Beringer (not a law firm) cite Orsay:

tag	ϵ	w	Q_{high}	Q_{low}
e	5.1 ± 0.4	$4.3 \pm 4.9 \pm 4.9$	0.036	0.055
μ	4.7 ± 0.4	$14.5 \pm 5.7 \pm 5.8$	0.015	0.035
K	36.1 ± 0.9	$20.6 \pm 2.6 \pm 2.9$	0.099	0.154
NT1	9.5 ± 0.5	$23.5 \pm 4.5 \pm 3.9$	0.009	0.038
NT2	18.2 ± 0.7	$32.4 \pm 3.4 \pm 2.4$	0.015	0.032
Total			0.174	0.314

$$\sqrt{Q} = 0.488 \pm 0.077$$

Too conservative.

How to assign systematic errors to dilutions?

Identify sources:

- Wrong time dependences (toy simulations)
- Difference between sideband and combinatorial
- Problems with wrong/fake leptons
- See new approach below

Possible Δz Dependence of Dilution

- Easy to believe dilution is smaller for small Δz
- Should check in data, binning in Δz
- Examine in toy MC simulation
  - "Exact" determination of shift in parameters
  - Uses Minuit + maximum likelihood, not MC
  - Example: set dilution=0 for $|z| < 100\mu$m.
  - True D=0.6

1	gamma	1.5600	constant
2	delta_m	0.42094	1.7438
3	sigma	151.58	472.82
4	dilu	0.37468	1.3302

- Small effect on Δm.
- Will there be effect on $\sin 2\beta$?
- Need to include in simulation

Charged Bs and Similar Background Problems

$$
\begin{aligned}
U &= f_{B^0}\tfrac{1}{2}[(1-w)(1+\cos\Delta mt)+w(1-\cos\Delta mt)]+f_{B^\pm}(1-w') \\
&= \tfrac{1}{2}[1+f_{B^\pm}D']+f_{B^0}D\cos\Delta mt \\
M &= f_{B^0}\tfrac{1}{2}[(1-w)(1-\cos\Delta mt)+w(1+\cos\Delta mt)]+f_{B^\pm}w' \\
&= \tfrac{1}{2}[1-f_{B^\pm}D']-f_{B^0}D\cos\Delta mt
\end{aligned}
$$

- Only the combinations $f_{B^0}D$ and $f_{B^\pm}D'$ are observable in TagMix
- The dilution is the coefficient of oscillating term
- Need only correct for
  - The fraction of the signal that oscillates
  - Oscillation due to other neutral B components
- Dilutions of non-oscilllating terms irrelevant

Alternatives for separating neutral and charged Bs

- Assume $D' = D$:

$$D = D^0 + (D_c^0 - D^0)(1 - f_{B^0})$$

- Assume $f_{B^0} = \tilde{f}$ known

$$D = D^0 f_{B^0}^0 / \tilde{f}$$

- Assume $\tilde{D}_c$ known, say from fully reco'd charged Bs

$$D = D^0 \left[1 - \frac{\tilde{D}_c - D_c^0}{D_c^0} \left(1 - \frac{1}{f_{B^0}} \right) \right]^{-1}$$

- Take $D_c - D = \Delta$ from Monte Carlo

$$D = D^0 + (1 - f_{B^0})(D_c^0 - D^0 - \Delta)$$

Errors here seem to be of order 0.02 in D.
Acceptable for now.

Problem is Generic

- All backgrounds arising from neutral or charged Bs have this feature
  - Most of combinatorial
  - Most of wrong/fake leptons
  - Charm not so different
  - Only continuum is different

If no continuum background, TagMix measures only

$$r = \sum_i f_0^i D_0^i; \qquad r' = \sum_i f_\pm^i D_\pm^i$$

To find the real dilution we take

$$D^{sig} = \frac{1}{f_{sig}}(r - \sum_{bkgd\ i} f_0^i D_0^i)$$

- Get $\sum_{bkgd\ i} f_0^i D_0^i$ from sidebands
- Need $f_{sig} = 1 - \sum_{bkgd\ i} f_0^i - \sum_i f_\pm^i$.
- Don't need charged B dilutions

Can We Ignore $\tau = 0$ (continuum) Component?

- "Fast-Exact MC" study
  - Fix all parameters except neutral, charged B dilutions
  - Ignore oscillatory combinatorial background
  - Fix combinatorial background to 20%
  - Fix "signal" to be 90% charged, 10% neutral
  - Fix $\tau_{comb} = 1.5$ ps
  - Fix amount of $\tau = 0$ but fit to wrong amount

$\tau = 0$ true	$\tau = 0$ assumed	true signal dilution	found signal dilution
0.0	0.0	0.600	0.600
0.25	0.0	0.600	0.618
0.50	0.0	0.600	0.638
1.00	0.0	0.600	0.688

- Encouraging evidence that effect of $\tau = 0$ is minor
- Generic MC results indicate small $\tau = 0$ contribution
- Tests in data require full (three-component) fit

If we ignore oscillating term in background?

	Input	Output
$D(B^0)$	0.6	0.635
$D(B^\pm)$	0.7	0.373
$f(B^0$	0.9, fixed	0.9, fixed
$f(comb)$	0.8, fixed	0.8
$f(\tau = 0)$	0.333	0.329
$f(\tau = 1.56 \text{ ps})$ no mixing	0.333	0.671
$f(\tau = 1.56 \text{ ps})$ mixing	0.333	0.0, fixed
$D(\tau = 0)$	0.4, fixed	0.4, fixed
$D(\tau = 1.56 \text{ ps})$	0.4, fixed	0.4, fixed
$D(\tau = 1.56 \text{ ps})$	0.4, fixed	0.4, fixed

Check above results:

Oscillating: $\sum f_0^i D_0^i$:

In:

$$0.9 \times 0.8 \times 0.6 + (1/3) \times 0.2 \times 0.4 = 0.459$$

Out:

$$0.9 \times 0.8 \times 0.635 = 0.457$$

Non-oscillating $\sum f_\pm^i D_\pm^i$:

In:

$$0.1 \times 0.8 \times 0.7 + (2/3) \times 0.2 \times 0.4 = 0.109$$

Out:

$$0.1 \times 0.8 \times 0.373 + (3/3) \times 0.2 \times 0.4 = 0.110$$

Simplified Picture

Note

$$D^{sig} = \frac{1}{1 - \sum_i f^i_\pm} (r - \sum_{bkgd\ i} f^i_0 (D^i_0 - D))$$

If backgrounds and signal from neutral Bs have same dilutions

$$D^{sig} = \frac{r}{1 - \sum_i f^i_\pm}$$

- Correct for $D_0^{i\ bkd} \neq D_0^{sig}$
- Correct for continuum
- Benefit from simplicity
- Systematic errors more evident

Proposed Simple Confirming Analysis

- Ignore sidebands
- Fit data in signal region as if just charged and neutral B
- Use measured or estimated charged B rate to correct
- Charm-originating events count as charged B

Conclusion: Uncertainty in "charged B" rate is dominating systematic

Comments

- No contradiction with standard methods
- Minuit knows all this...
- Must determine whether continuum is small
- Need to try simplified approach
- Study of errors needed - use ‘fast MC’
- Highest priority:
  - $f_{B^\pm}$, f_{charm}, $f_{B^\pm \ wrong \ leptons}$, $fake \ leptons...$
  - $f_{continuum}$: what is correction here? Evidently small.
- Next priority
  - Fractions and dilutions of oscillating backgrounds
- Please send me the answers!