A Determination of the CKM Parameter $|V_{us}|$

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We present a determination of the CKM parameter $|V_{us}|$ based on new measurements of the six largest K_L branching fractions and semileptonic form factors by the KTeV (E832) experiment at Fermilab. We find $|V_{us}| = 0.2252 \pm 0.0008_{\rm KTeV} \pm 0.0021_{\rm ext}$, where the errors are from KTeV measurements and from external sources. We also use the measured branching fractions to determine the CP violation parameter $|\eta_{+-}| = (2.228 \pm 0.005_{\rm KTeV} \pm 0.009_{\rm ext}) \times 10^{-3}$.

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The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2] describes the charged current couplings of the u, c, and t quarks to the d, s, and b quarks. The first row of this matrix provides the most stringent test of the unitarity of the matrix. Current measurements [3] deviate from unitarity at the 2.2 sigma level: $1-(|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2) = 0.0043 \pm 0.0019$. $|V_{us}|$, which contributes an uncertainty of 0.0010 to this unitarity test, has been determined from charged and neutral kaon semileptonic decay rates. This determination is based on the partial width for semileptonic K decay, $\Gamma_{K\ell 3}$:

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW}(1+\delta_K^\ell) C^2 \left|V_{us}\right|^2 f_+^2(0) I_K^\ell, \quad (1)$$

where ℓ refers to either e or μ, G_F is the Fermi constant, M_K is the kaon mass, S_{EW} is the short-distance radiative correction, δ_K^ℓ is the mode-dependent long-distance radiative correction, $f_+(0)$ is the calculated form factor at zero momentum transfer for the $\ell\nu$ system, and I_K^ℓ is the phase-space integral, which depends on measured semileptonic form factors. C^2 is 1 (1/2) for neutral (charged) kaon decays. The current PDG determination of $|V_{us}|$ is based only on $K \to \pi e \nu$ decays; $K \to \pi\mu\nu$ decays have not been used because of large uncertainties in I_K^μ.

In this Letter, we present a determination of $|V_{us}|$ by the KTeV (E832) experiment at Fermilab based on measurements of the $K_L \to \pi^\pm e^\mp \nu$ and $K_L \to \pi^\pm \mu^\mp \nu$ partial widths and form factors. These measurements are described in detail elsewhere [4, 5]; a brief summary is given here. Our $|V_{us}|$ determination also makes use of a new treatment of radiative corrections [6].

To determine the $K_L \to \pi^\pm e^\mp \nu$ and $K_L \to \pi^\pm \mu^\mp \nu$ partial widths, we measure the following five ratios:

$$\begin{aligned}
\Gamma_{K\mu 3}/\Gamma_{Ke3} &\equiv \Gamma(K_L \to \pi^\pm \mu^\mp \nu)/\Gamma(K_L \to \pi^\pm e^\mp \nu) && (2)\\
\Gamma_{+-0}/\Gamma_{Ke3} &\equiv \Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi^\pm e^\mp \nu) && (3)\\
\Gamma_{000}/\Gamma_{Ke3} &\equiv \Gamma(K_L \to \pi^0\pi^0\pi^0)/\Gamma(K_L \to \pi^\pm e^\mp \nu) && (4)\\
\Gamma_{+-}/\Gamma_{Ke3} &\equiv \Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_L \to \pi^\pm e^\mp \nu) && (5)\\
\Gamma_{00}/\Gamma_{000} &\equiv \Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_L \to \pi^0\pi^0\pi^0), && (6)
\end{aligned}$$

where internal bremsstrahlung contributions are included for all decay modes with charged particles. Since the six decay modes listed above account for more than 99.9% of the total decay rate, the five partial width ratios may be converted into measurements of the branching fractions for the six decay modes. The K_L lifetime is then used to convert these branching fractions into partial widths. The branching fraction measurements also can be used to determine the CP violation parameter $|\eta_{+-}|^2 \equiv \Gamma(K_L \to$

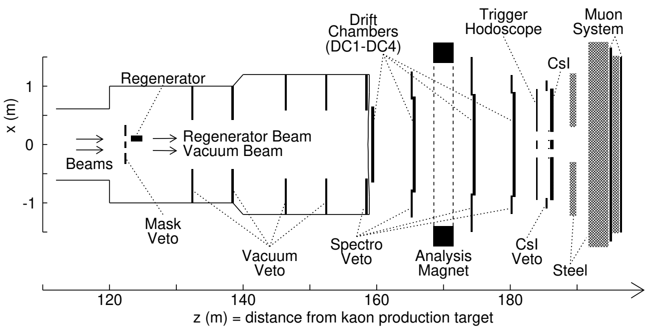

FIG. 1: The KTeV (E832) detector.

$\pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)$.

The semileptonic form factors describe the distribution of t, the momentum transfer to the $\ell\nu$ system. This t dependence increases the decay phase space integrals, I_K^e and I_K^μ, by about 10%. We use the following parametrization for the two independent semileptonic form factors:

$$\begin{aligned} f_+(t) &= f_+(0)\left(1 + \lambda'_+ \frac{t}{M_\pi^2} + \frac{1}{2}\lambda''_+ \frac{t^2}{M_\pi^4}\right) \\ f_0(t) &= f_+(0)\left(1 + \lambda_0 \frac{t}{M_\pi^2}\right), \end{aligned} \tag{7}$$

where $f_+(0)$ is obtained from theory, and we measure λ'_+, λ''_+, and λ_0.

In principle, the form factors can be measured directly from the t distribution. The undetected neutrino and unknown kaon momentum, however, result in a twofold ambiguity in the reconstructed value of t. To avoid systematic uncertainties associated with this ambiguity, we use a technique based only on components of particle momenta measured transverse to the kaon momentum [5].

The KTeV experiment (Fig. 1) and associated event reconstruction techniques have been described in detail elsewhere [7]. An 800 GeV/c proton beam striking a BeO target is used to produce two almost parallel neutral beams. The regenerator beam, which includes K_S, is not used in this analysis; the vacuum beam provides K_L decays used for these measurements. A large vacuum decay region surrounded by photon veto detectors extends to 159 m from the primary target. Following the vacuum region is a drift chamber spectrometer, trigger hodoscope, pure CsI electromagnetic calorimeter, and a muon system consisting of scintillator hodoscopes behind 4 m and 5 m of steel. The analyses presented in this Letter make use of the detector calibration and Monte Carlo simulation from the KTeV ϵ'/ϵ analysis [7].

Simple event reconstruction and selection may be used to distinguish different kaon decay modes from each other, and to reduce background to a negligible level for all decay modes. The reconstruction of charged decay modes ($K_L \to \pi^\pm e^\mp \nu$, $K_L \to \pi^\pm \mu^\mp \nu$, $K_L \to \pi^+\pi^-\pi^0$, and $K_L \to \pi^+\pi^-$) begins with the identification of two oppositely charged tracks coming from a single vertex. Note that for the $K_L \to \pi^+\pi^-\pi^0$ decay, we choose not to reconstruct the $\pi^0 \to \gamma\gamma$ decay to reduce the acceptance uncertainty in the $\Gamma_{+-0}/\Gamma_{Ke3}$ ratio.

The charged decay modes are separated from each other on the basis of particle identification and kinematic requirements. To select $K_L \to \pi^\pm e^\mp \nu$ decays, the electron is identified using the calorimeter energy measurement (E), combined with the spectrometer momentum (p). $K_L \to \pi^+\pi^-$ is separated from other two-track decays based on the two-track invariant mass, $m_{\pi\pi}$, and the square of the two-track momentum transverse to the K_L direction, p_t^2. To isolate $K_L \to \pi^+\pi^-\pi^0$ we use an additional variable, k_{+-0}, described in [4]. Note that the $K_L \to \pi^\pm\mu^\mp\nu$ background to each of these decay modes is suppressed by rejecting events with hits in the muon system. Two-track events that are not identified as $K_L \to \pi^\pm e^\mp \nu$, $K_L \to \pi^+\pi^-\pi^0$, or $K_L \to \pi^+\pi^-$, are selected as $K_L \to \pi^\pm\mu^\mp\nu$ candidates. To reduce the acceptance uncertainty in the $\Gamma_{K\mu3}/\Gamma_{Ke3}$ ratio, we do not require a signal in the muon hodoscope to identify $K_L \to \pi^\pm\mu^\mp\nu$ decays.

The reconstruction of the $K_L \to \pi^0\pi^0$ and $K_L \to \pi^0\pi^0\pi^0$ decay modes, where $\pi^0 \to \gamma\gamma$, is based on energies and positions of photons measured in the CsI electromagnetic calorimeter as described in [7]. Exactly four (six) clusters, each with a transverse profile consistent with a photon, are required for $K_L \to \pi^0\pi^0$ ($K_L \to \pi^0\pi^0\pi^0$). Photons are paired to reconstruct two or three neutral pions consistent with a single decay vertex.

All reconstructed decay modes are required to have kaon energy, E_K, between 40 and 120 GeV, and decay position, z, between 123 and 158 m from the target. For the reconstruction of semileptonic and $K_L \to \pi^+\pi^-\pi^0$ decays, there is a missing particle (ν or π^0), which results in multiple kaon energy solutions. Each of these solutions is required to be in the accepted range.

After all event selection requirements and background subtraction, we have between 10^5 and 10^6 events per decay mode. The background is 0.7% for $K_L \to \pi^0\pi^0$ and much smaller for the other decay modes. After correcting each of the ratios for acceptance differences between numerator and denominator, we find the partial width ratios given in Table I. The precision is 1.2% for $\Gamma_{000}/\Gamma_{Ke3}$ and about 0.5% for other ratios. A comparison of data and MC z-vertex distributions for the semileptonic decay modes (Fig. 2) demonstrates the quality of the MC simulation used for the acceptance correction.

The five partial width ratios may be combined to determine the branching fractions shown in Table II [4]. Using the PDG average for the neutral kaon lifetime [3], $\tau_L = (5.15 \pm 0.04) \times 10^{-8}$ s, our branching fraction measurements correspond to the partial decay widths shown in Table II.

Figure 3 shows a comparison of the KTeV and PDG

TABLE I: Measured partial width ratios. The first error is statistical and the second systematic. The five statistical errors are independent; correlations among the systematic errors are treated in [4].

Decay Modes	Partial Width Ratio
$\Gamma_{K\mu 3}/\Gamma_{Ke3}$	$0.6640 \pm 0.0014 \pm 0.0022$
$\Gamma_{000}/\Gamma_{Ke3}$	$0.4782 \pm 0.0014 \pm 0.0053$
$\Gamma_{+-0}/\Gamma_{Ke3}$	$0.3078 \pm 0.0005 \pm 0.0017$
Γ_{+-}/Γ_{Ke3}	$(4.856 \pm 0.017 \pm 0.023) \times 10^{-3}$
Γ_{00}/Γ_{000}	$(4.446 \pm 0.016 \pm 0.019) \times 10^{-3}$

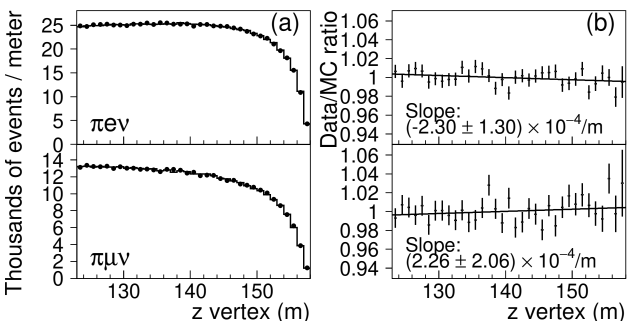

FIG. 2: (a) Comparison of the vacuum beam z distributions for data (dots) and MC (histogram); (b) Data-to-MC ratios with a linear fit.

values for the six branching fractions. The new KTeV measurements are on average a factor of two more precise than the current world average values, but are not in good agreement with these averages. Compared to the PDG fit [3], the KTeV measurement of $B(K_L \to \pi^\pm e^\mp \nu)$ is higher by 5%, $B(K_L \to \pi^0\pi^0\pi^0)$ is lower by 8%, $B(K_L \to \pi^+\pi^-)$ is lower by 5%, and $B(K_L \to \pi^0\pi^0)$ is lower by 8%. Our measurements of $B(K_L \to \pi^\pm \mu^\mp \nu)$ and $B(K_L \to \pi^+\pi^-\pi^0)$ are consistent with the PDG fit. A detailed comparison between the KTeV measurements and previous results is given in [4].

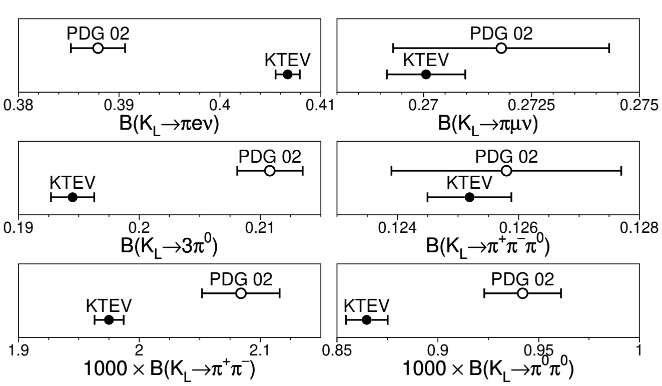

FIG. 3: K_L branching fractions measured by KTeV (dots) and from PDG fit (open circles).

TABLE II: K_L branching fractions and partial widths (Γ_i). Correlations among uncertainties in these measurements are given in [4].

Decay Mode	Branching fraction	Γ_i ($10^7 s^{-1}$)
$K_L \to \pi^\pm e^\mp \nu$	0.4067 ± 0.0011	0.7897 ± 0.0065
$K_L \to \pi^\pm \mu^\mp \nu$	0.2701 ± 0.0009	0.5244 ± 0.0044
$K_L \to \pi^+\pi^-\pi^0$	0.1252 ± 0.0007	0.2431 ± 0.0023
$K_L \to \pi^0\pi^0\pi^0$	0.1945 ± 0.0018	0.3777 ± 0.0045
$K_L \to \pi^+\pi^-$	$(1.975 \pm 0.012) \times 10^{-3}$	$(3.835 \pm 0.038) \times 10^{-3}$
$K_L \to \pi^0\pi^0$	$(0.865 \pm 0.010) \times 10^{-3}$	$(1.679 \pm 0.024) \times 10^{-3}$

Using the measured branching fractions for $K_L \to \pi^+\pi^-$ and $K_L \to \pi^0\pi^0$ together with τ_L, τ_S, $Re(\epsilon'/\epsilon)$, and $B(K_S \to \pi\ell\nu)$, we determine the CP violation parameter $|\eta_{+-}| = (2.228 \pm 0.005_{\rm KTeV} \pm 0.009_{\rm ext}) \times 10^{-3}$; most of the external error results from the uncertainty in τ_L. Our result is 2.6% lower than the PDG average. A comparison of $|\eta_{+-}|$ determinations is given in [4].

The $f_+(t)$ form factor is measured in both semileptonic decay modes; the effect of $f_0(t)$ is proportional to the lepton mass, so it is only measured in $K_L \to \pi^\pm\mu^\mp\nu$ decays. The measured parameters for the semileptonic form factors are $\lambda'_+ = (20.64 \pm 1.75) \times 10^{-3}$, $\lambda''_+ = (3.20 \pm 0.69) \times 10^{-3}$, and $\lambda_0 = (13.72 \pm 1.31) \times 10^{-3}$. The corresponding phase space integrals are $I_K^e = 0.15350 \pm 0.00105$ and $I_K^\mu = 0.10165 \pm 0.00080$, where the quoted errors include an additional uncertainty related to the form factor parameterization [5]. Compared to phase space integrals based on PDG form factors, KTeV's I_K^e and I_K^μ integrals are 1.7% and 4.2% lower, respectively. If we fit our data without the λ''_+ term, our I_K^e and I_K^μ integrals are increased by 1%, and are consistent with PDG averages that use only linear terms.

To check the consistency of our branching fraction and form factor measurements with lepton universality, we compare G_F for the two decay modes by taking the ratio of Eq. 1 for $K_L \to \pi^\pm\mu^\mp\nu$ and $K_L \to \pi^\pm e^\mp\nu$:

$$\left(\frac{G_F^\mu}{G_F^e}\right)^2 = \left[\frac{\Gamma(K_L \to \pi^\pm\mu^\mp\nu)}{\Gamma(K_L \to \pi^\pm e^\mp\nu)}\right] \Big/ \left(\frac{1+\delta_K^\mu}{1+\delta_K^e} \cdot \frac{I_K^\mu}{I_K^e}\right). \quad (8)$$

Many common uncertainties cancel in this ratio. The ratio of radiative corrections is calculated to be $(1+\delta_K^\mu)/(1+\delta_K^e) = 1.0058 \pm 0.0010$ [6], the ratio of the phase space integrals is $I_K^\mu/I_K^e = 0.6622 \pm 0.0018$, and $\Gamma_{K\mu3}/\Gamma_{Ke3}$ is from Table II. The resulting ratio of couplings squared is $(G_F^\mu/G_F^e)^2 = 0.9969 \pm 0.0048$, consistent with lepton universality. The same ratio calculated from PDG widths and form factors is $(G_F^\mu/G_F^e)^2 = 1.0270 \pm 0.0182$. Note that the 0.5% uncertainty in our universality test is much smaller than the 5% difference between the KTeV and PDG values of $\Gamma_{K\mu3}/\Gamma_{Ke3}$.

The measured partial widths and phase space integrals for semileptonic decays can be combined with theo-

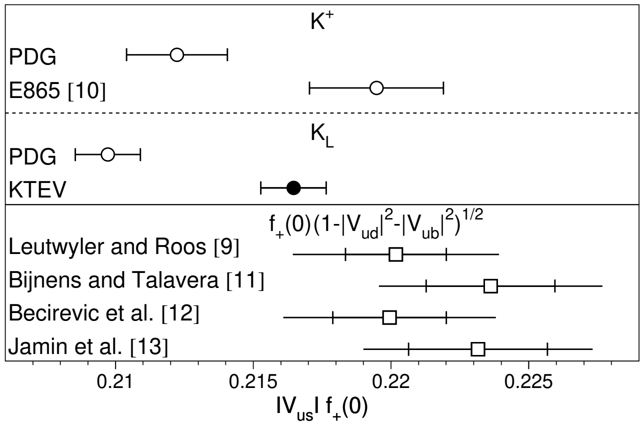

FIG. 4: Comparison of the KTeV measurement of $|V_{us}|f_+(0)$ with Brookhaven E865 [10], PDG, and also with determinations of $f_+(0)(1-|V_{ud}|^2-|V_{ub}|^2)^{1/2}$ based on different theoretical calculations of $f_+(0)$ [9, 11, 12, 13]. K^+ measurements have been divided by 1.022 ± 0.005, the ratio of $f_+(0)$ for charged and neutral kaons [14]. For K_L measurements, the uncertainties are mainly from τ_L. PDG refers to our evaluation based on PDG partial widths and form factors. For $f_+(0)(1-|V_{ud}|^2-|V_{ub}|^2)^{1/2}$, the inner error bars are from $f_+(0)$ uncertainty; the total uncertainties include the $|V_{ud}|$ and $|V_{ub}|$ errors.

retical corrections to calculate $|V_{us}|$ using Eq. 1. The short-distance radiative correction, $S_{EW} = 1.022$ [8], is evaluated with a cutoff at the proton mass. The long-distance radiative corrections are taken from [6]: $\delta_K^e = 0.013 \pm 0.003$ and $\delta_K^\mu = 0.019 \pm 0.003$. For $f_+(0)$, we use the same value used in the PDG evaluation of $|V_{us}|$: $f_+(0) = 0.961 \pm 0.008$ [9].

The resulting values of $|V_{us}|$ are 0.2253 ± 0.0023 for K_{e3} and 0.2250 ± 0.0023 for $K_{\mu 3}$, where the errors include an external uncertainty of 0.0021 from $f_+(0)$, the K_L lifetime, and radiative corrections. Assuming lepton universality, we average the $K_L \to \pi^\pm e^\mp \nu$ and $K_L \to \pi^\pm \mu^\mp \nu$ results (accounting for correlations):

$$|V_{us}| = 0.2252 \pm 0.0008_{\mathrm{KTeV}} \pm 0.0021_{\mathrm{ext}}. \qquad (9)$$

The KTeV error comes from uncertainties in the KTeV branching fraction and form factor measurements.

To compare our result with previous charged and neutral kaon measurements, we use the product of $|V_{us}|$ and $f_+(0)$ rather than $|V_{us}|$ to avoid significant common uncertainties from $f_+(0)$. Figure 4 shows a comparison of our measurement of

$$|V_{us}|f_+(0) = 0.2165 \pm 0.0012 \qquad (10)$$

with values from the PDG and Brookhaven E865 [10]. Our value of $|V_{us}|f_+(0)$ is inconsistent with previous K_L determinations, but is consistent with K^+ results (both E865 and earlier measurements). The figure also shows $f_+(0)(1-|V_{ud}|^2-|V_{ub}|^2)^{1/2}$, the expectation for $f_+(0)|V_{us}|$ assuming unitarity, based on $|V_{ud}| = 0.9734 \pm 0.0008$, $|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$, and several recent calculations of $f_+(0)$. Our value of $|V_{us}|$ (Eq. 9), based the Leutwyler and Roos calculation of $f_+(0)$, is consistent with unitarity: $1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 0.0018 \pm 0.0019$. For other calculations of $f_+(0)$, the consistency with unitarity ranges from 1 to 1.7 sigma, as shown in Fig. 4. Our improved form factor measurements may help to reduce theoretical uncertainties in $f_+(0)$ [11, 12, 13].

In summary, KTeV has made improved measurements of the six largest K_L branching fractions and the semileptonic form factors. We use these results to determine $|\eta_{+-}| = (2.228 \pm 0.010) \times 10^{-3}$ and $|V_{us}|f_+(0) = 0.2165 \pm 0.0012$. Using $f_+(0) = 0.961 \pm 0.008$ [9], we find $|V_{us}| = 0.2252 \pm 0.0022$, consistent with unitarity of the CKM matrix.

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[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[3] Particle Data Group, Phys. Ref. D **66**, 1 (2002).
[4] T. Alexopoulos et al. (KTeV), Phys. Rev. D (2004), submitted to Phys. Rev. D, hep-ex/0406002.
[5] T. Alexopoulos et al. (KTeV) (2004), submitted to Phys. Rev. D, hep-ex/0406003.
[6] T. Andre (2004), to be submitted to Phys. Rev. D, hep-ph/0406006.
[7] A. Alavi-Harati et al. (KTeV), Phys. Rev. **D67**, 012005 (2003), hep-ex/0208007.
[8] W. Marciano and A. Sirlin, Phys. Rev. Lett. **56**, 22 (1986).
[9] H. Leutwyler and M. Roos, Z. Phys. **C25**, 91 (1984).
[10] A. Sher et al. (BNL E865), Phys. Rev. Lett. **91**, 261802 (2003).
[11] J. Bijnens and P. Talavera, Nucl. Phys. **B669**, 341 (2003).
[12] D. Becirevic et al., hep-ph **0403217** (2004).
[13] M. Jamin, J. A. Oller, and A. Pich, JHEP **02**, 047 (2004), hep-ph/0401080.
[14] V. Cirigliano et al., hep-ph **0401173** (2004).