Review on the Bell-Steinberger relations

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CPT theorem is based on three assumptions: quantum field theory, locality and Lorentz invariance and thus it is a fundamental probe of our basic understanding of particle physics. Strangeness oscillation in $K^0 - \bar{K}^0$ system (see PDG reviews [1], [2, 3] and KLOE paper [4] for notations and previous literature), described by

$$i\frac{d}{dt}\left[\begin{array}{c} K^0 \\ \bar{K}^0 \end{array}\right] = [M - i\Gamma/2]\left[\begin{array}{c} K^0 \\ \bar{K}^0 \end{array}\right],$$

allows a very accurate test of the CPT symmetry. CPT requires M and Γ hermitian, $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. Then the mass and width eigenstates $K_{S,L}$ have a CPT violating piece, δ, in addition to the usual CPT conserving parameter ϵ:

$$K_{S,L} = \frac{1}{\sqrt{2\left(1+|\epsilon_{S,L}|^2\right)}}\left[\left(1+\epsilon_{S,L}\right)K^0 + \left(1-\epsilon_{S,L}\right)\bar{K}^0\right]$$

$$\epsilon_{S,L} = \frac{-i\Im\left(M_{12}\right) - \frac{1}{2}\Im\left(\Gamma_{12}\right) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}\left(\Gamma_{11} - \Gamma_{22}\right)\right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \equiv \epsilon \mp \delta. \qquad (1)$$

Unitarity allows to determine the phase of ϵ, found to be $\varphi_{SW} \equiv \arctan\frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$. Furthermore imposing unitarity to an arbitrary combination of K^0 and $\bar{K}^0$ wave functions leads to the Bell-Steinberger relation [5] connecting CP and CPT violation in the mass matrix to CP and CPT violation in the decay; in fact neglecting $\mathcal{O}(\epsilon)$ corrections to the coefficient of the CPT violating parameter, δ, we can write [4]

$$\left[\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i\tan\phi_{\rm SW}\right]\left[\frac{\Re(\epsilon)}{1+|\epsilon|^2} - i\Im(\delta)\right] = \frac{1}{\Gamma_S - \Gamma_L}\sum_f A_L(f)A_S^*(f), \qquad (2)$$

where $A_{L,S}(f) \equiv A(K_{L,S} \to f)$. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in (2); in fact defining for the hadronic modes

$$\alpha_i \equiv \frac{1}{\Gamma_S}\left\langle \mathcal{A}_L(i)\mathcal{A}_S^*(i)\right\rangle = \eta_i\,\mathcal{B}(K_S \to i)\;, \qquad i = \pi^0\pi^0\;,\;\pi^+\pi^-(\gamma), 3\pi^0\;,\;\pi^0\pi^+\pi^-(\gamma), \quad (3)$$

the recent data from CPLEAR, KLOE, KTeV and NA48 have led the following determinations (the analysis described in ref. [4] has been updated by using the recent measurements of K_L branching ratios from KTeV[6] and NA48[7])[8]

$$\alpha_{\pi^+\pi^-} \;=\; \Big((1.112 \pm 0.013) + i(1.061 \pm 0.014)\Big) \times 10^{-3}\,,$$

$$
\begin{aligned}
\alpha_{\pi^0\pi^0} &= \Big((0.493 \pm 0.007) + i(0.471 \pm 0.007)\Big) \times 10^{-3}\,, \\
\alpha_{\pi^+\pi^-\pi^0} &= \Big((0 \pm 2) + i(0 \pm 2)\Big) \times 10^{-6}, \\
|\alpha_{\pi^0\pi^0\pi^0}| &< 7 \times 10^{-6} \text{ at } 95\% \text{ CL}\,.
\end{aligned}
$$

The semileptonic contribution to the right-handed side of eq.(2) requires the determination of several observables: we define [2, 3]

$$
\begin{aligned}
\mathcal{A}(K^0 \to \pi^- l^+ \nu) &= \mathcal{A}_0(1-y)\ , \\
\mathcal{A}(K^0 \to \pi^+ l^- \nu) &= \mathcal{A}_0^*(1+y^*)(x_+ - x_-)^*\ , \\
\mathcal{A}(\overline{K}^0 \to \pi^+ l^- \nu) &= \mathcal{A}_0^*(1+y^*)\ , \\
\mathcal{A}(\overline{K}^0 \to \pi^- l^+ \nu) &= \mathcal{A}_0(1-y)(x_+ + x_-)\ ,
\end{aligned} \tag{4}
$$

where x_+ (x_-) describes the violation of the $\Delta S = \Delta Q$ rule in CPT conserving (violating) decay amplitudes, and y parametrizes CPT violation for $\Delta S = \Delta Q$ transitions. Taking advantage of their tagged $K^0(\bar{K}^0)$- beams CPLEAR has measured $\Im(x_+)$, $\Re(x_-)$, $\Im(\delta)$, $\Re(\delta)$ [10]. These determinations have been improved in Ref.[4] by including the information $A_S - A_L = 4[\Re(\delta) + \Re(x_-)]$, where $A_{L,S}$, are the K_L and K_S semileptonic charge asymmetries, respectively from the PDG [11] and KLOE [12]. Here we are also including the T-violating asymmetry measurement from CPLEAR [13].

	value	Correlation coefficients				
$\Re(\delta)$	$(3.0 \pm 2.3) \times 10^{-4}$	1				
$\Im(\delta)$	$(-0.66 \pm 0.65) \times 10^{-2}$	-0.21	1			
$\Re(x_-)$	$(-0.30 \pm 0.21) \times 10^{-2}$	-0.21	-0.60	1		
$\Im(x_+)$	$(0.02 \pm 0.22) \times 10^{-2}$	-0.38	-0.14	0.47	1	
$A_S + A_L$	$(-0.40 \pm 0.83) \times 10^{-2}$	-0.10	-0.63	0.99	0.43	1

Table 1: Values, errors, and correlation coefficients for $\Re(\delta)$, $\Im(\delta)$, $\Re(x_-)$, $\Im(x_+)$, and $A_S + A_L$ obtained from a combined fit, including KLOE [4] and CPLEAR [13].

The value $A_S + A_L$ in Tab. 1 can be directely included in the semileptonic contributions to the Bell Steinberger relations in (2)

$$
\begin{aligned}
\sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu)\mathcal{A}_S^*(\pi\ell\nu)\rangle = &\quad 2\Gamma(K_L \to \pi\ell\nu)\Big(\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\delta))\Big) \\
= &\ 2\Gamma(K_L \to \pi\ell\nu)\Big((A_S + A_L)/4 - i(\Im(x_+) + \Im(\delta))\Big)\ .
\end{aligned}
$$

Defining

$$
\alpha_{\pi\ell\nu} \equiv \frac{1}{\Gamma_S} \sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu)\mathcal{A}_S^*(\pi\ell\nu)\rangle + 2i\frac{\tau_{K_S}}{\tau_{K_L}}\, \mathcal{B}(K_L \to \pi\ell\nu)\Im(\delta) \tag{5}
$$

we find:

$$\alpha_{\pi\ell\nu} = \Big((-0.2 \pm 0.5) + i(0.1 \pm 0.5)\Big) \times 10^{-5}\,.$$

Inserting the values of the α parameters into (2) we find

$$\Re(\epsilon) = (161.2 \pm 0.6) \times 10^{-5}, \qquad \Im(\delta) = (-0.6 \pm 1.9) \times 10^{-5}\,, \tag{6}$$

The complete information on (6) is given in Tab.2.

	value	Correlation coefficients			
$\Re(\epsilon)$	$(161.2 \pm 0.6) \times 10^{-5}$	1			
$\Im(\delta)$	$(-0.6 \pm 1.9) \times 10^{-5}$	0.26	1		
$\Re(\delta)$	$(2.5 \pm 2.3) \times 10^{-4}$	0.08	-0.09	1	
$\Re(x_-)$	$(-4.2 \pm 1.7) \times 10^{-3}$	0.13	0.17	-0.43	1

Table 2: Summary of results: values, errors, and correlation coefficients for $\Re(\epsilon)$, $\Im(\delta)$, $\Re(\delta)$, and $\Re(x_-)$.

Now the agrement with CPT conservation, $\Im(\delta) = \Re(\delta) = \Re(x_-) = 0$, is at 30% C.L..

The allowed region in the $\Re(\epsilon)$, $\Im(\delta)$ plane at 68% CL and 95% CL is shown in the left panel of figure 1.

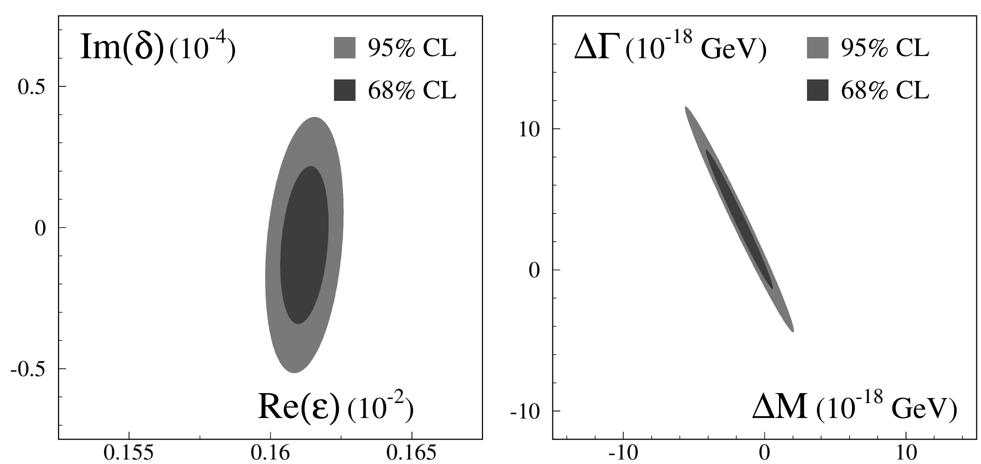

Figure 1: Left: allowed region at 68% and 95% CL in the $\Re(\epsilon)$, $\Im(\delta)$ plane. Right: allowed region at 68% and 95% CL in the $\Delta M, \Delta\Gamma$ plane.

The process giving the largest contribution to the size of the allowed region is $K_L \to \pi^+\pi^-$, through the uncertainty on ϕ_{+-}.

The limits on $\Im(\delta)$ and $\Re(\delta)$ can be used to constrain the mass and width difference between K^0 and $\overline{K}^0$ via

$$\delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos\phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)]\,.$$

The allowed region in the $\Delta M = (m_{K^0} - m_{\overline{K}^0}), \Delta\Gamma = (\Gamma_{K^0} - \Gamma_{\overline{K}^0})$ plane is shown in the right panel of figure 1. As a results we improve on the previous limits (see for instance P. Bloch in [11]) finding

$$-5.1 \times 10^{-19}\ \text{GeV} < m_{K^0} - m_{\overline{K}^0} < 5.1 \times 10^{-19}\ \text{GeV} \quad \text{at } 95\ \%\ \text{CL}\,.$$

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