Slide 1

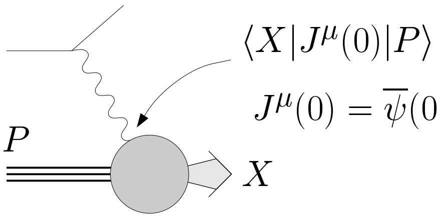
T-odd effects in hadronic scattering processes

Cedran Bomhof

- Introduction to PDFs.
- Introduction to gauge-links and their consequences.
- $p^{\uparrow}p \to \pi\pi X$
- Summary.

Slide 2

deep inelastic scattering



$$J^{\mu}(0) = \overline{\psi}(0)\gamma^{\mu}\psi(0)$$

$$\mathrm{d}\sigma^{\mathrm{Hadron}} \propto \sum_q \int \mathrm{d}x\ f(x)\ \mathrm{d}\hat{\sigma}^{\mathrm{parton}}(x)$$

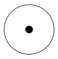
$$f_{\Gamma}(x) \propto \int \mathrm{d}k^{-}\mathrm{d}^2k_T\ \ \mathcal{FT}\ \langle P|\,\overline{\psi}(0)\,\mathcal{U}(0,\xi)\,\Gamma\psi(\xi)\,|P\rangle$$

Slide 3

quark-distribution functions

- $\Gamma = \gamma^+$: *Unpolarized distribution*

$$f_1^q(x) = q(x) =$$

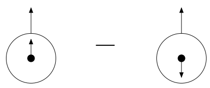


- $\Gamma = \gamma^+\gamma_5$: *Longitudinal polarization distribution* or *Helicity distribution*

$$g_1^q(x) = \Delta q(x) =$$

- $\Gamma = i\sigma^{S_T+}\gamma_5$: *Transverse polarization distribution* or *Transversity distribution*

$$h_1^q(x) = \delta q(x) =$$



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single spin asymmetries

One possible explanation for SSA is partonic intrinsic transverse momentum.

Examples:

- Sivers effect $\quad f_{1T}^{\perp(1)}$
- Collins effect $\quad H_1^{\perp(1)}$
- Boer-Mulders effect $\quad h_1^{\perp(1)}$

These effects are described by T-odd functions

$$f^{(T\text{-odd})}(x)$$
$$\propto \int \mathrm{d}k^- \mathrm{d}^2k_T\; k_T\; \mathcal{FT}\langle\, \overline{\psi}(0)\mathcal{U}(0,\xi)\Gamma\psi(\xi)\,\rangle$$

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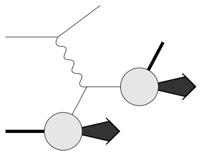
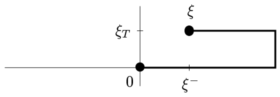
gauge-link

$$\mathcal{U}^C(0,\xi) = \mathcal{P}\exp ig\int_C \mathrm{d}z\cdot A(z)\ .$$

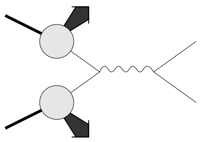
- SIDIS $(\ell + h_1 \to \ell' + h_2 + X)$

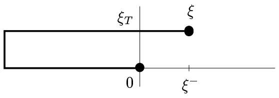




$\mathcal{U}^{[+]}$

- Drell-Yan $(h_1 + \overline{h}_2 \to \ell + \overline{\ell} + X)$





$\mathcal{U}^{[-]}$

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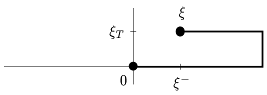
consequences of gauge-links for T-even functions

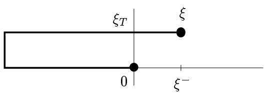
$\Rightarrow$ T-even functions, e.g. $f_1(x)$:

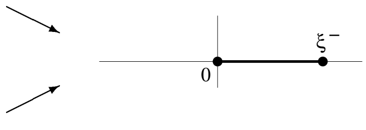
$$f_1^{[\pm]}(x) \propto \int \mathrm{d}k^-\,\mathrm{d}^2k_T\ \ \mathcal{FT}\langle\,\overline{\psi}(0)\,\mathcal{U}^{[\pm]}(0,\xi)\,\gamma^+\psi(\xi)\,\rangle \propto f_1(x)$$

$\mathcal{U}^{[+]}$:



$\mathcal{U}^{[-]}$:





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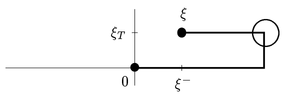
consequences of gauge-links for T-odd distribution functions

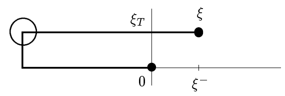
$\Rightarrow$ T-odd functions, e.g. Sivers $f_{1T}^{\perp(1)}(x)$:

$$(f_{1T}^{\perp(1)})^{[\pm]}(x) \propto \int \mathrm{d}k^-\,\mathrm{d}^2k_T \;\; k_T \;\; \mathcal{FT}\langle\,\overline{\psi}(0)\,\mathcal{U}^{[\pm]}(0,\xi)\,\gamma^+\psi(\xi)\,\rangle$$
$$\propto \pm\, f_{1T}^{\perp(1)}(x)$$



$\longrightarrow \;\langle\, A_T(+\infty) - A_T(-\infty)\,\rangle$
(in light-cone gauge)

$-\,\langle\, A_T(+\infty) - A_T(-\infty)\,\rangle \;\longleftarrow$



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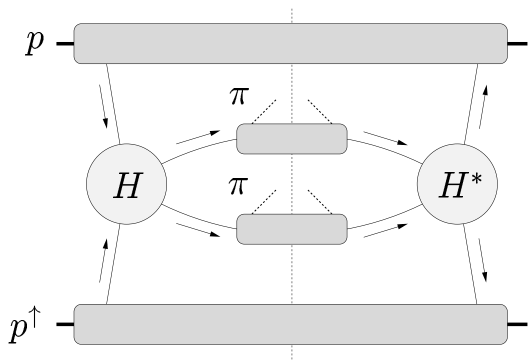
$p^\uparrow p \to \pi\pi X$ scattering



Complications

- Partonic scattering through strong interaction.
- Many different partonic scattering channels that contribute.

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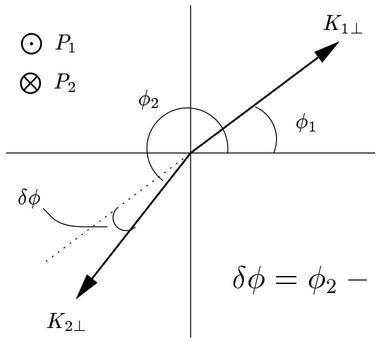
example of azimuthal asymmetry

Transverse plane:



$$\delta\phi = \phi_2 - \phi_1 - \pi$$

The weighed cross section

$$\langle\, \sin(\delta\phi)\, \mathrm{d}\sigma \,\rangle = \int \frac{\mathrm{d}\phi_2}{2\pi}\; \sin(\delta\phi)\; \mathrm{d}\sigma$$

is sensitive to intrinsic transverse momenta.

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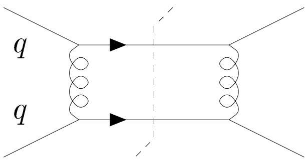
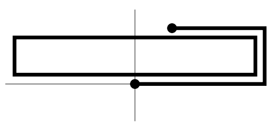
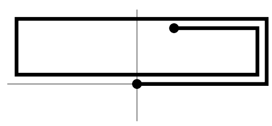
example of gauge-link in qq-scattering



tt^*-diagram

$\frac{5}{4} \times$  $- \frac{1}{4} \times$ 

$$= \frac{5}{4} \times \quad \mathrm{Tr}\big(\mathcal{U}^{[\Box]}\big)\mathcal{U}^{[+]}/3 \quad - \frac{1}{4} \times \quad \mathcal{U}^{[\Box]}\mathcal{U}^{[+]}$$

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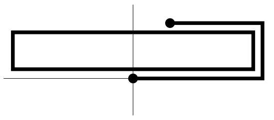
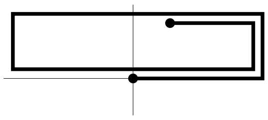
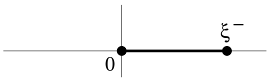
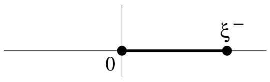
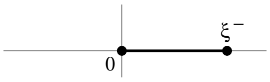
consequence for T-even functions

k_T-integration:

$\frac{5}{4} \times$  $-\frac{1}{4} \times$ 

$\frac{5}{4} \times$ 

$-\frac{1}{4} \times$ 

$=$ 

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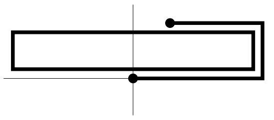
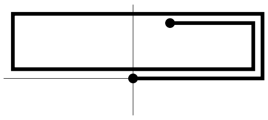
consequence for T-odd functions

k_T-weighing:

$\frac{5}{4} \times$  $-\frac{1}{4} \times$ 

$$\frac{5}{4} \times \langle\, A_T(\infty) - A_T(-\infty)\,\rangle \quad - \frac{1}{4} \times 3\,\langle\, A_T(\infty) - A_T(-\infty)\,\rangle$$

$$= \frac{1}{2} \times \langle\, A_T(\infty) - A_T(-\infty)\,\rangle$$

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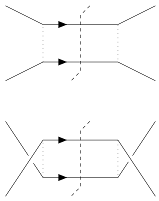
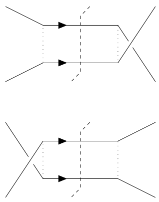
Sivers effects in identical-quark scattering



$$\mathcal{U} = \tfrac{5}{4}\,\mathrm{Tr}(\mathcal{U}^{[\Box]})\mathcal{U}^{[+]}/3 - \tfrac{1}{4}\,\mathcal{U}^{[\Box]}\mathcal{U}^{[-]}$$

$$\Rightarrow \quad f_{1T}^{\perp(1)}(tt^*) = f_{1T}^{\perp(1)}(uu^*) = \tfrac{1}{2}\,f_{1T}^{\perp(1)}$$



$$\mathcal{U} = \tfrac{9}{4}\,\mathrm{Tr}(\mathcal{U}^{[\Box]})\mathcal{U}^{[+]}/3 - \tfrac{5}{4}\,\mathcal{U}^{[\Box]}\mathcal{U}^{[-]}$$

$$\Rightarrow \quad f_{1T}^{\perp(1)}(tu^*) = f_{1T}^{\perp(1)}(ut^*) = -\tfrac{3}{2}\,f_{1T}^{\perp(1)}$$

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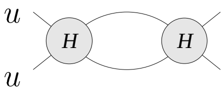
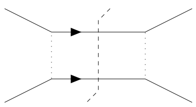
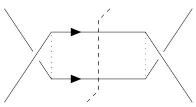
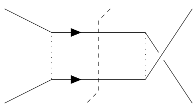
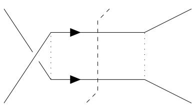
Identical quark scattering

partonic scattering process:

$\mathrm{d}\hat{\sigma}(uu\to uu) \propto$ 

$=$  $+$ 

$-$  $-$ 

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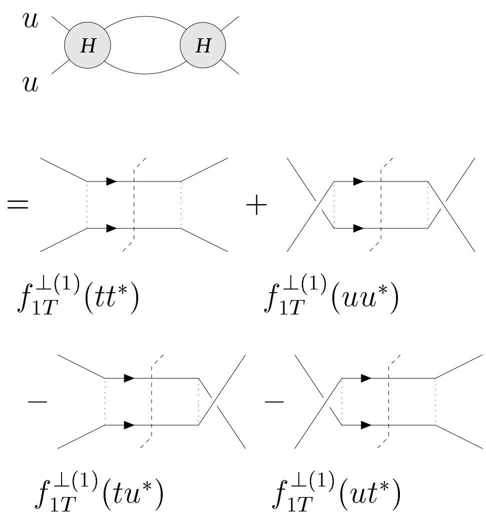
Identical quark scattering

hadronic scattering process:



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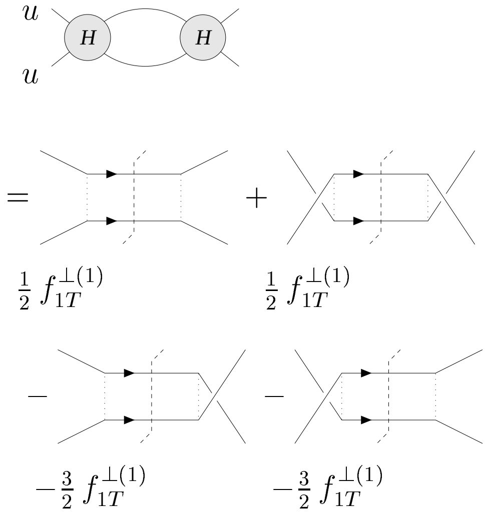
Identical quark scattering

using the ‘universal’ Sivers function



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Identical quark scattering

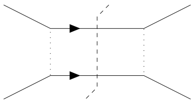
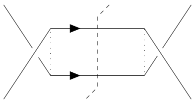
Sivers effect in identical-quark contribution to hadronic scattering

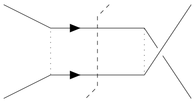
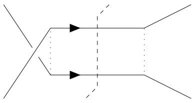
$\mathrm{d}\sigma^{\mathrm{HADRON}}$

$\propto f_{1T}^{\perp(1)}(x_1)\, f_1(x_2)\, \mathrm{d}S^{uu\to uu}\, D_1(z_1)\, D_1(z_2)$

with ‘modified’ or *gluonic pole cross section*

$\mathrm{d}S^{uu\to uu}$

$= \frac{1}{2}$  $+\ \frac{1}{2}$ 

$+\ \frac{3}{2}$  $+\ \frac{3}{2}$ 

$\neq \mathrm{d}\hat{\sigma}(uu\to uu)$

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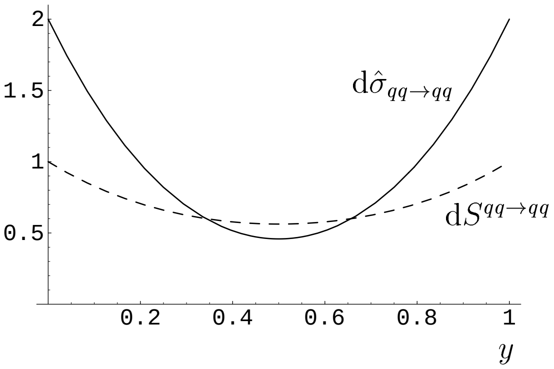
comparison of ‘true’ and gluonic pole cross section



$y = \cos^2(\frac{1}{2}\theta)$

θ : polar angle of outgoing pion in c.o.m.

weighted scattering cross section

[hep-ph/0505268]

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$$\langle \sin(\delta\phi)\, \mathrm{d}\sigma \rangle$$

$$\propto M_1 \sum\nolimits_{q_i} f_{1T}^{\perp(1)}(x_1) f_1(x_2) \frac{\mathrm{d}S^{\mathrm{SIVERS}}}{\mathrm{d}\hat{t}} D_1(z_1) D_1(z_2)$$

$$+ M_2 \sum\nolimits_{q_i} h_1(x_1) h_1^{\perp(1)}(x_2) \frac{\mathrm{d}S^{\mathrm{BOER\text{-}MULDERS}}}{\mathrm{d}\hat{t}} D_1(z_1) D_1(z_2)$$

+ Collins effect contributions

+ Gluon scattering contributions

summary, part I

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- Gauge-links are needed for a gauge-invariant description of PDFs and FFs.
- Gauge-links can be calculated by resumming all initial and/or final state interactions of collinear gluons.

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summary, part II

- Consequence of the gauge-links is that T-odd functions appear with different calculable strengths in the different scattering channels.
- Total cross section in $p^{\uparrow}p \to \pi\pi X$ can be written as a convolution of *universal* PDFs, FFs and process-dependent hard parts.
- These hard parts are, in general, different from the 'true' partonic cross sections.