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# CP Violation In and Beyond the Standard Model

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The special features of CP violation in the Standard Model are presented. The significance of measuring CP violation in $B$, $K$ and $D$ decays is explained. The predictions of the Standard Model for CP asymmetries in $B$ decays are analyzed in detail. Then, four frameworks of new physics are reviewed: (i) Supersymmetry provides an excellent demonstration of the power of CP violation as a probe of new physics. (ii) Left-right symmetric models are discussed as an example of an extension of the gauge sector. CP violation suggests that the scale of LRS breaking is low. (iii) The variety of extensions of the scalar sector are presented and their unique CP violating signatures are emphasized. (iv) Vector-like down quarks are presented as an example of an extension of the fermion sector. Their implications for CP asymmetries in $B$ decays are highly interesting.

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## 1. Introduction

CP violation arises naturally in the three generation Standard Model. The CP violation that has been measured in neutral $K$-meson decays ($\varepsilon_K$ and $\varepsilon'_K$) is accommodated in the Standard Model in a simple way [1]. Yet, CP violation is one of the least tested aspects of the Standard Model. The value of the $\varepsilon_K$ parameter [2] as well as bounds on other CP violating parameters (most noticeably, the electric dipole moments of the neutron, $d_N$, and of the electron, $d_e$) can be accounted for in models where CP violation has features that are very different from the Standard Model ones.

It is unlikely that the Standard Model provides the complete description of CP violation in nature. First, it is quite clear that there exists New Physics beyond the Standard Model. Almost any extension of the Standard Model has additional sources of CP violating effects (or effects that change the relationship of the measurable quantitites to the CP violating parameters of the Standard Model). In addition there is a great puzzle in cosmology that relates to CP violation, and that is the baryon asymmetry of the universe [3]. Theories that explain the observed asymmetry must include new sources of CP violation [4]: the Standard Model cannot generate a large enough matter-antimatter imbalance to produce the baryon number to entropy ratio observed in the universe today [5-7].

In the near future, significant new information on CP violation will be provided by various experiments. The main source of information will be measurements of CP violation in various $B$ decays, particularly neutral $B$ decays into final CP eigenstates [8-10]. First attempts have already been reported [11-13] and interpreted in the framework of various models of new physics [14-15]. Another piece of valuable information might come from a measurement of the $K_L \to \pi^0\nu\bar{\nu}$ decay [16-19]. For the first time, the pattern of CP violation that is predicted by the Standard Model will be tested. Basic questions such as whether CP is an approximate symmetry in nature will be answered.

It could be that the scale where new CP violating sources appear is too high above the Standard Model scale (e.g. the GUT scale) to give any observable deviations from the Standard Model predictions. In such a case, the outcome of the experiments will be a (frustratingly) successful test of the Standard Model and a significant improvement in our knowledge of the CKM matrix.

A much more interesting situation will arise if the new sources of CP violation appear at a scale that is not too high above the electroweak scale. Then they might be discovered in the forthcoming experiments. Once enough independent observations of CP violating effects are made, we will find that there is no single choice of CKM parameters that is consistent with all measurements. There may even be enough information in the pattern of the inconsistencies to tell us something about the nature of the new physics contributions [20-22].

The aim of these lectures is to explain the theoretical tools with which we will analyze new information about CP violation. The first part, chapters 2-6, deal with the Standard Model while the second, chapters 7-10, discuss physics beyond the Standard Model. In chapter 2, we present the Standard Model picture of CP violation. We emphasize the features that are unique to the Standard Model. In chapter 3, we give a brief, model-independent discussion of CP violating observables in $B$ meson decays. In chapter 4, we discuss CP violation in the $K$ system (particularly, the $\varepsilon_K$ and $\varepsilon'_K$ parameters) in a model independent way and in the framework of the Standard Model. We also describe CP violation in $K \to \pi\nu\bar{\nu}$. In chapter 5, we discuss CP violation in $D \to K\pi$ decays. In chapter 6, we present in detail the Standard Model predictions for CP asymmetries in $B$ decays. In chapter 7, the power of CP violation as a probe of new physics is explained. Then, we discuss specific frameworks of new physics: Supersymmetry (chapter 8), Left-Right symmetry as an example of extensions of the gauge sector (chapter 9), extensions of the scalar sector (chapter 10), and extra down singlet-quarks as an example of extensions of the fermion sector (chapter 11). Finally, we summarize our main points in chapter 12.

## 2. Theory of CP Violation in the Standard Model

### *2.1. Yukawa Interactions Are the Source of CP Violation*

A model of the basic interactions between elementary particles is defined by the following three ingredients:

(i) The symmetries of the Lagrangian;

(ii) The representations of fermions and scalars;

(iii) The pattern of spontaneous symmetry breaking.

The Standard Model (SM) is defined as follows:

(i) The gauge symmetry is

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}. \tag{2.1}$$

(ii) There are three fermion generations, each consisting of five representations:

$$Q^I_{Li}(3,2)_{+1/6}, \quad u^I_{Ri}(3,1)_{+2/3}, \quad d^I_{Ri}(3,1)_{-1/3}, \quad L^I_{Li}(1,2)_{-1/2}, \quad \ell^I_{Ri}(1,1)_{-1}. \tag{2.2}$$

Our notations mean that, for example, the left-handed quarks, $Q^I_L$, are in a triplet (3) of the $SU(3)_{\rm C}$ group, a doublet (2) of $SU(2)_{\rm L}$ and carry hypercharge $Y = Q_{\rm EM} - T_3 = +1/6$. The index $I$ denotes interaction eigenstates. The index $i = 1,2,3$ is the flavor (or generation) index. There is a single scalar multiplet:

$$\phi(1,2)_{+1/2}. \tag{2.3}$$

(iii) The $\phi$ scalar assumes a VEV,

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \tag{2.4}$$

so that the gauge group is spontaneously broken:

$$G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}. \tag{2.5}$$

The Standard Model Lagrangian, $\mathcal{L}_{\rm SM}$, is the most general renormalizable Lagrangian that is consistent with the gauge symmetry $G_{\rm SM}$ of eq. (2.1). It can be divided to three parts:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}. \tag{2.6}$$

As concerns the kinetic terms, to maintain gauge invariance, one has to replace the derivative with a covariant derivative:

$$D^\mu = \partial^\mu + ig_s G^\mu_a L_a + igW^\mu_b T_b + ig' B^\mu Y. \tag{2.7}$$

Here $G_a^\mu$ are the eight gluon fields, $W_b^\mu$ the three weak interaction bosons and $B^\mu$ the single hypercharge boson. The $L_a$'s are $SU(3)_{\rm C}$ generators (the $3\times3$ Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the $T_b$'s are $SU(2)_{\rm L}$ generators (the $2\times2$ Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and $Y$ are the $U(1)_{\rm Y}$ charges. For example, for the left-handed quarks $Q_L^I$, we have

$$\begin{aligned}\mathcal{L}_{\rm kinetic}(Q_L) &= i\overline{Q_{Li}^I}\gamma^\mu D_\mu Q_{Li}^I,\\ D^\mu Q_{Li}^I &= \left(\partial^\mu+\frac{i}{2}g_s G_a^\mu\lambda_a+\frac{i}{2}gW_b^\mu\tau_b+\frac{i}{6}g'B^\mu\right)Q_{Li}^I.\end{aligned} \tag{2.8}$$

This part of the interaction Lagrangian is always CP conserving.

The Higgs potential, which describes the scalar self interactions, is given by:

$$\mathcal{L}_{\rm Higgs}=\mu^2\phi^\dagger\phi-\lambda(\phi^\dagger\phi)^2. \tag{2.9}$$

For the Standard Model scalar sector, where there is a single doublet, this part of the Lagrangian is also CP conserving. For extended scalar sector, such as that of a two Higgs doublet model, $\mathcal{L}_{\rm Higgs}$ can be CP violating. Even in case that it is CP symmetric, it may lead to spontaneous CP violation.

The Yukawa interactions are given by

$$-\mathcal{L}_{\rm Yukawa}=Y_{ij}^d\overline{Q_{Li}^I}\phi d_{Rj}^I+Y_{ij}^u\overline{Q_{Li}^I}\tilde{\phi}u_{Rj}^I+Y_{ij}^\ell\overline{L_{Li}^I}\phi\ell_{Rj}^I+{\rm h.c.}. \tag{2.10}$$

This part of the Lagrangian is, in general, CP violating. More precisely, CP is violated if and only if [23]

$$\mathcal{I}m\left\{\det[Y^dY^{d\dagger},Y^uY^{u\dagger}]\right\}\neq0. \tag{2.11}$$

An intuitive explanation of why CP violation is related to *complex* Yukawa couplings goes as follows. The hermiticity of the Lagrangian implies that $\mathcal{L}_{\rm Yukawa}$ has its terms in pairs of the form

$$Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj}+Y_{ij}^*\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}. \tag{2.12}$$

A CP transformation exchanges the operators

$$\overline{\psi_{Li}}\phi\psi_{Rj}\leftrightarrow\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \tag{2.13}$$

but leaves their coefficients, $Y_{ij}$ and $Y^*_{ij}$, unchanged. This means that CP is a symmetry of $\mathcal{L}_{\rm Yukawa}$ if $Y_{ij} = Y^*_{ij}$.

How many independent CP violating parameters are there in $\mathcal{L}_{\rm Yukawa}$? Each of the three Yukawa matrices $Y^f$ is $3 \times 3$ and complex. Consequently, there are 27 real and 27 imaginary parameters in these matrices. Not all of them are, however, physical. If we switch off the Yukawa matrices, there is a global symmetry added to the Standard Model,

$$G^{\rm SM}_{\rm global}(Y^f = 0) = U(3)_Q \times U(3)_{\bar d} \times U(3)_{\bar u} \times U(3)_L \times U(3)_{\bar \ell}. \qquad (2.14)$$

A unitary rotation of the three generations for each of the five representations in (2.2) would leave the Standard Model Lagrangian invariant. This means that the physics described by a given set of Yukawa matrices $(Y^d, Y^u, Y^\ell)$, and the physics described by another set,

$$\tilde Y^d = V_Q^\dagger Y^d V_{\bar d}, \quad \tilde Y^u = V_Q^\dagger Y^u V_{\bar u}, \quad \tilde Y^\ell = V_L^\dagger Y^\ell V_{\bar \ell}, \qquad (2.15)$$

where $V$ are all unitary matrices, is the same. One can use this freedom to remove, at most, 15 real and 30 imaginary parameters (the number of parameters in five $3 \times 3$ unitary matrices). However, the fact that the Standard Model with the Yukawa matrices switched on has still a global symmetry of

$$G^{\rm SM}_{\rm global} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \qquad (2.16)$$

means that only 26 imaginary parameters can be removed. We conclude that there are 13 flavor parameters: 12 real ones and a single phase. This single phase is the source of CP violation.

### *2.2. Quark Mixing is the (Only!) Source of CP Violation*

Upon the replacement $\mathcal{R}e(\phi^0) \to (v + H^0)/\sqrt{2}$ (see eq. (2.4)), the Yukawa interactions (2.10) give rise to mass terms:

$$-\mathcal{L}_M = (M_d)_{ij} \overline{d^I_{Li}} d^I_{Rj} + (M_u)_{ij} \overline{u^I_{Li}} u^I_{Rj} + (M_\ell)_{ij} \overline{\ell^I_{Li}} \ell^I_{Rj} + {\rm h.c.}, \qquad (2.17)$$

where

$$M_f = \frac{v}{\sqrt{2}} Y^f, \qquad (2.18)$$

and we decomposed the $SU(2)_{\rm L}$ doublets into their components:

$$Q^I_{Li} = \begin{pmatrix} u^I_{Li} \\ d^I_{Li} \end{pmatrix}, \quad L^I_{Li} = \begin{pmatrix} \nu^I_{Li} \\ \ell^I_{Li} \end{pmatrix}. \tag{2.19}$$

Since the Standard Model neutrinos have no Yukawa interactions, they are predicted to be massless.

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices $V_{fL}$ and $V_{fR}$ such that

$$V_{fL} M_f V^\dagger_{fR} = M_f^{\rm diag}, \tag{2.20}$$

with $M_f^{\rm diag}$ diagonal and real. The mass eigenstates are then identified as

$$\begin{aligned} d_{Li} &= (V_{dL})_{ij} d^I_{Lj}, \quad d_{Ri} = (V_{dR})_{ij} d^I_{Rj}, \\ u_{Li} &= (V_{uL})_{ij} u^I_{Lj}, \quad u_{Ri} = (V_{uR})_{ij} u^I_{Rj}, \\ \ell_{Li} &= (V_{\ell L})_{ij} \ell^I_{Lj}, \quad \ell_{Ri} = (V_{\ell R})_{ij} \ell^I_{Rj}, \\ \nu_{Li} &= (V_{\nu L})_{ij} \nu^I_{Lj}. \end{aligned} \tag{2.21}$$

Since the Standard Model neutrinos are massless, $V_{\nu L}$ is arbitrary.

The charged current interactions (that is the interactions of the charged $SU(2)_{\rm L}$ gauge bosons $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)$) for quarks, which in the interaction basis are described by (2.8), have a complicated form in the mass basis:

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \overline{u_{Li}} \gamma^\mu (V_{uL} V^\dagger_{dL})_{ij} d_{Lj} W^+_\mu + {\rm h.c.}. \tag{2.22}$$

The unitary $3\times 3$ matrix,

$$V_{\rm CKM} = V_{uL} V^\dagger_{dL}, \quad (V_{\rm CKM} V^\dagger_{\rm CKM} = 1), \tag{2.23}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) *mixing matrix* for quarks [24,1]. A unitary $3\times 3$ matrix depends on nine parameters: three real angles and six phases.

The form of the matrix is not unique. Usually, the following two conventions are employed:

($i$) There is freedom in defining $V_{\rm CKM}$ in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, i.e. $m_{u_1} < m_{u_2} < m_{u_3}$ and $m_{d_1} < m_{d_2} < m_{d_3}$. Usually, we call $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$, and the elements of $V_{\rm CKM}$ are written as follows:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{2.24}$$

$(ii)$ There is further freedom in the phase structure of $V_{\rm CKM}$. Let us define $P_f$ ($f = u, d, \ell$) to be diagonal unitary (phase) matrices. Then, if instead of using $V_{fL}$ and $V_{fR}$ for the rotation (2.21) to the mass basis we use $\tilde{V}_{fL}$ and $\tilde{V}_{fR}$, defined by $\tilde{V}_{fL} = P_f V_{fL}$ and $\tilde{V}_{fR} = P_f V_{fR}$, we still maintain a legitimate mass basis since $M_f^{\rm diag}$ remains unchanged by such transformations. However, $V_{\rm CKM}$ does change:

$$V_{\rm CKM} \to P_u V_{\rm CKM} P_d^*. \tag{2.25}$$

This freedom is fixed by demanding that $V_{\rm CKM}$ will have the minimal number of phases. In the three generation case $V_{\rm CKM}$ has a single phase. (There are five phase differences between the elements of $P_u$ and $P_d$ and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of *CP violation* in the Standard Model [1]. For example, the elemnts of the CKM matrix can be written as follows (this is the standard parametrization [25,26]):

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm KM}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm KM}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm KM}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm KM}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm KM}} & c_{23}c_{13} \end{pmatrix}, \tag{2.26}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The three $\sin\theta_{ij}$ are the three real mixing parameters.

As a result of the fact that $V_{\rm CKM}$ is not diagonal, the $W^\pm$ gauge bosons couple to quark (mass eigenstates) of different generations. Within the Standard Model, this is the only source of *flavor changing* interactions. In principle, there could be additional sources of flavor mixing (and of CP violation) in the lepton sector and in $Z^0$ interactions. We now explain why, within the Standard Model, this does not happen.

*Mixing in the lepton sector*: An analysis similar to the above applies also to the left-handed leptons. The mixing matrix is [27] $V_{\rm MNS} = V_{\nu L} V_{\ell L}^\dagger$. However, we can use the arbitrariness of $V_{\nu L}$ (related to the masslessness of neutrinos) to choose $V_{\nu L} = V_{\ell L}$, and the mixing matrix becomes a unit matrix. We conclude that the masslessness of neutrinos (if true) implies that there is no mixing in the lepton sector. If neutrinos have masses then the leptonic charged current interactions will exhibit mixing and CP violation.

*Mixing in neutral current interactions*: We study the interactions of the neutral $Z$-boson, $Z^\mu = \cos\theta_W W_3^\mu - \sin\theta_W B^\mu$ ($\tan\theta_W \equiv g'/g$) with, for example, left-handed down quarks. The $W_3$-interactions are given in (2.8), while the $B$ interactions are given by

$$-\mathcal{L}_B(Q_L) = -\frac{g'}{6}\overline{Q_{Li}^I}\gamma^\mu Q_{Li}^I B_\mu. \tag{2.27}$$

In the mass basis, we have then

$$\begin{aligned}-\mathcal{L}_Z =& \frac{g}{\cos\theta_W}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\overline{d_{Li}}\gamma^\mu(V_{dL}^\dagger V_{dL})_{ij}d_{Lj}Z_\mu \\ =& \frac{g}{\cos\theta_W}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\overline{d_{Li}}\gamma^\mu d_{Li}Z_\mu.\end{aligned} \tag{2.28}$$

We learn that the neutral current interactions remain universal in the mass basis and there are no additional flavor parameters in their description. This situation goes beyond the Standard Model to all models where all left-handed quarks are in $SU(2)_{\rm L}$ doublets and all right-handed ones in singlets. The $Z$-boson does have flavor changing couplings in models where this is not the case.

Examining the mass basis one can easily identify the flavor parameters. In the quark sector, we have six quark masses, three mixing angles (the number of real parameters in $V_{\rm CKM}$) and the single phase $\delta_{\rm KM}$ mentioned above. In the lepton sector, we have the three charged lepton masses.

We have also learnt now some of the special features of CP violation in the Standard Model:

(i) CP is explicitly broken.

(ii) There is a single source of CP violation, that is $\delta_{\rm KM}$.

(iii) CP violation appears only in the charged current interactions of quarks.

(iv) CP violation is closely related to flavor changing interactions.

### *2.3. The CKM Matrix and the Unitarity Triangles*

In the mass basis, CP violation is related to the CKM matrix. The fact that there are only three real and one imaginary physical parameters in $V_{\rm CKM}$ can be made manifest by choosing an explicit parametrization. One example was given above, in eq. (2.26), with the four parameters $(s_{12}, s_{23}, s_{13}, \delta_{\rm KM})$. Another, very useful, example is the Wolfenstein parametrization of $V_{\rm CKM}$, where the four mixing parameters are $(\lambda, A, \rho, \eta)$ with $\lambda = |V_{us}| = 0.22$ playing the role of an expansion parameter and $\eta$ representing the CP violating phase [28]:

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{2.29}$$

Various parametrizations differ in the way that the freedom of phase rotation, eq. (2.25), is used to leave a single phase in $V_{\rm CKM}$. One can define, however, a CP violating quantity in $V_{\rm CKM}$ that is independent of the parametrization [23]. This quantity is called $J$ and defined through

$$\mathcal{I}m[V_{ij}V_{kl}V_{il}^*V_{kj}^*] = J \sum_{m,n=1}^{3} \epsilon_{ikm}\epsilon_{jln}, \quad (i,j,k,l = 1,2,3). \tag{2.30}$$

CP is violated in the Standard Model only if $J \neq 0$.

The usefulness of $J$ may not be clear from its formal definition in (2.30), but does give useful insights once the *unitarity triangles* are introduced. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{2.31}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{2.32}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{2.33}$$

Each of the three relations (2.31)-(2.33) requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (2.33) only. It is a surprising feature of the CKM matrix that all unitarity triangles are equal in area: the area of each unitarity triangle equals $|J|/2$ while the sign of $J$ gives the direction of the complex vectors around the triangles. The relation between Jarlskog's measure of CP violation $J$ and the Wolfenstein parameters is given by

$$J \simeq \lambda^6 A^2 \eta. \tag{2.34}$$

The rescaled unitarity triangle is derived from (2.33) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters $(\rho,\eta)$. The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the $(\rho,\eta)$ plane, the lengths of the two complex sides are

$$R_u \equiv \sqrt{\rho^2+\eta^2} = \frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|, \quad R_t \equiv \sqrt{(1-\rho)^2+\eta^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|. \tag{2.35}$$

The three angles of the unitarity triangle are denoted by $\alpha,\beta$ and $\gamma$ [29]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{2.36}$$

They are physical quantities and, we will soon see, can be independently measured by CP asymmetries in $B$ decays. It is also useful to define the two small angles of the unitarity triangles (2.32) and (2.31):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{2.37}$$

To make predictions for future measurements of CP violating observables, we need to find the allowed ranges for the CKM phases. There are three ways to determine the CKM parameters (see *e.g.* [30]):

(i) **Direct measurements** are related to SM tree level processes. At present, we have direct measurements of $|V_{ud}|$, $|V_{us}|$, $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$, $|V_{cb}|$ and $|V_{tb}|$.

(ii) **CKM Unitarity** ($V_{\rm CKM}^\dagger V_{\rm CKM} = \mathbf{1}$) relates the various matrix elements. At present, these relations are useful to constrain $|V_{td}|$, $|V_{ts}|$, $|V_{tb}|$ and $|V_{cs}|$.

(iii) **Indirect measurements** are related to SM loop processes. At present, we constrain in this way $|V_{tb}V_{td}|$ (from $\Delta m_B$ and $\Delta m_{B_s}$) and $\delta_{\rm KM}$ or, equivalently, $\eta$ (from $\varepsilon_K$).

When all available data is taken into account, we find [31-36]:

$$\lambda = 0.2205 \pm 0.0018, \quad A = 0.826 \pm 0.041, \tag{2.38}$$

$$-0.15 \leq \rho \leq +0.35, \quad +0.20 \leq \eta \leq +0.45, \tag{2.39}$$

$$0.4 \leq \sin 2\beta \leq 0.8, \quad -0.9 \leq \sin 2\alpha \leq 1.0, \quad 0.23 \leq \sin^2\gamma \leq 1.0. \tag{2.40}$$

Of course, there are correlations between the various parameters. The full information can be described by allowed regions in the $(\rho, \eta)$ or the $(\sin 2\alpha, \sin 2\beta)$ planes (see e.g. [35]). (Recently, it has been shown that $B \to \pi K$ decays can provide bounds on the angle $\gamma$ of the unitarity triangle [37-41]. A bound of $\cos\gamma \leq 0.32$ was derived in [40]. We did not incorporate these bounds into our analysis.)

Eqs. (2.39) and (2.40) show yet another important feature of CP violation in the Standard Model. The fact that $\eta/\rho = \mathcal{O}(1)$ or, equivalently, $\sin\gamma = \mathcal{O}(1)$, implies that *CP is not an approximate symmetry* within the Standard Model. This is not an obvious fact: after all, the two measured CP violating quantities, $\varepsilon_K$ and $\varepsilon'_K$, are very small (orders $10^{-3}$ and $10^{-6}$, respectively). The Standard Model accounts for their smallness by the smallness of the flavor violation, that is the mixing angles, and not by the smallness of CP violation, that is a small phase. Indeed, the Standard Model predicts that in some (yet unmeasured) processes, the CP asymmetry is of order one.

### *2.4. The Uniqueness of the Standard Model Picture of CP Violation*

In the previous subsections, we have learnt several features of CP violation as explained by the Standard Model:

(i) CP is explicitly broken.

(ii) $\delta_{\mathrm{KM}}$ is the only source of CP violation.

(iii) CP violation appears only in the charged current interactions of quarks.

(iv) CP violation would vanish in the absence of flavor changing interactions.

(v) CP is not an approximate symmetry ($\delta_{\mathrm{KM}} = \mathcal{O}(1)$).

(Non-perturbative corrections to the Standard Model tree-level Lagrangian are expected to induce $\theta_{\mathrm{QCD}}$, a CP violating parameter. This second possible source of CP violation is related to strong interactions and is flavor diagonal. The bounds on the electric dipole moment of the neutron imply that $\theta_{\mathrm{QCD}} \lesssim 10^{-9}$. The Standard Model offers no natural explanation to the smallness of $\theta_{\mathrm{QCD}}$. This puzzle is called 'the strong CP problem'. We assume that it is solved by some type of new physics, such as a Peccei-Quinn symmetry [42], which sets $\theta_{\rm QCD}$ to zero.)

It is important to realize that (a) none of features (i)-(v) is experimentally established and that (b) various reasonable extensions of the Standard Model provide examples where these features do not hold. In particular, it could be that CP violation in Nature has some or all of the following features:

(i) CP is spontaneously broken.

(ii) There are many independent sources of CP violation.

(iii) CP violation appears in lepton interactions and/or in neutral current interactions and/or in new sectors beyond the SM.

(iv) CP violation appears also in flavor diagonal interactions.

(v) CP is an approximate symmetry.

This situation, where the Standard Model has a very unique description of CP violation and experiments have not yet confirmed this description, is the basis for the strong interest, experimental and theoretical, in CP violation. There are two types of unambiguous tests concerning CP violation in the Standard Model: First, since there is a single source of CP violation, all observables are correlated with each other. For example, the CP asymmetries in $B \to \psi K_S$ and in $K \to \pi\nu\bar{\nu}$ are strongly correlated. Second, since CP violation is restricted to flavor changing quark processes, it is predicted to practically vanish in the lepton sector and in flavor diagonal processes. For example, the transverse lepton polarization in semileptonic meson decays, CP violation in $t\bar{t}$ production, and (assuming $\theta_{\rm QCD} = 0$) the electric dipole moment of the neutron are all predicted to be orders of magnitude below the (present and near future) experimental sensitivity.

The experimental investigation of CP violation in $B$ decays will shed light on some but not all of these questions. In particular, it will easily test the question of whether CP is an approximate symmetry: if any $\mathcal{O}(1)$ asymmetry is observed, for example in the $B \to \psi K_S$ mode, then we immediately learn that CP is not an approximate symmetry. It will also test (though probably in a less definitive way) the question of whether the Kobayashi-Maskawa phase is the only source of CP violation. On the other hand, we will learn little on flavor diagonal CP violation and on CP violation outside the quark sector.

It is therefore important to search for CP violation in many different systems.

## 3. CP Violation in Meson Decays

In the previous section, we understood how CP violation arises in the Standard Model. In the next three sections, we would like to understand the implications of this theory for the phenomenology of CP violation in meson decays. Our main focus will be on $B$-meson decays. To do so, we first present a model independent analysis of CP violation in meson decays.

There are three different types of CP violation in meson decays:

(i) CP violation in mixing, which occurs when the two neutral mass eigenstate admixtures cannot be chosen to be CP-eigenstates;

(ii) CP violation in decay, which occurs in both charged and neutral decays, when the amplitude for a decay and its CP-conjugate process have different magnitudes;

(iii) CP violation in the interference of decays with and without mixing, which occurs in decays into final states that are common to $B^0$ and $\bar{B}^0$.

### *3.1. Notations and Formalism*

To define the three types of CP violation in meson decays and to discuss their theoretical calculation and experimental measurement, we first introduce some notations and formalism. We refer specifically to $B$ meson mixing and decays, but most of our discussion applies equally well to $K$, $B_s$ and $D$ mesons.

Our phase convention for the CP transformation law of the neutral $B$ mesons is defined by

$$\mathrm{CP}|B^0\rangle = \omega_B|\bar{B}^0\rangle, \quad \mathrm{CP}|\bar{B}^0\rangle = \omega_B^*|B^0\rangle, \quad (|\omega_B| = 1). \tag{3.1}$$

Physical observables do not depend on the phase factor $\omega_B$. The time evolution of any linear combination of the neutral $B$-meson flavor eigenstates,

$$a|B^0\rangle + b|\bar{B}^0\rangle, \tag{3.2}$$

is governed by the Schrödinger equation,

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} \equiv \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} a \\ b \end{pmatrix}, \tag{3.3}$$

for which $M$ and $\Gamma$ are $2 \times 2$ Hermitian matrices.

The off-diagonal terms in these matrices, $M_{12}$ and $\Gamma_{12}$, are particularly important in the discussion of mixing and CP violation. $M_{12}$ is the dispersive part of the transition amplitude from $B^0$ to $\bar{B}^0$, while $\Gamma_{12}$ is the absorptive part of that amplitude.

The light $B_L$ and heavy $B_H$ mass eigenstates are given by

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle. \tag{3.4}$$

The complex coefficients $q$ and $p$ obey the normalization condition $|q|^2 + |p|^2 = 1$. Note that $\arg(q/p^*)$ is just an overall common phase for $|B_L\rangle$ and $|B_H\rangle$ and has no physical significance. The mass difference and the width difference between the physical states are given by

$$\Delta m \equiv M_H - M_L, \quad \Delta\Gamma \equiv \Gamma_H - \Gamma_L. \tag{3.5}$$

Solving the eigenvalue equation gives

$$(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \qquad \Delta m \Delta\Gamma = 4\mathcal{R}e(M_{12}\Gamma_{12}^*), \tag{3.6}$$

$$\frac{q}{p} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta\Gamma} = -\frac{\Delta m - \frac{i}{2}\Delta\Gamma}{2M_{12} - i\Gamma_{12}}. \tag{3.7}$$

In the $B$ system, $|\Gamma_{12}| \ll |M_{12}|$ (see discussion below), and then, to leading order in $|\Gamma_{12}/M_{12}|$, eqs. (3.6) and (3.7) can be written as

$$\Delta m_B = 2|M_{12}|, \quad \Delta\Gamma_B = 2\mathcal{R}e(M_{12}\Gamma_{12}^*)/|M_{12}|, \tag{3.8}$$

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}. \tag{3.9}$$

To discuss CP violation in mixing, it is useful to write eq. (3.7) to first order in $|\Gamma_{12}/M_{12}|$:

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}\left[1 - \frac{1}{2}\mathcal{I}m\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]. \tag{3.10}$$

To discuss CP violation in decay, we need to consider decay amplitudes. The CP transformation law for a final state $f$ is

$$\mathrm{CP}|f\rangle = \omega_f|\bar{f}\rangle, \quad \mathrm{CP}|\bar{f}\rangle = \omega_f^*|f\rangle, \quad (|\omega_f| = 1). \tag{3.11}$$

For a final CP eigenstate $f = \bar{f} = f_{\mathrm{CP}}$, the phase factor $\omega_f$ is replaced by $\eta_{f_{\mathrm{CP}}} = \pm 1$, the CP eigenvalue of the final state. We define the decay amplitudes $A_f$ and $\bar{A}_f$ according to

$$A_f = \langle f|\mathcal{H}_d|B^0\rangle, \quad \bar{A}_f = \langle f|\mathcal{H}_d|\bar{B}^0\rangle, \tag{3.12}$$

where $\mathcal{H}_d$ is the decay Hamiltonian.

CP relates $A_f$ and $\bar{A}_{\bar{f}}$. There are two types of phases that may appear in $A_f$ and $\bar{A}_{\bar{f}}$. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in $A_f$ and $\bar{A}_{\bar{f}}$ with opposite signs. In the Standard Model these phases occur only in the CKM matrix which is part of the electroweak sector of the theory, hence these are often called "weak phases". The weak phase of any single term is convention dependent. However the difference between the weak phases in two different terms in $A_f$ is convention independent because the phase rotations of the initial and final states are the same for every term. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Such phases do not violate CP and they appear in $A_f$ and $\bar{A}_{\bar{f}}$ with the same sign. Their origin is the possible contribution from intermediate on-shell states in the decay process, that is an absorptive part of an amplitude that has contributions from coupled channels. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again only the relative strong phases of different terms in a scattering amplitude have physical content, an overall phase rotation of the entire amplitude has no physical consequences. Thus it is useful to write each contribution to $A$ in three parts: its magnitude $A_i$; its weak phase term $e^{i\phi_i}$; and its strong phase term $e^{i\delta_i}$. Then, if several amplitudes contribute to $B \to f$, we have

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| = \left|\frac{\sum_i A_i e^{i(\delta_i - \phi_i)}}{\sum_i A_i e^{i(\delta_i + \phi_i)}}\right|. \tag{3.13}$$

To discuss CP violation in the interference of decays with and without mixing, we introduce a complex quantity $\lambda_f$ defined by

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}. \tag{3.14}$$

We further define the CP transformation law for the quark fields in the Hamiltonian (a careful treatment of CP conventions can be found in [43]):

$$q \to \omega_q \bar{q}, \quad \bar{q} \to \omega_q^* q, \quad (|\omega_q| = 1). \tag{3.15}$$

The effective Hamiltonian that is relevant to $M_{12}$ is of the form

$$H_{\rm eff}^{\Delta b=2} \propto e^{+2i\phi_B}\left[\bar{d}\gamma^\mu(1-\gamma_5)b\right]^2 + e^{-2i\phi_B}\left[\bar{b}\gamma^\mu(1-\gamma_5)d\right]^2, \tag{3.16}$$

where $2\phi_B$ is a CP violating (weak) phase. (We use the Standard Model $V-A$ amplitude, but the results can be generalized to any Dirac structure.) For the $B$ system, where $|\Gamma_{12}| \ll |M_{12}|$, this leads to

$$q/p = \omega_B \omega_b^* \omega_d e^{-2i\phi_B}. \tag{3.17}$$

(We implicitly assumed that the vacuum insertion approximation gives the correct sign for $M_{12}$. In general, there is a sign$(B_B)$ factor on the right hand side of eq. (3.17) [44].) To understand the phase structure of decay amplitudes, we take as an example the $b \to q\bar{q}d$ decay ($q = u$ or $c$). The decay Hamiltonian is of the form

$$H_d \propto e^{+i\phi_f}\left[\bar{q}\gamma^\mu(1-\gamma_5)d\right]\left[\bar{b}\gamma_\mu(1-\gamma_5)q\right] + e^{-i\phi_f}\left[\bar{q}\gamma^\mu(1-\gamma_5)b\right]\left[\bar{d}\gamma_\mu(1-\gamma_5)q\right], \tag{3.18}$$

where $\phi_f$ is the appropriate weak phase. (Again, for simplicity we use a $V-A$ structure, but the results hold for any Dirac structure.) Then

$$\bar{A}_{\bar{f}}/A_f = \omega_f \omega_B^* \omega_b \omega_d^* e^{-2i\phi_f}. \tag{3.19}$$

Eqs. (3.17) and (3.19) together imply that for a final CP eigenstate,

$$\lambda_{f_{\rm CP}} = \eta_{f_{\rm CP}} e^{-2i(\phi_B+\phi_f)}. \tag{3.20}$$

**(i) CP violation in mixing:**

$$|q/p| \neq 1. \tag{3.21}$$

This results from the mass eigenstates being different from the CP eigenstates, and requires a relative phase between $M_{12}$ and $\Gamma_{12}$. For the neutral $B$ system, this effect could be observed through the asymmetries in semileptonic decays:

$$a_{\rm SL} = \frac{\Gamma(\bar{B}^0_{\rm phys}(t) \to \ell^+\nu X) - \Gamma(B^0_{\rm phys}(t) \to \ell^-\nu X)}{\Gamma(\bar{B}^0_{\rm phys}(t) \to \ell^+\nu X) + \Gamma(B^0_{\rm phys}(t) \to \ell^-\nu X)}. \tag{3.22}$$

In terms of $q$ and $p$,

$$a_{\rm SL} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \tag{3.23}$$

CP violation in mixing has been observed in the neutral $K$ system ($\mathcal{R}e\ \varepsilon_K \neq 0$).

In the neutral $B$ system, the effect is expected to be small, $\lesssim \mathcal{O}(10^{-2})$. The reason is that, model independently, one expects that $a_{\rm SL} \lesssim \Delta\Gamma_B/\Delta m_B$. (We assume here that $\arg(\Gamma_{12}/M_{12})$ is not particularly close to $\pi/2$.) The difference in width is produced by decay channels common to $B^0$ and $\bar{B}^0$. The branching ratios for such channels are at or below the level of $10^{-3}$. Since various channels contribute with differing signs, one expects that their sum does not exceed the individual level. Hence, we can safely assume that $\Delta\Gamma_B/\Gamma_B = \mathcal{O}(10^{-2})$. On the other hand, it is experimentaly known that $\Delta m_B/\Gamma_B \approx 0.7$.

To calculate $a_{\rm SL}$, we use (3.23) and (3.10), and get:

$$a_{\rm SL} = \mathcal{I}m \frac{\Gamma_{12}}{M_{12}}. \tag{3.24}$$

To predict it in a given model, one needs to calculate $M_{12}$ and $\Gamma_{12}$. This involves large hadronic uncertainties, in particular in the hadronization models for $\Gamma_{12}$.

**(ii) CP violation in decay:**

$$|\bar{A}_{\bar{f}}/A_f| \neq 1. \tag{3.25}$$

This appears as a result of interference among various terms in the decay amplitude, and will not occur unless at least two terms have different weak phases and different strong phases. CP asymmetries in charged $B$ decays,

$$a_{f^\pm} = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)}, \tag{3.26}$$

are purely an effect of CP violation in decay. In terms of the decay amplitudes,

$$a_{f\pm} = \frac{1-|\bar{A}_{f-}/A_{f+}|^2}{1+|\bar{A}_{f-}/A_{f+}|^2}. \tag{3.27}$$

CP violation in decay has been observed in the neutral $K$ system ($\mathcal{R}e\ \varepsilon'_K \neq 0$).

To calculate $a_{f\pm}$, we use (3.27) and (3.13). For simplicity, we consider decays with contributions from two weak phases and with $A_2 \ll A_1$. We get:

$$a_{f\pm} = -2(A_2/A_1)\sin(\delta_2-\delta_1)\sin(\phi_2-\phi_1). \tag{3.28}$$

The magnitude and strong phase of any amplitude involve long distance strong interaction physics, and our ability to calculate these from first principles is limited. Thus quantities that depend only on the weak phases are much cleaner than those that require knowledge of the relative magnitudes or strong phases of various amplitude contributions, such as CP violation in decay.

**(iii) CP violation in the interference between decays with and without mixing:**

$$|\lambda_{f_{\rm CP}}| = 1, \quad \mathcal{I}m\ \lambda_{f_{\rm CP}} \neq 0. \tag{3.29}$$

Any $\lambda_{f_{\rm CP}} \neq \pm 1$ is a manifestation of CP violation. The special case (3.29) isolates the effects of interest since both CP violation in decay, eq. (3.25), and in mixing, eq. (3.21), lead to $|\lambda_{f_{\rm CP}}| \neq 1$. For the neutral $B$ system, this effect can be observed by comparing decays into final CP eigenstates of a time-evolving neutral $B$ state that begins at time zero as $B^0$ to those of the state that begins as $\bar{B}^0$:

$$a_{f_{\rm CP}} = \frac{\Gamma(\bar{B}^0_{\rm phys}(t)\to f_{\rm CP}) - \Gamma(B^0_{\rm phys}(t)\to f_{\rm CP})}{\Gamma(\bar{B}^0_{\rm phys}(t)\to f_{\rm CP}) + \Gamma(B^0_{\rm phys}(t)\to f_{\rm CP})}. \tag{3.30}$$

This time dependent asymmetry is given (for $|\lambda_{f_{\rm CP}}| = 1$) by

$$a_{f_{\rm CP}} = -\mathcal{I}m\lambda_{f_{\rm CP}}\sin(\Delta m_B t). \tag{3.31}$$

CP violation in the interference of decays with and without mixing has been observed for the neutral $K$ system ($\mathcal{I}m\ \varepsilon_K \neq 0$). It is expected to be an effect of $\mathcal{O}(1)$ in various $B$ decays. For such cases, the contribution from CP violation in mixing is clearly negligible.

For decays that are dominated by a single CP violating phase (for example, $B \to \psi K_S$ and $K_L \to \pi^0 \nu\bar{\nu}$), so that the contribution from CP violation in decay is also negligible, $a_{f_{\rm CP}}$ is cleanly interpreted in terms of purely electroweak parameters. Explicitly, $\mathcal{I}m\lambda_{f_{\rm CP}}$ gives the difference between the phase of the $B-\bar{B}$ mixing amplitude ($2\phi_B$) and twice the phase of the relevant decay amplitude ($2\phi_f$) (see eq. (3.20)):

$$\mathcal{I}m\lambda_{f_{\rm CP}} = -\eta_{f_{\rm CP}} \sin[2(\phi_B + \phi_f)]. \tag{3.32}$$

A summary of the main properties of the different types of CP violation in meson decays is given in the table I.

| Type | Exp. | Theory | Calculation | Uncertainties | Observed in |
|---|---|---|---|---|---|
| mixing | $a_{\rm SL}$ | $\frac{1-\lvert q/p\rvert^4}{1+\lvert q/p\rvert^4}$ | $\mathcal{I}m\frac{\Gamma_{12}}{M_{12}}$ | $\Gamma_{12} \Longrightarrow$ Large | $\mathcal{R}e\ \varepsilon_K$ |
| decay | $a_{f\pm}$ | $\frac{1-\lvert\bar{A}_{f-}/A_{f+}\rvert^2}{1+\lvert\bar{A}_{f-}/A_{f+}\rvert^2}$ | $-2\frac{A_2}{A_1}\sin(\delta_2-\delta_1)\sin(\phi_2-\phi_1)$ | $\delta_i, A_i \Longrightarrow$ Large | $\mathcal{R}e\ \varepsilon'_K$ |
| interference | $a_{f_{CP}}$ | $-\mathcal{I}m\lambda_{f_{CP}}$ | $\eta_{f_{CP}}\sin[2(\phi_B+\phi_f)]$ | Small | $\mathcal{I}m\ \varepsilon_K$ |

Table I. The three types of CP violation in meson decays.

## 4. CP Violation in $K$ Decays

The two CP violating observables that have been measured are related to $K$ meson decays. In this chapter we present these observables and their significance.

### *4.1. Direct and Indirect CP Violation*

The terms indirect CP violation and direct CP violation are commonly used in the literature. While various authors use these terms with different meanings, the most useful definition is the following:

**Indirect CP violation** refers to CP violation in meson decays where the CP violating phases can all be chosen to appear in $\Delta F = 2$ (mixing) amplitudes.

**Direct CP violation** refers to CP violation in meson decays where some CP violating phases necessarily appear in $\Delta F = 1$ (decay) amplitudes.

Examining eqs. (3.21) and (3.7), we learn that CP violation in mixing is a manifestation of indirect CP violation. Examining eqs. (3.25) and (3.12), we learn that CP violation in decay is a manifestation of direct CP violation. Examining eqs. (3.29) and (3.14), we learn that the situation concerning CP violation in the interference of decays with and without mixing is more subtle. For any single measurement of $\mathcal{I}m\lambda_f \neq 0$, the relevant CP violating phase can be chosen by convention to reside in the $\Delta F = 2$ amplitude ($\phi_f = 0$, $\phi_B \neq 0$ in the notation of eq. (3.20)), and then we would call it indirect CP violation. Consider, however, the CP asymmetries for two different final CP eigenstates (for the same decaying meson), $f_a$ and $f_b$. Then, a non-zero difference between $\mathcal{I}m\lambda_{f_a}$ and $\mathcal{I}m\lambda_{f_b}$ requires that there exists CP violation in $\Delta F = 1$ processes ($\phi_{f_a} - \phi_{f_b} \neq 0$), namely direct CP violation.

Experimentally, both direct and indirect CP violation have been established. Below we will see that $\varepsilon_K$ signifies indirect CP violation while $\varepsilon'_K$ signifies direct CP violation.

Theoretically, most models of CP violation (including the Standard Model) have predicted that both types of CP violation exist. There is, however, one class of models, that is *superweak models*, that predict only indirect CP violation. The measurement of $\varepsilon'_K \neq 0$ has excluded this class of models.

### *4.2. The $\varepsilon_K$ and $\varepsilon'_K$ Parameters*

Historically, a different language from the one used by us has been employed to describe CP violation in $K \to \pi\pi$ and $K \to \pi\ell\nu$ decays. In this section we 'translate' the language of $\varepsilon_K$ and $\varepsilon'_K$ to our notations. Doing so will make it easy to understand which type of CP violation is related to each quantity.

The two CP violating quantities measured in neutral $K$ decays are

$$\eta_{00} = \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle}, \quad \eta_{+-} = \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle}. \tag{4.1}$$

Define, for $(ij) = (00)$ or $(+-)$,

$$A_{ij} = \langle \pi^i\pi^j|\mathcal{H}|K^0\rangle, \quad \bar{A}_{ij} = \langle \pi^i\pi^j|\mathcal{H}|\bar{K}^0\rangle, \tag{4.2}$$

$$\lambda_{ij} = \left(\frac{q}{p}\right)_K \frac{\bar{A}_{ij}}{A_{ij}}. \tag{4.3}$$

Then

$$\eta_{00}=\frac{1-\lambda_{00}}{1+\lambda_{00}},\quad \eta_{+-}=\frac{1-\lambda_{+-}}{1+\lambda_{+-}}. \tag{4.4}$$

The $\eta_{00}$ and $\eta_{+-}$ parameters get contributions from CP violation in mixing ($|(q/p)|_K \neq 1$) and from the interference of decays with and without mixing ($\mathcal{I}m\lambda_{ij} \neq 0$) at $\mathcal{O}(10^{-3})$ and from CP violation in decay ($|\bar{A}_{ij}/A_{ij}| \neq 1$) at $\mathcal{O}(10^{-6})$.

There are two isospin channels in $K \to \pi\pi$ leading to final $(2\pi)_{I=0}$ and $(2\pi)_{I=2}$ states:

$$\begin{aligned}\langle\pi^0\pi^0| &= \sqrt{\frac{1}{3}}\langle(\pi\pi)_{I=0}| - \sqrt{\frac{2}{3}}\langle(\pi\pi)_{I=2}|,\\ \langle\pi^+\pi^-| &= \sqrt{\frac{2}{3}}\langle(\pi\pi)_{I=0}| + \sqrt{\frac{1}{3}}\langle(\pi\pi)_{I=2}|.\end{aligned} \tag{4.5}$$

The fact that there are two strong phases allows for CP violation in decay. The possible effects are, however, small (on top of the smallness of the relevant CP violating phases) because the final $I=0$ state is dominant (this is the $\Delta I = 1/2$ rule). Defining

$$A_I = \langle(\pi\pi)_I|\mathcal{H}|K^0\rangle,\quad \bar{A}_I = \langle(\pi\pi)_I|\mathcal{H}|\bar{K}^0\rangle, \tag{4.6}$$

we have, experimentally, $|A_2/A_0| \approx 1/20$. Instead of $\eta_{00}$ and $\eta_{+-}$ we may define two combinations, $\varepsilon_K$ and $\varepsilon'_K$, in such a way that the possible effects of CP violation in decay (mixing) are isolated into $\varepsilon'_K$ ($\varepsilon_K$).

The experimental definition of the $\varepsilon_K$ parameter is

$$\varepsilon_K \equiv \frac{1}{3}(\eta_{00}+2\eta_{+-}). \tag{4.7}$$

To zeroth order in $A_2/A_0$, we have $\eta_{00}=\eta_{+-}=\varepsilon_K$. However, the specific combination (4.7) is chosen in such a way that the following relation holds to *first* order in $A_2/A_0$:

$$\varepsilon_K = \frac{1-\lambda_0}{1+\lambda_0}, \tag{4.8}$$

where

$$\lambda_0 = \left(\frac{q}{p}\right)_K \left(\frac{\bar{A}_0}{A_0}\right). \tag{4.9}$$

Since, by definition, only one strong channel contributes to $\lambda_0$, there is indeed no CP violation in decay in (4.8). It is simple to show that $\mathcal{R}e\ \varepsilon_K \neq 0$ is a manifestation of CP violation in mixing while $\mathcal{I}m\ \varepsilon_K \neq 0$ is a manifestation of CP violation in the interference between decays with and without mixing. Since experimentally $\arg\varepsilon_K \approx \pi/4$, the two contributions are comparable. It is also clear that $\varepsilon_K \neq 0$ is a manifestation of indirect CP violation: it could be described entirely in terms of a CP violating phase in the $M_{12}$ amplitude.

The experimental value of $\varepsilon_K$ is given by [26]

$$|\varepsilon_K| = (2.280 \pm 0.013) \times 10^{-3}. \tag{4.10}$$

The experimental definition of the $\varepsilon'_K$ parameter is

$$\varepsilon'_K \equiv \frac{1}{3}(\eta_{+-} - \eta_{00}). \tag{4.11}$$

The theoretical expression is

$$\varepsilon'_K \approx \frac{1}{6}(\lambda_{00} - \lambda_{+-}). \tag{4.12}$$

Obviously, any type of CP violation which is independent of the final state does not contribute to $\varepsilon'_K$. Consequently, there is no contribution from CP violation in mixing to (4.12). It is simple to show that $\mathcal{R}e\ \varepsilon'_K \neq 0$ is a manifestation of CP violation in decay while $\mathcal{I}m\ \varepsilon'_K \neq 0$ is a manifestation of CP violation in the interference between decays with and without mixing. Following our explanations in the previous section, we learn that $\varepsilon'_K \neq 0$ is a manifestation of direct CP violation: it requires $\phi_2 - \phi_0 \neq 0$ (where $\phi_I$ is the CP violating phase in the $A_I$ amplitude defined in (4.6)).

The quantity that is actually measured in experiment is

$$1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = 6\mathcal{R}e(\varepsilon'/\varepsilon). \tag{4.13}$$

The average over the experimental measurements of $\varepsilon'/\varepsilon$ [45-49] is given by:

$$\mathcal{R}e(\varepsilon'/\varepsilon) = (2.11 \pm 0.46) \times 10^{-3}. \tag{4.14}$$

### *4.3. The $\varepsilon_K$ Parameter in the Standard Model*

An approximate expression for $\varepsilon_K$, that is convenient for calculating it, is given by

$$\varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}}\frac{\mathcal{I}mM_{12}}{\Delta m_K}. \tag{4.15}$$

A few points concerning this expression are worth emphasizing:

(i) Eq. (4.15) is given in a specific phase convention, where $A_2$ is real. Within the Standard Model, this is a phase convention where $V_{ud}V_{us}^*$ is real, a condition fulfilled in both the standard parametrization of eq. (2.26) and the Wolfenstein parametrization of eq. (2.29).

(ii) The phase of $\pi/4$ is approximate. It is determined by hadronic parameters and therefore is independent of the electroweak model. Specifically,

$$\Delta\Gamma_K \approx -2\Delta m_K \implies \arg(\varepsilon_K) \approx \arctan(-2\Delta m_K/\Delta\Gamma_K) \approx \pi/4. \tag{4.16}$$

(iii) A term of order $2\frac{\mathcal{I}m\ A_0}{\mathcal{R}e\ A_0}\frac{\mathcal{R}e\ M_{12}}{\mathcal{I}m\ M_{12}} \lesssim 0.02$ is neglected when (4.15) is derived.

(iv) There is a large hadronic uncertainty in the calculation of $M_{12}$ coming from long distance contributions. There are, however, good reasons to believe that the long distance contributions are important in $\mathcal{R}e\ M_{12}$ (where they could be even comparable to the short distance contributions), but negligible in $\mathcal{I}m\ M_{12}$. To avoid this uncertainty, one uses $\mathcal{I}mM_{12}/\Delta m_K$, with the experimentally measured value of $\Delta m_K$, instead of $\mathcal{I}mM_{12}/2\mathcal{R}e\ M_{12}$ with the theoretically calculated value of $\mathcal{R}e\ M_{12}$.

(v) The matrix element $\langle \bar{K}^0|(\bar{s}d)_{V-A}(\bar{s}d)_{V-A}|K^0\rangle$ is yet another source of hadronic uncertainty. If both $\mathcal{I}m\ M_{12}$ and $\mathcal{R}e\ M_{12}$ were dominated by short distance contributions, the matrix element would have cancelled in the ratio between them. However, as explained above, this is not the case.

Within the Standard Model, $\mathcal{I}m\ M_{12}$ is accounted for by box diagrams. One gets:

$$\varepsilon_K = e^{i\pi/4}C_\varepsilon B_K \mathcal{I}m\lambda_t\{\mathcal{R}e\lambda_c[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathcal{R}e\lambda_t\eta_2 S_0(x_t)\}, \tag{4.17}$$

where the CKM parameters are $\lambda_i = V_{is}^*V_{id}$, the constant $C_\varepsilon$ is a well known parameter,

$$C_\varepsilon = \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2\Delta m_K} = 3.8\times 10^4, \tag{4.18}$$

the $\eta_i$ are QCD correction factors [50],

$$\eta_1 = 1.38\pm 0.20, \quad \eta_2 = 0.57\pm 0.01, \quad \eta_3 = 0.47\pm 0.04, \tag{4.19}$$

$S_0$ is a kinematic factor, given approximately by [36]

$$\begin{aligned} S_0(x_t) &= 2.4 \left(\frac{m_t}{170\ GeV}\right)^{1.52}, \quad S_0(x_c) = x_x = 2.6 \times 10^{-4}, \\ S_0(x_t, x_c) &= x_c \left[\ln \frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2}\right] = 2.3 \times 10^{-3}, \end{aligned} \tag{4.20}$$

and $B_K$ is the ratio between the matrix element of the four quark operator and its value in the vacuum insertion approximation (see [36] for a precise definition),

$$B_K = 0.85 \pm 0.15. \tag{4.21}$$

Note that $\varepsilon_K$ is proportional to $\mathcal{I}m\lambda_t$ and, consequently, to $\eta$. The $\varepsilon_K$ constraint on the Wolfenstein parameters can be written as follows:

$$\eta \left[(1-\rho)|V_{cb}|^2 \eta_2 S_0(x_t) + \eta_3 S_0(x_c, x_t) - \eta_1 S_0(x_c)\right] |V_{cb}|^2 B_K = 1.24 \times 10^{-6}. \tag{4.22}$$

Eq. (4.22) gives hyperbolae in the $\rho - \eta$ plane. The main sources of uncertainty are in the $B_K$ parameter and in the $|V_{cb}|^4$ dependence. When the theoretically reasonable range for the first and the experimentally allowed range for the second are taken into account, one finds that $\eta \gtrsim 0.2$ is required to explain $\varepsilon_K$ (see (2.39)). Hence our statement that CP is not an approximate symmetry of the Standard Model.

### *4.4. The $\varepsilon'_K$ Parameter in the Standard Model*

A convenient approximate expression for $\varepsilon'_K$ is given by:

$$\varepsilon'_K = \frac{i}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| e^{i(\delta_2 - \delta_0)} \sin(\phi_2 - \phi_0). \tag{4.23}$$

We would like to emphasize a few points:

(i) The approximations used in (4.23) are $\lambda_0 = 1$ and $|A_2/A_0| \ll 1$.

(ii) The phase of $\varepsilon'_K$ is determined by hadronic parameters and, therefore, model independent:

$$\arg(\varepsilon'_K) = \pi/2 + \delta_2 - \delta_0 \approx \pi/4. \tag{4.24}$$

The fact that, accidentally, $\arg(\varepsilon_K) \approx \arg(\varepsilon'_K)$, means that

$$\mathcal{R}e(\varepsilon'/\varepsilon) \approx \varepsilon'/\varepsilon. \tag{4.25}$$

(iii) $\mathcal{R}e\ \varepsilon'_K \neq 0$ requires $\delta_2 - \delta_0 \neq 0$, consistent with our statement that it is a manifestation of CP violation in decay. $\varepsilon'_K \neq 0$ requires $\phi_2 - \phi_0 \neq 0$, consistent with our statement that it is a manifestation of direct CP violation.

The calculation of $\varepsilon'/\varepsilon$ within the Standard Model is complicated and suffers from large hadronic uncertainties. Let us first try a very naive order of magnitude estimate. The relevant quark decay process is $s \to d\bar{u}u$. All tree diagrams contribute with the same weak phase, $\phi_T = \arg(V_{ud}^* V_{us})$. Spectator diagrams contribute to both $I = 0$ and $I = 2$ final states. Penguin diagrams with an intermediate $q = u, c, t$ quarks, contribute with a weak phase $\phi_P^q = \arg(V_{qd}^* V_{qs})$. Strong penguin contribute to the final $I = 0$ states only. Electroweak penguins contribute also to final $I = 2$ states, but we will ignore them for the moment. (The fact that the top quark is heavy means that the electroweak penguins are important.) A difference in the weak phase between $A_0$ and $A_2$ is then a result of the fact that $A_0$ has contributions from penguin diagrams with intermediate $c$ and $t$ quarks. Consequently, $\varepsilon'_K$ is suppressed by the following factors:

a. $|A_2/A_0| \sim 0.045$;

b. $|A_0^{\rm penguin}/A_0^{\rm tree}| \sim 0.05$.

c. $\sin\beta_K \sim 10^{-3}$.

The last factor appears, however, also in $\varepsilon_K$. Therefore, it cancels in the ratio $\varepsilon'/\varepsilon$. A very rough order of magnitude estimate is then $\varepsilon'/\varepsilon \sim 10^{-3}$. Note that $\varepsilon'/\varepsilon$ is not small because of small CP violating parameters but because of hadronic parameters. Actually, it is independent of $\sin\delta_{\rm KM}$. (In most calculations one uses the experimental value of $\varepsilon_K$ and the theoretical expression for $\varepsilon'_K$. Then the expression for $\varepsilon'/\varepsilon$ depends on $\sin\delta_{\rm KM}$.)

The actual calculation of $\varepsilon'/\varepsilon$ is very complicated. There are several comparable contributions with differing signs. The final result can be written in the form (for recent work, see [51-54,36] and references therein):

$$\varepsilon'/\varepsilon = \mathcal{I}m(V_{td}V_{ts}^*)\left[P^{(1/2)} - P^{(3/2)}\right], \tag{4.26}$$

where we omitted a phase factor using the approximation $\arg(\varepsilon_K) = \arg(\varepsilon'_K)$, and where

$$\begin{aligned} P^{(1/2)} &= \frac{G_F \mathcal{R}e A_2}{2|\varepsilon_K|(\mathcal{R}e A_0)^2} \sum y_i \langle Q_i \rangle_0 (1 - \Omega_{\eta+\eta'}), \\ P^{(3/2)} &= \frac{G_F}{2|\varepsilon_K| \mathcal{R}e A_0} \sum y_i \langle Q_i \rangle_2. \end{aligned} \tag{4.27}$$

$Q_i$ are four quark operators, and $y_i$ are the Wilson coefficient functions. The most important operators are

$$\begin{aligned} Q_6 &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}, \\ Q_8 &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A}. \end{aligned} \tag{4.28}$$

$P^{(3/2)}$ is dominated by electroweak penguin contributions while $P^{(1/2)}$ is dominated by QCD penguin contributions. The latter are suppressed by isospin breaking effects ($m_u \neq m_d$), parametrized by

$$\Omega_{\eta+\eta'} = \frac{\mathcal{R}e A_0}{\mathcal{R}e A_2} \frac{(\mathcal{I}m A_2)_{\text{I.B.}}}{\mathcal{I}m A_0} \approx 0.25 \pm 0.08. \tag{4.29}$$

A crude approximation to (4.26) which emphasizes the sources of hadronic uncertainty is given by

$$\begin{aligned} \varepsilon'/\varepsilon \approx 13\, \mathcal{I}m(V_{td}V_{ts}^*) &\left(\frac{110\ MeV}{m_s(2\ GeV)}\right)^2 \left(\frac{\Lambda_{\overline{\text{MS}}}^{(4)}}{340\ MeV}\right) \\ &\times \left[B_6^{(1/2)}(1 - \Omega_{\eta+\eta'}) - 0.4 B_8^{(3/2)} \left(\frac{m_t}{165\ GeV}\right)^{2.5}\right]. \end{aligned} \tag{4.30}$$

$B_6^{(1/2)}$ and $B_8^{(3/2)}$ parametrize the hadronic matrix elements:

$$\begin{aligned} \langle Q_6 \rangle_0 &\equiv B_6^{(1/2)} \langle Q_6 \rangle_0^{(\text{vac})}, \quad B_6^{(1/2)} \approx 1.0 \pm 0.3, \\ \langle Q_8 \rangle_2 &\equiv B_8^{(3/2)} \langle Q_8 \rangle_2^{(\text{vac})}, \quad B_8^{(3/2)} \approx 0.8 \pm 0.2. \end{aligned} \tag{4.31}$$

The main sources of uncertainties lie then in the parameters $m_s$, $B_6^{(1/2)}$, $B_8^{(3/2)}$, $\Omega_{\eta+\eta'}$ and $\Lambda_{\overline{\text{MS}}}^{(4)}$. The importance of these uncertainties is increased because of the cancellation between the two contributions in (4.30). Taking the above reasonable ranges for the hadronic parameters (from lattice calculations and $1/N_c$ expansion), one can estimate [53]:

$$\mathcal{R}e(\varepsilon'/\varepsilon)^{\text{SM}} = (7.7^{+6.0}_{-3.5}) \times 10^{-4}. \tag{4.32}$$

The fact that the range in (4.32) is lower than the experimentally allowed range in (4.14) cannot be taken as evidence for new physics. With a more conservative treatment of the theoretical uncertainties, one can stretch the theoretical upper bound to values consistent with the experimental lower bound [55-58,52-53].

### *4.5. CP violation in $K \to \pi\nu\bar\nu$*

CP violation in the rare $K \to \pi\nu\bar\nu$ decays is very interesting. It is very different from the CP violation that has been observed in $K \to \pi\pi$ decays which is small and involves theoretical uncertainties. Similar to the CP asymmetry in $B \to \psi K_S$, it is predicted to be large and can be cleanly interpreted. Furthermore, observation of the $K_L \to \pi^0\nu\bar\nu$ decay at the rate predicted by the Standard Model will provide further evidence that CP violation cannot be attributed to mixing ($\Delta S = 2$) processes only, as in superweak models.

Define the decay amplitudes

$$A_{\pi^0\nu\bar\nu} = \langle \pi^0\nu\bar\nu|\mathcal{H}|K^0\rangle, \quad \bar A_{\pi^0\nu\bar\nu} = \langle \pi^0\nu\bar\nu|\mathcal{H}|\bar K^0\rangle, \tag{4.33}$$

and the related $\lambda_{\pi\nu\bar\nu}$ quantity:

$$\lambda_{\pi\nu\bar\nu} = \left(\frac{q}{p}\right)_K \frac{\bar A_{\pi^0\nu\bar\nu}}{A_{\pi^0\nu\bar\nu}}. \tag{4.34}$$

The decay amplitudes of $K_{L,S}$ into a final $\pi^0\nu\bar\nu$ state are then

$$\langle \pi^0\nu\bar\nu|\mathcal{H}|K_{L,S}\rangle = pA_{\pi^0\nu\bar\nu} \mp q\bar A_{\pi^0\nu\bar\nu}, \tag{4.35}$$

and the ratio between the corresponding decay rates is

$$\frac{\Gamma(K_L \to \pi^0\nu\bar\nu)}{\Gamma(K_S \to \pi^0\nu\bar\nu)} = \frac{1 + |\lambda_{\pi\nu\bar\nu}|^2 - 2\mathcal{R}e\lambda_{\pi\nu\bar\nu}}{1 + |\lambda_{\pi\nu\bar\nu}|^2 + 2\mathcal{R}e\lambda_{\pi\nu\bar\nu}}. \tag{4.36}$$

We learn that the $K_L \to \pi^0\nu\bar\nu$ decay rate vanishes in the CP limit ($\lambda_{\pi\nu\bar\nu} = 1$), as expected on general grounds [16]. (The CP conserving contributions were explicitly calculated within the Standard Model [59] and within its extension with massive neutrinos [60], and found to be negligible.)

Since the effects of CP violation in decay and in mixing are expected to be negligibly small, $\lambda_{\pi\nu\bar\nu}$ is, to an excellent approximation, a pure phase. Defining $\theta_K$ to be the relative phase between the $K-\bar{K}$ mixing amplitude and the $s\to d\nu\bar{\nu}$ decay amplitude, namely $\lambda_{\pi\nu\bar{\nu}}=e^{2i\theta_K}$, we get from (4.36):

$$\frac{\Gamma(K_L\to\pi^0\nu\bar{\nu})}{\Gamma(K_S\to\pi^0\nu\bar{\nu})}=\frac{1-\cos 2\theta_K}{1+\cos 2\theta_K}=\tan^2\theta_K. \tag{4.37}$$

Using the isospin relation $A(K^0\to\pi^0\nu\bar{\nu})/A(K^+\to\pi^+\nu\bar{\nu})=1/\sqrt{2}$, we get

$$a_{\pi\nu\bar{\nu}}\equiv\frac{\Gamma(K_L\to\pi^0\nu\bar{\nu})}{\Gamma(K^+\to\pi^+\nu\bar{\nu})}=\frac{1-\cos 2\theta_K}{2}=\sin^2\theta_K. \tag{4.38}$$

Note that $a_{\pi\nu\bar{\nu}}\leq 1$, and consequently a measurement of $\Gamma(K^+\to\pi^+\nu\bar{\nu})$ can be used to set a model independent upper limit on $\Gamma(K_L\to\pi^0\nu\bar{\nu})$ [19].

Within the Standard Model, the $K\to\pi\nu\bar{\nu}$ decays are dominated by short distance $Z$-penguins and box diagrams. The branching ratio for $K^+\to\pi^+\nu\bar{\nu}$ can be expressed in terms of $\rho$ and $\eta$ [36]:

$$BR(K^+\to\pi^+\nu\bar{\nu})=4.11\times 10^{-11}A^4[X(x_t)]^2\left[\eta^2+(\rho_0-\rho)^2\right], \tag{4.39}$$

where

$$\rho_0=1+\frac{P_0(X)}{A^2X(x_t)}, \tag{4.40}$$

and $X(x_t)$ and $P_0(X)$ represent the electroweak loop contributions in NLO for the top quark and for the charm quark, respectively. The main theoretical uncertainty is related to the strong dependence of the charm contribution on the renormalization scale and the QCD scale, $P_0(X)=0.42\pm 0.06$. First evidence for $K^+\to\pi^+\nu\bar{\nu}$ was presented recently [61]. The large experimental error does not yet give a useful CKM constraint and is consistent with the Standard Model prediction.

The branching ratio for the $K_L\to\pi^0\nu\bar{\nu}$ decay can be expressed in terms of $\eta$ [36]:

$$BR(K_L\to\pi^0\nu\bar{\nu})=1.80\times 10^{-10}[X(x_t)]^2A^4\eta^2. \tag{4.41}$$

The present experimental bound, $BR(K_L\to\pi^0\nu\bar{\nu})\leq 1.6\times 10^{-6}$ [62] lies about five orders of magnitude above the Standard Model prediction [36] and about two orders of magnitude above the bound that can be deduced using model independent isospin relations [19] from the experimental upper bound on the charged mode.

Note that if the charm contribution to $K^+ \to \pi^+ \nu\bar{\nu}$ were negligible, so that both the charged and the neutral decays were dominated by the intermediate top contributions, then $a_{\pi\nu\bar{\nu}}$ would simply measure $\sin^2\beta$. While the charm contribution makes the evaluation of $a_{\pi\nu\bar{\nu}}$ more complicated, a reasonable order of magnitude estimate is still given by $\sin^2\beta$.

## 5. CP Violation in $D \to K\pi$ Decays

Within the Standard Model, $D-\bar{D}$ mixing is expected to be well below the experimental bound. Furthermore, effects related to CP violation in $D-\bar{D}$ mixing are expected to be negligibly small since this mixing is described to a good approximation by physics of the first two generations. An experimental observation of $D-\bar{D}$ mixing close to the present bound (and, even more convincingly, of related CP violation) will then be evidence for New Physics. The most sensitive experimental searches for $D-\bar{D}$ mixing use $D \to K\pi$ decays [63-67]. We now give the formalism of neutral $D$ decays into final $K^\pm\pi^\mp$ states.

### *5.1. Formalism*

We define the neutral $D$ mass eigenstates:

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle. \tag{5.1}$$

We define the following four decay amplitudes:

$$\begin{aligned} A_{K^+\pi^-} &= \langle K^+\pi^-|\mathcal{H}|D^0\rangle, \quad \bar{A}_{K^+\pi^-} = \langle K^+\pi^-|\mathcal{H}|\bar{D}^0\rangle, \\ A_{K^-\pi^+} &= \langle K^-\pi^+|\mathcal{H}|D^0\rangle, \quad \bar{A}_{K^-\pi^+} = \langle K^-\pi^+|\mathcal{H}|\bar{D}^0\rangle. \end{aligned} \tag{5.2}$$

We introduce the following two quantities:

$$\lambda_{K^+\pi^-} = \left(\frac{q}{p}\right)_D \frac{\bar{A}_{K^+\pi^-}}{A_{K^+\pi^-}}, \quad \lambda_{K^-\pi^+} = \left(\frac{q}{p}\right)_D \frac{\bar{A}_{K^-\pi^+}}{A_{K^-\pi^+}}. \tag{5.3}$$

The following approximations are all experimentally confirmed:

$$x \equiv \frac{\Delta m_D}{\Gamma_D} \ll 1; \quad y \equiv \frac{\Delta\Gamma_D}{2\Gamma_D} \ll 1; \quad |\lambda^{-1}_{K^+\pi^-}| \ll 1; \quad |\lambda_{K^-\pi^+}| \ll 1. \tag{5.4}$$

Using these approximations, the decay rates for the doubly Cabibbo suppressed (DCS) decays are given by

$$
\begin{aligned}
\Gamma[D^0(t) \to K^+\pi^-] &= e^{-t} \left|\bar{A}_{K^+\pi^-}\right|^2 |q/p|^2 \times \\
&\left[ \left|\lambda^{-1}_{K^+\pi^-}\right|^2 + \mathcal{R}e(\lambda^{-1}_{K^+\pi^-})yt + \mathcal{I}m(\lambda^{-1}_{K^+\pi^-})xt + \frac{1}{4}(x^2+y^2)t^2 \right], \\
\Gamma[\bar{D}^0(t) \to K^-\pi^+] &= e^{-t} \left|A_{K^-\pi^+}\right|^2 |p/q|^2 \times \\
&\left[ \left|\lambda_{K^-\pi^+}\right|^2 + \mathcal{R}e(\lambda_{K^-\pi^+})yt + \mathcal{I}m(\lambda_{K^-\pi^+})xt + \frac{1}{4}(x^2+y^2)t^2 \right].
\end{aligned}
\tag{5.5}
$$

The time $t$ is given here in units of the $D$-lifetime, $\tau_D = 1/\Gamma_D$. Eqs. (5.5) are valid for times $\lesssim \tau_D$.

The decay rates for the Cabibbo favored (CF) modes are given to a good approximation by

$$
\begin{aligned}
\Gamma[D^0(t) \to K^-\pi^+] &= e^{-t} \left|A_{K^-\pi^+}\right|^2 , \\
\Gamma[\bar{D}^0(t) \to K^+\pi^-] &= e^{-t} \left|\bar{A}_{K^+\pi^-}\right|^2 .
\end{aligned}
\tag{5.6}
$$

*5.2. CP Violation*

We will assume that the CF decays are unaffected by CP violation, that is,

$$
\left|A_{K^-\pi^+}\right| = \left|\bar{A}_{K^+\pi^-}\right| \equiv A_{\mathrm{CF}} . \tag{5.7}
$$

In general, $|q/p|$ is a positive real number and $\lambda^{-1}_{K^+\pi^-}$ and $\lambda_{K^-\pi^+}$ are two independent complex numbers. We now parametrize these quantities in a way that is convenient to separate the three types of CP violation:

$$
\begin{aligned}
|q/p| &= R_m , \\
\lambda^{-1}_{K^+\pi^-} &= \frac{R}{R_d R_m} e^{-i(\delta_{K\pi}+\phi_{K\pi})} , \\
\lambda_{K^-\pi^+} &= R R_d R_m e^{-i(\delta_{K\pi}-\phi_{K\pi})} .
\end{aligned}
\tag{5.8}
$$

CP violation in mixing, that is violation of $|q/p| = 1$, is related to $R_m \neq 1$. CP violation in decay, that is violation of $\left|\frac{A_{K^+\pi^-}}{\bar{A}_{K^-\pi^+}}\right| = \left|\frac{\bar{A}_{K^+\pi^-}}{A_{K^-\pi^+}}\right| = 1$, is related to $R_d \neq 1$. CP violation in interference of decays with and without mixing, that is violation of $\frac{\mathcal{I}m(\lambda^{-1}_{K^+\pi^-})}{|\lambda^{-1}_{K^+\pi^-}|} = \frac{\mathcal{I}m(\lambda_{K^-\pi^+})}{|\lambda_{K^-\pi^+}|}$, is related to $\phi_{K\pi} \neq 0$.

We also define

$$\begin{aligned} x' &\equiv x\cos\delta_{K\pi} + y\sin\delta_{K\pi}, \\ y' &\equiv y\cos\delta_{K\pi} - x\sin\delta_{K\pi}. \end{aligned} \tag{5.9}$$

In the language of eqs. (5.7), (5.8) and (5.9), we can rewrite eq. (5.5) as follows:

$$\begin{aligned} \Gamma[D^0(t)\to K^+\pi^-] &= e^{-t}A_{\rm CF}^2\left[\frac{R^2}{R_d^2} + \frac{RR_m}{R_d}(y'c_\phi - x's_\phi)t + \frac{R_m^2}{4}(x^2+y^2)t^2\right], \\ \Gamma[\bar D^0(t)\to K^-\pi^+] &= e^{-t}A_{\rm CF}^2\left[R^2R_d^2 + \frac{RR_d}{R_m}(y'c_\phi + x's_\phi)t + \frac{1}{4R_m^2}(x^2+y^2)t^2\right], \end{aligned} \tag{5.10}$$

where $s_\phi \equiv \sin\phi_{K\pi}$ and $c_\phi \equiv \cos\phi_{K\pi}$. Note that the three different mechanisms of CP violation can be distinguished if the time dependent rates (5.10) are measured:

(i) A different $t^2e^{-t}$ term in the DCS decays of $D^0$ and $\bar D^0$ is an unambiguous signal of CP violation in mixing.

(ii) A different $e^{-t}$ term in the DCS decays of $D^0$ and $\bar D^0$ is an unambiguous signal of CP violation in decay.

(iii) A measurement of all three terms for each of the two decay rates can provide an unambiguous signal of CP violation in interference.

In the presence of new physics, the most likely situation is that we have observable CP violation in interference between decays with and without mixing, while CP violation in mixing and in decays [68] remain unobservably small. In this case, $R_m = 1$, $R_d = 1$, but $\phi_{K\pi} \neq 0$. Furthermore, while $\Delta m_D$ can be enhanced to the level of present experimental sensitivity, $\Delta\Gamma_D$ is likely to remain close to the SM prediction. Neglecting $y$, we have in this case

$$\begin{aligned} \Gamma[D^0(t)\to K^+\pi^-] &= e^{-t}A_{\rm CF}^2\left[R^2 - Rx(s_\delta c_\phi + c_\delta s_\phi)t + \frac{x^2}{4}t^2\right], \\ \Gamma[\bar D^0(t)\to K^-\pi^+] &= e^{-t}A_{\rm CF}^2\left[R^2 - Rx(s_\delta c_\phi - c_\delta s_\phi)t + \frac{x^2}{4}t^2\right]. \end{aligned} \tag{5.11}$$

The $te^{-t}$ terms in (5.11) are potentially CP violating. There are four possibilities concerning these terms [69,70]:

(i) They vanish: both strong and weak phases play no role.

(ii) They are equal in magnitude and in sign: weak phases play no role.

(iii) They are equal in magnitude but have opposite signs: strong phases play no role.

(iv) They have different magnitudes: both strong and weak phases play a role.

## 6. CP Violation in $B$ Decays in the Standard Model

### *6.1.* $|q/p| \neq 1$

As explained in the previous section, in the $B_d$ system we expect model independently that $\Gamma_{12} \ll M_{12}$. Within any given model we can actually calculate the two quantities from quark diagrams. Within the Standard Model, $M_{12}$ is given by box diagrams. For both the $B_d$ and $B_s$ systems, the long distance contributions are expected to be negligible and the calculation of these diagrams with a high loop momentum is a very good approximation. $\Gamma_{12}$ is calculated from a cut of box diagrams [71]. Since the cut of a diagram always involves on-shell particles and thus long distance physics, the cut of the quark box diagram is a poor approximation to $\Gamma_{12}$. However, it does correctly give the suppression from small electroweak parameters such as the weak coupling. In other words, though the hadronic uncertainties are large and could change the result by order 50%, the cut in the box diagram is expected to give a reasonable order of magnitude estimate of $\Gamma_{12}$. (For $\Gamma_{12}(B_s)$ it has been shown that local quark-hadron duality holds exactly in the simultaneous limit of small velocity and large number of colors. We thus expect an uncertainty of $\mathcal{O}(1/N_C) \sim 30\%$ [72,73]. For $\Gamma_{12}(B_d)$ the small velocity limit is not as good an approximation but an uncertainty of order 50% still seems a reasonable estimate [74].)

Within the Standard Model, $M_{12}$ is dominated by top-mediated box diagrams [75]:

$$M_{12} = \frac{G_F^2}{12\pi^2} m_B m_W^2 \eta_B B_B f_B^2 (V_{tb}V_{td}^*)^2 S_0(x_t), \tag{6.1}$$

where $S_0(x_t)$ is given in eq. (4.20), $\eta_B = 0.55$ is a QCD correction, and $B_B f_B^2$ parametrizes the hadronic matrix element. For $\Gamma_{12}$, we have [76-78]

$$\begin{aligned} \Gamma_{12} \;=\; & -\frac{G_F^2}{24\pi} m_B m_b^2 B_B f_B^2 (V_{tb}V_{td}^*)^2 \\ & \times \left[\frac{5}{3}\frac{m_B^2}{(m_b+m_d)^2}\frac{B_S}{B_B}(K_2-K_1) + \frac{4}{3}(2K_1+K_2) + 8(K_1+K_2)\frac{m_c^2}{m_b^2}\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*}\right], \end{aligned} \tag{6.2}$$

where $K_1 = -0.39$ and $K_2 = 1.25$ [78] are combinations of Wilson coefficients and $B_S$ parametrizes the $(S-P)^2$ matrix element. Note that new physics is not expected to affect $\Gamma_{12}$ significantly because it usually takes place at a high energy scale and is relevant to the short distance part only. Therefore, the Standard Model estimate in eq. (6.2) remains valid model independently. Combining (6.1) and (6.2), one gets

$$\frac{\Gamma_{12}}{M_{12}} = -5.0\times 10^{-3}\left(1.4\frac{B_S}{B_B} + 0.24 + 2.5\frac{m_c^2}{m_b^2}\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*}\right). \tag{6.3}$$

We learn that $|\Gamma_{12}/M_{12}| = \mathcal{O}(m_b^2/m_t^2)$, which confirms our model independent order of magnitude estimate, $|\Gamma_{12}/M_{12}| \lesssim 10^{-2}$. As concerns the imaginary part of this ratio, we have

$$a_{\rm SL} = \mathcal{I}m\frac{\Gamma_{12}}{M_{12}} = -1.1\times 10^{-3}\sin\beta = -(2-5)\times 10^{-4}. \tag{6.4}$$

The strong suppression of $a_{\rm SL}$ compared to $|\Gamma_{12}/M_{12}|$ comes from the fact that the leading contribution to $\Gamma_{12}$ has the same phase as $M_{12}$. Consequently, $a_{\rm SL} = \mathcal{O}(m_c^2/m_t^2)$. The CKM factor, $\mathcal{I}m\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*} = \sin\beta$, is of order one. In contrast, for the $B_s$ system, where (6.3) holds except that $V_{td}$ ($V_{cd}$) is replaced by $V_{ts}$ ($V_{cs}$), there is an additional suppression from $\mathcal{I}m\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*} = \sin\beta_s \sim 10^{-2}$ (see the corresponding unitarity triangle).

### *6.2.* $|\bar{A}_{\bar f}/A_f| \neq 1$

In the previous subsection we estimated the effect of CP violation in mixing to be of $\mathcal{O}(10^{-3})$ within the Standard Model, and $\leq \mathcal{O}(|\Gamma_{12}/M_{12}|) \sim 10^{-2}$ model independently (for recent discussions, see [79-80,14]). In semileptonic decays, CP violation in mixing is the leading effect and therefore it can be measured through $a_{\rm SL}$. In purely hadronic $B$ decays, however, CP violation in decay and in the interference of decays with and without mixing is $\geq \mathcal{O}(10^{-2})$. We can therefore safely neglect CP violation in mixing in the following discussion and use

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\omega_B. \tag{6.5}$$

(From here on we omit the convention-dependent quark phases $\omega_q$ defined in eq. (3.15). Our final expressions for physical quantities are of course unaffected by such omission.)

A crucial question is then whether CP violation in decay is comparable to the CP violation in the interference of decays with and without mixing or negligible. In the first case, we can use the corresponding charged $B$ decays to observe effects of CP violation in decay. In the latter case, CP asymmetries in neutral $B$ decays are subject to clean theoretical interpretation: we will either have precise measurements of CKM parameters or be provided with unambiguous evidence for new physics. The question of the relative size of CP violation in decay can only be answered on a channel by channel basis, which is what we do in this section.

Channels that have contributions from tree diagrams only depend each on a single CKM combination:

$$\begin{aligned} A(c\bar{u}d) &= T_{c\bar{u}d} V_{cb} V_{ud}^*, \\ A(c\bar{u}s) &= T_{c\bar{u}s} V_{cb} V_{us}^*, \\ A(u\bar{c}d) &= T_{u\bar{c}d} V_{ub} V_{cd}^*, \\ A(u\bar{c}s) &= T_{u\bar{c}s} V_{ub} V_{cs}^*. \end{aligned} \tag{6.6}$$

The subdivision of tree processes into spectator, exchange and annihilation diagrams is unimportant in this respect since they all carry the same weak phase. For such modes, $|\bar{A}/A| = 1$. It is possible that $\mathcal{I}m\lambda \neq 0$, but since the final states are not CP eigenstates, a clean theoretical interpretation is difficult. We do not discuss these modes here any further.

Most channels have contributions from both tree- and three types of penguin-diagrams, the latter classified according to the identity of the quark in the loop, as diagrams with different intermediate quarks may have both different strong phases and different weak phases [81]. Consider first $b \to q\bar{q}s$ decays, with $q = c$ or $u$. Using (2.32), we can write the CKM dependence of these amplitudes as follows:

$$\begin{aligned} A(c\bar{c}s) &= (T_{c\bar{c}s} + P_s^c - P_s^t) V_{cb} V_{cs}^* + (P_s^u - P_s^t) V_{ub} V_{us}^*, \\ A(u\bar{u}s) &= (P_s^c - P_s^t) V_{cb} V_{cs}^* + (T_{u\bar{u}s} + P_s^u - P_s^t) V_{ub} V_{us}^*, \end{aligned} \tag{6.7}$$

where $T$ stands for the tree amplitude and $P^q$ for a penguin diagram with an intermediate $q$-quark. Next, consider $b \to q\bar{q}d$ decays, with $q = c$ or $u$. Using (2.33), we can write the CKM dependence of these amplitudes as follows:

$$\begin{aligned} A(c\bar{c}d) &= (P_d^t - P_d^u) V_{tb} V_{td}^* + (T_{c\bar{c}d} + P_d^c - P_d^u) V_{cb} V_{cd}^*, \\ A(u\bar{u}d) &= (P_d^t - P_d^c) V_{tb} V_{td}^* + (T_{u\bar{u}d} + P_d^u - P_d^c) V_{ub} V_{ud}^*. \end{aligned} \tag{6.8}$$

Note that in both (6.7) and (6.8) only differences of penguin contributions occur, which makes the cancellation of the ultraviolet divergences of these diagrams explicit.

To estimate the size of CP violation in decay for these channels, we need to know the ratio of the contribution from the difference between a top and light quark strong penguin diagram to the contribution from a tree diagram (with the CKM combination factored out):

$$r_{PT} = \frac{P^t - P^{\mathrm{light}}}{T_{q\bar{q}q'}} \approx \frac{\alpha_s}{12\pi} \ln\frac{m_t^2}{m_b^2} = \mathcal{O}(0.03). \tag{6.9}$$

However, this estimate does not include the effect of hadronic matrix elements, which are the probability factor to produce a particular final state particle content from a particular quark content. Since this probability differs for different kinematics, color flow and spin structures, it can be different for tree and penguin contributions and may partially compensate the coupling constant suppression of the penguin term. Recent CLEO results on $BR(B \to K\pi)$ and $BR(B \to \pi\pi)$ [82] suggest that the matrix element of penguin operators is indeed enhanced compared to that of tree operators. The enhancement could be by a factor of a few, leading to

$$r_{PT} \sim \lambda^2 - \lambda. \tag{6.10}$$

(Note that $r_{PT}$ does not depend on the CKM parameters. We use powers of the Wolfenstein parameter $\lambda$ to quantify our estimate for $r_{PT}$ in order to simplify the comparison between the size of CP violation in decay and CP violation in the interference between decays with and without mixing.) Take, for example, the $b \to c\bar{c}s$ decays. Using eqs. (3.28) and (6.7), the size of CP violation in decay is then estimated as follows:

$$1 - \left|\frac{\bar{A}_{c\bar{c}s}}{A_{c\bar{c}\bar{s}}}\right| \lesssim r_{PT}\ \mathcal{I}m\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*} = \mathcal{O}(\lambda^4 - \lambda^3). \tag{6.11}$$

Note that, in the language of eq. (3.28), this estimate includes $(A_2/A_2)\sin(\phi_2 - \phi_1)$ but not $\sin(\delta_2 - \delta_1)$. We only used $\sin(\delta_2 - \delta_1) \leq 1$ but if, for some reason, the difference in strong phases is small, the effect of CP violation in decay will be accordingly suppressed compared to (6.11).

Finally, processes involving only down-type quarks have no contributions from tree diagrams:

$$\begin{aligned} A(s\bar{s}s) &= (P_s^c - P_s^t)V_{cb}V_{cs}^* + (P_s^u - P_s^t)V_{ub}V_{us}^*, \\ A(s\bar{s}d) &= (P_d^t - P_d^u)V_{tb}V_{td}^* + (P_d^c - P_d^u)V_{cb}V_{cd}^*. \end{aligned} \tag{6.12}$$

For the estimate of CP violation in decay for these modes, we need to consider two types of ratios between penguin diagrams:

$$
\begin{aligned}
r_{P^c P^u} &= \frac{P^u - P^t}{P^c - P^t} \approx 1, \\
r_{P^l P^t} &= \frac{P^c - P^u}{P^t - P^{\text{light}}} \approx 0.1.
\end{aligned}
\tag{6.13}
$$

In the $m_c = m_u$ limit, we would have $r_{P^c P^u} = 1$ and $r_{P^l P^t} = 0$. The deviations from these limiting values should then be GIM suppressed [83]. The estimate of the somewhat surprisingly large $r_{P^l P^t}$ is based on refs. [84,85]. We get:

$$
1 - \left|\frac{\bar{A}_{s\bar{s}s}}{A_{s\bar{s}\bar{s}}}\right| \lesssim r_{P^l P^t}\ \mathcal{I}m\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*} = \mathcal{O}(\lambda^2),
\tag{6.14}
$$

$$
1 - \left|\frac{\bar{A}_{s\bar{s}d}}{A_{s\bar{s}\bar{d}}}\right| \lesssim r_{P^u P^c}\ \mathcal{I}m\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*} = \mathcal{O}(0.1).
\tag{6.15}
$$

As concerns the $b \to d\bar{d}s$ and $b \to d\bar{d}d$ processes, they mix strongly through rescattering effects with the tree mediated $b \to u\bar{u}s$ and $b \to u\bar{u}d$ decays, respectively. It is difficult to estimate these soft rescattering effects and we do not consider these modes here any further.

We thus classify the $B$ decays described in eqs. (6.7), (6.8) and (6.12) into four classes. Classes (i) and (ii) are expected to have relatively small CP violation in decay and hence are particularly interesting for extracting CKM parameters from interference of decays with and without mixing. In classes (iii) and (iv), CP violation in decay could be significant and might be observable in charged $B$ decays.

(i) Decays dominated by a single term: $b \to c\bar{c}s$ and $b \to s\bar{s}s$. The Standard Model predicts very small CP violation in decay: $\mathcal{O}(\lambda^4 - \lambda^3)$ for $b \to c\bar{c}s$ and $\mathcal{O}(\lambda^2)$ for $b \to s\bar{s}s$. Any observation of large CP asymmetries in charged $B$ decays for these channels would be a clue to physics beyond the Standard Model. The corresponding neutral modes have cleanly predicted relationships between CKM parameters and the measured asymmetry from interference between decays with and without mixing. The modes $B \to \psi K$ and $B \to \phi K$ are examples of this class.

(ii) Decays with a small second term: $b \to c\bar{c}d$ and $b \to u\bar{u}d$. The expectation that penguin-only contributions are suppressed compared to tree contributions suggests that these modes will have small effects of CP violation in decay, of $\mathcal{O}(\lambda^2 - \lambda)$, and an approximate prediction for the relationship between measured asymmetries in neutral decays and CKM phases can be made. Examples here are $B \to DD$ and $B \to \pi\pi$.

(iii) Decays with a suppressed tree contribution: $b \to u\bar{u}s$. The tree amplitude is suppressed by small mixing angles, $V_{ub}V_{us}$. The no-tree term may be comparable or even dominate and give large interference effects. An example is $B \to \rho K$.

(iv) Decays with no tree contribution and a small second term: $b \to s\bar{s}d$. Here the interference comes from penguin contributions with different charge 2/3 quarks in the loop and gives CP violation in decay that could be as large as 10%. An example is $B \to KK$.

Note that if the penguin enhancement is significant, then some of the decay modes listed in class (ii) might actually fit better in class (iii). For example, it is possible that $b \to u\bar{u}d$ decays have comparable contributions from tree and penguin amplitudes. On the other hand, this would also mean that some modes listed in class (iii) could be dominated by a single penguin term. For such cases an approximate relationship between measured asymmetries in neutral decays and CKM phases can be made.

A summary of our discussion in this section is given in the table II.

| Quark process | Sample $B^\pm$ mode | $\mathcal{O}(1 - \lvert\bar{A}/A\rvert)$ |
|---|---|---|
| $b \to c\bar{c}s$ | $\psi K^\pm$ | $r_{PT} \sin\beta_s \sim \lambda^4 - \lambda^3$ |
| $b \to s\bar{s}s$ | $\phi K^\pm$ | $\sin\beta_s \sim \lambda^2$ |
| $b \to u\bar{u}d$ | $\pi^0\pi^\pm$ | $r_{PT} \sin\alpha \sim \lambda^2 - \lambda$ |
| $b \to c\bar{c}d$ | $DD^\pm$ | $r_{PT} \sin\gamma \sim \lambda^2 - \lambda$ |
| $b \to u\bar{u}s$ | $\pi^0 K^\pm$ | $r_{PT}^{-1} \sin\beta_s \sim \lambda - 1$ |
| $b \to s\bar{s}d$ | $\phi\pi^\pm$ | $r_{PP} \sin\beta \sim 0.1$ |

Table II. CP violation in $B$ decays.

### *6.3.* $\mathcal{I}m\lambda_{f_{\rm CP}} \neq 0$

Let us first discuss an example of class (i), $B \to \psi K_S$. A new ingredient in the analysis is the effect of $K - \bar{K}$ mixing. For decays with a single $K_S$ in the final state,

$K-\bar{K}$ mixing is essential because $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$, and interference is possible only due to $K-\bar{K}$ mixing. This adds a factor of

$$\left(\frac{p}{q}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\omega_K^* \tag{6.16}$$

into $(\bar{A}/A)$. The quark subprocess in $\bar{B}^0 \to \psi\bar{K}^0$ is $b \to c\bar{c}s$ which is dominated by the $W$-mediated tree diagram:

$$\frac{\bar{A}_{\psi K_S}}{A_{\psi K_S}} = \eta_{\psi K_S}\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right)\omega_B^*. \tag{6.17}$$

The CP-eigenvalue of the state is $\eta_{\psi K_S} = -1$. Combining eqs. (6.5) and (6.17), we find

$$\lambda(B \to \psi K_S) = -\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}\right) \Longrightarrow \mathcal{I}m\lambda_{\psi K_S} = \sin(2\beta). \tag{6.18}$$

We have seen in the previous section that, for $b \to c\bar{c}s$ decays, we have a very small CP violation in decay, $1 - |\bar{A}/A| \sim \lambda^2 r_{PT}$. Consequently, eq. (6.18) is clean of hadronic uncertainties to better than $\mathcal{O}(10^{-2})$. This means that the measuremnet of $a_{\psi K_S}$ can give the theoretically cleanest determination of a CKM parameter, even cleaner than the determination of $|V_{us}|$ from $K \to \pi\ell\nu$. (If BR$(K_L \to \pi\nu\bar{\nu})$ is measured, it will give a comparably clean determination of $\eta$.)

A second example of a theoretically clean mode in class (i) is $B \to \phi K_S$. We showed in the previous section that, for $b \to s\bar{s}s$ decays, we have small CP violation in decay, $1 - |\bar{A}/A| = \mathcal{O}(\lambda^2) = \mathcal{O}(0.05)$. We can neglect this effect. The analysis is similar to the $\psi K_S$ case, and the asymmetry is proportional to $\sin(2\beta)$.

The same quark subprocesses give theoretically clean CP asymmetries also in $B_s$ decays. These asymmetries are, however, very small since the relative phase between the mixing amplitude and the decay amplitudes ($\beta_s$ defined in (2.37)) is very small.

The best known example of class (ii) is $B \to \pi\pi$. The quark subprocess is $b \to u\bar{u}d$ which is dominated by the $W$-mediated tree diagram. Neglecting for the moment the second, pure penguin, term we find

$$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = \eta_{\pi\pi}\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}\omega_B^*. \tag{6.19}$$

The CP eigenvalue for two pions is +1. Combining eqs. (6.5) and (6.19), we get

$$\lambda(B \to \pi^+\pi^-) = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*}\right) \implies \mathcal{I}m\lambda_{\pi\pi} = \sin(2\alpha). \tag{6.20}$$

The pure penguin term in $A(u\bar{u}d)$ in eq. (6.8) has a weak phase, $\arg(V_{td}^* V_{tb})$, different from the term with the tree contribution, so it modifies both $\mathcal{I}m\lambda_{\pi\pi}$ and (if there are non-trivial strong phases) $|\lambda_{\pi\pi}|$. The recent CLEO results mentioned above suggest that the penguin contribution to $B \to \pi\pi$ channel is significant, probably 10% or more. This then introduces CP violation in decay, unless the strong phases cancel (or are zero, as suggested by factorization arguments). The resulting hadronic uncertainty can be eliminated using isospin analysis [86]. This requires a measurement of the rates for the isospin-related channels $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$ as well as the corresponding CP-conjugate processes. The rate for $\pi^0\pi^0$ is expected to be small and the measurement is difficult, but even an upper bound on this rate can be used to limit the magnitude of hadronic uncertainties [87].

Related but slightly more complicated channels with the same underlying quark structure are $B \to \rho^0\pi^0$ and $B \to a_1^0\pi^0$. Again an analysis involving the isospin-related channels can be used to help eliminate hadronic uncertainties from CP violations in the decays [88,89]. Channels such as $\rho\rho$ and $a_1\rho$ could in principle also be studied, using angular analysis to determine the mixture of CP-even and CP-odd contributions.

The analysis of $B \to D^+D^-$ proceeds along very similar lines. The quark subprocess here is $b \to c\bar{c}d$, and so the tree contribution gives

$$\lambda(B \to D^+D^-) = \eta_{D^+D^-} \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{cd}^* V_{cb}}{V_{cd} V_{cb}^*}\right) \implies \mathcal{I}m\lambda_{DD} = -\sin(2\beta), \tag{6.21}$$

where we used $\eta_{D^+D^-} = +1$. Again, there are hadronic uncertainties due to the pure penguin term in (6.8), but they are estimated to be small.

A summary of our results for CP violation in the interference of decays with and without mixing in $B_q \to f_{\rm CP}$ is given in table III. For each mode, we give the asymmetry that would arise if the dominat contribution were the *only* contribution.

| Quark process | Sample $B_d$ mode | $\mathcal{I}m\lambda_{B_d\to f_{\rm CP}}$ | Sample $B_s$ mode | $\mathcal{I}m\lambda_{B_s\to f_{\rm CP}}$ |
|---|---|---|---|---|
| $b\to c\bar c s$ | $\psi K_S$ | $\sin 2\beta$ | $D_s\overline{D_s}$ | $\sin 2\beta_s$ |
| $b\to s\bar s s$ | $\phi K_S$ | $\sin 2\beta$ | $\phi\eta'$ | 0 |
| $b\to u\bar u d$ | $\pi\pi$ | $\sin 2\alpha$ | $\pi^0 K_S$ | $\sin 2\gamma$ |
| $b\to c\bar c d$ | $D^+D^-$ | $\sin 2\beta$ | $\psi K_S$ | $\sin 2\beta_s$ |
| $b\to u\bar u s$ | $\pi^0 K_S$ | $\sin 2\beta$ | $\phi\pi^0$ | 0 |
| $b\to s\bar s d$ | $\phi\pi$ | 0 | $\phi K_S$ | $\sin 2\beta$ |

Table III. $\mathcal{I}m\lambda(B_q\to f_{\rm CP})$.

In all cases the above discussions have neglected the distinction between strong penguins and electroweak penguins. The CKM phase structure of both types of penguins is the same. The only place where this distinction becomes important is when an isospin argument is used to remove hadronic uncertainties due to penguin contributions. These arguments are based on the fact that gluons have isospin zero, and hence strong penguin processes have definite $\Delta I$. Photons and $Z$-bosons on the other hand contribute to more than one $\Delta I$ transition and hence cannot be separated from tree terms by isospin analysis. In most cases electroweak penguins are small, typically no more than ten percent of the corresponding strong penguins and so their effects can safely be neglected. However in cases (iii) and (iv), where tree contributions are small or absent, their effects may need to be considered. (A full review of the role of electroweak penguins in $B$ decays has been given in ref. [90].)

## 7. CP Violation Can Probe New Physics

We have argued that the Standard Model picture of CP violation is rather unique and highly predictive. We have also stated that reasonable extensions of the Standard Model have a very different picture of CP violation. Experimental results are too few to decide between the various possibilities. But in the near future, we expect many new measurements of CP violating observables. Our discussion of CP violation in the presence of new physics is aimed to demonstrate that, indeed, models of new physics can significantly modify the

Standard Model predictions and that the near future measurements will therefore have a strong impact on the theoretical understanding of CP violation.

To understand how the Standard Model predictions could be modified by New Physics, we focus on CP violation in the interference between decays with and without mixing. As explained above, this type of CP violation may give, due to its theoretical cleanliness, unambiguous evidence for New Physics most easily.

Let us consider five specific CP violating observables.

(i) $\mathcal{I}m\lambda_{\psi K_S}$, the CP asymmetry in $B \to \psi K_S$. This measurement will cleanly determine the relative phase between the $B-\bar{B}$ mixing amplitude and the $b \to c\bar{c}s$ decay amplitude ($\sin 2\beta$ in the Standard Model). The $b \to c\bar{c}s$ decay has Standard Model tree contributions and therefore is very unlikely to be significantly affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. We parametrize such a modification by a phase $\theta_d$:

$$2\theta_d = \arg(M_{12}/M_{12}^{\rm SM}) \quad \Longrightarrow \quad \mathcal{I}m\lambda_{\psi K_S} = \sin[2(\beta+\theta_d)]. \tag{7.1}$$

(ii) $\mathcal{I}m\lambda_{\phi K_S}$, the CP asymmetry in $B \to \phi K_S$. This measurement will cleanly determine the relative phase between the $B-\bar{B}$ mixing amplitude and the $b \to s\bar{s}s$ decay amplitude. The $b \to s\bar{s}s$ decay has only Standard Model penguin contributions and therefore is sensitive to new physics. We parametrize the modification of the decay amplitude by a phase $\theta_A$ [91]:

$$\theta_A = \arg(\bar{A}_{\phi K_S}/\bar{A}_{\phi K_S}^{\rm SM}) \quad \Longrightarrow \quad \mathcal{I}m\lambda_{\phi K_S} = \sin[2(\beta+\theta_d+\theta_A)]. \tag{7.2}$$

(iii) $a_{\pi\nu\bar{\nu}}$, the CP violating ratio of $K \to \pi\nu\bar{\nu}$ decays, defined in (4.38). This measurement will cleanly determine the relative phase between the $K-\bar{K}$ mixing amplitude and the $s \to d\nu\bar{\nu}$ decay amplitude. The experimentally measured small value of $\varepsilon_K$ requires that the phase of the $K-\bar{K}$ mixing amplitude is not modified from the Standard Model prediction. (More precisely, it requires that the phase in the mixing amplitude is very close to the phase in the $s \to d\bar{u}u$ decay amplitude.) On the other hand, the decay, which in the Standard Model is a loop process with small mixing angles, can be easily modified by new physics.

(iv) $a_{D\to K\pi}$, the CP violating quantity in $D \to K^\pm\pi^\mp$ decays (see (5.5) and (5.11)):

$$a_{D\to K\pi} = \frac{\mathcal{I}m(\lambda_{K^-\pi^+}) - \mathcal{I}m(\lambda^{-1}_{K^+\pi^-})}{|\lambda_{K^-\pi^+}|} \quad\Longrightarrow\quad a_{D\to K\pi} = 2\cos\delta_{K\pi}\sin\phi_{K\pi}. \tag{7.3}$$

It depends on the relative phase between the $D-\bar{D}$ mixing amplitude and the $c \to d\bar{s}u$ and $c \to s\bar{d}u$ decay amplitudes. The two decay channels are tree level and therefore unlikely to be affected by new physics [68]. On the other hand, the mixing amplitude can be easily modified by new physics [69].

(v) $d_N$, the electric dipole moment of the neutron. We did not discuss this quantity so far because, unlike CP violation in meson decays, flavor changing couplings are not necessary for $d_N$. In other words, the CP violation that induces $d_N$ is *flavor diagonal.* It does in general get contributions from flavor changing physics, but it could be induced by sectors that are flavor blind. Within the Standard Model (and ignoring the strong CP angle $\theta_{\rm QCD}$), the contribution from $\delta_{\rm KM}$ arises at the three loop level and is at least six orders of magnitude below the experimental bound. We denote the present 90% C.L upper bound on $d_N$ by $d_N^{\rm exp}$. It is given by [92]

$$d_N^{\rm exp} = 6.3 \times 10^{-26}\ e\ {\rm cm}. \tag{7.4}$$

The main features of the observables that we chose are summarized in Table IV.

| Process | Observable | Mixing | Decay | SM | NP |
|---|---|---|---|---|---|
| $B \to \psi K_S$ | $\mathcal{I}m\lambda_{\psi K_S}$ | $B-\bar{B}$ | $b \to c\bar{c}s$ | $\sin 2\beta$ | $\sin 2(\beta+\theta_d)$ |
| $B \to \phi K_S$ | $\mathcal{I}m\lambda_{\phi K_S}$ | $B-\bar{B}$ | $b \to s\bar{s}s$ | $\sin 2\beta$ | $\sin 2(\beta+\theta_d+\theta_A)$ |
| $K \to \pi\nu\bar{\nu}$ | $a_{\pi\nu\bar{\nu}}$ | $K-\bar{K}$ | $s \to d\nu\bar{\nu}$ | $\sim \sin^2\beta$ | $\sin^2\theta_K$ |
| $D \to K\pi$ | $a_{D\to K\pi}$ | $D-\bar{D}$ | $c \to d\bar{s}u$ | 0 | $\sim \sin\phi_{K\pi}$ |
| $d_N$ | | | | $\lesssim 10^{-6} d_N^{\rm exp}$ | FD phases |

Table IV. Features of various CP violating observables.

The various CP violating observables discussed above are sensitive then to new physics in the mixing amplitudes for the $B-\bar{B}$ and $D-\bar{D}$ systems, in the decay amplitudes for $b \to s\bar{s}s$ and $s \to d\nu\bar{\nu}$ channels and to flavor diagonal CP violation. If information about all these processes becomes available and deviations from the Standard Model predictions are found, we can ask rather detailed questions about the nature of the new physics that is responsible to these deviations:

(i) Is the new physics related to the down sector? the up sector? both?

(ii) Is the new physics related to $\Delta B = 1$ processes? $\Delta B = 2$? both?

(iii) Is the new physics related to the third generation? to all generations?

(iv) Are the new sources of CP violation flavor changing? flavor diagonal? both?

It is no wonder then that with such rich information, flavor and CP violation provide an excellent probe of new physics.

## 8. Supersymmetry

A generic supersymmetric extension of the Standard Model contains a host of new flavor and CP violating parameters. (For recent reviews on supersymmetry see refs. [93-97]. The following chapter is based on [98].) It is an amusing exercise to count the number of parameters. The supersymmetric part of the Lagrangian depends, in addition to the three gauge couplings of $G_{\rm SM}$, on the parameters of the superpotential $W$, which can be written as a function of the scalar matter fields:

$$W = \sum_{i,j} \left( Y^u_{ij} h_u \tilde{q}_{Li} \tilde{u}_{Rj} + Y^d_{ij} h_d \tilde{q}_{Li} \tilde{d}_{Rj} + Y^\ell_{ij} h_d \tilde{L}_{Li} \tilde{\ell}_{Rj} \right) + \mu h_u h_d. \tag{8.1}$$

In addition, we have to add soft supersymmetry breaking terms:

$$\begin{aligned} \mathcal{L}_{\rm soft} \;=\; & -\left( a^u_{ij} h_u \tilde{q}_{Li} \tilde{u}_{Rj} + a^d_{ij} h_d \tilde{q}_{Li} \tilde{d}_{Rj} + a^\ell_{ij} h_d \tilde{L}_{Li} \tilde{\ell}_{Rj} + b h_u h_d + \text{h.c.} \right) \\ & - \sum_{\text{all scalars}} m^{S2}_{ij} A_i \bar{A}_j - \frac{1}{2} \sum_{(a)=1}^{3} \left( \tilde{m}_{(a)} (\lambda\lambda)_{(a)} + \text{h.c.} \right). \end{aligned} \tag{8.2}$$

The three Yukawa matrices $Y^f$ depend on 27 real and 27 imaginary parameters. Similarly, the three $a^f$-matrices depend on 27 real and 27 imaginary parameters. The five $m^{S2}$ hermitian $3 \times 3$ mass-squared matrices for sfermions ($S = \tilde{Q}, \tilde{d}_R, \tilde{u}_R, \tilde{L}, \tilde{\ell}_R$) have 30 real parameters and 15 phases. The gauge and Higgs sectors depend on

$$\theta_{\rm QCD}, \tilde{m}_{(1)}, \tilde{m}_{(2)}, \tilde{m}_{(3)}, g_1, g_2, g_3, \mu, b, m^2_{h_u}, m^2_{h_d}, \tag{8.3}$$

that is 11 real and 5 imaginary parameters. Summing over all sectors, we get 95 real and 74 imaginary parameters. If we switch off all the above parameters but the gauge couplings, we gain global symmetries:

$$G^{\rm SUSY}_{\rm global}(Y^f, \mu, a^f, b, m^2, \tilde{m} = 0) = U(3)^5 \times U(1)_{\rm PQ} \times U(1)_R, \tag{8.4}$$

where the $U(1)_{\rm PQ} \times U(1)_R$ charge assignments are:

$$\begin{array}{cccccc} & h_u & h_d & Q\bar{u} & Q\bar{d} & L\bar{\ell} \\ U(1)_{\rm PQ} & 1 & 1 & -1 & -1 & -1\,. \\ U(1)_{\rm R} & 1 & 1 & 1 & 1 & 1 \end{array} \tag{8.5}$$

Consequently, we can remove at most 15 real and 32 imaginary parameters. But even when all the couplings are switched on, there is a global symmtery, that is

$$G^{\rm SUSY}_{\rm global} = U(1)_B \times U(1)_L, \tag{8.6}$$

so that 2 of the 32 imaginary parameters cannot be removed. We are left then with

$$124 \; = \; \begin{cases} 80 & \text{real} \\ 44 & \text{imaginary} \end{cases} \text{physical parameters.} \tag{8.7}$$

In particular, there are 43 new CP violating phases! In addition to the single Kobayashi-Maskawa of the SM, we can put 3 phases in $M_1, M_2, \mu$ (we used the $U(1)_{\rm PQ}$ and $U(1)_{\rm R}$ to remove the phases from $\mu B^*$ and $M_3$, respectively) and the other 40 phases appear in the mixing matrices of the fermion-sfermion-gaugino couplings. (Of the 80 real parameters, there are 11 absolute values of the parameters in (8.3), 9 fermion masses, 21 sfermion masses, 3 CKM angles and 36 SCKM angles.) Supersymmetry provides a nice example to our statement that reasonable extensions of the Standard Model may have more than one source of CP violation.

The requirement of consistency with experimental data provides strong constraints on many of these parameters. For this reason, the physics of flavor and CP violation has had a profound impact on supersymmetric model building. A discussion of CP violation in this context can hardly avoid addressing the flavor problem itself. Indeed, many of the supersymmetric models that we analyze below were originally aimed at solving flavor problems.

As concerns CP violation, one can distinguish two classes of experimental constraints. First, bounds on nuclear and atomic electric dipole moments determine what is usually called the *supersymmetric CP problem.* Second, the physics of neutral mesons and, most importantly, the small experimental value of $\varepsilon_K$ pose the *supersymmetric* $\varepsilon_K$ *problem.* In the next two subsections we describe the two problems. Then we describe various supersymmetric flavor models and the ways in which they address the supersymmetric CP problem.

Before turning to a detailed discussion, we define two scales that play an important role in supersymmetry: $\Lambda_S$, where the soft supersymmetry breaking terms are generated, and $\Lambda_F$, where flavor dynamics takes place. When $\Lambda_F \gg \Lambda_S$, it is possible that there are no genuinely new sources of flavor and CP violation. This leads to models with exact universality, which we discuss in section 8.3. When $\Lambda_F \lesssim \Lambda_S$, we do not expect, in general, that flavor and CP violation are limited to the Yukawa matrices. One way to suppress CP violation would be to assume that, similarly to the Standard Model, CP violating phases are large, but their effects are screened, possibly by the same physics that explains the various flavor puzzles. Such models, with Abelian or non-Abelian horizontal symmetries, are described in section 8.4. It is also possible that CP violating effects are suppressed because squarks are heavy. This scenario is also discussed in section 8.4. Another option is to assume that CP is an approximate symmetry of the full theory (namely, CP violating phases are all small). We discuss this scenario in section 8.5. A brief discussion of the implications of $\varepsilon'/\varepsilon$ is included in this subsection. Some concluding comments regarding CP violation as a probe of supersymmetric flavor models are given in section 8.6.

### *8.1. The Supersymmetric CP Problem*

One aspect of supersymmetric CP violation involves effects that are flavor preserving. Then, for simplicity, we describe this aspect in a supersymmetric model without additional flavor mixings, *i.e.* the minimal supersymmetric standard model (MSSM) with universal sfermion masses and with the trilinear SUSY-breaking scalar couplings proportional to the corresponding Yukawa couplings. (The generalization to the case of non-universal soft terms is straightforward.) In such a constrained framework, there are four new phases beyond the two phases of the Standard Model ($\delta_{\rm KM}$ and $\theta_{\rm QCD}$). One arises in the bilinear $\mu$-term of the superpotential (8.1), while the other three arise in the soft supersymmetry breaking parameters of (8.2): $\tilde m$ (the gaugino mass), $a$ (the trilinear scalar coupling) and $b$ (the bilinear scalar coupling). Only two combinations of the four phases are physical [99,100]. To see this, note that one could treat the various dimensionful parameters in (8.1) and (8.2) as spurions which break the $U(1)_{\rm PQ}\times U(1)_{\rm R}$ symmetry, thus deriving selection rules:

$$\begin{array}{ccccc} & \tilde m & A & b & \mu \\ U(1)_{\rm PQ} & 0 & 0 & -2 & -2 \\ U(1)_{\rm R} & -2 & -2 & -2 & 0 \end{array} \qquad (8.8)$$

(where we defined $A$ through $a^f = AY^f$). Physical observables can only depend on combinations of the dimensionful parameters that are neutral under both $U(1)$'s. There are three such independent combinations: $\tilde m\mu b^*$, $A\mu b^*$ and $A^*\tilde m$. However, only two of their phases are independent, say

$$\phi_A = \arg(A^*\tilde m), \quad \phi_B = \arg(\tilde m\mu b^*). \qquad (8.9)$$

In the more general case of non-universal soft terms there is one independent phase $\phi_{A_i}$ for each quark and lepton flavor. Moreover, complex off-diagonal entries in the sfermion mass-squared matrices may represent additional sources of CP violation.

The most significant effect of $\phi_A$ and $\phi_B$ is their contribution to electric dipole moments (EDMs). The electric dipole moment of a fermion $\psi$ is defined as the coefficient $d_\psi$ of the operator

$$\mathcal{L}_{d_\psi} = -\frac{i}{2} d_\psi \bar\psi \sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu}. \qquad (8.10)$$

For example, the contribution from one-loop gluino diagrams to the down quark EDM is given by [101,102]:

$$d_d = M_d \frac{e\alpha_3}{18\pi\tilde m^3}\left(|A|\sin\phi_A + \tan\beta|\mu|\sin\phi_B\right), \qquad (8.11)$$

where we have taken $m_Q^2 \sim m_D^2 \sim m_{\tilde g}^2 \sim \tilde m^2$, for left- and right-handed squark and gluino masses. We define, as usual, $\tan\beta = \langle H_u\rangle/\langle H_d\rangle$. Similar one-loop diagrams give rise to chromoelectric dipole moments. The electric and chromoelectric dipole moments of the light quarks $(u, d, s)$ are the main source of $d_N$ (the EDM of the neutron), giving [103]

$$d_N \sim 2 \left( \frac{100\, GeV}{\tilde{m}} \right)^2 \sin\phi_{A,B} \times 10^{-23} \ e\,\mathrm{cm}, \tag{8.12}$$

where, as above, $\tilde{m}$ represents the overall SUSY scale. In a generic supersymmetric framework, we expect $\tilde{m} = \mathcal{O}(m_Z)$ and $\sin\phi_{A,B} = \mathcal{O}(1)$. Then the constraint (7.4) is generically violated by about two orders of magnitude. This is *the Supersymmetric CP Problem* [101-105].

Eq. (8.12) shows what are the possible ways to solve the supersymmetric CP problem:

(i) Heavy squarks: $\tilde{m} \gtrsim 1\ TeV$;

(ii) Approximate CP (or left-right symmetry [106]): $\sin\phi_{A,B} \ll 1$.

Recently, a third way has been investigated, that is cancellations between various contributions to the electric dipole moments [107-113]. However, there seems to be no symmetry that can guarantee such a cancellation. This is in contrast to the other two mechanisms mentioned above that were shown to arise naturally in specific models. We therefore do not discuss any further this third mechanism.

Finally, we mention that the electric dipole moment of the electron is also a sensitive probe of flavor diagonal CP phases. The present experimental bound, [114-115],

$$|d_e| \leq 4 \times 10^{-27} \ e \ \mathrm{cm}, \tag{8.13}$$

is also violated by about two orders of magnitue for 'natural' values of supersymmetric parameters.

### *8.2. The Supersymmetric $\varepsilon_K$ Problem*

The contribution to the CP violating $\varepsilon_K$ parameter in the neutral $K$ system is dominated by diagrams involving $Q$ and $\bar{d}$ squarks in the same loop [116-120]. The corresponding effective four-fermi operator involves fermions of both chiralities, so that its matrix elements are enhanced by $\mathcal{O}(m_K/m_s)^2$ compared to the chirality conserving operators. For $m_{\tilde{g}} \simeq m_Q \simeq m_D = \tilde{m}$ (our results depend only weakly on this assumption) and focusing on the contribution from the first two squark families, one gets [120]:

$$\varepsilon_K = \frac{5\ \alpha_3^2}{162\sqrt{2}} \frac{f_K^2 m_K}{\tilde{m}^2 \Delta m_K} \left[ \left( \frac{m_K}{m_s + m_d} \right)^2 + \frac{3}{25} \right] \mathcal{I}m \left\{ \frac{(\delta m_Q^2)_{12}}{m_Q^2} \frac{(\delta m_D^2)_{12}}{m_D^2} \right\}, \tag{8.14}$$

where $(\delta m^2_{Q,D})_{12}$ are the off diagonal entries in the squark mass matrices in a basis where the down quark mass matrix and the gluino couplings are diagonal. These flavor violating quantities are often written as $(\delta m^2_{Q,D})_{12} = V^{Q,D}_{11} \delta m^2_{Q,D} V^{Q,D*}_{21}$, where $\delta m^2_{Q,D}$ is the mass splitting among the squarks and $V^{Q,D}$ are the gluino coupling mixing matrices in the mass eigenbasis of quarks and squarks. Note that CP would be violated even if there were two families only [121]. Using the experimental value of $\varepsilon_K$, we get

$$\frac{(\Delta m_K \varepsilon_K)^{\rm SUSY}}{(\Delta m_K \varepsilon_K)^{\rm EXP}} \sim 10^7 \left(\frac{300\ GeV}{\tilde{m}}\right)^2 \left(\frac{m^2_{Q_2} - m^2_{Q_1}}{m^2_Q}\right) \left(\frac{m^2_{D_2} - m^2_{D_1}}{m^2_D}\right) |K^{dL}_{12} K^{dR}_{12}| \sin\phi, \tag{8.15}$$

where $\phi$ is the CP violating phase. In a generic supersymmetric framework, we expect $\tilde{m} = \mathcal{O}(m_Z)$, $\delta m^2_{Q,D}/m^2_{Q,D} = \mathcal{O}(1)$, $K^{Q,D}_{ij} = \mathcal{O}(1)$ and $\sin\phi = \mathcal{O}(1)$. Then the constraint (8.15) is generically violated by about seven orders of magnitude.

Eq. (8.15) also shows what are the possible ways to solve the supersymmetric $\varepsilon_K$ problem:

(i) Heavy squarks: $\tilde{m} \gg 300\ GeV$;

(ii) Universality: $\delta m^2_{Q,D} \ll m^2_{Q,D}$;

(iii) Alignment: $|K^d_{12}| \ll 1$;

(iv) Approximate CP: $\sin\phi \ll 1$.

### *8.3. Exact Universality*

Both supersymmetric CP problems are solved if, at the scale $\Lambda_S$, the soft supersymmetry breaking terms are universal and the genuine SUSY CP phases $\phi_{A,B}$ vanish. Then the Yukawa matrices represent the only source of flavor and CP violation which is relevant in low energy physics. This situation can naturally arise when supersymmetry breaking is mediated by gauge interactions at a scale $\Lambda_S \ll \Lambda_F$ [122]. In the simplest scenarios, the $A$-terms and the gaugino masses are generated by the same SUSY and $U(1)_R$ breaking source (see eq. (8.8)). Thus, up to very small effects due to the *standard* Yukawa matrices, $\arg(A) = \arg(m_{\tilde{g}})$ so that $\phi_A$ vanishes. In specific models also $\phi_B$ vanishes in a similar way [123,124]. It is also possible that similar boundary conditions occur when supersymmetry breaking is communicated to the observable sector up at the Planck scale.

The situation in this case seems to be less under control from the theoretical point of view. Dilaton dominance in SUSY breaking, though, seems a very interesting direction to explore [125,126].

The most important implication of this type of boundary conditions for soft terms, which we refer to as *exact universality*, is the existence of the SUSY analogue of the GIM mechanism which operates in the SM. The CP violating phase of the CKM matrix can feed into the soft terms via Renormalization Group (RG) evolution only with a strong suppression from light quark masses [99].

With regard to the supersymmetric CP problem, gluino diagrams contribute to quark EDMs as in eq. (8.11), but with a highly suppressed effective phase, *e.g.*

$$\phi_{A_d} \sim (t_S/16\pi^2)^4 Y_t^4 Y_c^2 Y_b^2 J. \tag{8.16}$$

Here $t_S = \log(\Lambda_S/M_W)$ arises from the RG evolution from $\Lambda_S$ to the electroweak scale, the $Y_i$'s are quark Yukawa couplings (in the mass basis), and $J \simeq 2 \times 10^{-5}$ is defined in eq. (2.30). A similar contribution comes from chargino diagrams. The resulting EDM is $d_N \lesssim 10^{-31}$ $e$ cm. This maximum can be reached only for very large $\tan\beta \sim 60$ while, for small $\tan\beta \sim 1$, $d_N$ is about 5 orders of magnitude smaller. This range of values for $d_N$ is much below the present ($\sim 10^{-25}$ $e$ cm) and foreseen ($\sim 10^{-28}$ $e$ cm) experimental sensitivities [127-130].

With regard to the supersymmetric $\varepsilon_K$ problem, the contribution to $\varepsilon_K$ is proportional to $\mathcal{I}m(V_{td}V_{ts}^*)^2 Y_t^4 (t_S/16\pi^2)^2$, giving the same GIM suppression as in the SM. This contribution turns out to be negligibly small [99,131]. The supersymmetric contribution to $D - \bar{D}$ mixing is similarly small and we expect no observable effects. For the $B_d$ and $B_s$ systems, the largest SUSY contribution to the mixing comes from box diagrams with intermediate charged Higgs and the up quarks. It can be up to $\mathcal{O}(0.2)$ of the SM amplitude for $\Lambda_S = M_{\rm Pl}$ and $\tan\beta = \mathcal{O}(1)$ [132-135], and much smaller for large $\tan\beta$. The contribution is smaller in models of gauge mediated SUSY breaking where the mass of the charged Higgs boson is typically $\gtrsim 300\ GeV$ [122] and $t_S \sim 5$. The SUSY contributions to $B_s - \bar{B}_s$ and $B_d - \bar{B}_d$ mixing are, to a good approximation, proportional to $(V_{tb}V_{ts}^*)^2$ and $(V_{tb}V_{td}^*)^2$, respectively, like in the SM. Then, regardless of the size of these contributions, the relation $\Delta m_{B_d}/\Delta m_{B_s} \sim |V_{td}/V_{ts}|^2$ and the CP asymmetries in neutral $B$ decays into final CP eigenstates are the same as in the SM.

### *8.4. Approximate Horizontal Symmetries*

In the class of supersymmetric models with $\Lambda_F \lesssim \Lambda_S$, the soft masses are generically not universal, and we do not expect flavor and CP violation to be limited to the Yukawa matrices. Most models where soft terms arise at the Planck scale ($\Lambda_S \sim M_{\rm Pl}$) belong to this class. It is possible that, similarly to the Standard Model, CP violating phases are large, but their effects are screened, possibly by the same physics that explains the various flavor puzzles. This usually requires Abelian or non-Abelian horizontal symmetries. Two ingredients play a major role here: selection rules that come from the symmetry and holomorphy of Yukawa and $A$-terms that comes from the supersymmetry. With Abelian symmetries, the screening mechanism is provided by *alignment* [136-139], whereby the mixing matrices for gaugino couplings have very small mixing angles, particularly for the first two down squark generations. With non-Abelian symmetries, the screening mechanism is *approximate universality*, where squarks of the two families fit into an irreducible doublet and are, therefore, approximately degenerate [140-147]. In all of these models, it is difficult to avoid $d_N \gtrsim 10^{-28}$ e cm.

As far as the third generation is concerned, the signatures of Abelian and non-Abelian models are similar. In particular, they allow observable deviations from the SM predictions for CP asymmetries in $B$ decays. The recent measurement of $a_{\psi K_S}$ gives first constraints on these contributions [14]. In some cases, non-Abelian models give relations between CKM parameters and consequently predict strong constraints on these CP asymmetries.

For the two light generations, only alignment allows interesting effects. In particular, it predicts large CP violating effects in $D-\bar{D}$ mixing [136,137]. Thus, it allows $a_{D\to K\pi} = \mathcal{O}(1)$.

Finally, it is possible that CP violating effects are suppressed because squarks are heavy. If the masses of the first and second generations squarks $m_i$ are larger than the other soft masses, $m_i^2 \sim 100\,\tilde{m}^2$ then the Supersymmetric CP problem is solved and the $\varepsilon_K$ problem is relaxed (but not eliminated) [143,148]. This does not necessarily lead to naturalness problems, since these two generations are almost decoupled from the Higgs sector.

Notice though that, with the possible exception of $m^2_{\tilde{b}_R}$, third family squark masses cannot naturally be much above $m^2_Z$. If the relevant phases are of $O(1)$, the main contribution to $d_N$ comes from the third family via the two-loop induced three-gluon operator [149], and it is roughly at the present experimental bound when $m_{\tilde{t}_{L,R}} \sim 100\ GeV$.

Models with the first two squark generations heavy have their own signatures of CP violation in neutral meson mixing [150]. The mixing angles relevant to $D-\bar{D}$ mixing are similar, in general, to those of models of alignment (if alignment is invoked to explain $\Delta m_K$ with $m^2_{Q,D} \lesssim 20\ TeV$). However, since the $\tilde{u}$ and $\tilde{c}$ squarks are heavy, the contribution to $D-\bar{D}$ mixing is one to two orders of magnitude below the experimental bound. This may lead to the interesting situation that $D-\bar{D}$ mixing will first be observed through its CP violating part [70]. In the neutral $B$ system, $\mathcal{O}(1)$ shifts from the Standard Model predictions of CP asymmetries in the decays to final CP eigenstates are possible. This can occur even when squark masses of the third family are $\sim 1\ TeV$ [151], since now mixing angles can naturally be larger than in the case of horizontal symmetries (alignment or approximate universality).

### *8.5. Approximate CP Symmetry*

Both supersymmetric CP problems are solved if CP is an approximate symmetry, broken by a small parameter of order $10^{-3}$. This is another possible solution to CP problems in the class of supersymmetric models with $\Lambda_F \lesssim \Lambda_S$. (Of course, some mechanism has also to suppress the real part of the $\Delta S = 2$ amplitude by a sufficient amount.)

If CP is an approximate symmetry, we expect also the SM phase $\delta_{\rm KM}$ to be $\ll 1$. Then the standard box diagrams cannot account for $\varepsilon_K$ which should arise from another source. In supersymmetry with non-universal soft terms, the source could be diagrams involving virtual superpartners, mainly squark-gluino box diagrams. Let us call $(M^K_{12})^{\rm SUSY}$ the supersymmetric contribution to the $K-\bar{K}$ mixing amplitude. Then the requirements $\mathcal{R}e(M^K_{12})^{\rm SUSY} \lesssim \Delta m_K$ and $\mathcal{I}m(M^K_{12})^{\rm SUSY} \sim \varepsilon_K \Delta m_K$ imply that the generic CP phases are $\geq \mathcal{O}(\varepsilon_K) \sim 10^{-3}$.

Of course, $d_N$ constrains the relevant CP violating phases to be $\lesssim 10^{-2}$. If all phases are of the same order, then $d_N$ must be just below or barely compatible with the present experimental bound. A signal should definitely be found if the accuracy is increased by two orders of magnitude.

The main phenomenological implication of these scenarios is that CP asymmetries in $B$ meson decays are small, perhaps $\mathcal{O}(\varepsilon_K)$, rather than $\mathcal{O}(1)$ as expected in the SM. Also the ratio $a_{\pi\nu\bar{\nu}}$ of eq. (4.38) is very small, in contrast to the Standard Model where it is expected to be of $\mathcal{O}(\sin^2\beta)$. Explicit models of approximate CP were presented in refs. [152-155].

The experimntal value of $\varepsilon'/\varepsilon$ has particularly interesting implications on models of approximate CP [156]. In this framework, the standard model cannot account for $\varepsilon'/\varepsilon$. A model of approximate CP would then be excluded if it does not provide sufficiently large contributions from new physics to this parameter. A generic supersymmetric model where the $a^q$ terms in (8.2) are not proportional to the $Y^q$ terms in (8.1) can provide a large contribution [157] related to imaginary part of

$$a^d_{12} \sim m_s |V_{us}|/\tilde{m}. \tag{8.17}$$

In models of non-Abelian flavor symmetries, the contribution is typically not large enough because of cancellation between the $a^d_{12}$ and $a^d_{21}$ terms [158,159]. In models of heavy $\tilde{d}$ and $\tilde{s}$ squarks, the contribution is highly suppressed by the heavy mass scale [156]. Models of alignment can give a contribution that is not much smaller than the estimate in (8.17). If, however, the related CP violating phase is small, then the model can account for $\varepsilon'/\varepsilon$ only if both the model parameters and the hadronic parameters assume rather extreme values [156]. We conclude that most existing models of supersymmetry with approximate CP are excluded (or, at least, strongly disfavored) by the experimental measurement of $\varepsilon'/\varepsilon$. (For other recent works on $\varepsilon'/\varepsilon$ in the supersymmetric framework, see [119-120,54,160-163].

The fact that the Standard Model and the models of approximate CP are both viable at present shows that the mechanism of CP violation has not really been tested experimentally. The only measured CP violating observales, that is $\varepsilon_K$ and $\varepsilon'_K$, are small. Their smallness could be related to the 'accidental' smallness of CP violation for the first two quark generations, as is the case in the Standard Model, or to CP being an approximate symmetry, as is the case in the models discussed here. Future measurements, particularly of processes where the third generation plays a dominant role (such as $a_{\psi K_S}$ or $a_{\pi\nu\bar{\nu}}$), will easily distinguish between the two scenarios. While the Standard Model predicts large CP violating effects for these processes, approximate CP would suppress them too.

### *8.6. Some Concluding Remarks*

We can get an intuitive understanding of how the various supersymmetric flavor models discussed in this chapter solve the supersymmetric flavor and CP problems by presenting the general form of the squark mass-squared matrices for each framework. This is summarized in Table V. The implications of each flavor model for the various CP violating observables presented in the previous chapter are given in Table VI.

| Flavor Model | Theory | $\frac{1}{\tilde{m}^2}\tilde{M}^2_{LL} \sim$ | Physical Parameters |
|---|---|---|---|
| Exact Universality | GMSB | diag$(a,a,a)$ | $\Delta\tilde{m}^2_{12} \sim m_c^2/m_W^2$ |
| Approx. Universality | Non-Abelian $H$ | diag$(a,a,b)$ | $\Delta\tilde{m}^2_{12} \sim \sin^2\theta_C$ |
| Alignment | Abelian $H$ | diag$(a,b,c)$ | $(K^d_L)_{12} \ll \sin\theta_C$ |
| Heavy Squarks | Comp.; Anom. $U(1)$ | diag$(A,B,c)$ | $\tilde{m}^2_{1,2} \sim 100\tilde{m}^2$ |
| Approximate CP | SCPV | | $10^{-3} \lesssim \phi_{\rm CP} \ll 1$ |

Table V. Supersymmetric flavor models.

| Model | $\frac{d_N}{10^{-25}\ e\ \rm cm}$ | $\theta_d$ | $\theta_A$ | $a_{D\to K\pi}$ | $a_{K\to\pi\nu\bar{\nu}}$ |
|---|---|---|---|---|---|
| Standard Model | $\lesssim 10^{-6}$ | 0 | 0 | 0 | $\mathcal{O}(1)$ |
| Exact Universality | $\lesssim 10^{-6}$ | 0 | 0 | 0 | =SM |
| Approx. Universality | $\gtrsim 10^{-2}$ | $\mathcal{O}(0.2)$ | $\mathcal{O}(1)$ | 0 | ≈SM |
| Alignment | $\gtrsim 10^{-3}$ | $\mathcal{O}(0.2)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | ≈SM |
| Heavy Squarks | $\sim 10^{-1}$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(10^{-2})$ | ≈SM |
| Approximate CP | $\sim 10^{-1}$ | $-\beta$ | 0 | $\mathcal{O}(10^{-3})$ | $\mathcal{O}(10^{-5})$ |

Table VI. Phenomenological implicatons of supersymmetric flavor models.

We would like to emphasize the following points:

(i) For supersymmetry to be established, a direct observation of supersymmetric particles is necessary. Once it is discovered, then measurements of CP violating observables will be a very sensitive probe of its flavor structure and, consequently, of the mechanism of dynamical supersymmetry breaking.

(ii) It is easy to distinguish between models of exact universality and models with genuine supersymmetric flavor and CP violation. This can be done through searches of $d_N$ and of CP asymmetries in $B$ decays.

(iii) The neutral $D$ system provides a stringent test of alignment.

(iv) The fact that $K \to \pi\nu\bar{\nu}$ decays are not affected by most supersymmetric flavor models [164,165] is actually an advantage. The Standard Model correlation between $a_{\pi\nu\bar{\nu}}$ and $a_{\psi K_S}$ is a much cleaner test than a comparison of $a_{\psi K_S}$ to the CKM constraints.

(v) Approximate CP has dramatic effects on all observables. My guess is that in lectures given a year from now, it will not appear in the Table as a viable option.

## 9. Left Right Symmetric Models of Spontaneous CP Violation

### *9.1. The Model*

We consider models with a symmetry $G_{\rm LRS} \times D_{\rm LRS}$, where $G_{\rm LRS}$ is the gauge group,

$$G_{\rm LRS} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}, \tag{9.1}$$

and $D_{\rm LRS}$ is a discrete group,

$$D_{\rm LRS} = P \times C. \tag{9.2}$$

Various versions of left-right symmetric models differ in $D_{\rm LRS}$. We are interested here in models where CP is only spontaneously broken, hence our choice of (9.2) [166-171].

The fermion representations consist of three generations of

$$Q_{Li}(3,2,1)_{1/3}, \quad Q_{Ri}(3,1,2)_{1/3}, \quad L_{Li}(1,2,1)_{-1}, \quad L_{Ri}(1,1,2)_{-1}. \tag{9.3}$$

Under $D_{\rm LRS}$, the fermion fields transform as follows:

$$\begin{array}{lcc} P: & Q_L \leftrightarrow Q_R & L_L \leftrightarrow L_R \\ C: & Q_L \leftrightarrow i\sigma_2(Q_R)^* & L_L \leftrightarrow i\sigma_2(L_R)^* \end{array} \tag{9.4}$$

The scalar representations consist of three multiplets [172],

$$\Delta_R(1,1,3)_2, \quad \Delta_L(1,3,1)_2, \quad \Phi(1,2,2)_0. \tag{9.5}$$

Under $D_{\rm LRS}$, the scalar fields transform as follows:

$$\begin{array}{ccc} P: & \Delta_L \leftrightarrow \Delta_R & \Phi \leftrightarrow \Phi^\dagger \\ C: & \Delta_L \leftrightarrow (\Delta_R)^* & \Phi \leftrightarrow \Phi^T \end{array} \tag{9.6}$$

It is often convenient to write $\Phi$ in a $2\times 2$ matrix form:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}. \tag{9.7}$$

The spontaneous symmetry breaking occurs in two stages,

$$G_{\rm LRS} \times D_{\rm LRS} \to G_{\rm SM} \to SU(3)_C \times U(1)_{\rm EM}, \tag{9.8}$$

due to the VEVs of the neutral members of the scalar fields:

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 \\ 0 \\ v_R e^{i\beta} \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 \\ 0 \\ v_L \end{pmatrix}, \quad \langle \Phi \rangle = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 e^{i\alpha} \end{pmatrix}. \tag{9.9}$$

(In general, all four VEVs are complex. There is, however, freedom of rotations by $U(1)_{B-L}$ for $\Delta_L$ and $\Delta_R$ and by $U(1)_{T_{3L}} \times U(1)_{T_{3R}}$ for $\phi_1$ and $\phi_2$, so that only two phases are physical.) These VEVs are assumed to satisfy the hierarchy

$$v_R \gg k_1, k_2 \gg v_L. \tag{9.10}$$

The first stage of symmetry breaking in (9.8) takes place at the scale $v_R$ and the second at $k = \sqrt{k_1^2 + k_2^2}$.

*9.2. Flavor Parameters*

The quark Yukawa couplings have the following form:

$$\mathcal{L}_{\rm Yuk} = f\overline{Q_L}\Phi Q_R + h\overline{Q_L}\widetilde{\Phi}Q_R + {\rm h.c.}, \tag{9.11}$$

where $\widetilde{\Phi} = \tau_2 \Phi^* \tau_2$. As a consequence of $D_{\rm LRS}$, the Yukawa matrices $f$ and $h$ are *symmetric and real*: $P$ requires that they are hermitian, $C$ requires that they are symmetric, and CP requires that they are real. The resulting mass matrices,

$$\begin{aligned} M_u &= f k_1 + h k_2 e^{-i\alpha}, \\ M_d &= h k_1 + f k_2 e^{i\alpha}, \end{aligned} \tag{9.12}$$

are complex symmetric matrices.

How many independent physical flavor parameters (and, in particular, phases) does this model have? We have two symmetric and complex mass matrices, that is twelve real and twelve imaginary Yukawa parameters. If we set $h = f = 0$, we gain a global $U(3)$ symmetry ($D_{\rm LRS}$ does not allow independent $U(3)_L$ and $U(3)_R$ rotations). This means that we can remove three real and six imaginary parameters. When $h$ and $f$ are different from zero, there is no global symmetry ($U(1)_{B-L}$ is part of the gauge symmetry). We conclude that there are nine real and six imaginary flavor parameters. Six of the real parameters are the six quark masses. To identify the other flavor parameters, note that the symmetric mass matrices can be diagonalized by a unitary transformation of the form

$$V_u M_u V_u^T = M_u^{\rm diag}, \quad V_d M_d V_d^T = M_d^{\rm diag}. \tag{9.13}$$

Consequently, the mixing matrices $V_L$ and $V_R$ describing, respectively, the $W_L$ and $W_R$ interactions,

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \left( W^+_{L\mu} \overline{u_L} V_L \gamma^\mu d_L + W^+_{R\mu} \overline{u_R} V_R \gamma^\mu d_R \right) + \text{h.c.}, \tag{9.14}$$

are related:

$$V_L = P^u V_R^* P^d, \tag{9.15}$$

where $P^u$ and $P^d$ are diagonal phase matrices. The three real parameters are then the three mixing angles, which are equal in $V_L$ and $V_R$. The six phases can be arranged in various ways. A convenient choice is to have a single phase in $V_L$, which is then just $\delta_{\rm KM}$ of $V_{\rm CKM}$ of the Standard Model, and five phases in $V_R$. (It is also possible to have $V_L = V_R^*$ with six phases in each.)

*9.3. What is the Low Energy Effective Theory of the LRS Model?*

It is interesting to ask what is the low enrgy effective theory below the scale $v_R$. It is straightforward to show that the light fields are precisely those of the Standard Model: the fermions are chiral under $SU(2)_L$ except for the right-handed neutrinos in $L_{Ri}$ which acquire Majorana masses at the scale $v_R$ due to their coupling to $\Delta_R$. (The left-handed neutrinos acquire very light masses from both the see-saw mechanism and their direct coupling to $\Delta_L$.) Only the one Higgs doublet related to $G_{\rm SM}$ breaking, that is $k_1\phi_1 + k_2 e^{-i\alpha}\phi_2$, remains light. The question is then whether the left-right symmetry constrains Standard Model parameters.

To answer this question, we first argue that phenomenological constraints forbid $r \equiv k_2/k_1 = \mathcal{O}(1)$. (More precisely, it is $r\sin\alpha$ which is constrained to be very small.) Consider eqs. (9.12). They lead to the following equations:

$$\begin{aligned} M_u r e^{i\alpha} - M_d \ &= k_1 h(r^2-1), \\ M_u - M_d r e^{-i\alpha} \ &= k_1 f(1-r^2). \end{aligned} \tag{9.16}$$

The right hand side of these equations is real. Then, the imaginary part of the left-hand side should vanish. Let us put all quark masses to zero, except for $m_t$ and $m_b$. We take then $(M_u)_{33} = m_t e^{i\theta_t}$ and $(M_d)_{33} = m_b e^{i\theta_b}$. We get:

$$\begin{aligned} r m_t \sin(\theta_t+\alpha) - m_b \sin\theta_b \ &=0, \\ m_t \sin\theta_t - m_b \sin(\theta_b - \alpha) \ &=0. \end{aligned} \tag{9.17}$$

The second equation implies that $\theta_t \lesssim m_b/m_t$. Then, the first equation gives [173]

$$r\sin\alpha \leq m_b/m_t. \tag{9.18}$$

(Note that the model is symmetric under $r \to 1/r$ and $\alpha \to -\alpha$. Therefore, $r\sin\alpha \geq m_t/m_b$ is acceptable and physically equivalent.)

The only source of CP violation in the quark mass matrices is the phase $\alpha$. (The phase $\beta$ in $\langle\Delta_R\rangle$ does not affect quark masses, though it may affect neutrino masses.) Moreover, if one of the $\langle\phi_i^0\rangle$ vanished, then again there would be no CP violation in the quark mass matrices. As a consequence of these two facts, all CP violating phases in the mixing matrices $V_L$ and $V_R$ are proportional to $r \sin\alpha$. Hence the importance of (9.18). In particular, for the Kobayashi-Maskawa phase, one finds [173]

$$\delta_{\rm KM} \sim r \sin\alpha (m_c/m_s) \leq \mathcal{O}(0.1). \tag{9.19}$$

We learn that the low energy effective theory of the left-right symmetric model is the Standard Model with a small value for $\delta_{\rm KM}$.

Phenomenologically, it is difficult, though not impossible, to account for $\varepsilon_K$ with just the Standard Model contribution and a small KM phase. There are then two possibilities:

(i) The left-right symmetry is broken at a very high scale. The low energy theory is to a good approximation just the Standard Model. CP is, however, an approximate symmetry in the kaon sector. The hadronic parameters playing a role in the calculation of $\varepsilon_K$ have to assume rather extreme values.

(ii) The left-right symmetry is broken at low enough scale so that there are significant new contributions to various rare processes. In particular, box diagrams with intermediate $W_R$-boson and tree diagrams with a heavy neutral Higgs dominate $\varepsilon_K$. This sets up an upper bound on the scale $v_R$, of order tens of $TeV$.

### *9.4. Phenomenology of CP Violation*

The smallness of $r \sin\alpha$ does not necessarily mean that CP is an approximate symmetry in the quark sector; the phases in the mixing matrices depend, in addition to $r \sin\alpha$, on quark mass ratios, some of which are large. An explicit calculation shows that the six phases actually divide to two groups: the KM phase and the three phases that appear in $V_R$ in a two generation model (usually denoted by $\delta_1$, $\delta_2$ and $\gamma$) are all small [173], while the two extra phases that appear in the three generation $V_R$ (denoted by $\sigma_1$, $\sigma_2$) are not [174]:

$$\begin{aligned} \delta, \delta_1, \delta_2, \gamma \ &\propto r \sin\alpha (m_c/m_s) \leq \mathcal{O}(0.1), \\ \sigma_1, \sigma_2 \ &\propto r \sin\alpha (m_t/m_b) \leq \mathcal{O}(1). \end{aligned} \tag{9.20}$$

In $\varepsilon_K$, it is mainly $\delta_1$ and $\delta_2$ which play a role. (We here assume that the hadronic parameters are close to their present theoretical estimate and therefore $\varepsilon_K$ cannot be explained in this framework by the Standard Model contribution alone.) Assuming that the $W_L - W_R$ box diagram gives the dominant contribution, one is led to conclude that [173]

$$M(W_R) \lesssim 20\ TeV \tag{9.21}$$

is favored. Note that CP conserving processes provide a lower bound [175,176],

$$M(W_R) \gtrsim 1.6\ TeV. \tag{9.22}$$

The favored range for $M(W_R)$ is then very constrained in this framework.

Taking into account this upper bound and the fact that the $\sigma_i$ phases are enhanced by a factor of about 10 compared to the $\delta_i$ phases, one finds that the left-right symmetric contributions compete with or even dominate over the Standard Model contributions to $B - \bar{B}$ mixing and to $B_s - \bar{B}_s$ mixing [174,177-179]. This means that CP asymmetries in $B$ or $B_s$ decays into final CP eigenstates could be substantially different from the Standard Model prediction. Moreover, the phases in the left-right symmetric contributions to $B - \bar{B}$ and $B_s - \bar{B}_s$ mixing are closely related, predicting correlations between the deviations. The CP asymmetry in semileptonic $B$ decays could also be significantly enhanced [180]. The recent measurement of $a_{\psi K_S}$ gives first constraints on $\sigma_1$ leading to new bounds on $a_{\rm SL}$ [14].

Finally, LRS models could enhance the electric dipole moments of the neutron and of the electron [181-184].

## 10. Multi-Scalar Models

The Standard Model has a single scalar field, $\phi(1,2)_{1/2}$, that is responsible for the spontaneous symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$. Within the framework of the Standard Model, the complex Yukawa couplings of the scalar doublet to fermions are the only source of flavor physics and of CP violation. However, in the mass basis, the interactions of the Higgs particle are flavor diagonal and CP conserving.

There are several good reasons for the interest in multi-scalar models in the context of flavor and CP violation:

a. If there exist additional scalars and, in particular, $SU(2)_L$-doublets, then not only there are new sources of CP violation, but also the Yukawa interactions in the mass basis as well as the scalar self-interactions may violate CP.

b. CP violation in scalar interactions has very different features from the $W$-mediated CP violation of the Standard Model. For example, it could lead to observable *flavor diagonal* CP violation in top physics or in electric dipole moments, or it could induce transverse lepton polarization in semileptonic meson decays.

c. With more than a single scalar doublet, CP violation could be spontaneous.

Indeed, there is no good reason to assume that the Standard Model doublet is the only scalar in Nature. Most extensions of the Standard Model predict that there exist additional scalars. For example, models with an extended gauge symmetry (such as GUTs and left-right symmetric models) need extra scalars to break the symmetry down to $G_{\rm SM}$; Supersymmetry requires that there exists a scalar partner to each Standard Model fermion. However, scalar masses are generically not protected by a symmetry. Consequently, in models where the low energy effective theory is the Standard Model, we expect in general that the only light scalar is the Standard Model doublet.

The study of multi-scalar models is then best motivated in the following cases:

(i) The scale of new physics is not very high above the electroweak scale. One has to remember, however, that in such cases there is more to the new physics than just extending the scalar sector.

(ii) The scalar is related to the spontaneous breaking of a global symmetry. In some cases, a discrete symmetry is enough to make a scalar light.

We will discuss scalar $SU(2)_L$-doublets and -singlets only. There are two reasons for that. First, the VEVs of higher multiplets need to be very small in order to avoid large deviations from the experimentally successful relation $\rho = 1$. Second, higher multiplets do not couple to the known fermions. (The only exception is an $SU(2)_L$-triplet that can couple to the left-handed leptons.)

### *10.1. Multi Higgs Doublet Models*

The most popular extension of the Higgs sector is the multi Higgs doublet model

(MHDM) and, in particular, the two Higgs doublet model (2HDM). These models have, in addition to the Kobayashi-Maskawa phase of the quark mixing matrix, several new sources of CP violation [185]:

(i) A complex mixing matrix for charged scalars [186].

(ii) Mixing of CP-even and CP-odd neutral scalars [187].

(iii) CP-odd Yukawa couplings (in the quark mass basis).

(iv) Complex quartic scalar couplings.

The CP violation that is relevant to near future experiments always involves fermions. Therefore, we will only discuss the new sources (i), (ii) and (iii).

A generic MHDM, with all dimensionful parameters at the electroweak scale and all dimensionless parameters of order one, leads to severe phenomenological problems. In particular, some of the physical scalars have flavor changing (and CP violating) couplings at tree level, violating bounds on rare processes such as $\Delta m_K$ and $\varepsilon_K$ by several orders of magnitude. There are three possible solutions to these problems:

(I) *Natural flavor conservation (NFC)* [188]: only a single scalar doublet couples to each fermion sector. 2HDM where the same (a different) scalar doublet couples to the up and the down quarks are called type I (II). The absence of flavor changing and/or CP violating Yukawa interactions in this case is based on the same mechanism as within the Standard Model.

(II) *Approximate flavor symmetries (AFS)* [189]: it is quite likely that the smallness and hierarchy in the fermion masses and mixing angles are related to an approximate flavor symmetry, broken by a small parameter. If so, then it is unavoidable that the Yukawa couplings of all scalar doublets are affected by the selection rules related to the flavor symmetry. In such a case, couplings to the light generations and, in particular, off-diagonal couplings, are suppressed.

(III) *Heavy scalars* [190]: all dimensionful parameters that are not constrained by the requirement that the spontaneous breaking of $G_{\rm SM}$ occurs at the electroweak scale are actually higher than this scale, $\Lambda_{\rm NP} \gg \Lambda_{\rm EW}$. Then all the new sources of flavor and CP violation in the scalar sector are suppressed by $\mathcal{O}(\Lambda^2_{\rm EW}/\Lambda^2_{\rm NP})$.

In table VII we summarize the implications of the various multi-scalar models for

CP violation. Note that, if we impose NFC, spontaneous CP violation (SCPV) [187] is impossible in 2HDM [186] and (since the combination of SCPV and NFC leads to $\delta_{\rm KM}=0$ [191]) is phenomenologically excluded in MHDM [192,193]. Explicit models of spontaneous CP violation have been constructed within the frameworks of approximate NFC [194], approximate flavor symmetries [139,154] and heavy scalars [184]. In the supersymmetric framework, one has to add at least two scalar singlets to allow for spontaneous CP violation [195]. Entries marked with '$*$' mean that the number of scalar doublets should be larger than 2 (that is, the answer is 'No' in 2HDM).

| Framework | Model (Example) | SCPV | (i) | (ii) | (iii) |
|---|---|---|---|---|---|
| NFC | MSSM | Excluded | Yes* | Yes* | No |
| AFS | Horizontal Sym. | Yes | Yes* | Yes | $\mathcal{O}(m_q/m_Z)$ |
| Heavy | LRS | Yes | $\mathcal{O}(\frac{\Lambda^2_{\rm EW}}{\Lambda^2_{\rm NP}})^*$ | $\mathcal{O}(\frac{\Lambda^2_{\rm EW}}{\Lambda^2_{\rm NP}})$ | $\mathcal{O}(\frac{\Lambda^2_{\rm EW}}{\Lambda^2_{\rm NP}})$ |

Table VII. Multi Higgs Doublet Models

*10.2. (i) Charged Scalar Exchange*

We investigate a multi Higgs doublet model (with $n\geq 3$ doublets) with NFC and assume that a different doublet couples to each of the the down, up and lepton sectors:

$$-\mathcal{L}_Y=-\frac{\phi_1^+}{v_1}\overline{U}V_{\rm CKM}M_d^{\rm diag}P_R\,D+\frac{\phi_2^+}{v_2}\overline{U}M_u^{\rm diag}V_{\rm CKM}P_L\,D-\frac{\phi_3^+}{v_3}\overline{\nu}M_\ell P_R\,\ell+{\rm h.c.},\quad(10.1)$$

where $P_{L,R}=(1\mp\gamma_5)/2$. We denote the physical charged scalars by $H_i^+$ ($i=1,2,\ldots,n-1$), and the would-be Goldstone boson (eaten by the $W^+$) by $H_n^+$. We define $K$ to be the matrix that rotates the charged scalars from the interaction- to the mass-eigenbasis. Then the Yukawa Lagrangian in the mass basis (for both fermions and scalars) is

$$\mathcal{L}_Y=\frac{G_F^{1/2}}{2^{1/4}}\sum_{i=1}^{n-1}\{H_i^+\overline{U}[Y_iM_u^{\rm diag}V_{\rm CKM}P_L+X_iV_{\rm CKM}M_d^{\rm diag}P_R]D+H_i^+\overline{\nu}[Z_iM_\ell P_R]\ell\}+{\rm h.c.},\quad(10.2)$$

where

$$X_i=-\frac{K_{i1}^*}{K_{n1}^*},\quad Y_i=-\frac{K_{i2}^*}{K_{n2}^*},\quad Z_i=-\frac{K_{i3}^*}{K_{n3}^*}.\quad(10.3)$$

CP violation in the charged scalar sector comes from phases in $K$. CP violating effects are largest when the lightest charged scalar is much lighter than the heavier ones [196,197]. Here we assume that all but the lightest charged scalar ($H_1^+$) effectively decouple from the fermions. Then, CP violating observables depend on three parameters:

$$
\begin{aligned}
\frac{\mathcal{I}m(XY^*)}{m_H^2} &\equiv \frac{\mathcal{I}m(X_1Y_1^*)}{m_{H_1}^2} \approx \sum_{i=1}^{n-1} \frac{\mathcal{I}m(X_iY_i^*)}{m_{H_i}^2}, \\
\frac{\mathcal{I}m(XZ^*)}{m_H^2} &\equiv \frac{\mathcal{I}m(X_1Z_1^*)}{m_{H_1}^2} \approx \sum_{i=1}^{n-1} \frac{\mathcal{I}m(X_iZ_i^*)}{m_{H_i}^2}, \\
\frac{\mathcal{I}m(YZ^*)}{m_H^2} &\equiv \frac{\mathcal{I}m(Y_1Z_1^*)}{m_{H_1}^2} \approx \sum_{i=1}^{n-1} \frac{\mathcal{I}m(Y_iZ_i^*)}{m_{H_i}^2}.
\end{aligned}
\tag{10.4}
$$

$\mathcal{I}m(XY^*)$ induces CP violation in the quarks sector, while $\mathcal{I}m(XZ^*)$ and $\mathcal{I}m(YZ^*)$ give CP violation that is observable in semi-leptonic processes.

There is an interesting question of whether charged scalar exchange could be the *only* source of CP violation. In other words, we would like to know whether a model of extended scalar sector with spontaneous CP violation and NFC is viable. In these models, $\delta_{\rm KM} = 0$ and $\varepsilon_K$ has to be accounted for by charged Higgs exchange. This requires very large long distance contributions. The CP violating coupling should fulfill [198-199]

$$\mathcal{I}m(XY^*) \geq \mathcal{O}(40). \tag{10.5}$$

However, the upper bounds on $d_N$ [192] and on BR($b \to s\gamma$) [193] require

$$\mathcal{I}m(XY^*) \leq \mathcal{O}(1). \tag{10.6}$$

We conclude that models of SCPV and NFC are excluded. It is, of course, still a viable possibility that CP is explicitly broken, in which case both quark and Higgs mixings provide CP violation.

The bound (10.6) implies that the charged Higgs contribution to $B - \bar{B}$ mixing is numerically small and would modify the Standard Model predictions for CP asymmetries in $B$ decays by no more than $\mathcal{O}(0.02)$ [193]. On the other hand, the contribution to $d_N$ can still be close to the experimental upper bound.

The lepton transverse polarization cannot be generated by vector or axial-vector interactions only [200,201], so it is particularly suited for searching for CP violating scalar contributions. As triple-vector correlation is odd under time-reversal, the experimental observation of such correlation would signal T and – assuming CPT symmetry – CP violation. (It is possible to get non-vanishing T$-$odd observables even without CP violation (see e.g. [202]). Such "fake" asymmetries can arise from final state interactions (FSI). They can be removed by comparing the measurements in two CP conjugate channels.) The muon transverse polarization in $K \to \pi\, \mu\, \nu$ decays and the tau transverse polarization in $B \to X \tau \nu$ are examples of such observables. The lepton transverse polarization, $P_\perp$, in semileptonic decays is defined as the lepton polarization component along the normal vector of the decay plane,

$$P_\perp = \frac{\vec{s}_\ell \cdot (\vec{p}_\ell \times \vec{p}_X)}{|\vec{p}_\ell \times \vec{p}_X|}\,, \tag{10.7}$$

where $\vec{s}_\ell$ is the lepton spin three-vector and $\vec{p}_\ell$ ($\vec{p}_X$) is the three-momentum of the lepton (hadron). Experimentally, it is useful to define the integrated CP violating asymmetry

$$a_{CP} \equiv \langle P_\perp \rangle = \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-}, \tag{10.8}$$

where $\Gamma^+$ ($\Gamma^-$) is the rate of finding the lepton spin parallel (anti-parallel) to the normal vector of the decay plane. A non-zero $a_{CP}$ can arise in our model from the interference between the $W$-mediated and the $H^+$-mediated tree diagrams. For example, in the semi-taonic bottom quark decay, the asymmetry is given by $a_{CP} = C_{ps}\frac{\mathcal{I}m(XZ^*)}{m_H^2}$ and could be as large as 0.3 (see *e.g.* [203-205]).

*10.3. (ii) Effects of CP-even and CP-odd Scalar Mixing in Top Physics*

It is possible that the neutral scalars are mixtures of CP-even and CP-odd scalar fields [187,206-209,196-197]. Such a scalar couples to both scalar and pseudoscalar currents:

$$\mathcal{L}_Y = H_i \bar{f}(a_i^f + i b_i^f \gamma_5) f, \tag{10.9}$$

where $H_i$ is the physical Higgs boson and $a_i^f, b_i^f$ are functions of mixing angles in the matrix that diagonalizes the neutral scalar mass matrix. (Specifically, they are proportional to the components of, respectively, $\mathcal{R}e\phi_u$ and $\mathcal{I}m\phi_u$ in $H_i$.) CP violation in processes involving fermions is proportional to $a_i^f b_i^{f*}$. The natural place to look for manifestations of this type of CP violation is top physics, where the large Yukawa couplings allow large asymmetries (see e.g. [210]). Note that unlike our discussion above, the asymmetries here have nothing to do with FCNC processes. Actually, in models with NFC (even if softly broken [207]), the effects discussed here contribute negligibly to $\varepsilon_K$ and to CP asymmetries in $B$ decays. On the other hand, two loop diagrams with intermediate neutral scalar and top quark can induce a CP violating three gluon operator [211,212] that would give $d_N$ close to the experimental bound [212-215].

*10.4. (iii) Flavor Changing Neutral Scalar Exchange*

Natural flavor conservation needs not be exact in models of extended scalar sector [194,207,216,217]. In particular, it is quite likely that the existence of the additional scalars is related to flavor symmetries that explain the smallness and hierarchy in the Yukawa couplings. In this case, the new flavor changing couplings of these scalars are suppressed by the same selection rules as those that are responsible to the smallness of fermion masses and mixing, and there is no need to impose NFC [189,218-222,185]. An explicit framework, with Abelian horizontal symmetries, was presented in [223,137,98]. (For another related study, see [224].) We explain the general idea using these models. We emphasize that in this example the scalar with flavor changing couplings is a Standard Model singlet, and not an extra doublet, but the idea that these couplings are suppressed by approximate horizontal symmetries works in the same way.

The simplest model of ref. [223] extends the SM by supersymmetry and by an Abelian horizontal symmetry $\mathcal{H} = U(1)$ (or $Z_N$). The symmetry $\mathcal{H}$ is broken by a VEV of a single scalar $S$ that is a singlet of the SM gauge group. Consequently, Yukawa couplings that violate $\mathcal{H}$ arise only from nonrenormalizable terms and are therefore suppressed. Explicitly, the quark Yukawa terms have the form

$$\mathcal{L}_Y = X_{ij}^d \left(\frac{S}{M}\right)^{n_{ij}^d} Q_i \bar{d}_j \phi_d + X_{ij}^u \left(\frac{S}{M}\right)^{n_{ij}^u} Q_i \bar{u}_j \phi_u, \tag{10.10}$$

where $M$ is some high energy scale and $n_{ij}^q$ is the horizontal charge of the combination $Q_i \bar{q}_j \phi_q$ (in units of the charge of $S$). The terms (10.10) lead to quark masses and mixing as well as to flavor changing couplings, $Z^q_{ij}$, for the scalar $S$. The magnitude of the latter is then related to that of the effective Yukawa couplings $Y^q_{ij}$:

$$Z^q_{ij} \sim \frac{M^q_{ij}}{\langle S \rangle}. \tag{10.11}$$

Since the order of magnitude of each entry in the quark mass matrices is fixed in these models in terms of quark masses and mixing, the $Z^q_{ij}$ couplings can be estimated in terms of these physical parameters and the scale $\langle S \rangle$. For example, these couplings contribute to $K - \bar{K}$ mixing proportionally to

$$Z^d_{12} Z^{d*}_{21} \sim \frac{m_d m_s}{\langle S \rangle^2}. \tag{10.12}$$

With arbitrary phase factors in the various $Z^q_{ij}$ couplings, the contributions to neutral meson mixing are, in general, CP violating. In particular, there will be a contribution to $\varepsilon_K$ from $\mathcal{I}m(Z^d_{12} Z^{d*}_{21})$. Requiring that the $S$-mediated tree level contribution does not exceed the experimental value of $\varepsilon_K$ gives, for $\mathcal{O}(1)$ phases,

$$M_S \langle S \rangle \gtrsim 1.8 \ TeV^2. \tag{10.13}$$

We learn that (for $M_S \sim \langle S \rangle$) the mass of the $S$-scalar could be as low as 1.5 TeV, some 4 orders of magnitude below the bound corresponding to $\mathcal{O}(1)$ flavor changing couplings.

The flavor changing couplings of the $S$-scalar lead also to a tree level contribution to $B - \bar{B}$ mixing proportional to

$$Z^d_{13} Z^{d*}_{31} \sim \frac{m_d m_b}{\langle S \rangle^2}. \tag{10.14}$$

This means that, for phases of order 1, the neutral scalar exchange accounts for at most a few percent of $B - \bar{B}$ mixing. This cannot be signaled in $\Delta m_B$ (because of the hadronic uncertainties in the calculation) but could be signalled (if $\langle S \rangle$ is at the lower bound) in CP asymmetries in $B^0$ decays.

Finally, the contribution to $D - \bar{D}$ mixing, proportional to

$$Z^u_{12} Z^{u*}_{21} \sim \frac{m_u m_c}{\langle S \rangle^2}, \tag{10.15}$$

is below a percent of the current experimental bound. This is unlikely to be discovered in near-future experiments, even if the new phases maximize the interference effects in the $D^0 \to K^-\pi^+$ decay.

To summarize, models with horizontal symmetries naturally suppress flavor changing couplings of extra scalars. There is no need to invoke NFC even for new scalars at the TeV scale. Furthermore, the magnitude of the flavor changing couplings is related to the observed fermion parameters. Typically, contributions from neutral scalars with flavor changing couplings could dominate $\varepsilon_K$. If they do, then a signal at the few percent level in CP asymmetries in neutral $B$ decays is quite likely

*10.5. The Superweak Scenario*

The original *superweak* scenario [225] stated that CP violation appears in a new $\Delta S = 2$ interaction while there is no CP violation in the SM $\Delta S = 1$ transitions. Consequently, the only large observable CP violating effect is $\varepsilon_K$, while $\varepsilon'/\varepsilon \sim 10^{-8}$ and EDMs are negligibly small. At present, the term "superweak CP violation" has been used for many different types of models. There are several reasons for this situation:

(i) The work of ref. [225] was concerned only with CP violation in $K$ decays. In extending the idea to other mesons, one may interpret the idea in various ways. On one side, it is possible that the superweak interaction is significant only in $K - \bar{K}$ mixing and (apart from the relaxation of the $\varepsilon_K$-bounds on the CKM parameters) has no effect on mixing of heavier mesons. On the other extreme, one may take the superweak scenario to imply that CP violation comes from $\Delta F = 2$ processes only for all mesons. The most common use of the term 'superweak' refers to the latter option, namely that there is no direct CP violation.

(ii) The scenario proposed in [225] did not employ any specific model. It was actually proposed even before the formulation of the Standard Model. To extend the idea to, for example, the neutral $B$ system, a model is required. Various models give very different predictions for CP asymmetries in $B$ decays.

If one extends the superweak scenario to the $B$ system by assuming that there is CP violation in $\Delta B = 2$ but not in $\Delta B = 1$ transitions, the prediction for CP asymmetries in $B$ decays into final CP eigenstates is that they are equal for all final states [226-228]. Whether these asymmetries are all small or could be large is model dependent. In addition, the asymmetries in charged $B$ decays vanish.

CP violation via neutral scalar exchange is the most commonly studied realization of the superweak idea. In particular, if the complex $Z^q_{ij}$ couplings presented in the previous section were the only source of CP violation, then this model would be superweak. The smallness of the $Z^q_{ij}$ couplings would make the contribution from neutral Higgs mediated diagrams negligible compared to the Standard Model diagrams in $\Delta S = 1$ processes, but the fact that they contribute to mixing at tree level would allow them to dominate the $\Delta S = 2$ processes. Various models (or scenarios) that realize the main features of the superweak idea can be found in refs. [229-230,194,216]. As mentioned above, there is a considerable variation in their predictions for $\varepsilon'/\varepsilon$, $d_N$ and other quantities. If we take the term 'superweak CP violation' to imply that there is only indirect CP violation, or at least that there is no direct CP violation in $K$ decays, then $\varepsilon'/\varepsilon \neq 0$ is inconsistent with this scenario which is therefore excluded.

## 11. Extensions of the Fermion Sector: Down Singlet Quarks

The fermion sector of the Standard Model is described in eq. (2.2). It can be extended by either a fourth, sequential generation or by non-sequential fermions, namely 'exotic' representations, different from those of (2.2). (The four generation model can only be viable if it is further extended to evade bounds related to the neutrino sector [231] and to electroweak precision data [26].) Our discussion in this chapter is focused on non-sequential fermions and their implications on CP asymmetries in neutral $B$ decays and on $K_L \to \pi\nu\bar{\nu}$.

### *11.1. The Theoretical Framework*

We consider a model with extra quarks in vector-like representations of the Standard Model gauge group,

$$d_4(3,1)_{-1/3} \ + \ \bar{d}_4(\bar{3},1)_{+1/3}. \tag{11.1}$$

Such (three pairs of) quark representations appear, for example, in $E_6$ GUTs. The mass matrix in the down sector, $M^d$, is now $4 \times 4$. (Note that the $M^d_{4i}$ entries do not violate $G_{\rm SM}$ and are, therefore, bare mass terms.)

How many independent CP violating parameters are there in $M^d$ and $M^u$? Since $M^d$ ($M^u$) is $4 \times 4$ ($3 \times 3$) and complex, there are 25 real and 25 imaginary parameters in these matrices. If we switch off the mass matrices, there is a global symmetry added to the model,

$$G^{\rm extra\ d}_{\rm global}(M^{d,u} = 0) = U(3)_Q \times U(4)_{\bar d} \times U(3)_{\bar u} \times U(1)_{d_4}. \tag{11.2}$$

One can remove, at most, 12 real and 23 imaginary parameters. However, the model with the quark mass matrices switched on has still a global symmetry of $U(1)_B$, so one of the imaginary parameters cannot be removed. We conclude that there are 16 flavor parameters: 13 real ones, that is seven masses and six mixing angles, and 3 phases. These three phases are independent sources of CP violation.

### *11.2. Z-Mediated FCNC*

The most important feature of this model for our purposes is that it allows CP violating $Z$-mediated Flavor Changing Neutral Currents (FCNC). To understand how these FCNC arise, it is convenient to work in a basis where the up sector interaction eigenstates are identified with the mass eigenstates. The down sector interaction eigenstates are then related to the mass eigenstates by a $4 \times 4$ unitary matrix $K$. Charged current interactions are described by

$$\mathcal{L}^W_{\rm int} = \frac{g}{\sqrt{2}}(W^+_\mu V_{ij} \bar u_{iL} \gamma^\mu d_{jL} + {\rm h.c.}). \tag{11.3}$$

The charged current mixing matrix $V$ is a $3 \times 4$ sub-matrix of $K$:

$$V_{ij} = K_{ij} \quad {\rm for} \quad i = 1,2,3;\ j = 1,2,3,4. \tag{11.4}$$

The $V$ matrix is parameterized, as anticipated above, by six real angles and three phases, instead of three angles and one phase in the original CKM matrix. All three phases may affect CP asymmetries in $B^0$ decays. Neutral current interactions are described by

$$\begin{aligned} \mathcal{L}^Z_{\rm int} &= \frac{g}{\cos\theta_W} Z_\mu (J^{\mu 3} - \sin^2\theta_W J^\mu_{\rm EM}), \\ J^{\mu 3} &= -\frac{1}{2} U_{pq} \bar d_{pL} \gamma^\mu d_{qL} + \frac{1}{2}\delta_{ij} \bar u_{iL} \gamma^\mu u_{jL}. \end{aligned} \tag{11.5}$$

The neutral current mixing matrix for the down sector is $U=V^\dagger V$. As $V$ is not unitary, $U\neq\mathbf{1}$. In particular, its non-diagonal elements do not vanish:

$$U_{pq}=-K^*_{4p}K_{4q}\quad\text{for}\quad p\neq q. \tag{11.6}$$

The three elements which are most relevant to our study are

$$\begin{aligned} U_{ds} &= V^*_{ud}V_{us}+V^*_{cd}V_{cs}+V^*_{td}V_{ts},\\ U_{db} &= V^*_{ud}V_{ub}+V^*_{cd}V_{cb}+V^*_{td}V_{tb},\\ U_{sb} &= V^*_{us}V_{ub}+V^*_{cs}V_{cb}+V^*_{ts}V_{tb}. \end{aligned} \tag{11.7}$$

The fact that, in contrast to the Standard Model, the various $U_{pq}$ do not necessarily vanish, allows FCNC at tree level. This may substantially modify the predictions for CP asymmetries.

The flavor changing couplings of the $Z$ contribute to various FCNC processes. Relevant constraints arise from semileptonic FCNC $B$ decays:

$$\frac{\Gamma(B\to\ell^+\ell^-X)_Z}{\Gamma(B\to\ell^+\nu X)}=\left[(1/2-\sin^2\theta_W)^2+\sin^4\theta_W\right]\frac{|U_{db}|^2+|U_{sb}|^2}{|V_{ub}|^2+F_{\rm ps}|V_{cb}|^2}, \tag{11.8}$$

where $F_{\rm ps}\sim 0.5$ is a phase space factor. The experimental upper bound on $\Gamma(B\to\ell^+\ell^-X)$ gives

$$\left|\frac{U_{db}}{V_{cb}}\right|\leq 0.04,\qquad \left|\frac{U_{sb}}{V_{cb}}\right|\leq 0.04. \tag{11.9}$$

Additional constraints come from neutral $B$ mixing:

$$(\Delta m_B)_Z=\frac{\sqrt{2}G_FB_Bf_B^2m_B\eta_B}{3}|U_{db}|^2. \tag{11.10}$$

Using $\sqrt{B_B}f_B\gtrsim 0.16\ GeV$, we get

$$|U_{db}|\lesssim 9\times10^{-4}. \tag{11.11}$$

As concerns $\Delta m_{B_s}$, only lower bounds exist and consequently there is no analog bound on $|U_{sb}|$.

Bounds on $U_{ds}$ can be derived from the measurements of BR$(K_L \to \mu^+\mu^-)$, BR$(K^+ \to \pi^+\nu\bar{\nu})$, $\varepsilon_K$ and $\varepsilon'/\varepsilon$ yielding, respectively (for recent derivations, see [160,54]),

$$\begin{aligned} |\mathcal{R}e(U_{ds})| &\lesssim\ 10^{-5}, \\ |U_{ds}| &\leq 3\times 10^{-5}, \\ |\mathcal{R}e(U_{ds})\mathcal{I}m(U_{ds})| &\lesssim\ 1.3\times 10^{-9}, \\ |\mathcal{I}m(U_{ds})| &\lesssim\ 10^{-5}. \end{aligned} \tag{11.12}$$

(Note that the combination of bounds from BR$(K_L \to \mu^+\mu^-)$ and from the recently improved $\varepsilon'/\varepsilon$ is stronger than the bounds from BR$(K^+ \to \pi^+\nu\bar{\nu})$ and $\varepsilon_K$ [54]. The latter bounds are, however, subject to smaller hadronic uncertainties.)

### *11.3. CP Asymmetries in B Decays*

The most interesting effects in this model concern CP asymmetries in neutral $B$ decays into final CP eigenstates [232-239,15]. We describe these effects in detail as they illustrate the type of new ingredients that are likely to affect CP asymmetries in neutral $B$ decays and the way in which the SM predictions might be modified. (If there exist light up quarks in exotic representations, they may introduce similar, interesting effects in neutral $D$ decays [69].)

If the $U_{qb}$ elements are not much smaller than the bounds (11.9) and (11.11), they will affect several aspects of physics related to CP asymmetries in $B$ decays.

(i) *Neutral B mixing*: The experimentally measured value of $\Delta m_{B_d}$ (and the lower bound on $\Delta m_{B_s}$) can be explained by Standard Model processes, namely box diagrams with intermediate top quarks. Still, the uncertainties in the theoretical calculations, such as the values of $f_B$ and $V_{td}$ (and the absence of an upper bound on $\Delta m_{B_s}$) allow a situation where SM processes do not give the dominant contributions to either or both of $\Delta m_{B_d}$ and $\Delta m_{B_s}$ [14]. The ratio between the $Z$-mediated tree diagram and the Standard Model box diagram is given by $(q=d,s)$

$$\frac{(\Delta m_{B_q})_{\rm tree}}{(\Delta m_{B_q})_{\rm box}} = \frac{2\sqrt{2}\pi^2}{G_F m_W^2 S_0(x_t)}\left|\frac{U_{qb}}{V_{tq}V_{tb}^*}\right|^2 \approx 150\left|\frac{U_{qb}}{V_{tq}V_{tb}^*}\right|^2 \lesssim \begin{cases} 5 & q=d \\ 0.25 & q=s \end{cases}. \tag{11.13}$$

(The last inequality is derived under the assumption that the violation of CKM unitarity is not strong. The bound on $(\Delta m_{B_d})_{\rm tree}/(\Delta m_{B_d})_{\rm box}$ is higher if $|V_{td}V_{tb}^*| < 0.005$ holds.)

From (11.9) and (11.13) we learn that the $Z$-mediated tree diagram could give the dominant contribution to $\Delta m_{B_d}$ but at most 25% of $\Delta m_{B_s}$.

(ii) *Unitarity of the* $3\times 3$ *CKM matrix*: Within the SM, unitarity of the three generation CKM matrix gives:

$$\begin{aligned}\mathcal{U}_{ds} &\equiv V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0,\\ \mathcal{U}_{db} &\equiv V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0,\\ \mathcal{U}_{sb} &\equiv V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0.\end{aligned} \tag{11.14}$$

Eq. (11.7), however, implies that now (11.14) is replaced by

$$\mathcal{U}_{ds} = U_{ds}, \quad \mathcal{U}_{db} = U_{db}, \quad \mathcal{U}_{sb} = U_{sb}. \tag{11.15}$$

A measure of the violation of (11.14) is given by

$$\left|\frac{U_{ds}}{V_{ud}V_{us}^*}\right| \lesssim 5\times 10^{-4}, \quad \left|\frac{U_{db}}{V_{td}V_{tb}^*}\right| \lesssim 0.18, \quad \left|\frac{U_{sb}}{V_{ts}V_{tb}^*}\right| \lesssim 0.04. \tag{11.16}$$

The bound on $|U_{db}/(V_{td}V_{tb}^*)|$ is even weaker if $|V_{td}|$ is lower than the three generation unitarity bound. We learn that the first of the SM relations in (11.14) is practically maintained, while the third can be violated by at most 4%. However, the $\mathcal{U}_{db}=0$ constraint may be violated by $\mathcal{O}(0.2)$ effects. The Standard Model unitarity triangle should be replaced by a unitarity *quadrangle*. After the recent measurement of $a_{\psi K_S}$ [13], not only the magnitude of $U_{db}$ but also the phases $\bar\alpha$ and $\bar\beta$,

$$\bar\alpha \equiv \arg\left(\frac{V_{ud}V_{ub}^*}{U_{db}^*}\right), \qquad \bar\beta \equiv \arg\left(\frac{U_{db}^*}{V_{cd}V_{cb}^*}\right), \tag{11.17}$$

are constrained [15], but the constraints are not very strong.

(iii) $Z$-mediated $B$ decays: Our main interest in this chapter is in hadronic $B^0$ decays to CP eigenstates, where the quark sub-process is $\bar b \to \bar u_i u_i \bar d_j$, with $u_i = u, c$ and $d_j = d, s$. These decays get new contributions from $Z$-mediated tree diagrams, in addition to the standard $W$-mediated ones. The ratio between the amplitudes is

$$\frac{A_Z}{A_W} = \left[\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right]\left|\frac{U_{jb}^*}{V_{ij}V_{ib}^*}\right|. \tag{11.18}$$

We find that the $Z$ contributions can be safely neglected in $\bar{b} \to \bar{c}c\bar{s}$ ( $\lesssim 0.013$) and $\bar{b} \to \bar{c}c\bar{d}$ ( $\lesssim 0.03$). On the other hand, it may be significant in $\bar{b} \to \bar{u}u\bar{d}$ ( $\lesssim 0.12$), and in processes with no SM tree contributions, e.g. $\bar{b} \to \bar{s}s\bar{s}$, that may have comparable contributions from penguin and $Z$-mediated tree diagrams.

(iv) *New contributions to* $\Gamma_{12}(B_q)$: The difference in width comes from modes that are common to $B_q$ and $\bar{B}_q$. As discussed above, there are new contributions to such modes from $Z$-mediated FCNC. However, while the new contributions to $M_{12}$ are from tree level diagrams, *i.e.* $\mathcal{O}(g^2)$, those to $\Gamma_{12}$ are still coming from a box-diagram, *i.e.* $\mathcal{O}(g^4)$. Consequently, no significant enhancement of the SM value of $\Gamma_{12}$ is expected, and the relation $\Gamma_{12} \ll M_{12}$ is maintained. (The new contribution could significantly modify the leptonic asymmetry in neutral $B$ decays [240,15] though the asymmetry remains small.)

The fact that $M_{12}(B^0)$ could be dominated by the $Z$-mediated FCNC together with the fact that this new amplitude depends on new CP violating phases means that large deviations from the Standard Model predictions for CP asymmetries are possible. As $\Gamma_{12} \ll M_{12}$ is maintained, future measurements of certain modes will still be subject to a clean theoretical interpretation in terms of the extended electroweak sector parameters.

Let us assume that, indeed, $M_{12}$ is dominated by the new physics. (Generalization to the case that the new contribution is comparable to (but not necessarily dominant over) the Standard Model one is straightforward [234,238].) Then

$$\left(\frac{p}{q}\right)_B \approx \frac{U^*_{db}}{U_{db}}. \tag{11.19}$$

We argued above that $b \to c\bar{c}s$ is still dominated by the $W$ mediated diagram. Furthermore, the first unitarity constraint in (11.14) is practically maintained. Then it is straightforward to evaluate the CP asymmetry in $B \to \psi K_S$. We find that it simply measures an angle of the unitarity quadrangle:

$$a_{CP}(B \to \psi K_S) = -\sin 2\bar{\beta}. \tag{11.20}$$

The new contribution to $b \to c\bar{c}d$ is $\mathcal{O}(3\%)$, which is somewhat smaller than the SM penguins. So we still have, to a good approximation, (taking into account CP-parities)

$$a_{CP}(B \to \psi K_S) \approx -a_{CP}(B \to DD). \tag{11.21}$$

Care has to be taken regarding $b \to u\bar{u}d$ decays. Here, direct CP violation may be large [237] and prevent a clean theoretical interpretation of the asymmetry. Only if the asymmetry is large, so that the shift from the $Z$-mediated contribution to the decay is small, we get

$$a_{CP}(B \to \pi\pi) = -\sin 2\bar{\alpha}. \tag{11.22}$$

The important point about the modification of the SM predictions is then not that the angles $\alpha, \beta$ and $\gamma$ may have very different values from those predicted by the SM, but rather that the CP asymmetries do not measure these angles anymore. As the experimental constraints on $\bar{\alpha}$ and $\bar{\beta}$ are still rather weak [15], a large range is possible for each of the asymmetries. This model demonstrates that there exist extensions of the SM where dramatic deviations from its predictions for CP asymmetries in $B$ decays are not unlikely.

Another interesting point concerns $B_s$ decays. If $B_s - \bar{B}_s$ mixing as well as the $b \to c\bar{c}s$ decay are dominated by the SM diagrams, we have, similar to the SM,

$$a_{CP}(B_s \to \psi\phi) \approx 0. \tag{11.23}$$

As shown in ref. [20], this is a sufficient condition for the angles extracted from $B \to \psi K_S$, $B \to \pi\pi$ and the relative phase between the $B_s - \bar{B}_s$ mixing amplitude and the $b \to u\bar{u}d$ decay amplitude (if it can be deduced from experiment) to sum up to $\pi$ (up to possible effects of direct CP violation). This happens in spite of the fact that the first two asymmetries do not correspond to $\beta$ and $\alpha$ of the unitarity triangle.

### *11.4. The $K_L \to \pi\nu\bar{\nu}$ Decay*

In chapter 7 we argued that the only potentially significant new contribution to $a_{\pi\nu\nu}$ can come from the decay amplitude. $Z$-mediated FCNC provide an explicit example of New Physics that may modify the SM prediction for $a_{\pi\nu\nu}$ of eq. (4.38). Assuming that the $Z$-mediated tree diagram dominates $K \to \pi\nu\bar{\nu}$, we get [232]

$$\sin\theta_K = \mathcal{I}m U_{ds}/|U_{ds}|. \tag{11.24}$$

Bounds on the relevant couplings were given in eq. (11.12) above. We learn that large effects are possible. When $|\mathcal{R}e(U_{ds})|$ and $|\mathcal{I}m(U_{ds})|$ are close to their upper bounds, the branching ratios $BR(K^+ \to \pi^+\nu\bar{\nu})$ and $BR(K_L \to \pi^0\nu\bar{\nu})$ are both $O(10^{-10})$ and $a_{\pi\nu\nu} = \mathcal{O}(1)$. Furthermore, as in this case the SM contribution is small, the measurement of $BR(K^+ \to \pi^+\nu\bar{\nu})$ approximately determines $|U_{ds}|$, and with the additional measurement of $BR(K_L \to \pi^0\nu\bar{\nu})$, $\arg(U_{ds})$ is approximately determined as well.

## 12. Conclusions

Experiments have not yet probed in a significant way the mechanism of CP violation. There is a large number of open questions concerning CP violation. Here are some examples:

• **Why are the measured parameters, $\varepsilon_K$ and $\varepsilon'_K$, small?**

The answer in the Standard Model is that CP violation is screened in processes that are dominated by the first two quark generations by small mixing angles. We have seen examples of new physics, that is supersymmetry with approximate CP, where the reason is the smallness of all CP violating phases.

Observing CP asymmetries of order one, as expected in processes that involve the first and third generations such as $B \to \psi K_S$, $K \to \pi\nu\bar{\nu}$ or even in charged $B$ decays, $B^\pm \to K^\pm\pi^0$, will exclude the approximate CP scenario.

• **What is the number of independent CP violating phases?**

The answer in the Standard Model is *one*, the Kobayashi-Maskawa phase. We have encountered models with a larger number, *e.g.* forty four in the supersymmetric standard model.

If the pattern predicted by the Standard Model, *e.g.* small CP asymmetries in $B_s \to \psi\phi$ and in $D \to K\pi$, a strong correlation between CP violation in $B \to \psi K_S$ and in $K_L \to \pi\nu\bar{\nu}$, equal asymmetries in $B \to \psi K_S$ and in $B \to \phi K_S$, etc., is inconsistent with measurements, then probably there are several independent phases.

• **Why is CP violated?**

The answer in the Standard Model is explicit breaking by complex Yukawa couplings. In left-right-symmetric models, the Lagrangian can be CP symmetric and the breaking is spontaneous.

It will be difficult to answer this question by experimental measurements, unless the correlations predicted by a specific model of spontaneous CP violation will be experimentally confirmed.

• **Is CP violation restricted to flavor changing interactions?**

This is indeed the case in the Standard Model. But in many of its extensions, such as supersymmetry, there is flavor diagonal CP violation.

Observation of an electric dipole moment or of CP violation in $t\bar{t}$ production will provide strong hints for flavor diagonal CP violation.

• **Is CP violation restricted to quark interactions?**

This is the case in the Standard Model but not if neutrinos have masses.

Observation of CP asymmetries in neutrino oscillation experiments will be a direct evidence of CP violation in the lepton sector.

• **Is CP violation restricted to the weak interactions?**

In the Standard Model, CP violation appears in charged current (that is, $W$-mediated) weak interactions only. In multi-scalar models, it appears in scalar interactions. In supersymmetry, it appears in strong interactions.

Observation of transverse lepton polarization in meson decays will provide evidence for CP violation in interactions that are not mediated by vector bosons.

There are more questions that we can ask and answers that we will learn in the near future. But the list above is enough to demonstrate how unique the Standard Model picture of CP violation is, how sensitive is CP violation to new physics, and how important are present and future experiments that will search for CP violation.


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