Cochannel FM Signal Separation

Jet Propulsion Laboratory
California Institute of Technology
Jon Hamkins

Talk Outline

- Cochannel interference (CCI) Introduction
  - Examples of CCI
  - FM CCI signal model
- Receivers
  - Conventional single user receivers
    - Phase-locked loop
    - Ideal phase tracker
  - Joint phase trackers
    - Analytical technique
    - Joint Viterbi approach
- Performance comparisons
  - Simulations
  - Field data
- Conclusions

Cochannel Interference (CCI)

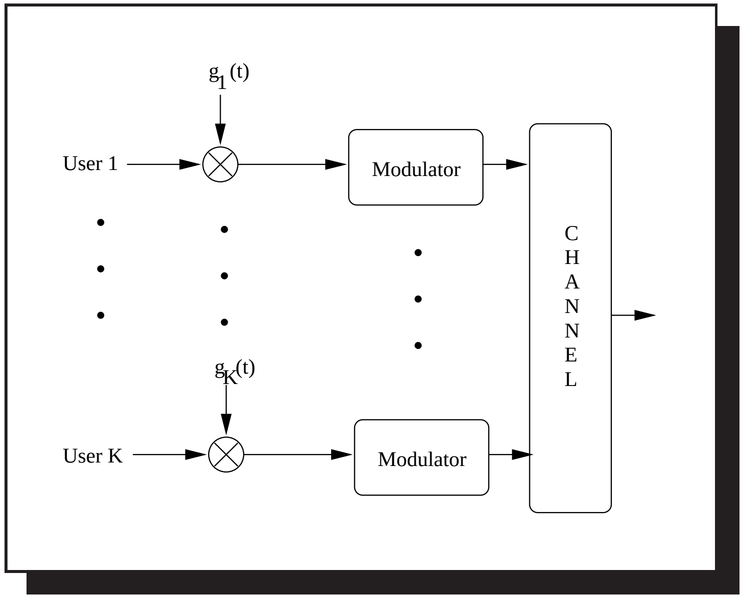
Two or more signals occupying the same frequency band produce *cochannel interference*.

Examples:

- Airborne observation

CCI Examples (continued)

- CDMA



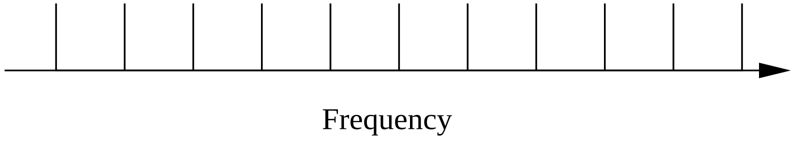
where

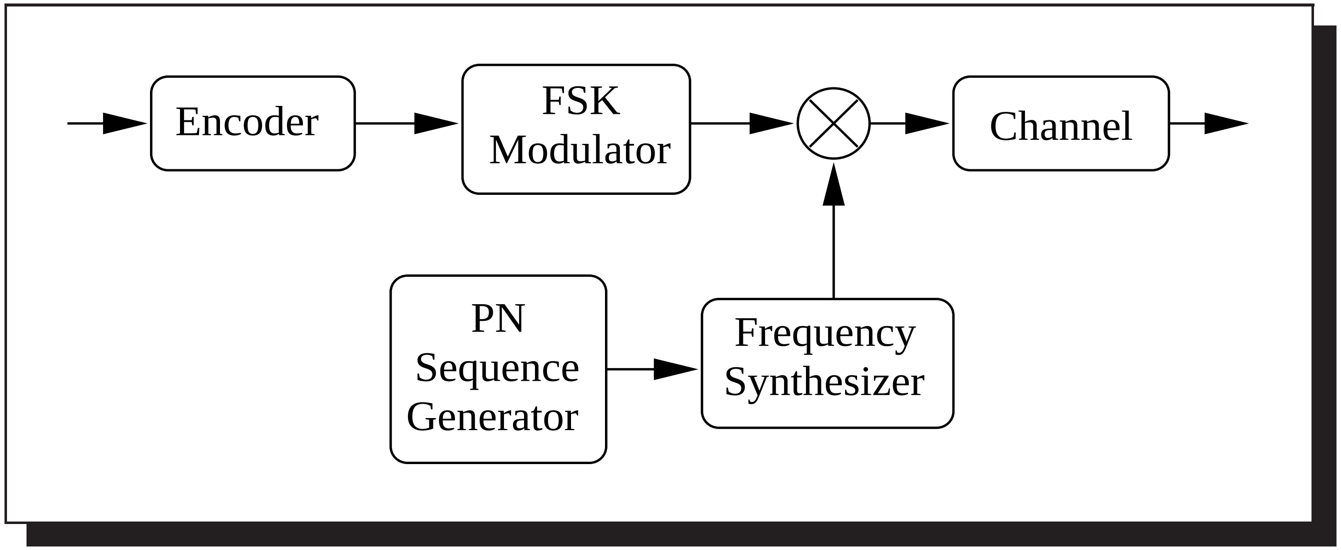
$g_i(t) = \sum_{n=0}^{L-1} a_k(n)p(t - nT_c)$	is the signature waveform of the ith user
$a_k(n)$	is a PN code sequence
$p(t)$	is a Pulse of duration T_c
T_c	is the chip duration

Advantage: can design $\{a_i\}$ sequences for good cross-correlation properties

CCI Examples (continued)

- Frequency hopping

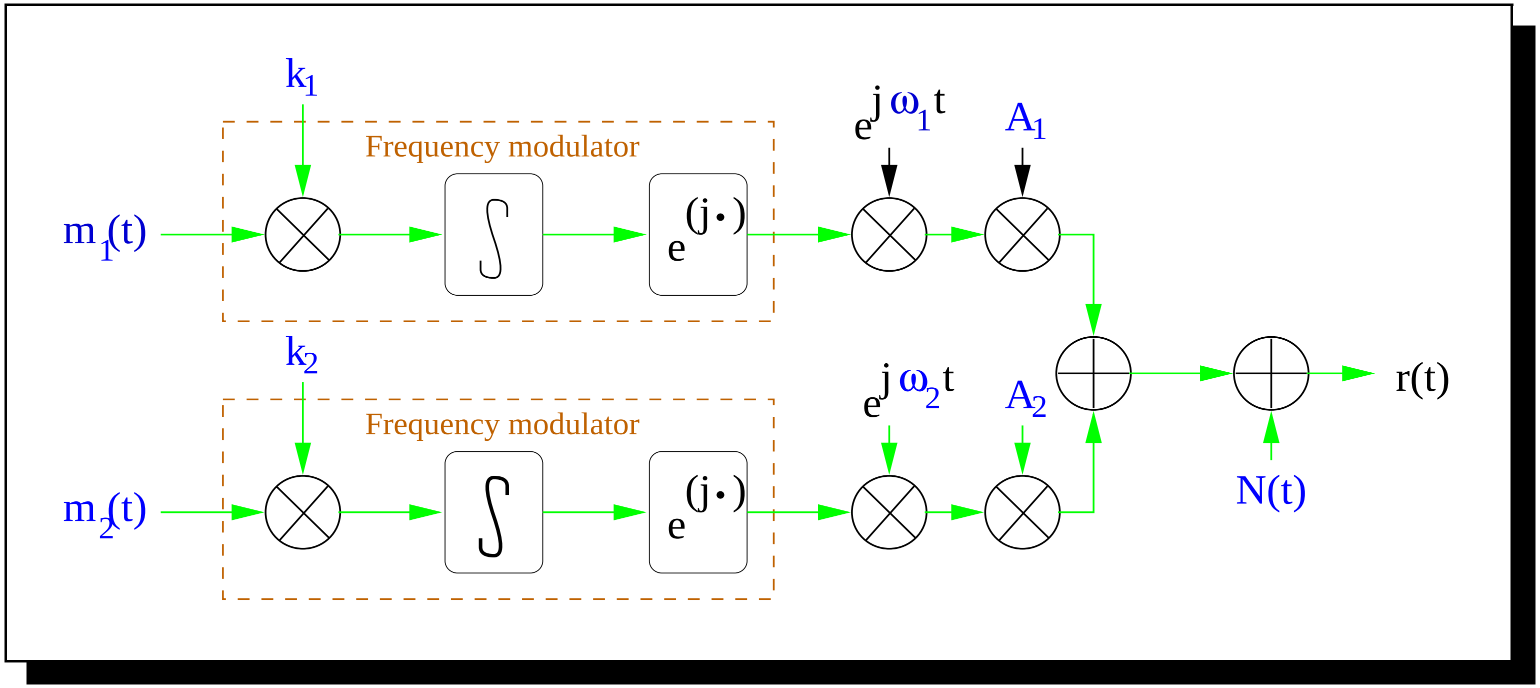




CCI Examples (continued)

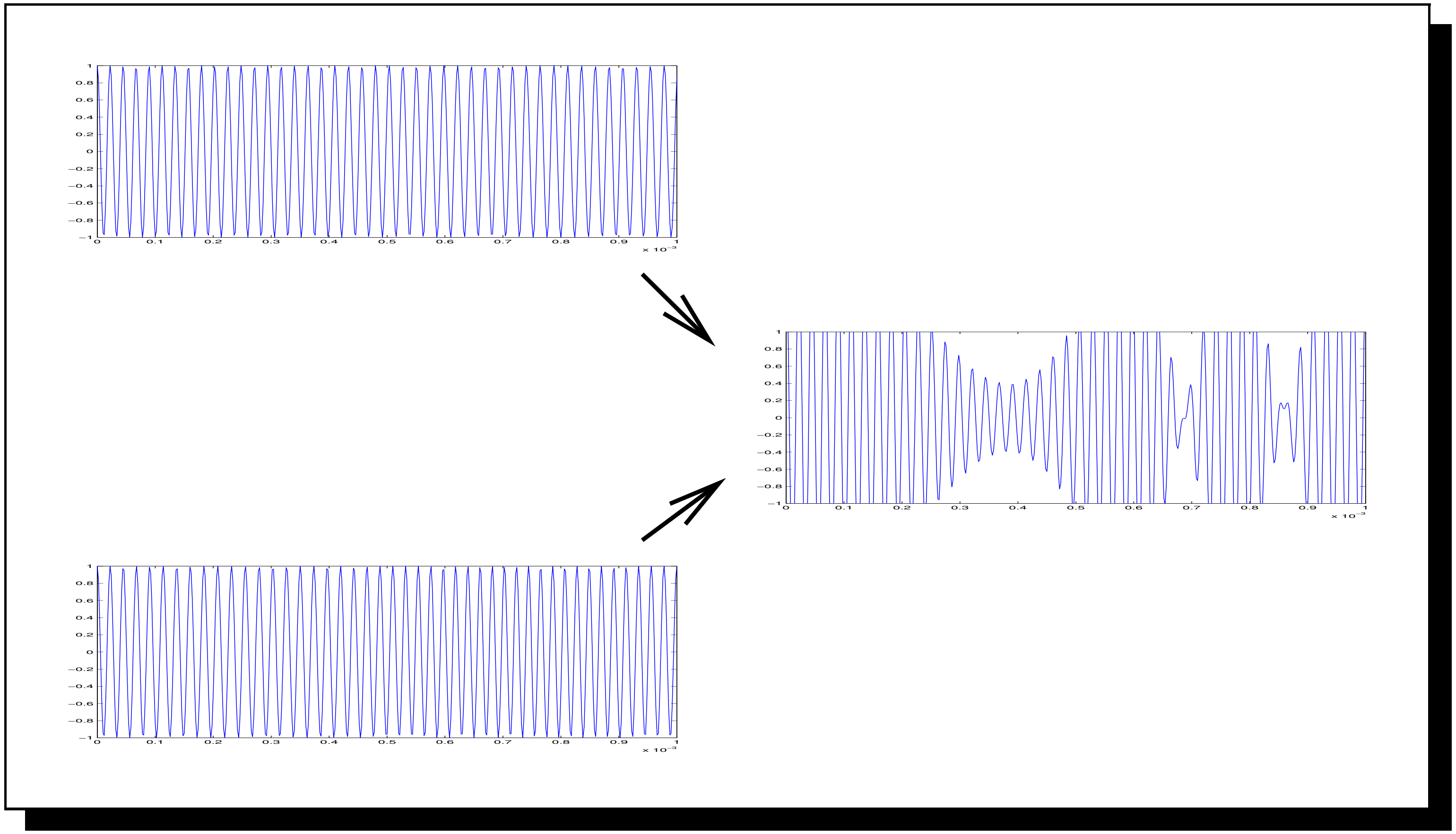
- Random access (ALOHA)
  - send packet as soon as it is generated
  - collision with other packets is detected
  - protocol specifies retransmission of lost packets
- Jamming
- Multipath

FM CCI Signal Model



- Model easily extended to more signals: $r(t) = A_1 e^{j\theta_1} + A_2 e^{j\theta_2} + \cdots$
- Can model a variety of signals
  - FM: m_1, m_2 are baseband audio
  - CW: $m_i = 0$ or $k_i = 0$
  - CPFSK: $m_i(t) = \sum_i a_i P_t(t)$
- Does not cover multi-channels—one receiver input only

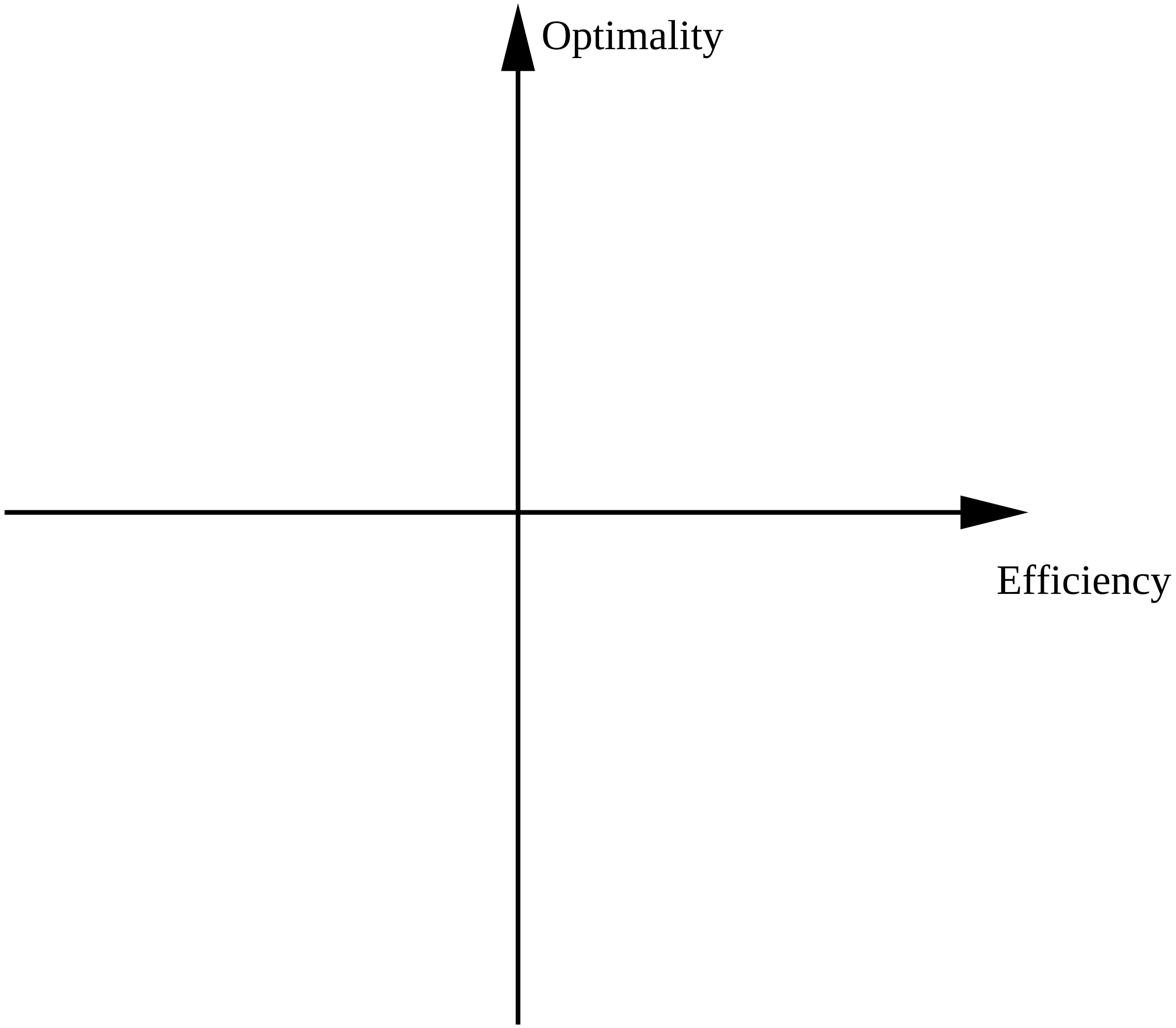
How CCI changes things



- FM signals individually have constant envelope
- CCI FM signal can have a widely varying envelope
- This hints that there is no "demodulation of the sum is the sum of the demodulations" type rule.

Cochannel FM Signal Separation

Efficiency vs. Optimality



Cochannel FM Signal Separation

Slowly but Surely

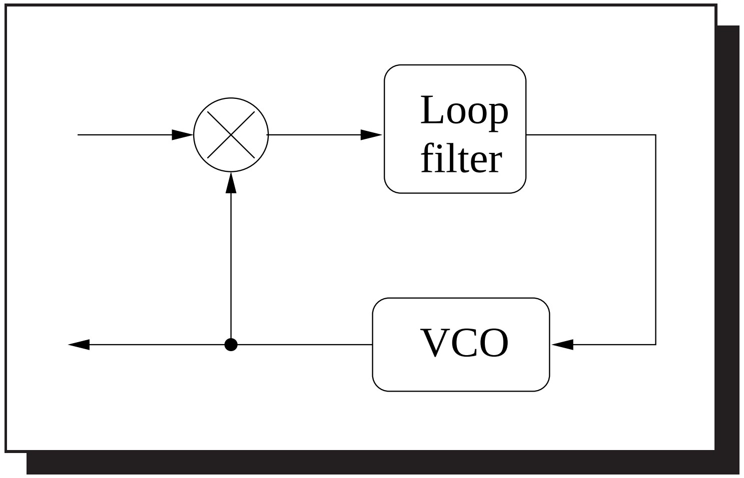
Short and Sweet

Too little, Too late

Quick and Dirty

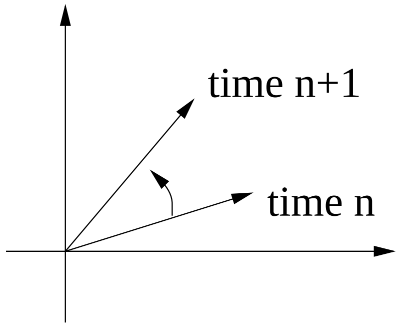
Conventional single-user receivers

- Phase-locked loop (PLL)



FM output is the time derivative of the phase of the VCO.

- Differential Phase Detector (DPD)



$r[n] = Ce^{j\phi[n]}$

Instantaneous frequency $\approx \frac{\phi[n]-\phi[n-1]}{T_s}$

Conventional single-user receivers

- Ideal Phase Tracker
  - Gives idea of best-case performance among all single-phase tracking receivers
  - If this fails, then all single-user receivers likely to fail in a similar fashion

The received signal is:

$$\begin{aligned} r &= A_1 e^{j\theta_1} + A_2 e^{j\theta_2} \\ &= C e^{j\phi} \end{aligned}$$

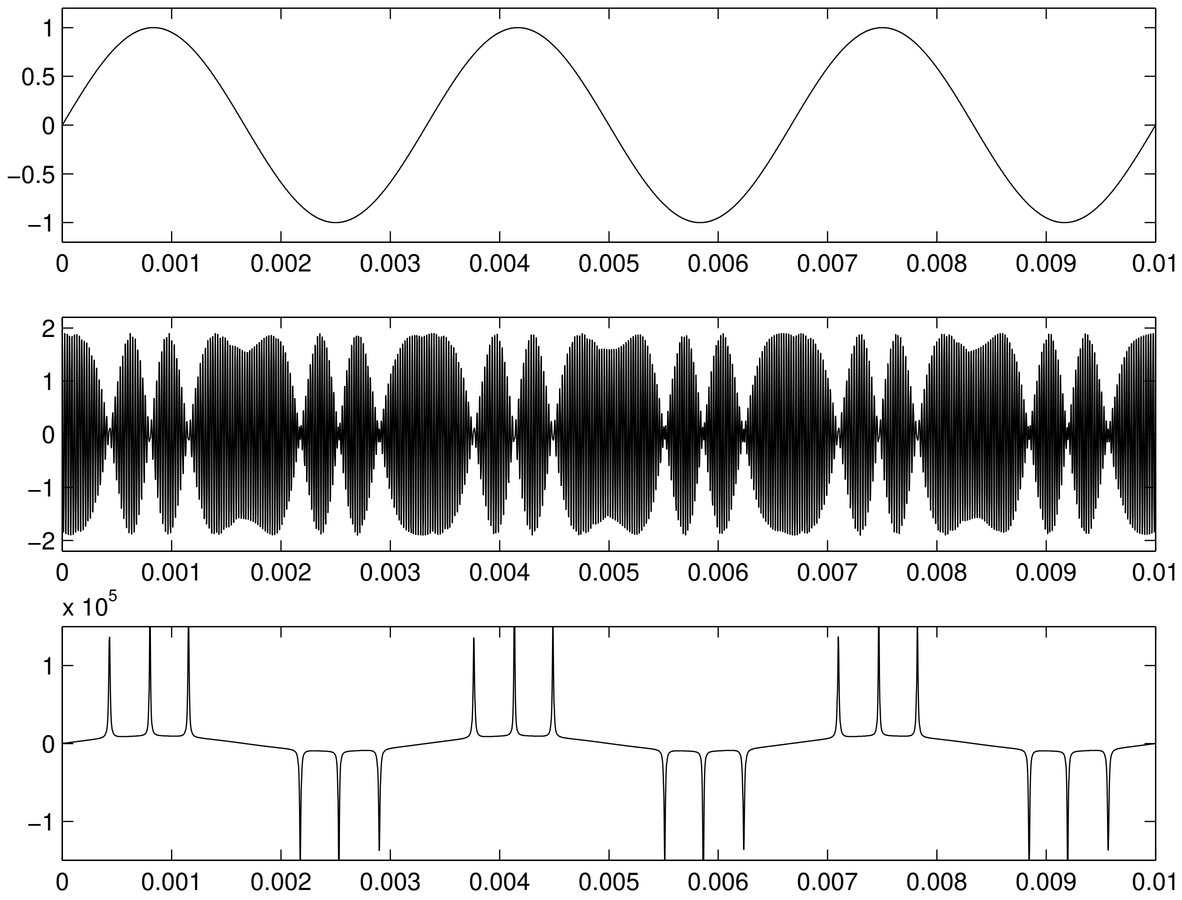
where

$$C = \sqrt{A_1^2 + 2A_1 A_2 \cos(\theta_1 - \theta_2) + A_2^2} \tag{1}$$

$$\phi = \tan^{-1}\left[\frac{A_1 \sin\theta_1 + A_2 \sin\theta_2}{A_1 \cos\theta_1 + A_2 \cos\theta_2}\right] \tag{2}$$

$$\Rightarrow \phi' = \frac{A_1^2\theta_1' + A_1 A_2(\theta_1' + \theta_2')\cos(\theta_1 - \theta_2) + A_2^2\theta_2')}{C^2}$$

Example Performance of Ideal Single Phase Tracker



A single tone, shown at top, is FM modulated and CW interference is added to it; this results in the middle figure. An Ideal phase tracker output is shown at bottom.

Conclusion: correcting for varying envelope is not good enough; the true phase itself contains unwanted spikes.

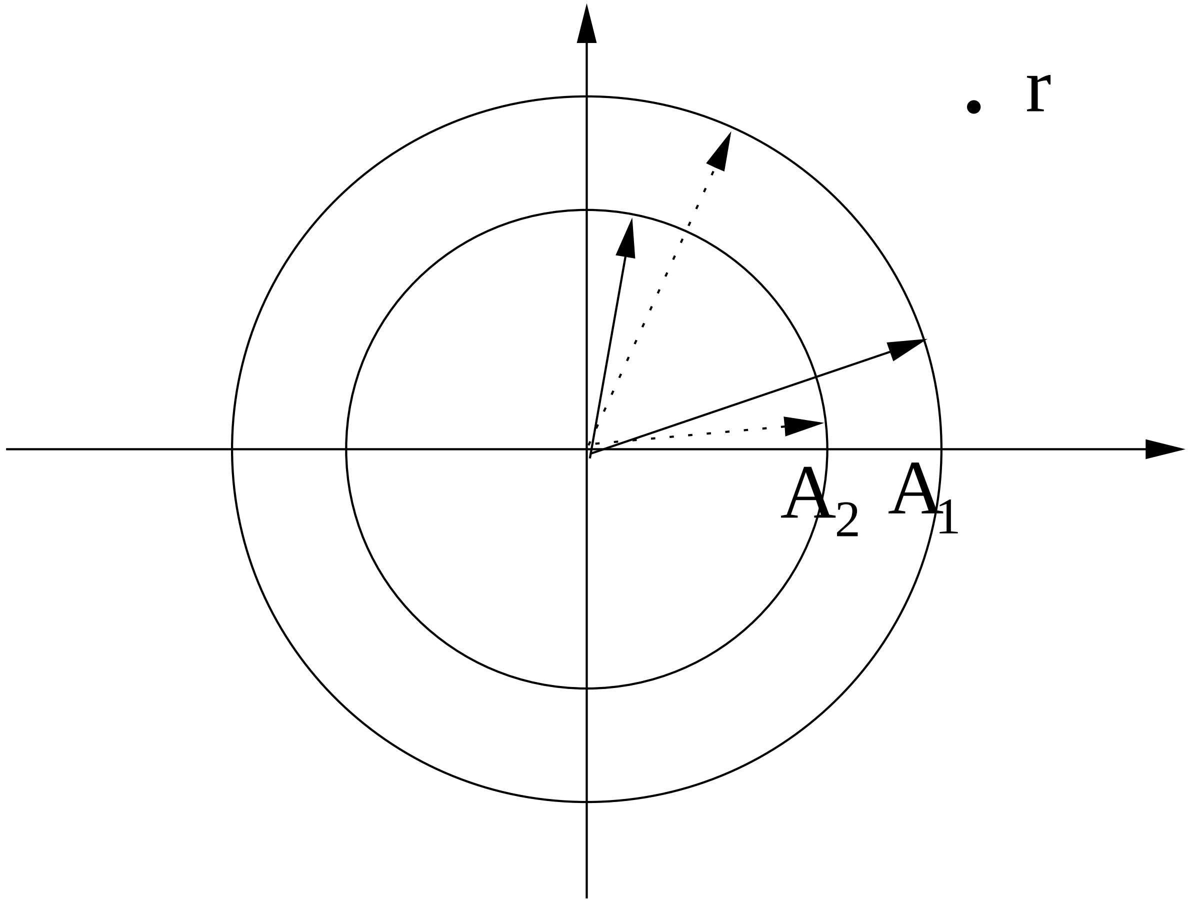
Cochannel Rejection vs. Cochannel Separation

	Rejection	Separation
Goal	Get better estimate of dominant signal by suppressing the interference.	Jointly estimate dominant and subdominant signals.
Method	Raise the SNR and SIR as much as possible, e.g. by LPF or notch filter.	Exploit signal power wherever it is. Re-mod estimates and compare to received signal.
Examples	A single PLL; a phase discriminator.	Cross-coupled PLL; Joint Viterbi phase estimator.

Analytical Technique

Assume we have external knowledge of the amplitudes A_1 and A_2.

$$r = A_1 e^{j\theta_1} + A_2 e^{j\theta_2}$$



It appears that we can solve for the individual phases, down to one of two possibilities.

Analytical Technique

Try Solving for θ_1 and θ_2:

$$r = A_1 e^{j\theta_1} + A_2 e^{j\theta_2}$$

$$\Rightarrow \|r\|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\theta_1 - \theta_2)$$

$$\Rightarrow \cos(\theta_1 - \theta_2) = \frac{\|r\|^2 - A_1^2 - A_2^2}{2A_1A_2}$$

$$\Rightarrow e^{j\theta_1} = \frac{r}{A_1 + A_2 e^{j(\theta_2 - \theta_1)}} = \frac{r(A_1 + A_2 e^{j(\theta_1 - \theta_2)})}{\|r\|^2}$$

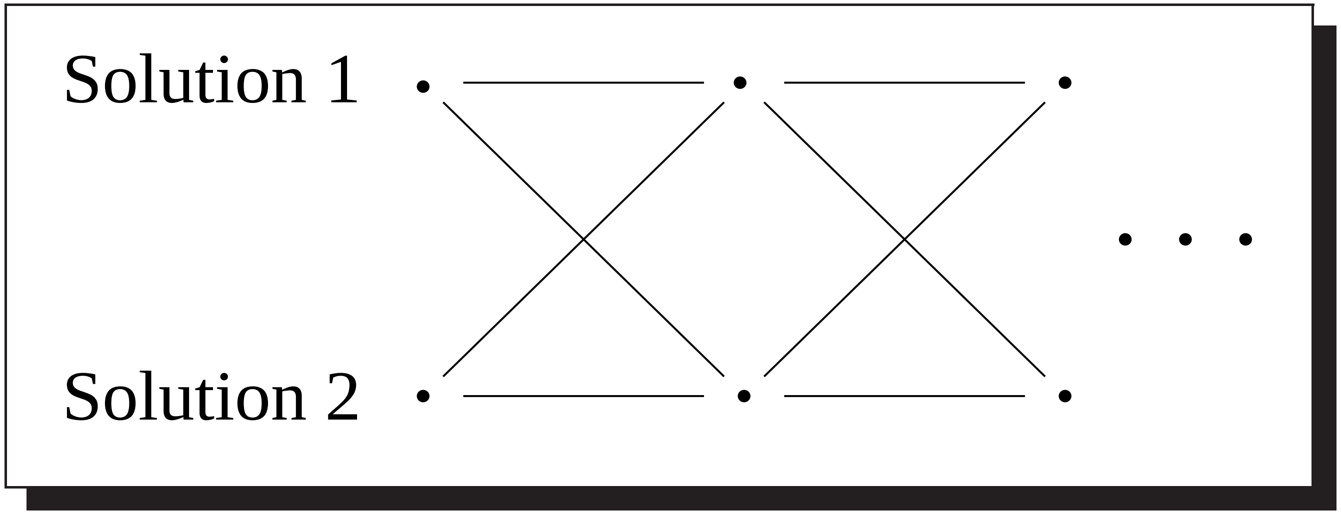
$$\theta_1 = \arg[r(A_1 + A_2\cos(\theta_1 - \theta_2) + jA_2\sin(\theta_1 - \theta_2)]$$
$$\theta_2 = \arg[r(A_2 + A_1\cos(\theta_1 - \theta_2) - jA_1\sin(\theta_1 - \theta_2)]$$

- A_1, A_2, $\cos(\theta_1 - \theta_2)$, and $\|r\|^2$ are all known.
- The unknown sign of $\sin(\theta_1 - \theta_2)$ gives rise to two possible solutions, as we expected from the figure.

Analytical Technique

- For a single sample, there is no reason to prefer one solution over the other.
- BUT, the sequence of solutions should make sense from an audio bandwidth or power spectral density point of view. For example, we don't expect the phase to jump too much from one sample to the next– this would imply a bandwidth wider than we expect for an audio signal.

Set up trellis



Choose path through trellis giving the "most likely" sequence of joint solutions.

Analytical Technique

Each state stores $\hat{\theta}_1'[n]$ and $\hat{\theta}_1'[n]$, as well as a 1-step linear minimum mean-square error prediction for θ_1' and θ_2' at time $n+1$:

$$\boxed{\hat{\theta}_i'[n+1] = \sum_{i=1}^{m} c_i \hat{\theta}_i'[n+1-i]}$$

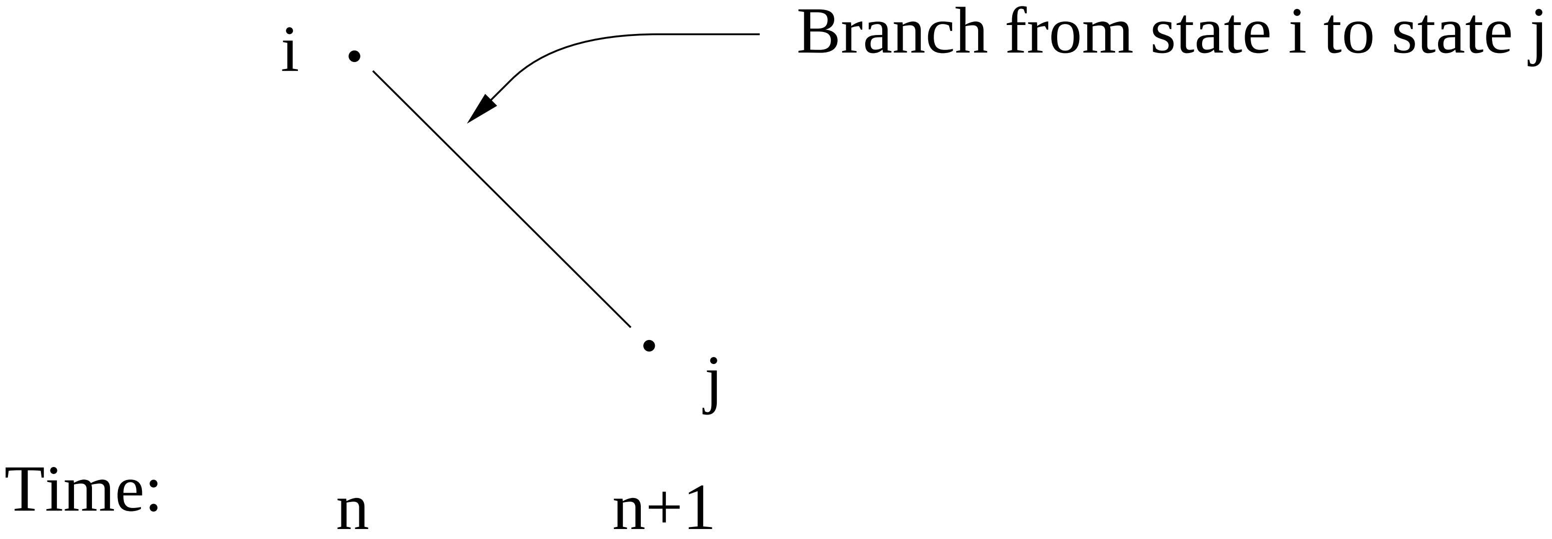
where

- $c_i = S_m^{-1} v$ are coefficients determined from Levinson-Durbin algorithm, where
- $v = (s_1, \ldots, s_m)^T$
- $S_m = \begin{bmatrix} s_0 & s_1 & \cdots & s_{m-1} \\ s_1 & s_0 & \cdots & s_{m-2} \\ \vdots & \vdots & \ddots & \vdots \\ s_{m-1} & s_{m-2} & \cdots & s_0 \end{bmatrix}.$
- $s_i = E[\theta_1'[n]\theta_1'[n-i]]$, which can be determined from the IFT of the PSD

If the PSD is unknown, we can use some default PSD for audio signals, or at least a flat PSD of some assumed audio bandwidth, to determine the s_i's.

Analytical Technique

The branch metric is squared difference between Levinson-Durbin prediction of instantaneous frequency and the hypothesized solution at a given state:

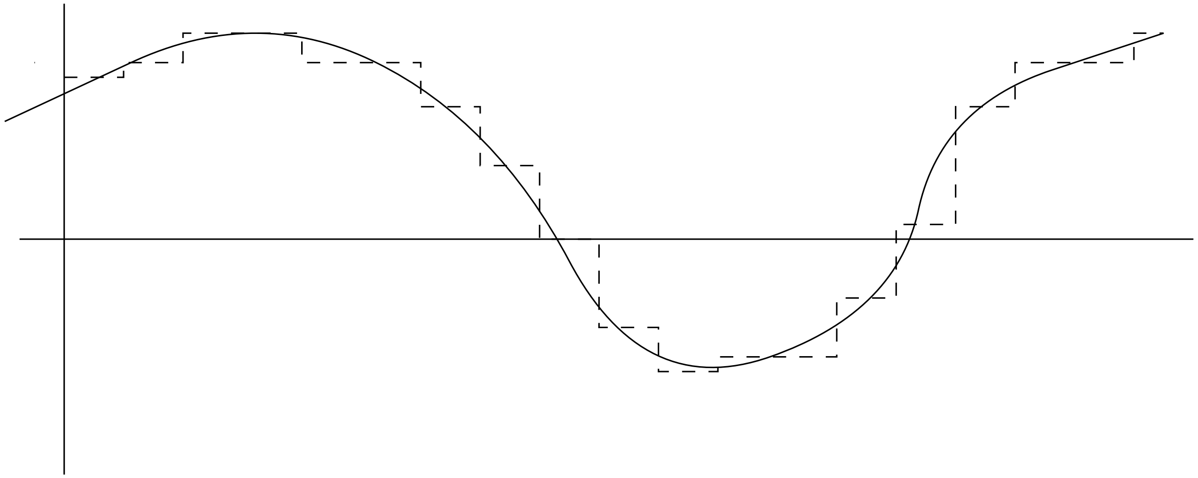


Branch metric $= (\hat{\theta}_1'^{\,i,LD}[n+1] - \hat{\theta}_1'^{\,j}[n+1])^2 + (\hat{\theta}_2'^{\,i,LD}[n+1] - \hat{\theta}_2'^{\,j}[n+1])^2$

Joint Viterbi Approach

Can we use Viterbi approach for analog demodulation?

Yes. Basic idea due to Cahn (IT Trans., '74). Introduce an artificial constraint:



$$\hat{\theta}_i'[n] = \hat{\theta}_i'[n-1] - a_i[n]C$$

where

- C is a constant
- $a_i[n] \in \{-1, +1\}$
- Use MLSE to find $\{a_i\}$.

Joint Viterbi Estimation

Approximate $r[n]$ by

$$\hat{r}[n] = \hat{A}_1[n]e^{j\hat{\theta}_1[n]} + \hat{A}_2[n]e^{j\hat{\theta}_2[n]}$$

Choose $\hat{A}_i[n]$ and $\hat{\theta}_i[n]$ to minimize

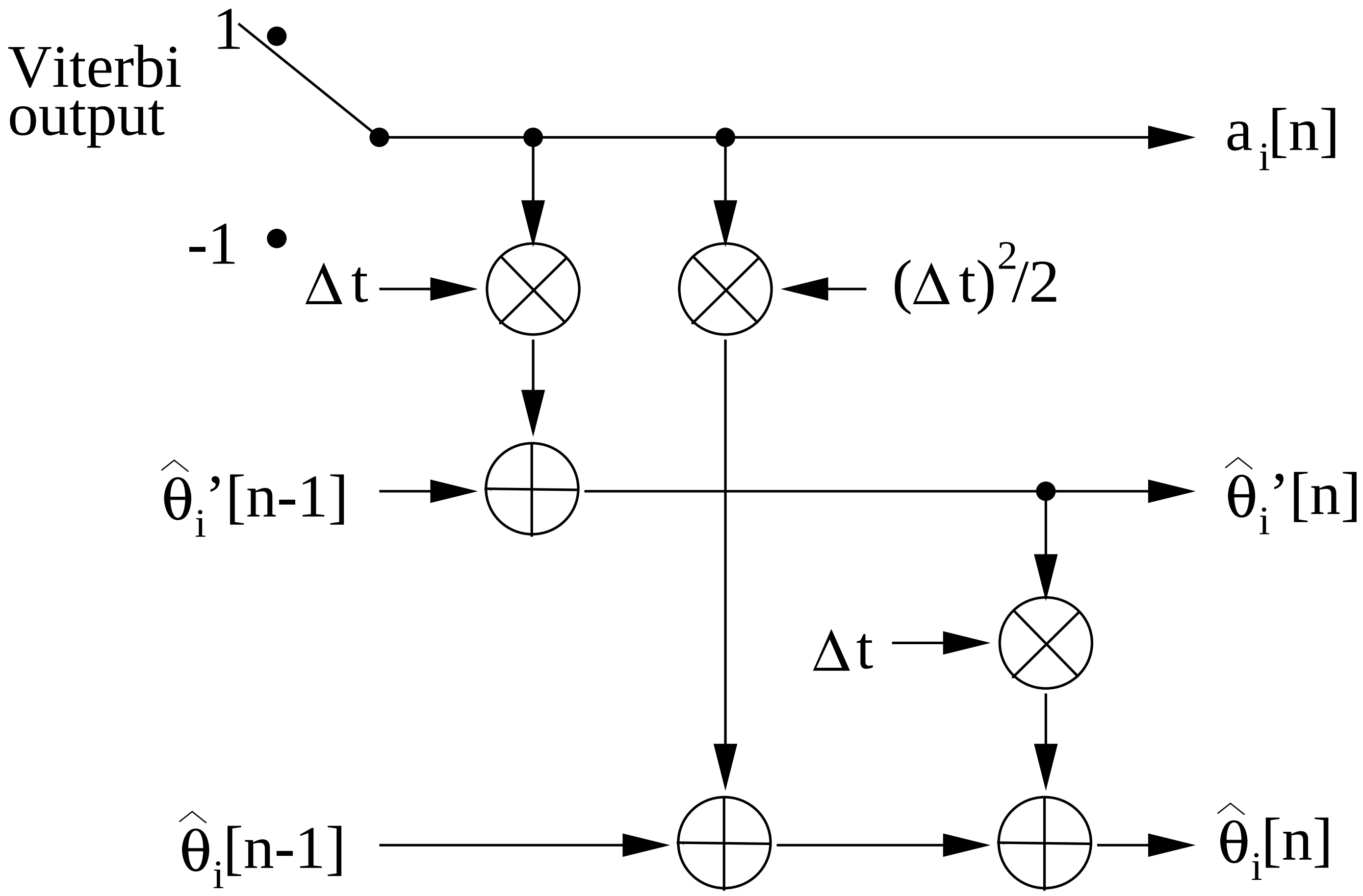
$$M[n] = \sum_{l=0}^{n} \|r[l] - \hat{r}[l]\|^2$$

Introduce artificial constraint

$$\hat{\theta}'_i[n] = \hat{\theta}'_i[n-1] + a_i[n]C$$

where $a_i[n] = \pm 1$, and use a joint MLSE algorithm is to determine $a_i[n]$.

Phase Updates

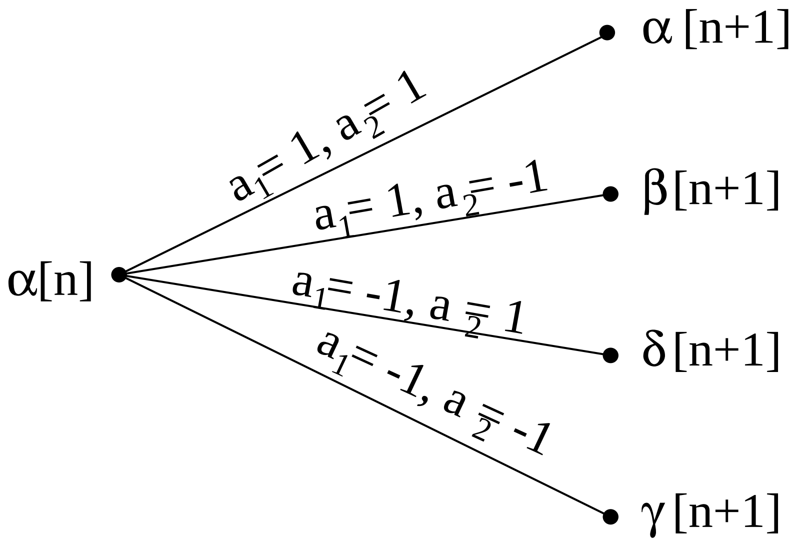


State description

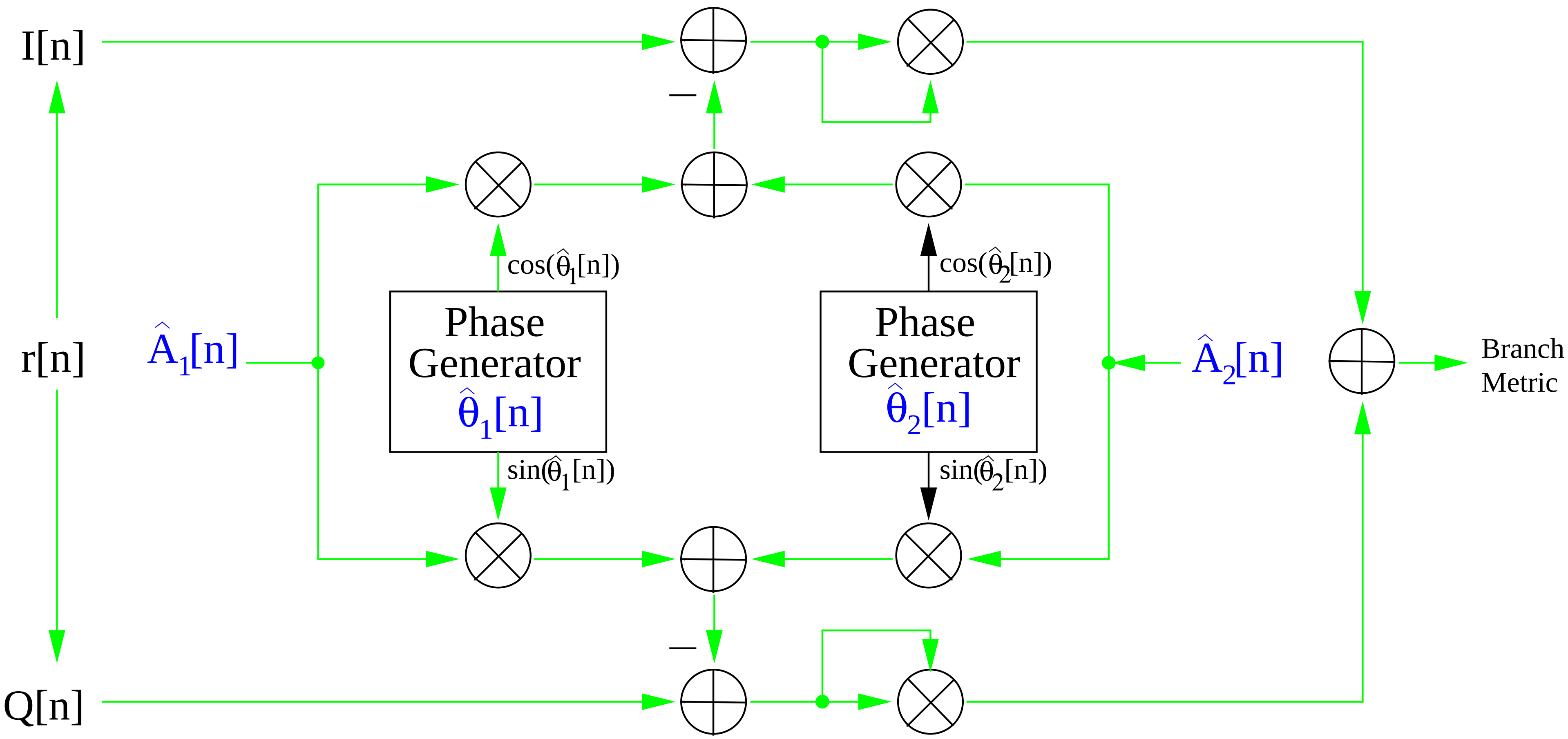
At time n, each state of the trellis stores:

$\alpha[n] = (\quad a_1[n-3], a_1[n-2], a_1[n-1],$	Last three "bits" of dominant signal
$a_2[n-3], a_2[n-2], a_2[n-1],$	Last three "bits" of subdominant signal
$\hat{\theta}'_1[n], \hat{\theta}'_2[n],$	Instantaneous frequency estimates
$\hat{\theta}_1[n], \hat{\theta}_2[n],$	Phase estimates
$\hat{A}_1[n], \hat{A}_2[n],$	Amplitude estimates (PSP)
$M[n])$	Metric for best path to this state

Trellis update:

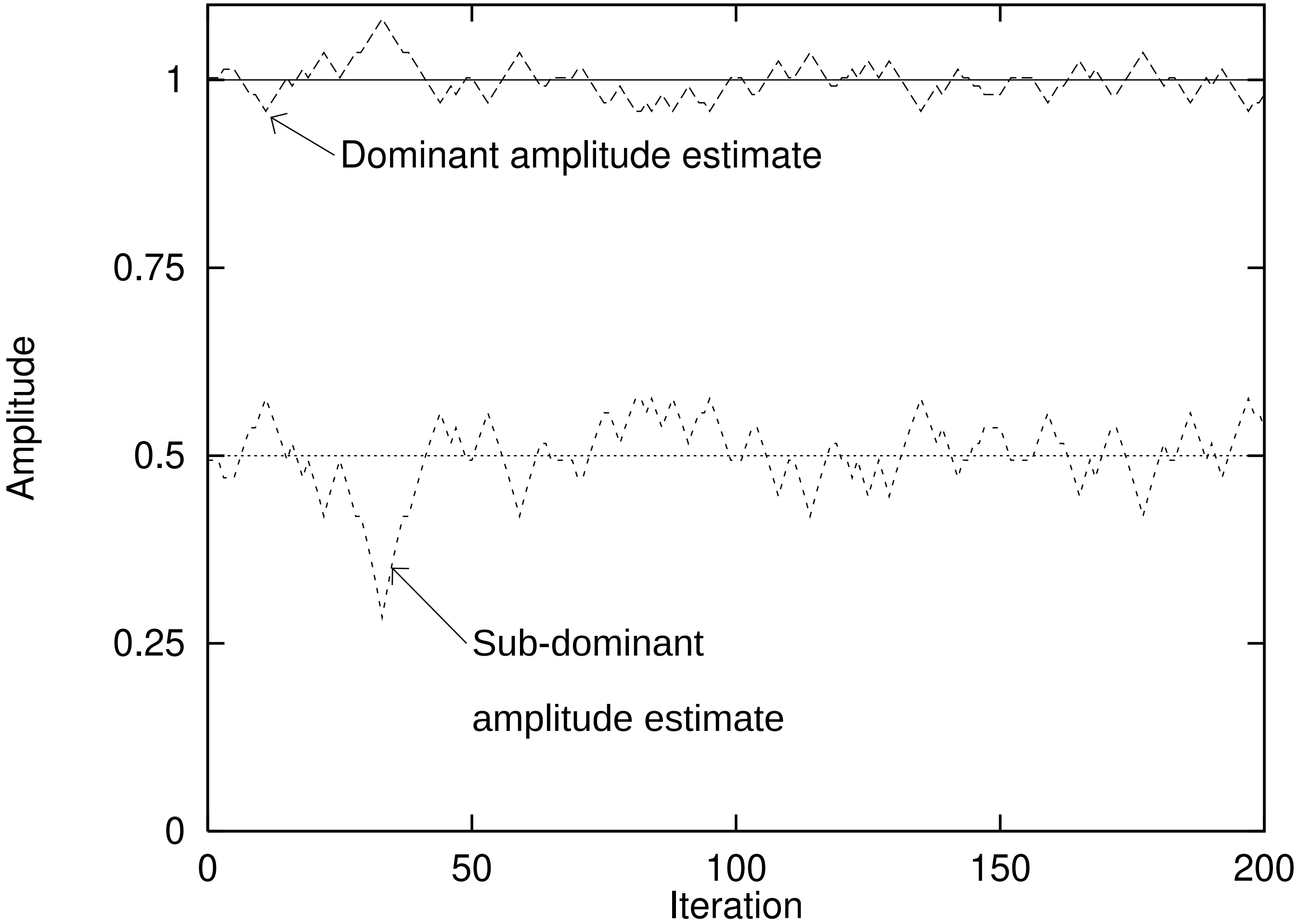


Branch metric computation

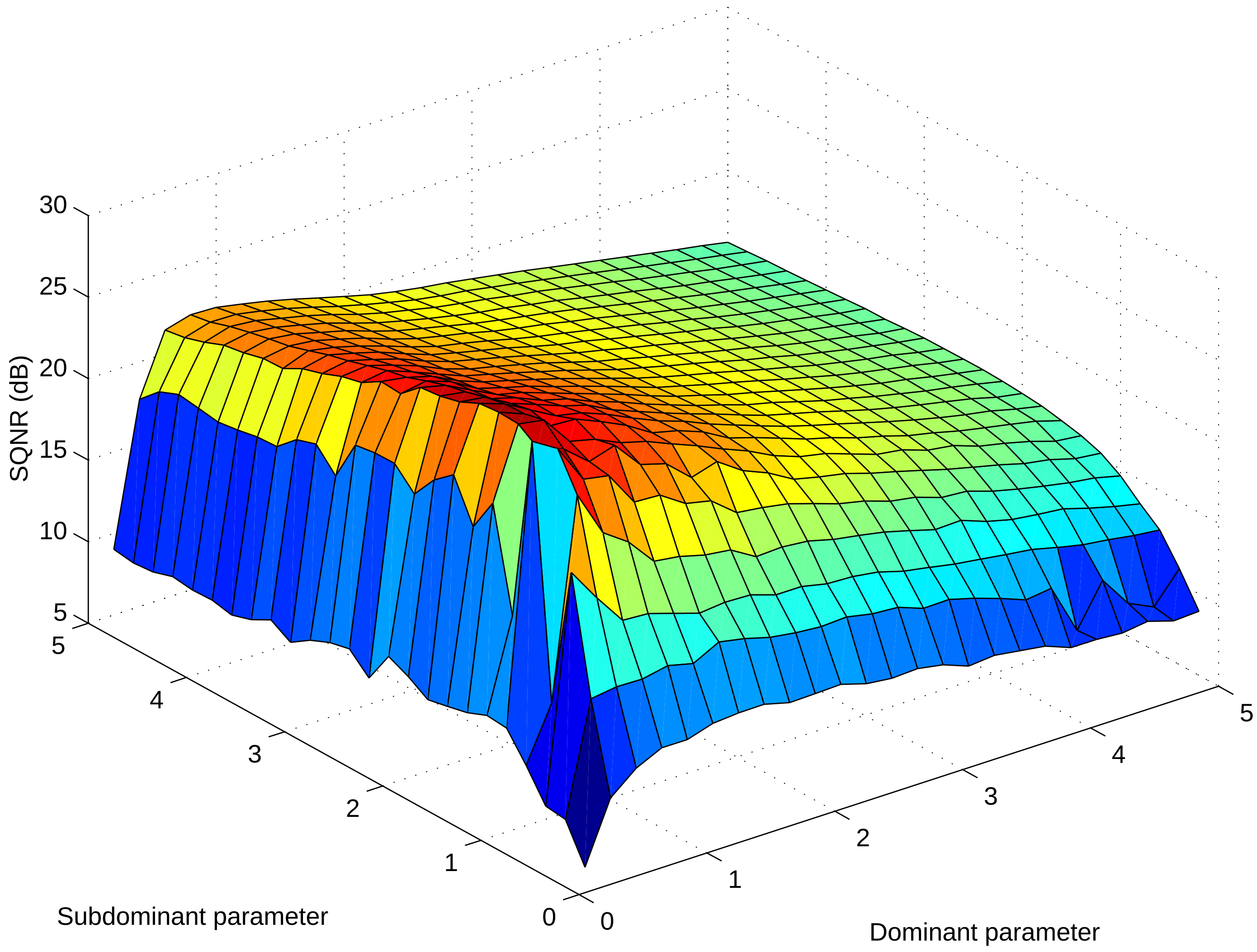


Amplitude Updates

Least Mean Square (LMS) algorithm and per-surviving processing (PSP)



Joint Viterbi Performance Sensitivity w.r.t. Choice of Quantizer Constants



Performance: Simulated signals

Two main cases:

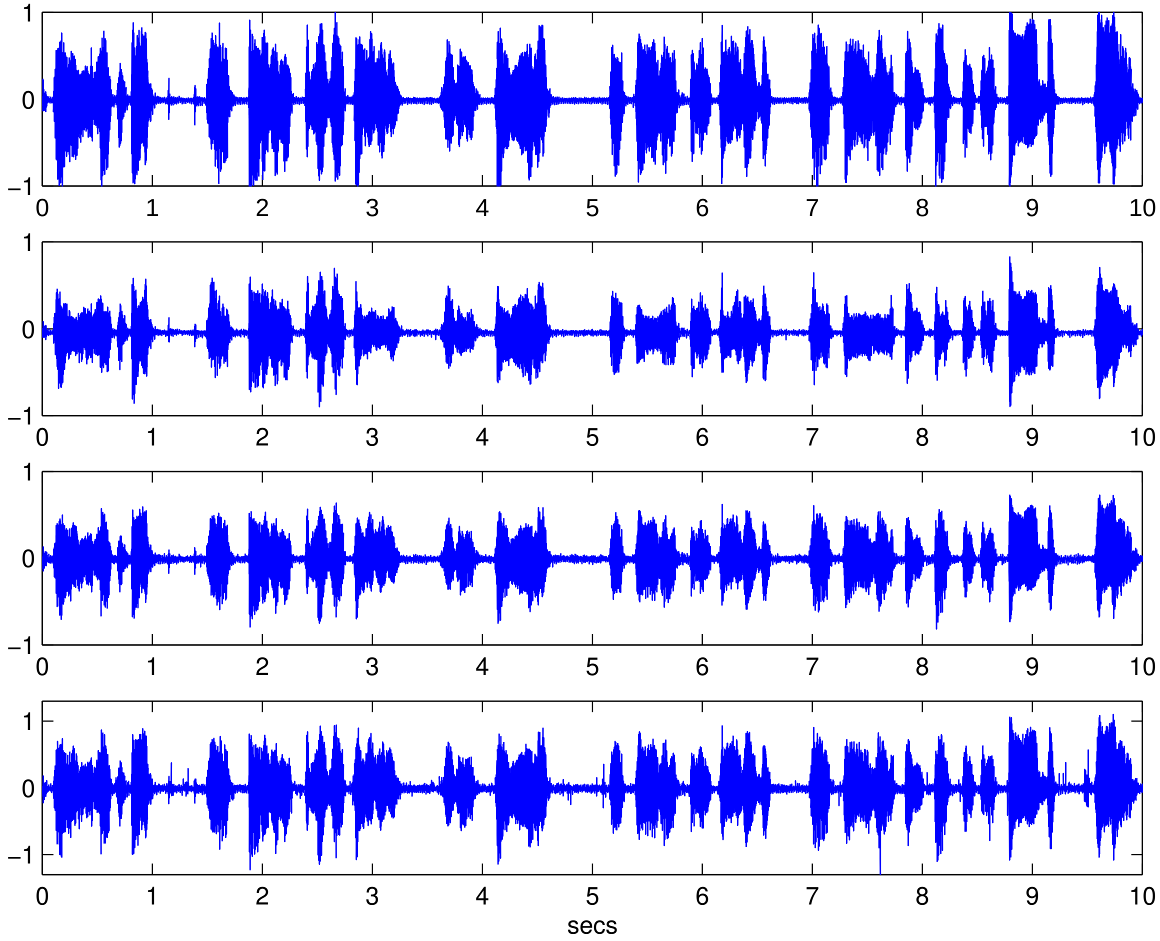
- Male dominant voice, female subdominant voice
- Male dominant voice, male subdominant voice

Five sub-cases:

Sub-case	Frequency Deviation (kHz)	SIR (dB)	SNR (dB)
A	12	6	∞
B	12	1	∞
C	8	6	∞
D	8	1	∞
E	12	6	10

- Sample rate of 132300Hz.
- Carrier frequencies set equal, and set to DC
- Each modulating signal has approximate bandwidth of 3.6kHz.
- Tests performed in non-real-time

Case 2 Joint Viterbi Dominant Estimates



"Here are your messages: you have 30 minutes to move your car; you have 10 minutes; your car has been impounded; your car has been crushed into a cube; you have 30 minutes to move your cube."

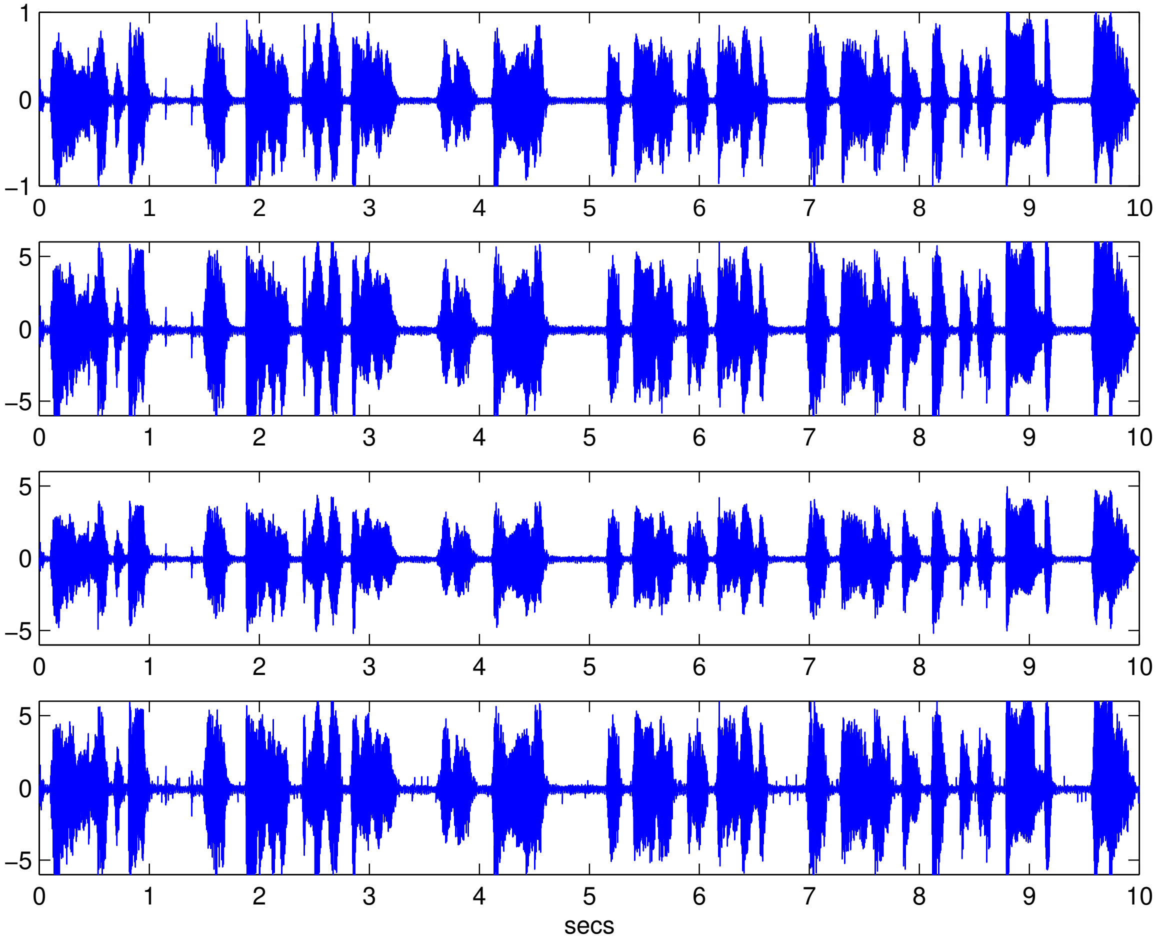
Joint Viterbi Estimate
(dev. = 12kHz, SIR = 6, SNR=∞)

Joint Viterbi Estimate
(dev. = 8kHz, SIR = 6, SNR=∞)

Joint Viterbi Estimate
(dev. = 12kHz, SIR = 6, SNR=10)

Conclusion: Intelligible in all sub-cases.

Case 2 Discriminator Estimates



"Here are your messages: you have 30 minutes to move your car; you have 10 minutes; your car has been impounded; your car has been crushed into a cube; you have 30 minutes to move your cube."

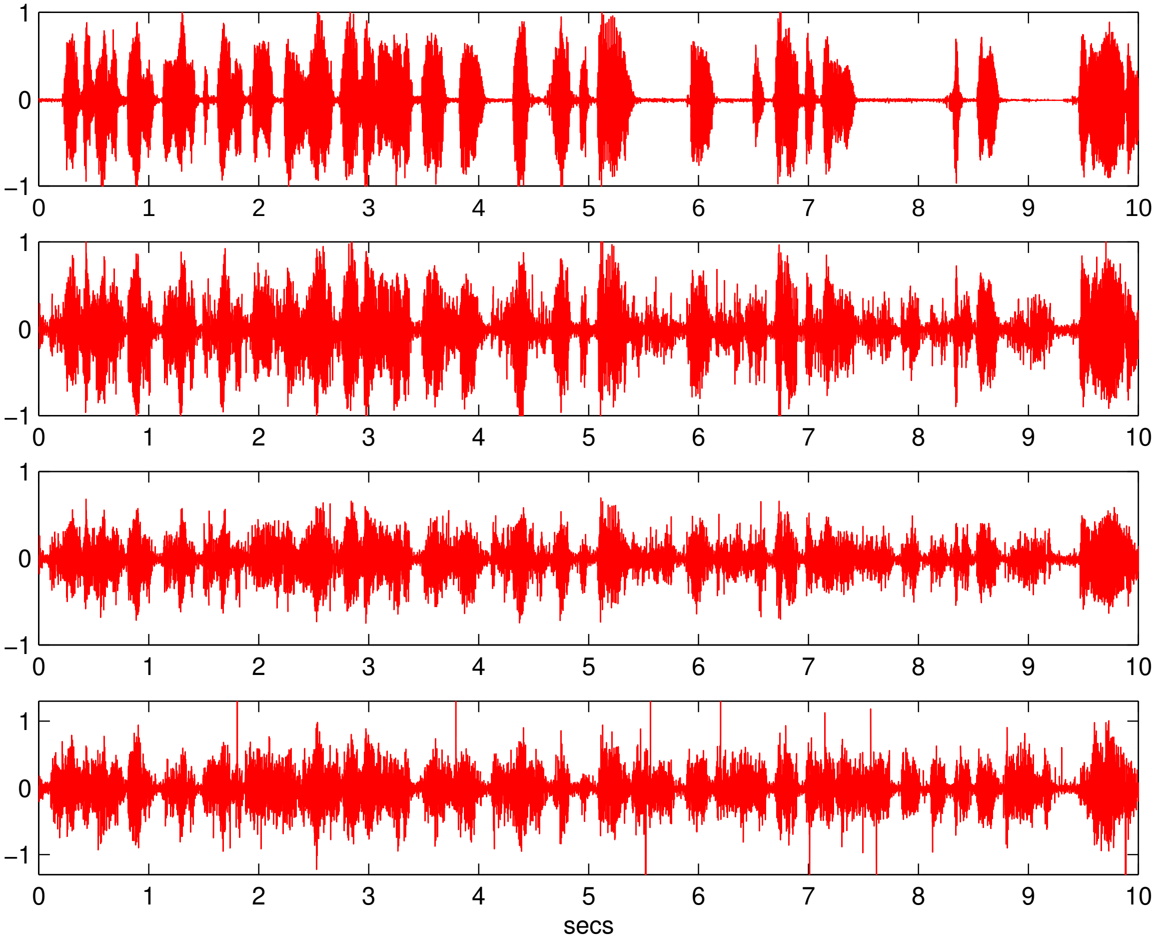
Discriminator Estimate
(dev. = 12kHz, SIR = 6, SNR=∞)

Discriminator Estimate
(dev. = 8kHz, SIR = 6, SNR=∞)

Discriminator Estimate
(dev. = 12kHz, SIR = 6, SNR=10)

Conclusion: Intelligible (dominant only) in all sub-cases.

Case 2 Joint Viterbi Subdominant Estimates



"Is it likely that the U.N. security council will order military force, if necessary, to obtain compliance? – The first priority has to be to resolve this..."

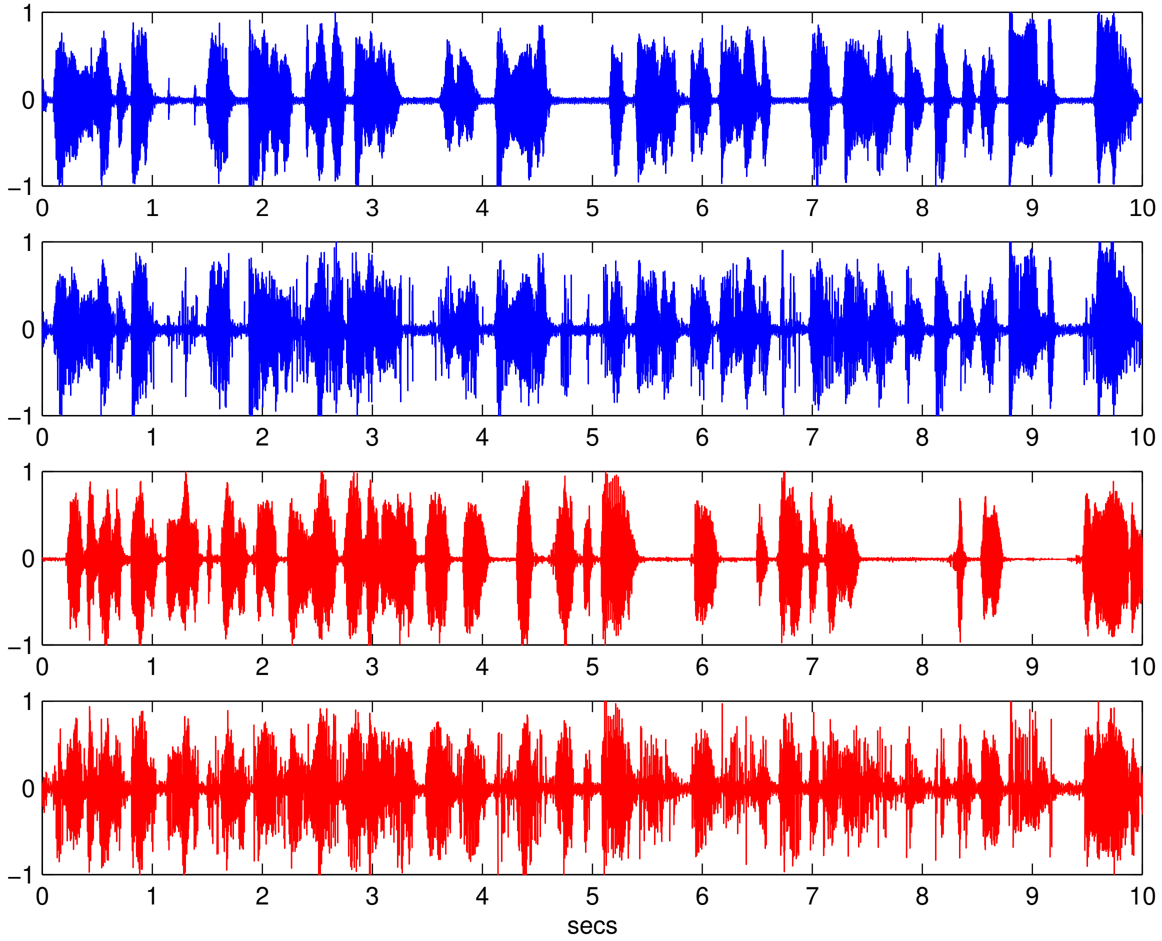
Joint Viterbi Estimate
(dev. = 12kHz, SIR = 6, SNR=∞)

Joint Viterbi Estimate
(dev. = 8kHz, SIR = 6, SNR=∞)

Joint Viterbi Estimate
(dev. = 12kHz, SIR = 6, SNR=10)

Conclusion: Lower SNR makes output almost unintelligible.

Case 2 Joint Viterbi Estimates– Low SIR



"Here are your messages: you have 30 minutes to move your car; you have 10 minutes; your car has been impounded; your car has been crushed into a cube; you have 30 minutes to move your cube."

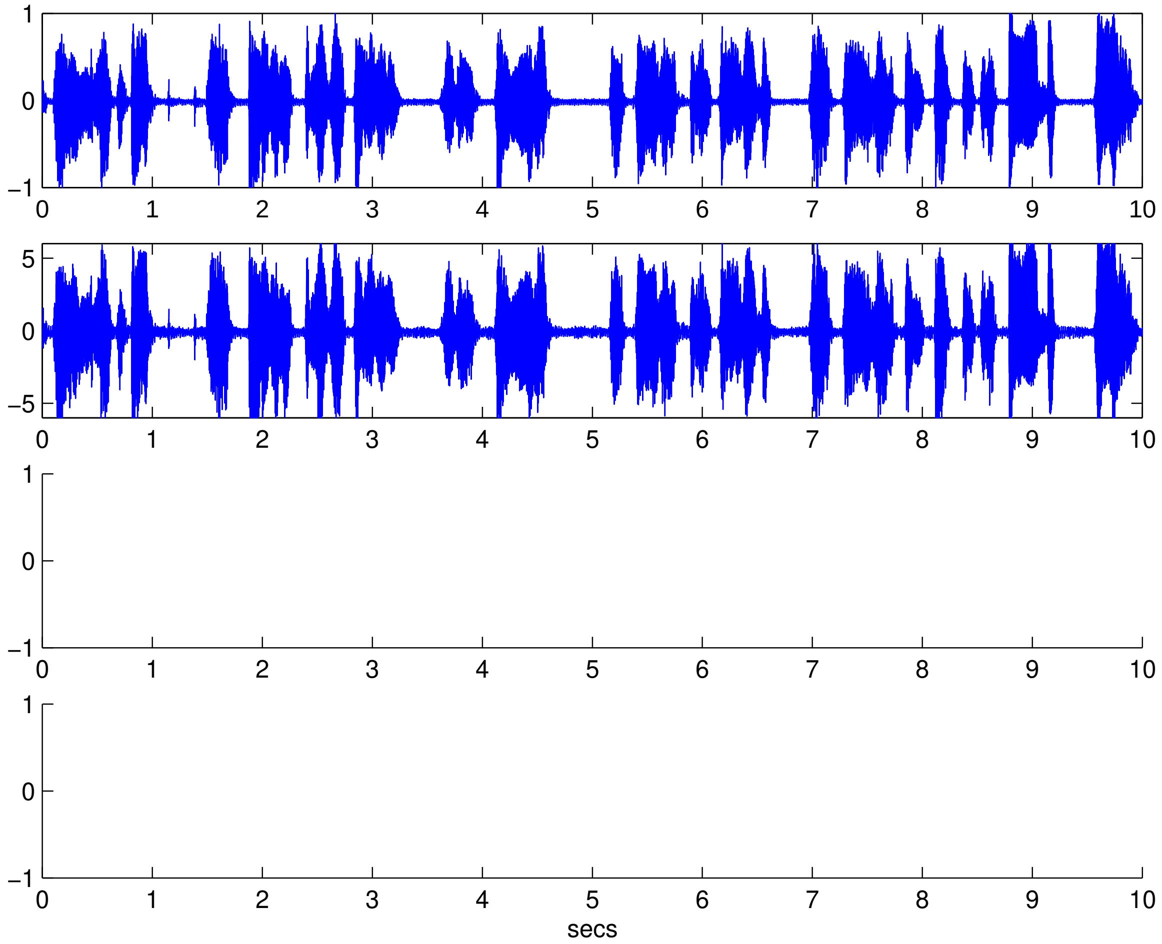
Joint Viterbi Estimate
(dev. = 12kHz, SIR = 1, SNR=∞)

"Is it likely that the U.N. security council will order military force, if necessary, to obtain compliance? – The first priority has to be to resolve this..."

Joint Viterbi Estimate
(dev. = 12kHz, SIR = 1, SNR=10)

Conclusion: Intelligible in all sub-cases.

Case 2 Discriminator Estimates– Low SIR



"Here are your messages: you have 30 minutes to move your car; you have 10 minutes; your car has been impounded; your car has been crushed into a cube; you have 30 minutes to move your cube."

Discriminator Estimate
(dev. = 12kHz, SIR = 1, SNR=∞)

Conclusion: Intelligible (dominant only) in all sub-cases.

Conclusions and Future Work

Conclusions

- Analytical solution
  - Excellent separation
  - Low complexity – can operate in real time
  - Solves for phases directly
  - Requires external amplitude estimates and very high SNR
- Joint Viterbi Demodulation
  - High complexity– 50 times real time
  - Can embed amplitude estimation in trellis
  - Good results

Future Work

- Analytical: amplitude estimation, noise robustness techniques
- Extension to multiple signals.