Emittanace Exchange with Super FOFO Lattice

DRAFT

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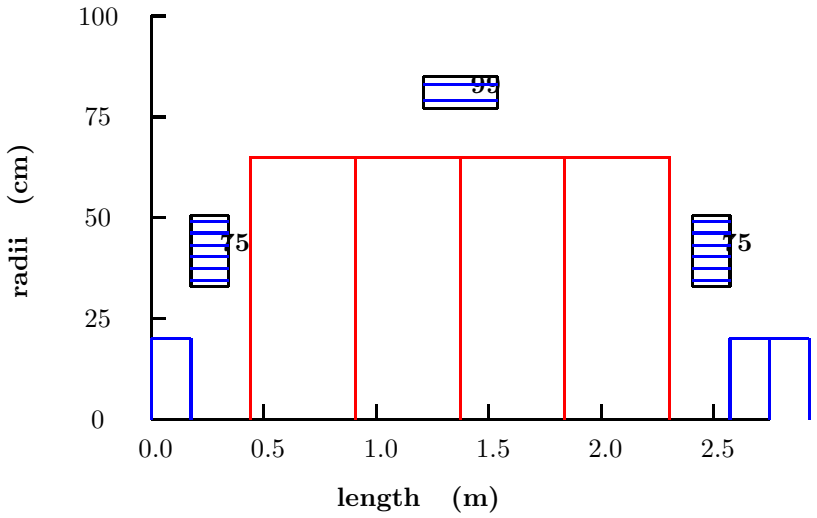
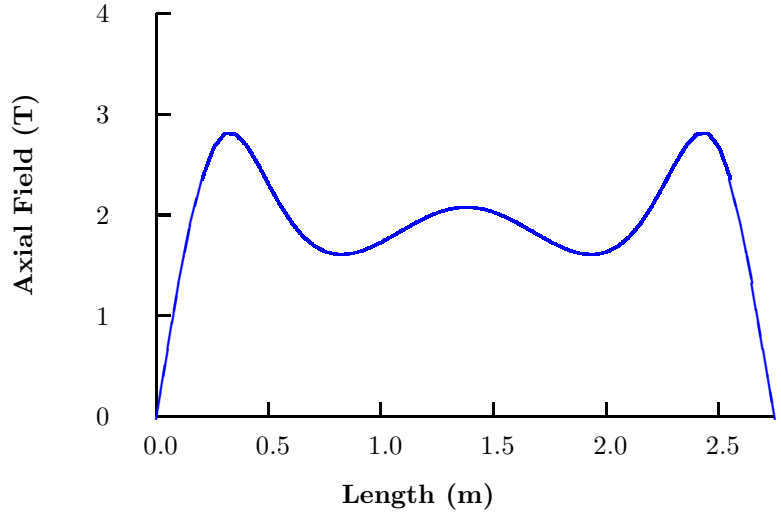
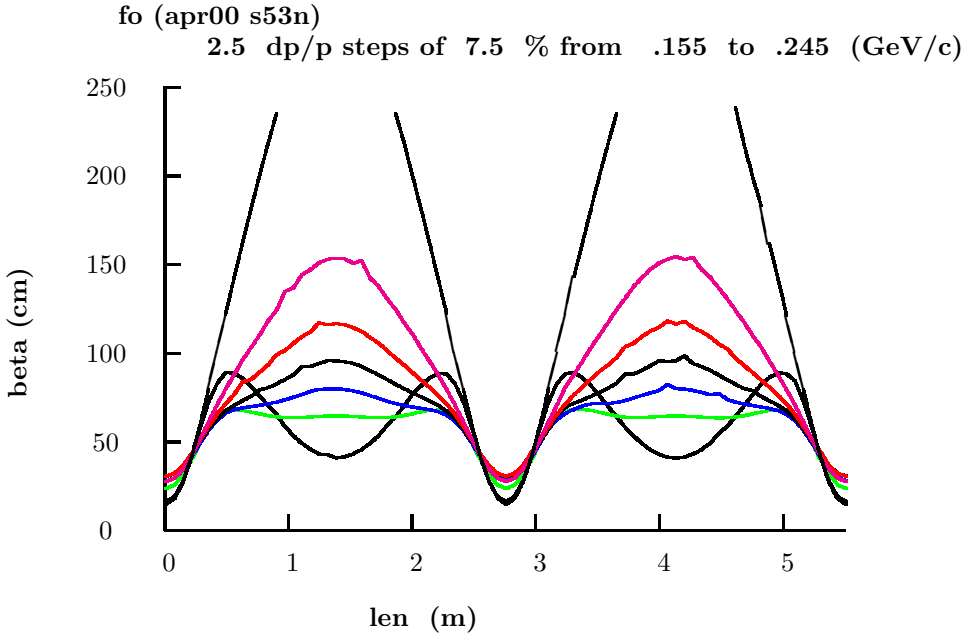
0.1 Super FOFO

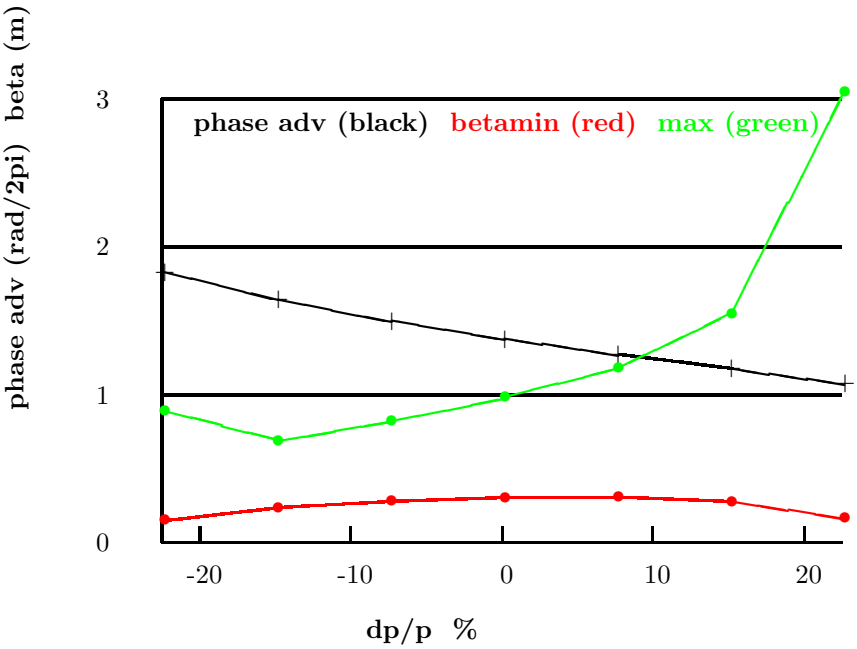
Consider modifying the initial super FOFO lattice as used now in the Study 2 simulations. The cell length is 2.75 m, the β at the absorber is $\approx$ .3 m and it accepts an emittance ($\epsilon_n \approx 10$ mm rad), and a momentum spread of +/− 22 %.

Figs (a) and (b) show the coils and axial field. Fig (c) shows the beta functions for a range of momenta, as a function of position down the cell. Fib (d) shows the phase advance per cell vs momentum (it is realtively linear between limits at π and 2 π where there are resonaces), and the maximum and minimum β's. It is seen that the *beta* at the absorber (red) is realtively flat, but the β at the center is strongly mometum dependent, rising from about .4 m at one limit and 3 m at the other (this will turn out ot be important).



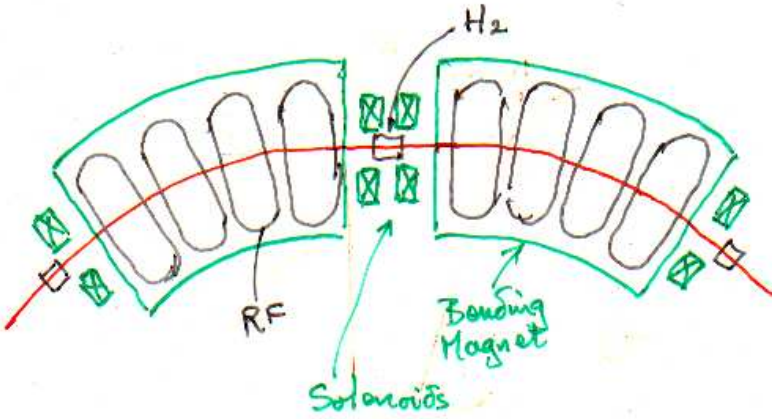

Axial Field (T)
4
3
2
1
0
0.0
0.5
1.0
1.5
2.0
2.5
Length (m)


fo (apr00 s53n)
2.5 dp/p steps of 7.5 % from .155 to .245 (GeV/c)
beta (cm)
250
200
150
100
50
0
0
1
2
3
4
5
len (m)



0.2 Add a Bend

In order to use this lattice for emittance exchange we must add some bending to generate dispersion at the absorber. The absorber can then be made wedge like and the required exchange obtained. Consider putting the bend in the center of the cell¿ It now falls on top of the coupling solenoid, which is uncomfortable. But bening magnets have focussing: perhaps it can replace this solenoid.

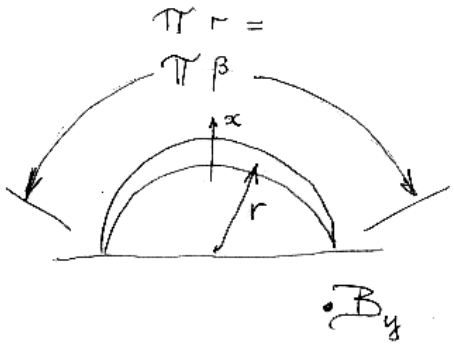


So the questions are:

1. How strong is the dipole to give focussing equivalent to that from the coupling solenoid?
2. Is the dispersion generated enough?
3. What is the resulting momentum compaction and what has it done to the bucket?
4. Can we mix x and y so as to exchange longitudinal emittance with both transvers emittances.
5. how linear is the dispersion. i.e. how independent of momentum is it?
6. does it work.

The answer to the last question must await simulation, but we can draw some preliminary conclusions from a simple theoretical analysis.

0.3 Gradient focussing



If there is no gradient, then the horizontal focussing β_x in a dipole is equal to the radius of bend, and there is no vertical focussing. If a gradient is chosen to give equal horizontal and vertical focussing, then the focussing $\beta_{x,y}$ will be double the bending radius:

$$\beta_{x,y} = \frac{2\ p}{B_\perp\ c}$$

Surprisingly this has the same form as the focussing from a long solenoid:

$$\beta_{x,y} = \frac{2\ p}{B_z\ c}$$

Thus the bending field needed for focussing is of the same as the solenoid in the unmodified lattice: about 2 T in this case.

0.4 Dispersion

To answer the second question we must estimate the required bend to give the required dispersion. We can do this by considering a normalized phase diagram in dispersion $\mathbf{D}$ and angular dispersion $\mathbf{D}'$. In the absence of a bend, the normalized dispersion moves on a circle. A local bend changes the angular dispersion $\mathbf{D}'$, but not $\mathbf{D}$. So for our central phase advance $\chi \approx 3\pi/4$ and a thin dipole bend, the diagram is:

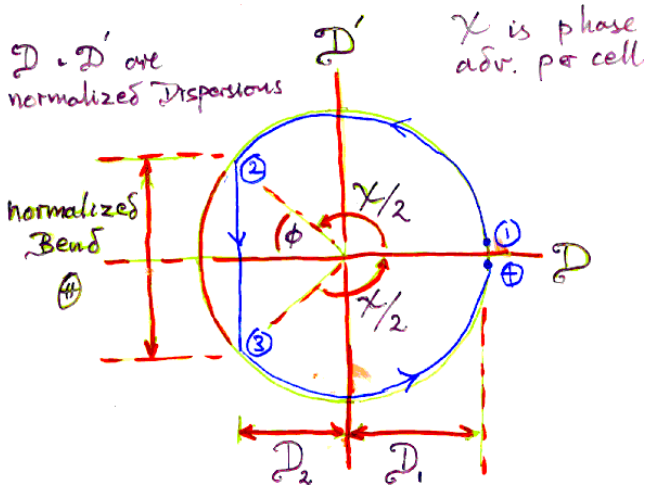


Fig 1

where the un-normalized dispersions are:

$$D_i = \mathbf{D}_i \sqrt{\beta_i}$$

$$D'_i = \mathbf{D}'_i \sqrt{\frac{1}{\beta_i}}$$

A thin bend through an angle θ changes D' by θ, so we can define a normalized bend angle Θ:

$$\Delta D' = \theta = \Theta \sqrt{\beta_\theta}$$

where β_θ is the β at the bend.

Note that the normalized variables all have the dimensions of $\sqrt{length}$.
from the diagram:

$$\frac{\Theta}{2} = \mathbf{D}_1 \sin(\chi)$$

so

$$\mathbf{D}_1 = \frac{\Theta}{2\sin(\chi)}$$

and the un-normalized dispersion:

$$\mathbf{D}_1 \;=\; \frac{\Theta}{2\,\sin(\chi)}\;\sqrt{\beta_1\;\beta_2}$$

For significant emittance exchange, we require:

$$|D_1\;\delta| \;=\; k\;\sigma_1$$

where $\sigma_1 = \sqrt{\frac{\beta_1\;\epsilon_n}{\beta_v\gamma}}$, and k is of order 1 from which we obtain:

$$\theta \;=\; 2\sin(\chi) \qquad \frac{k}{\delta}\;\;\sigma_\theta \qquad \sqrt{\frac{\beta_1}{\beta_2}}$$

where

$$\sigma_\theta \;=\; \sqrt{\frac{\epsilon_n}{\beta_1\;\;\beta_v\gamma}}$$

is the rms divergence of the beam at the absorber.

For good transverse cooling, we require $\sigma_\theta \;\approx\; 0.1$. Consider a fractional momentum spread $\delta = 0.07$, and $\sqrt{\frac{\beta_1}{\beta_2}} \;\approx\; 0.5$.

then:

$$\Delta\theta \;=\; \sqrt{2} \qquad \frac{0.7}{.07} \qquad 0.1 \qquad .5$$
$$\approx\; 0.7 \;\;\text{(radians)} \quad \approx 45 \quad \text{(deg)}$$

The available length for bending is about .5 m, so for 200 MeV/c, the required field is :

$$B \;=\; \frac{\Delta\theta\;p}{c\;L} \;\;=\; \frac{.7\;\;.2}{.3\;\;0.5} \;\approx\; 1 \quad \text{(T)}$$

which is close enough to the field ($\approx$ 2 T) needed for coupling that a solution may be found.

Note: the angle of bend $\Delta\theta$ is independent of emittance.
But the required field rises inversely with the falling emittance.

We could address the question of linearity of dispersion here, but we will put that off and do it for the more general case with mixing between x and y.

0.5 Add a twist

In the above concept, the emittance exchange occurs only with the transverse direction in the plane of the bend. In order to exchange with both directions, some kind of x-y mixing is needed. A net axial solenoidal field would do this, skew quads would probably also work, but the easiest solution (from Scott Berg) is to introduce a twist about the z axis at each absorber. In this case the lattice (which was, in fact a ring) becomes a spiral.

We must now find the solution for the dispersions in this case. We now find finite dispersions in both x and y and finite angular dispersion at the cell ends. The object is to obtain the dispertion at the absorber, as a function of the phase advance χ; and thus the displacements of the beam as a function of the momentum dp/p, including any nonlinearities.

Fig 2 shows the dispersions and angular dispersions at the beginning and end of a cell. The requirement is that the angles beteen beginning and end have rotated by the twist angle (ψ):

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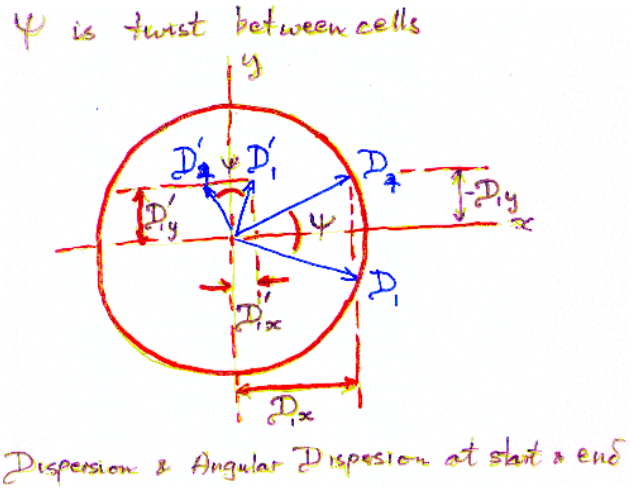

Fig 2

$$\tan\left(\frac{\psi}{2}\right) = \left(\frac{\mathbf{D}_y}{\mathbf{D}_x}\right) = \left(\frac{\mathbf{D}'_x}{\mathbf{D}'_y}\right) = \sqrt{\frac{\mathbf{D}'_x}{\mathbf{D}_x}}\sqrt{\frac{\mathbf{D}_y}{\mathbf{D}'_y}} \qquad (1)$$

The relations between dispersions in x are similar to those obtaines above and shown in fig 1, but now the initial and final dispersions are modified as shown in fig 3:

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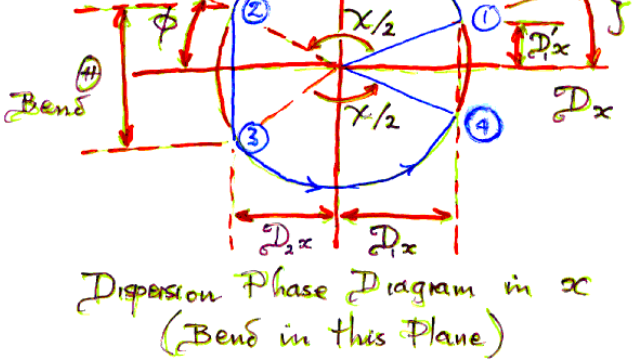


Fig 3

Defining ζ and ϕ:

$$\tan(\zeta) = \frac{\mathbf{D}'_x}{\mathbf{D}_x} \tag{2}$$

$$\phi + \frac{\chi}{2} + \zeta = \pi$$

and the radius of the circle a is:

$$a = \frac{\mathbf{D}_x}{\cos(\zeta)} = \frac{\Theta/2}{\sin(\phi)}$$

In the y plane, there is no bend and the dispersion diagram is simpler: .

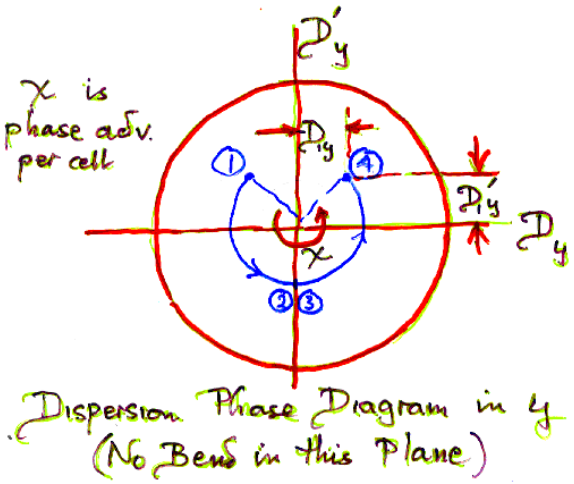


Fig 4

$$\tan\left(\frac{\chi}{2}\right) = \frac{\mathbf{D}_y}{\mathbf{D}'_y}$$

This together with eqn. 2 when substituted into eqn. 1 gives:

$$\tan^2\left(\frac{\psi}{2}\right) = \tan\left(\frac{\chi}{2}\right)\ \tan\left(\zeta\right)$$

So we have:

$$\tan\left(\zeta\right) = \frac{\tan^2\left(\frac{\psi}{2}\right)}{\tan\left(\frac{\chi}{2}\right)}$$

$$\phi = \pi - \frac{\chi}{2} - \zeta$$

$$\mathbf{D} = \frac{\Theta}{2}\,\frac{\cos(\zeta)}{\sin(\phi)}$$

ψ is the fixed twist angle, so we now have the normalized dispersion $\mathbf{D}$ as a function of the phase advance χ, which is, in turn a function of the momentum displacement dp/p as obtined for the particulars of the lattice.

$$\mathbf{D} = fn(dp/p)$$

recalling:

$$D_i = \mathbf{D}_i\ \sqrt{\beta_i}$$

$$D'_i = \mathbf{D}'_i\ \sqrt{\frac{1}{\beta_i}}$$

$$\Delta \mathbf{D} \;=\; \Theta \;=\; \theta\,\sqrt{\beta_\theta}$$

The displacement d for a given momentum diference dp is then:

$$d \;=\; \frac{dp}{p}\,\sqrt{\beta_{1,2}\beta_\theta}\;\mathbf{D}(dp/p)$$

Note: as the twist $\psi \;\rightarrow\; 0$, then $\zeta \rightarrow 0$, $\eta \rightarrow \pi \;-\; \chi$, so:

$$\mathbf{D} \;\rightarrow\; \frac{\Theta}{\sin\left(\pi \;-\; \frac{\chi}{2}\right)}$$

as in the non twist case above.

The lower bound on the phase advance is given by the betatron resonance at $\chi = \pi$. The upper bound that, in the flat case was at $\chi = 2\;\pi$, is now:

$$\chi_{upper} \;=\; 2\;\pi \;-\; 2\;\zeta$$

Thus the momentum acceptance falls as the twist (and thus ζ) increase, but for twist angles less than about 20 degrees, there is little reduction of acceptance.

0.6 Calculations

In the following figures we give values of various parameters as a function of the momentum offests dp/p for:

Bend angle:

$$\theta \;=\; 45 \text{ deg.}$$

twist angle:

$$\psi \;=\; 18 \text{ deg.}$$

and assuming, as are approximately the case here,

$$\chi \;=\; \pi\;\left(1.5 \;+\; 0.5\;\frac{dp/p}{.25}\right)$$

$$\beta_1 \;=\; 0.4\;\sqrt{1 \;-\; \left(\frac{dp/p}{.25}\right)^2}$$

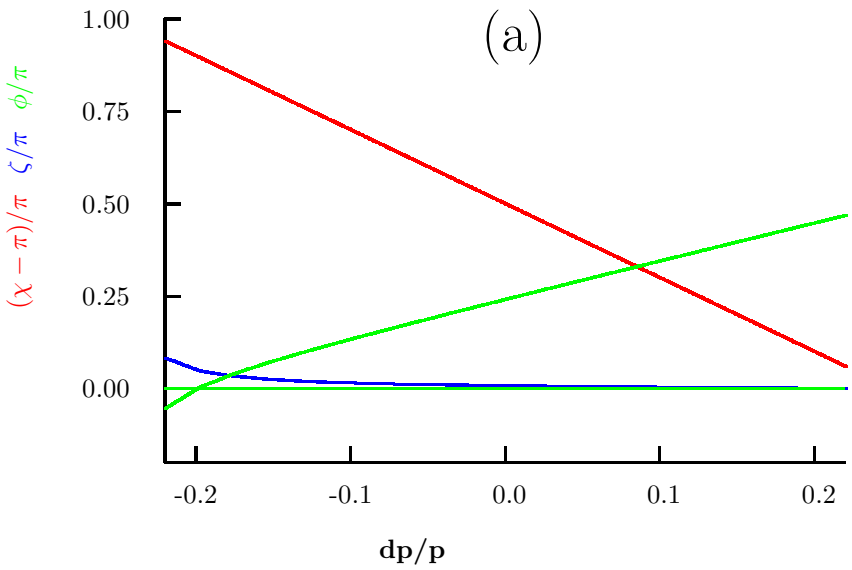
$$\beta_2 \;=\; 2.5\;(dp/p \;+\; .25) \;+\; 10\;\;(dp/p \;+\; .25)^2$$

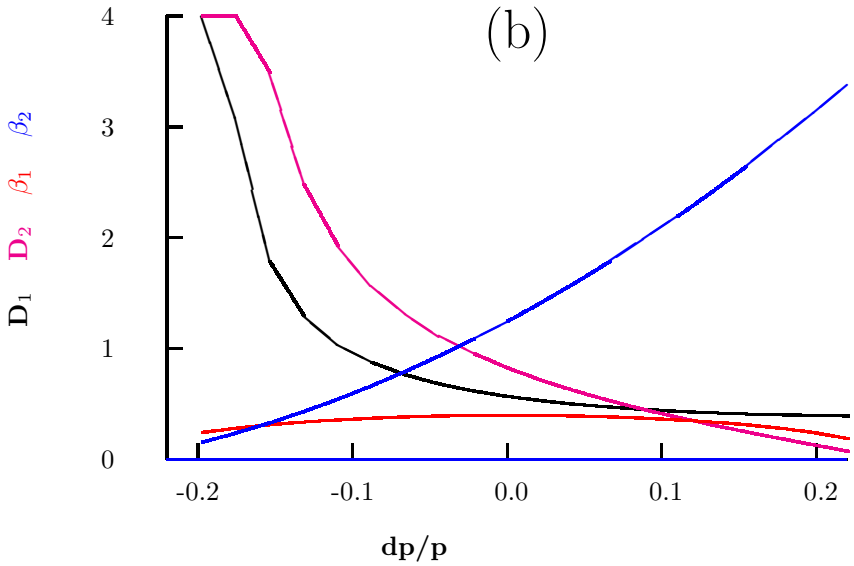
Fig (a) shows the angles (χ, ζ and ϕ). It is seen that ϕ goes through zero at dp/p of -20%. The dispersion will go to infinity at this point, thus limiting the lower bound of the momentum acceptance.

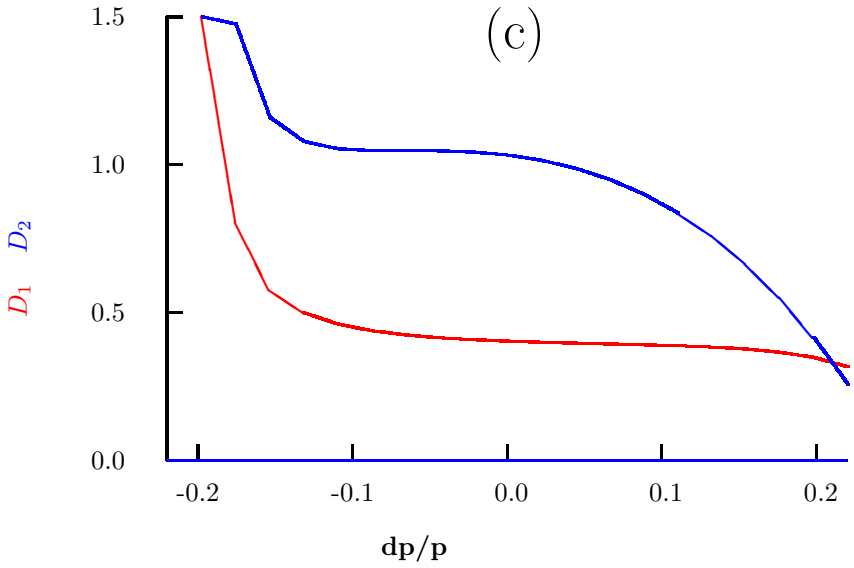
Fig (b) gives the β at the absorber (1) and bend (2) as given by the above approximate fomulae, together with gives the normalized dispersions at the absorber (1) and bend (2). They are seen to vary strongly with momentum.

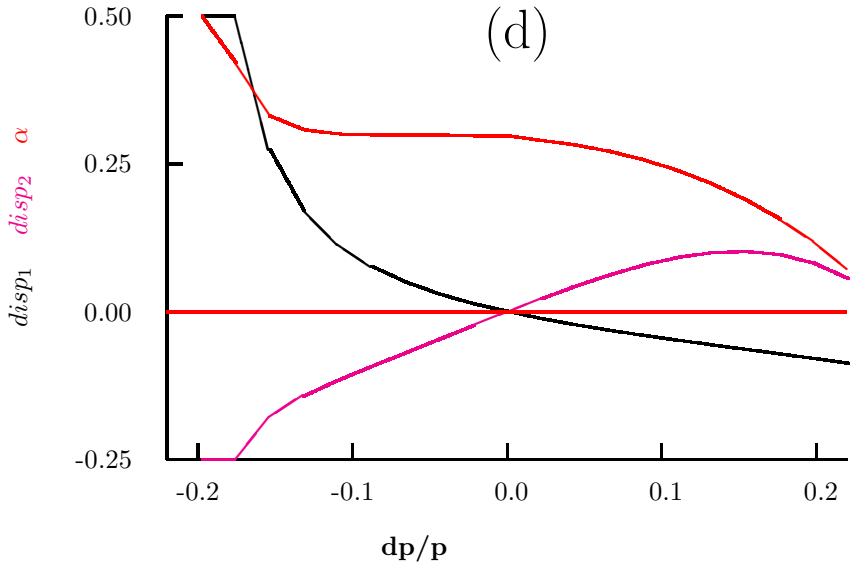
Fig (c) gives the un-normalized dispersions at the absorber (1) and bend (2). These are much more constant than the normalized values due to the strong momentum dependence of β_2.

fig (d) gives the displacements the absorber (1) and bend (2), and the momentum compaction α. The displacemnts are seen to be reasonably linear over a range of $+/-$ 15 %. The momentum compaction α is seen to be quite large ($\approx .25$) which, for $\gamma \approx 2$ puts the lattice at transition (forward velocity independent of momentum). This has the advantage that little rf is needed to maintain the bunch, but, if $\eta = 1/\gamma^2 - \alpha$) is too small, will be unstable. Clearly, some fine tuning is needed here.




(b)
4
3
2
1
0
$\mathbf{D}_1$ $\mathbf{D}_2$ β_1 β_2
-0.2
-0.1
0.0
0.1
0.2
dp/p


(c)
1.5
1.0
0.5
0.0
D_1 D_2
-0.2
-0.1
0.0
0.1
0.2
dp/p



1 Conclusion

So far this solution is looking good, but much work remains to be done.