Hop Timing Estimation for Noncoherent Frequency-Hopped M-FSK Intercept Receivers

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Abstract

The optimum hop timing estimator (based on likelihood-ratio (LR) theory) is derived for noncoherent slow and fast frequency-hopped M-FSK intercept receivers. Such receivers have no a priori knowledge of the hopping code and thus the solution to this estimation problem differs considerably from the more commonly considered case of the friendly receiver. The implementation and performance of the LR hop timing structures are presented and compared with that of other suboptimum schemes that have been discussed in the literature.

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1.0 Introduction

Over the last decade, there has been considerable interest in obtaining optimum structures for coherent and noncoherent frequency-hopped (FH) *M*-FSK intercept receivers, and analyzing their detection performance [1-8]. The structures have been derived based on average-likelihood ratio (ALR) and maximum-likelihood ratio (MLR) tests, and the performance has been obtained from various combinations of analytical and simulation statistical models. In arriving at the results, one of the assumptions that was made in all of the cases reported was that the hop timing, i.e., the location of the time epoch of each hop interval within the observation, was known to the receiver. As such, the optimum structures and their associated performances were ideal in that they represented the best that could be achieved in a theoretical sense and thus serve as a standard against which more practical structures and performances could be compared.

In this paper, we deviate from the above idealized contributions by considering the important practical problem of how one goes about providing an estimate of hop timing to the receiver. In particular, we shall primarily be interested in theoretically optimum hop timing estimation structures since their performance provides a benchmark against which more practical but suboptimum structures can be compared, Two such suboptimum (ad *hoc*) structures will be considered in this paper for the purpose of comparison with the optimum schemes. As in the above-referenced papers, we shall again define optimum in the context of structures derived from ALR and MLR tests. In fact, because of this underlying theme, we shall see that there is a strong similarity between the optimum hop timing structures so derived and the idealized optimum detector structures previously obtained, The measure of performance used in evaluating the behavior of these optimum hop timing stuctures will be the rms error between the true received hop epoch and the receiver's estimate of it. Such a performance measure is typical of analyses of timing synchronization.

Our interest here is in both the cases of slow frequency-hopped (SFH) and fast frequency-hopped (FFH) noncoherent M-FSK. Because of the similarity of the theory used to derive the optimum structures here and in the above-mentioned papers, we shall be somewhat brief in our presentation and make direct reference to the previous work on optimum FH detectors wherever appropriate. We begin by presenting the system model from which we shall derive the structure of the optimum hop timing estimator based on an ALR test.

2.0 The Optimum Hop Timing Estimator Based on an ALR Test

Consider a received signal of the form

$$r(t) = s(t;\alpha) + n(t) \tag{1}$$

which is observed for a T-see interval defined as $0 \le t \le T$ and which covers N_h hops each of duration $T_h = T/N_h$. In (l), $n(t)$ is a zero mean additive white Gaussian noise (AWGN) process with single-sided power spectral density (PSD) N_0 and $s(t;\alpha)$ represents the noncoherent FH signal parameterized by the normalized timing epoch α which is taken to be a continuous random variable uniformly distributed in the interval (O, 1). In the case of SFH, each of the hops contains N_s data symbols of duration $T_s = T_h/N_s$. For the case of FFH, a data symbol contains one or more hops. For FFH, it is sufficient to consider the case where there is only a single data symbol per hop[1] and thus from both the signal detection and hop timing estimation standpoints, we may equivalently consider the signal as being unmodulated.

In the view of the above, for FFH the signal $s(t;\alpha)$ can be represented as

$$s(t;\alpha) = \begin{cases} \sqrt{2S}\cos\left(2\pi f_{j_0} t - \theta_0\right); & 0 \le t \le \alpha T_h \\ \sqrt{2S}\cos\left(2\pi f_{j_1} t - \theta_1\right); & \alpha T_h \le t \le (1+\alpha)T_h \\ \vdots & \\ \sqrt{2S}\cos\left(2\pi f_{j_{N_h}} t - \theta_N\right); & (N_h - 1 + \alpha)T_h \le t \le N_h T_h \end{cases} \tag{2}$$

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[1] It is a widely accepted terminology to distinguish between SFH and FFH according to whether there are multiple data symbols per hop (SFH) or a single data symbol per hop (FFH).

where S denotes the signal power, f_{j_i} is equally likely to be one of G possible hop frequencies chosen from the uniformly spaced set $\{f_1, f_2, \ldots, f_G\}$, and *because* of the assumption of noncoherence, the carrier phases associated with each hop within the observation interval, **i.e., 8.,81,...,** θ_N are statistically independent (S1), uniformly distributed on (O, 2π) random variables. For orthogonality, a minimum spacing of $1 / T_h$ must be provided between adjacent frequencies, i.e.,

$$f_k - f_j = \frac{k-j}{T_h} \tag{3}$$

Similarly, for SFH the signal $s(t;\alpha)$ can be represented as

$$s(t;\alpha) = \begin{cases} \sqrt{2S}\cos\left[2\pi\left(f_{j_0} + d_{l,0}R_s\right)t - \theta_0\right]; \max\left[0,(\alpha-1)T_h + lT_s\right] \le t \le (\alpha-1)T_h + (\text{f}+1)T_s, \\ \qquad l = N_s - N_\alpha, N_s - N_\alpha + 1, \ldots, N_s - 1 \\ \sqrt{2S}\cos\left[2\pi\left(f_{j_1} + d_{l,1}R_s\right)t - \theta_1\right]; \alpha T_h + lT_s \le t \le \alpha T_h + (l+1)T_s, \\ \qquad l = 0,1,2,\ldots,N_s - 1 \\ \vdots \\ \sqrt{2S}\cos\left[2\pi\left(f_{j_{N_h}} + d_{l,N_h}R_s\right)t - \text{ON,]}\ (N_h - 1 + \alpha)T_h + lT_s \le t \le \min\left[(N_h - 1 + \alpha)T_h + (l+1)T_s, T\right], \\ \qquad l = 0,1,2,\ldots,N_s - N_a \end{cases} \tag{4}$$

where $d_{l,i}$ represents the lth bit on the ith hop which for the binary case takes on values of *1,2,...,* M with equal probability, R_s is the data symbol rate, and $N_\alpha \triangleq \lceil \alpha T_h / T_s \rceil = \lceil \alpha N_s \rceil$ where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

In either case, the optimum estimator of a is derived from the likelihood functional of the received signal $r(t)$ conditioned on the parameters $\mathbf{j} \triangleq (j_0, j_{1,\ldots,} j_{N_h})$, $\boldsymbol{\theta} \triangleq (\theta_0, \theta_1, \ldots, \theta_{N_h})$, α, and additionally in the SFH case, the data sequence $\mathbf{d} \triangleq (\mathbf{d}_0, \mathbf{d}_1 \ldots, \mathbf{d}_{N_h})$ where d_i denotes the sequence of data symbols $d_{l,i}$ for the ith hop.

It is well known that the likelihood functional for an observation as in (1) takes the form[2]

$$p\left(r(t)\middle|\mathbf{j},\boldsymbol{\theta},\alpha,\mathbf{d}\right) = C\exp\left\{\frac{2}{N_0}\int_0^T r(t)s(t;\alpha)dt\right\} \tag{5}$$

where C is a constant that does not depend on the conditioning parameters. Considering first the simpler FFH case, the correlation required in the argument of the exponential in (5) can be obtained by substituting (2) into this integral with the result

$$\int_0^T r(t)s(t;\alpha)dt = \sqrt{2S}\sum_{i=0}^{N_h} Z_{i,j_i}\cos\left(\theta_i - \phi_{i,j_i}\right) \tag{6}$$

where

$$Z_{i,j_i} \triangleq \sqrt{X_{i,j_i}^2 + Y_{i,j_i}^2},\ \phi_{i,j_i} \triangleq \tan^{-1}\frac{Y_{i,j_i}}{X_{i,j_i}} \tag{7}$$

with

$$\begin{aligned}
X_{0,j_0} &= \int_0^{\alpha T_h} r(t)\cos 2\pi f_{j_0} t\,dt,\ Y_{0,j_0} = \int_0^{\alpha T_h} r(t)\sin 2\pi f_{j_0} t\,dt \\
X_{i,j_i} &= \int_{(i-1+\alpha)T_h}^{(i+\alpha)T_h} r(t)\cos 2\pi f_{j_i} t\,dt,\quad Y_{i,j_i} = \int_{(i-1+\alpha)T_h}^{(i+\alpha)T_h} r(t)\sin 2\pi f_{j_i} t\,dt; \\
&\qquad\qquad i=1,2,\ldots,N_h-1,\ j_i = 1,2,\ldots,G \\
X_{N_h,j_{N_h}} &= \int_{(N_h-1+\alpha)T_h}^{N_hT_h} r(t)\cos 2\pi f_{j_{N_h}} t\,dt,\ Y_{N_h,j_{N_h}} = \int_{(N_h-1+\alpha)T_h}^{N_hT_h} r(t)\sin 2\pi f_{j_{N_h}} t\,dt
\end{aligned} \tag{8}$$

Since the components of $\boldsymbol{\theta}$ are uniformly independent and identically distributed (i.i.d.) random variables, then so are the members of the set $\{\theta_i - \phi_{i,j_i}\}$ in (6). Thus, averaging the likelihood functional over $\boldsymbol{\theta}$ yields

$$\begin{aligned}
p\left(r(t)\middle|\mathbf{j},\alpha\right) &= C\overline{\exp\left\{\frac{2\sqrt{2S}}{N_0}\sum_{i=0}^{N_h} Z_{i,j_i}\cos\left(\theta_i - \phi_{i,j_i}\right)\right\}}^{\theta} \\
&= C\prod_{i=0}^{N_h}\overline{\exp\left\{\frac{2\sqrt{2S}}{N_0}Z_{i,j_i}\cos\left(\theta_i - \phi_{i,j_i}\right)\right\}}^{\theta_i} = C'\prod_{i=0}^{N_h} I_0\left(\frac{2\sqrt{2S}}{N_0}Z_{i,j_i}\right)
\end{aligned} \tag{9}$$

where C’ is another constant of proportionality. Finally, averaging over the i.i.d. components of *j, we* get

[2] In the FFH case, wc would merely ignore the conditioning dependence on d in the notation.

$$p(r(t)|\alpha) = C'\prod_{i=0}^{N_h}\overline{I_0\left(\frac{2\sqrt{2S}}{N_0}Z_{i,j_i}\right)}^{j_i} = C'\prod_{i=0}^{N_h}\left(\frac{1}{G}\sum_{j_i=1}^{G} I_0\left(\frac{2\sqrt{2S}}{N_0}Z_{i,j_i}\right)\right) \triangleq \Lambda_{ALR}(\alpha) \quad (10)$$

where the subscript ALR denotes the fact that the likelihood functional, A, is based on an ALR test. The optimum estimate of *a*, denoted by $\hat{\alpha}_{ALR}$ is the value of α that maximizes A_{ALR} *(a)*. We denote this as follows:

$$\hat{\alpha}_{ALR} = \max_{\alpha}{}^{-1}\prod_{i=0}^{N_h}\left(\frac{1}{G}\sum_{j_i=1}^{G} I_0\left(\frac{2\sqrt{2S}}{N_0}Z_{i,j_i}\right)\right) \qquad (11)$$

Note that the dependence of the right hand side of (11) on the timing offset α is imbedded in Z_{i,j_i} in accordance with (7) and (8). In principle, a solution to (11) can be obtained by differentiating *Au(a)* of (1 O) with respect to α and equating the result to zero. Unfortunately, this leads to a transcendental equation for which an explicit solution for $\hat{\alpha}_{ALR}$ cannot be determined. Thus, there is no advantage to this approach and we shall resign ourselves to solving (11) by numerical methods. An implementation of (11) is illustrated in Figure 1. Comparing this structure with the optimum FFH/*M*-FSK detector (assuming perfect hop timing) derived from an ALR test in [8] (see Figure 5 of [8] with N_C = G) , we observe that the primary difference is that the comparison of *A* with a threshold required for detection (hypothesis testing) is replaced by a maximization over the unknown parameter *a* for hop timing estimation. The only other difference between the two is that the integration limits for each hop correlation in Figure 1 are synchronous with the assumed value of hop timing epoch whereas in Figure 5 of [8] they are fixed because of the assumption of perfect hop timing.

By similar reasoning and analogy with the results in [8], the optimum estimate of *a* for SFH is given by[3]

$$\hat{\alpha}_{ALR} = \max_{\alpha}{}^{-1}\prod_{i=0}^{N_h}\left(\frac{1}{G}\sum_{j_i=1}^{G}\prod_{l}\frac{1}{M}\sum_{m=1}^{M} I_0\left(\frac{2\sqrt{2S}}{N_0}Z_{i,j_i,l,m}\right)\right) \qquad (12)$$

where $Z_{i,j_i,l,m} \triangleq \sqrt{X^2_{i,j_i,l,m} + Y^2_{i,j_i,l,m}}$ with

[3]We assume that the phase of the hop carrier is discontinuous from data symbol to data symbol. By analogy with the results given in [8], it is also possible to derive the optimum hop timing estimator for the case where the hop carrier is continuous from data symbol to data symbol along a given hop, i.e., continuous phase FSK (CPFSK). For the sake of brevity, we consider only the discontinuous phase case in this paper.

$$X_{0,j_0,l,m} = \int_{t_{\min}}^{(\alpha-1)T_h+(l+1)T_s} r(t)\cos 2\pi\left(f_{j_0} + mR_s\right)t\,dt,\ Y_{0,j_0,l,m} = \int_{t_{\min}}^{(\alpha-1)T_h+(l+1)T_s} r(t)\sin 2\pi\left(f_{j_0} + mR_s\right)t\,dt,$$

$$l = N_s - N_a, N_s - N_a + 1, \ldots, N_s - 1$$

$$X_{i,j_i,l,m} = \int_{(\alpha+i-1)T_h+lT_s}^{(\alpha+i-1)T_h+(l+1)T_s} r(t)\cos 2\pi\left(f_{j_i} + mR_s\right)t\,dt,\ Y_{i,j_i,l,m} = \int_{(\alpha+i-1)T_h+lT_s}^{(\alpha+i-1)T_h+(l+1)T_s} r(t)\sin 2\pi\left(f_{j_i} + mR_s\right)t\,dt,$$

$$l = 0,1,2,\ldots,N_s - 1$$

$$X_{N_h,j_i,l,m} = \int_{(\alpha+N_h-1)T_h+lT_s}^{t_{\max}} r(t)\cos 2\pi\left(f_{j_{N_h}} + mR_s\right)t\,dt,\ Y_{N_h,j_i,l,m} = \int_{(\alpha+N_h-1)T_h+lT_s}^{t_{\max}} r(t)\sin 2\pi\left(f_{j_{N_h}} + mR_s\right)t\,dt,$$

$$l = 0,1,2,\ldots,N_s - N_a \tag{13}$$

and

$$t_{\min} = \max\left(0, (\alpha-1)T_h + lT_s\right), t_{\max} = \min\left((\alpha + N_h - 1)T_h + (l+1)T, T\right) \tag{14}$$

An implementation of (12) is illustrated in Figure 2.

Because of the symmetry of the likelihood functional with α, the estimators in (11) and (12) are unbiased, i.e., $E\{\hat{\alpha}|\alpha\} = a$ for all values of the system parameters. Similarly, the conditional variance $\sigma^2_{\hat{\alpha}|\alpha} = E\left\{\left[\hat{\alpha} - E\{\hat{\alpha}|\alpha\}\right]^2|\alpha\right\}$ is independent of a but does depend on the system parameters.

2.1 <u>The Optimum Hop Timing Estimator Based on an MLR Test</u>

In certain situations it might be desirable to *jointly* determine the hop frequency sequence $f_{j_0}, f_{j_1}, \ldots, f_{j_{N_h}}$ (equivalently, the vector $\mathbf{j} = (j_0, j_1, \ldots, j_{N_h})$) and the normalized timing offset α. In this case, we *maximize* (rather than average) the likelihood functional of (5) over $\mathbf{j}$. Starting with (9) for FFH, we now get

$$p(r(t)|\alpha) = C'\prod_{i=0}^{N_h}\max_{j_i} I_0\left(\frac{2\sqrt{2S}}{N_0}Z_{i,j_i}\right) = C'\prod_{i=0}^{N_h} I_0\left(\frac{2\sqrt{2S}}{N_0}\max_{j_i} Z_{i,j_i}\right) \triangleq \Lambda_{MLR}(\alpha) \tag{15}$$

where the subscript MLR now denotes the fact that the likelihood function, A, is based on an MLR test. As before, the optimum estimate of a, denoted by $\hat{\alpha}_{MLR}$ is the value of a that maximizes $\Lambda_{MLR}(\alpha)$. We denote this as follows:

$$\hat{\alpha}_{MLR} = \max_{\alpha}{}^{-1}\prod_{i=0}^{N_h} I_0\left(\frac{2\sqrt{2S}}{N_0}\max_{j_i} Z_{i,j_i}\right) \tag{16}$$

where the maximization was allowed to be carried inside the argument of the Bessel function because of the monotonicity of $I_0(x)$ with x in the interval $0 < x \leq \infty$.

Correspondingly the ML estimate of j, denoted by $\hat{\mathbf{j}}_{MLR}$ has components determined from

$$\hat{j}_i = \max_{j_i}{}^{-1} Z_{i,j_i} \tag{17}$$

The quantities $\hat{\mathbf{j}}_{MLR}$ and $\hat{\alpha}_{MLR}$ are the optimum *joint* estimates of the hop sequence and the hop timing offset based on an observation of $r(t)$ over the interval $0 \le t \le T$. Of course, if we are only interested in hop timing, then $\hat{\alpha}_{ALR}$ is preferred over $\hat{\alpha}_{MLR}$.

By similar reasoning and analogy with (12), the optimum MLR estimate of α for SFH is given by

$$\hat{\alpha}_{MLR} = \max_{\alpha}{}^{-1} \prod_{i=0}^{N_h} \left(\max_{j_i} \prod_{l} \frac{1}{M} \sum_{m=1}^{M} I_0\left(\frac{2\sqrt{2S}}{N_0} Z_{i,j_i,l,m} \right) \right) \tag{18}$$

For the sake of brevity, we do not draw the implementations of (16) and (18) since they are easily envisioned as modifications of Figs. 1 and 2.

3.0 Suboptimum HoP Timing Estimators

In order to assess the performance benefits of the optimum ALR and MLR hop timing estimates, we shall compare them with two other schemes, both of which are theoretically suboptimum but more practical from an implementation point of view. The first of these was suggested by Chung and Polydoros [9] and is based on autocorrelation techniques analogous to those used for the LPI detection technique described by Polydoros and Woo in [10]. In fact, many of the analysis results used in [9] to describe the performance of the hop timing estimator are taken from [10], Here we briefly summarize the results obtained in [9] with emphasis on expressing them in a form that allows comparison with the optimum hop timing estimators considered in this paper.

3.1 Hop Timing Estimation Using Autocorrelation Techniques

Figure 3 is an illustration of a maximum-likelihood hop timing estimator based on using a single-hop autocorrelation (SHAC) device as a preprocessor. The observed signal plus noise is characterized by (1) where $s(t;\alpha)$ is model led as an unmodulated random FH signal. After bandpass filtering[4] (bandwidth W_{SS}), the signal is correlated with itself over a hop interval producing

[4]For analytical simplicity, a filter with ideal brick-wall frequency response is assumed in [9].

$$y(\tau) \triangleq \int_{\tau}^{T_h} r_{BP}(t) r_{BP}(t - \tau) dt \tag{19}$$

This signal is comprised of SXS, SXN, and NxN terms. Assuming that the hop frequencies for successive signal hops are sufficiently far apart, then the SXS term is composed of two terms consisting of harmonics at the two (because of the hop misalignment) signal hop frequencies present in the hop interval. Each of these harmonics is weighted by a triangular correlation function whose duration is proportional to the fraction of time that each occupies the hop interval, i.e., αT_h and $(1-\alpha)T_h$. For values of input SNR $\gamma = S / N_0 W_{ss}$ much less than unity, the SXN term can be ignored with little loss in accuracy [9,10]. With this in mind, $y(\tau)$ is next power sampled[5] at a rate W_{SS} samples/see producing the set of samples $W_k = y^2(\tau_k)\big|_{LP} = y^2(kW_{ss}^{-1})\big|_{LP}; k = 1,2,\ldots,G_h$ where $G_h = W_{ss}T_h$ is the bandwidth-hop time product. The process of power sampling in the SHAC domain suppresses the dependence on the actual hopping pattern, the candidate hop frequencies, and the carrier phases (herein lies the simplicity of this scheme) while at the same maintaining the hop timing information in the signal.

Next, a weighted sum of a fraction, λ, of the total number of power samples in a hop time, G_h, is formulated as

$$Y = \sum_{k=1}^{\lambda G_h} \frac{W_k}{(T_h - \tau_k)^2} \tag{20}$$

and based on the behavior of the conditional mean $E\{Y|\alpha\}$ when $\lambda G_h >> 1$ (see [9] for the details), a new statistic linearly related to Y is defined as

$$Z \triangleq \frac{1}{4} + \frac{1}{a(\lambda, G_h, S)} - p(\lambda, G_h, S) - q(\lambda, G_h, N_0, W_{BP})) \tag{21}$$

where

$$\begin{aligned} a(\lambda, G_h, S) &\triangleq 2G_h S^2 \frac{\lambda}{1-\lambda} \\ p(\lambda, G_h, S) &\triangleq S^2 G_h \left\{ 2\mathrm{A} + 2\ln(1-\lambda) + \frac{\lambda}{1-\lambda} \right\} \\ q(\lambda, G_h, N_0, W_{ss}) &\triangleq \sum_{k=1}^{\lambda G_h} 2N_0^2 W_{ss}^2 \int_0^1 (1-\rho) \left(\frac{\sin \pi (G_h - k)}{\pi (G_h - k)} \right)^2 d\rho \end{aligned} \tag{22}$$

[5] By "power" sampling is meant the process of producing a set of samples proportional to the power of the signal. This is accomplished by sampling, squaring the resulting samples, and then lowpass filtering to remove harmonics of the hop frequencies,

Based on the properties of a similar statistic in [10], for $\lambda G_h \le 0.1$ and fixed α, the statistic Z is approximately Gaussian [9] with conditional mean and variance approximately given by

$$E\{Z|\alpha\} \triangleq \mu_Z(\alpha) \cong \left(\alpha - \frac{1}{2}\right)^2 = \frac{1}{4} + \alpha(\alpha - 1)$$
$$\mathrm{var}\{Z\} \triangleq \sigma_Z^2 \cong \left(\frac{1-\lambda}{4\lambda}\right)\left(\frac{1}{\gamma^4 G_h^3}\right) \tag{23}$$

Finally, appending the parenthetical argument n to Y and Z in (20) and (21), respective y, to denote these power sum statistics corresponding to the n th observation cell $(n-1)T_h \le t \le nT_h$ (Y_0 and Z_0 would then by Y and Z as defined above in view of the limits of integration in (19)), then an N_h-hop ML estimator of a is given by

$$\hat{\alpha}_{AC} = \begin{cases} \frac{1}{2}, & \overline{Z} \le 0 \\ \frac{1}{2} - \sqrt{\overline{Z}}, & 0 < \overline{Z} < \frac{1}{4} \\ 0, & \overline{Z} \ge \frac{1}{4} \end{cases} ; \quad \overline{Z} \triangleq \frac{1}{N_h}\sum_{n=1}^{N_h} Z(n) \tag{24}$$

It should be emphasized that the ML hop timing estimator in (24) is only maximum-likelihood *conditioned on the assumption of an SHAC preprocessor* (which removes some relevant timing information from the input observable) and is thus suboptimum relative to the (unconditional) ML hop timing estimator derived in Section 2.1. Also, since $\mu_Z(a)$ of (23) is symmetric around the value $a = 0.5$, the estimator in (24) possesses an $(\alpha, 1-a)$ ambiguity in that values of a and 1- a cannot be distinguished by this estimator. Various approaches for resolving this ambiguity are mentioned in [9]. For our purposes here, it is sufficient to note that in view of this ambiguity, the statistical behavior of $\hat{\alpha}_{AC}$ as a function of a is limited to the interval 0< *as 0,5,*

The conditional moments of $\hat{\alpha}_{AC}$ are given by [9]

$$E\{\hat{\alpha}_{AC}|\alpha\} = \frac{1}{2}Q\left(\frac{\mu_{\overline{Z}}}{\sigma_{\overline{Z}}}\right) + V_1(\mu_{\overline{Z}}, \sigma_{\overline{Z}})$$
$$\sigma^2_{\hat{\alpha}_{AC}|\alpha} = \frac{1}{4}Q\left(\frac{\mu_{\overline{Z}}}{\sigma_{\overline{Z}}}\right) + V_2(\mu_{\overline{Z}}, \sigma_{\overline{Z}}) - \left(E\{\hat{\alpha}_{AC}|\alpha\}\right)^2 \tag{25}$$

where

$$\mu_{\bar{Z}} = \mu_Z = \left(\alpha - \frac{1}{2}\right)^2; \quad \sigma_{\bar{Z}} = \frac{1}{\sqrt{N_h}}\sigma_Z = \sqrt{\frac{1}{N_h}\left(\frac{1-\lambda}{4\lambda}\right)\left(\frac{1}{\gamma^4 G_h^3}\right)} \tag{26}$$

and

$$V_i(t,s) \triangleq \frac{1}{\sqrt{2\pi s^2}} \int_0^{\frac{1}{2}} x^i (1-2x) \exp\left\{-\frac{1}{2s^2}\left[(1 \ -2.)^2 - t^2\right]\right\} dx; \ i = 1, 2$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{y^2}{2}\right) dy \tag{27}$$

Note from (25) that $\hat{\alpha}_{AC}$ is a biased estimator of *a*. Thus, the performance of this estimator as measured by its conditional rms value [the square root of $\sigma^2_{\hat{\alpha}_{AC}|\alpha}$ in *(25)]* depends on *a*.

3.2 HoP Time Estimation Using a "Ping-Pong" Approach

Another suboptimum scheme for performing hop timing estimation is based on a "ping-pong" approach which is discussed in [11] in connection with a technique primarily developed in the context of hop *rate* estimation. In particular, with reference to Figure 4, the received FH signal plus noise is passed through two adjoint BPF's (assumed to have ideal rectangular frequency responses) which split the W_{SS} Hz input spread spectrum passband into two contiguous segments each of bandwidth W_{SS} / 2. As such, in each hop interval, the output of one of these two BPF's will be the sum of signal plus noise and the other will contain only noise. The particular BPF which contains the signal plus noise bounces (at the hop rate) between BPF_1 and BPF2 in accordance with whether the corresponding hop is in the upper or lower half of the input spread spectrum band, hence the colloquial term "ping-pong". The outputs of the two BPF's are squared and difference producing a signal whose SXS term ideally has a rectangular envelope with potential transitions that occur at multiples of the hop time T_h and are synchronous with the hopping carrier transition instants. Passing this signal through a lowpass filter extracts this envelope. Next, a delay (by half a hop time) -and-multiply type operation produces a dc biased square wave at the hop rate whose first harmonic has zero crossings that are synchronous with the hop transition instants. Extracting this harmonic with a narrowband bandpass filter and then processing this signal by a MAP phase estimator (based on a Gaussian assumption at the narrowband BPF output) produces the desired hop timing epoch estimate, Figure 4 is a discrete version of the MAP estimator. It is straightforward to show (and has been verified by computer simulation) that the "ping-pong " hop timing estimator is unbiased. Finally, it should be noted that, in principle, the "ping-pong " hop timing scheme is analogous to a closed loop *cross-spectrum bit synchronizer* [12] which tracks the zero crossings of a binary (± 1) equiprobable rectangular data waveform in additive white Gaussian noise,

4.0 Numerical Results

Computer simulations have been written for evaluating the performance of the optimum (ALR and MLR) and suboptimum ("ping-pong") schemes.[6] To allow a fair comparison, we have assumed the case of no modulation in all evaluations. For numerical expediency, the region for $\hat{\alpha}$ has been quantized into 20 steps (5% of the hop interval), Such quantization introduces an error floor into the calculation of the conditional rms error $\sigma_{\hat{\alpha}|\alpha}$, that is, in the limit of infinite input SNR $\sigma_{\hat{\alpha}|\alpha}$ approaches the value $\sigma_{\infty} = \sqrt{(.05)^{-1}\int_{-.025}^{.025}\hat{\alpha}^2 d\hat{\alpha}} = 1.02 \times 10^{-2}$. In order for this level of quantization to have negligible effect on the results corresponding to the true situation where $\hat{\alpha}$ can take on a continuum of values, i.e., $0< \hat{\alpha} \le 1$, we must require that in the input SNR region of interest, the numerical values of $\sigma_{\hat{\alpha}|\alpha}$ found from the simulation are well above this quantization error floor. Shortly, we shall demonstrate that this is indeed the case. In the limit of zero input SNR, the conditional pdf of $\hat{\alpha}$ given a approaches a uniform distribution in the interval $-0.5< \hat{\alpha} \le 0.5$ and hence $\sigma_{\hat{\alpha}|\alpha}$ approaches the value $\sigma_0 = \sqrt{\int_{-.5}^{.5}\hat{\alpha}^2 d\hat{\alpha}} =.289$.

Figure 5 illustrates results obtained from a computer simulation of the ALR FFH timing estimation "scheme. The set of system parameters chosen for these simulation are similar to those used in arriving at the comparable signal detection results in [7], namely, G = 100, $N_h = 20$, $1/T_h = 100$ hops/see, and $W_{ss} = 10^4$ Hz. Also shown are analogous results for the suboptimum MHAC scheme discussed in Section 3.1, A value of A = 0.1 was chosen for these curves which, as discussed in [9], assures that the SHAC power sum behaves as a Gaussian random variable. As was shown in [9] and can be determined from (25), the MHAC estimator is biased; hence we must decide on a value of α to use for computing the rms timing error. In this regard, we considered three different cases. The curve labelled "best α" corresponds to the value of α that yields the minimum conditional rms error $\sigma_{\hat{\alpha}|\alpha}$ at each SNR. From the results in [9] it can be deterermined that, independent of SNR, the value of a that yields the minimum $\sigma_{\hat{\alpha}|\alpha}$ is a = O which is intuitively satisfying. The curve labelled "worst a"

[6] Recall that the suboptimum autocorrelation approach lends itself to analytical results and thus a computer simulation was not necessary here.

corresponds to the value of α that yields the maximum conditional rms error $\sigma_{\hat{\alpha}|\alpha}$ at each SNR. Finally, the curve labelled "average a" assumes a uniform distribution for a and averages the conditional rms error $\sigma_{\hat{\alpha}|\alpha}$ over this distribution. Clearly, its performance lies between the curves corresponding to the best and worst values of a, We note that the limiting value of all three of these MHAC curves as input SNR approaches zero is identical but different in value than that achieved by the ALR scheme. The reason for this goes back to the observation made in Section 3.1 concerning the $(\alpha, 1-a)$ ambiguity associated with the MHAC estimator. It can be shown that in addition to a being restricted to lie in the interval $0 \le a \le 0.5$, the limiting form of the conditional pdf of $\hat{\alpha}$ given a approaches the two-point discrete distribution $p(\hat{\alpha}|\alpha) = 0.5$, $a = 0, 0.5$. Thus, in the limit as input SNR approaches zero, $\sigma_{\hat{\alpha}|\alpha}$ approaches 0.25.

Comparing the MHAC curve labelled "average a" with the ALR results for detection-independent performance, we see that there is about a 15 dB difference between the two! At first it might be conjectured that this large difference in performance stems from the fact that the optimum ALR scheme requires complete knowledge of the set of hopping frequencies and indeed exploits this knowledge in its channelized structure, whereas the MHAC scheme neglects this information. To demonstrate the degree to which this conjecture is valid, we evaluated the performance of the optimum ALR scheme for the case when the actual received hopping frequencies are displaced from those assumed by the receiver implementation by one-half the hop rate. Since the frequency spacing between the G hop channels in Figure 1 is equal to the hop rate, $1/T_h$, then a frequency offset of $1/2T_h$ represents a worst case situation with regard to the receiver's knowledge of the true hop frequency set. The numerical results are illustrated in Figure 5 by the curve labelled FFH / ALR - $(\Delta f = 1/2T_h)$. Comparing this curve with that corresponding to exact knowledge of the hop frequency set (the curve simply labelled FFH / ALR), we see that the worst case lack of hop frequency knowledge only results in a degradation of about 1.8 dB. If the optimum ALR receiver had no knowledge whatseover of the set of hop frequencies, then the *relative* offset between the receiver's G hop frequency channels and the actual hop frequencies in the received waveform would always lie between zero (no offset) and one-half the hop rate. As such, the average performance in the complete absence of hop frequency information would lie between that corresponding to the ideal case of exact knowledge of the hop frequency set (as has already been discussed) and that corresponding to the worst case described above.

Figure 6 illustrates analogous computer simulation results to those in Figure 5 for the ALR SFH timing estimation scheme. The set of system parameters chosen for these simulations is identical to that in Figure 5 with the addition of $N_s = 100$ corresponding to 100 symbols per hop. When the SFH results of Figure 6 are compared with those for FFH in Figure 5, we see that the presence of the data modulation causes about a 2 dB performance penalty asymptotically as SNR becomes large.

Figures 7 and 8 illustrate the computer simulation results for the MLR hop timing estimator of Section 2.1 corresponding respectively to FFH and SFH modulations. Shown for comparison are the results for the corresponding ALR structures. We note that MLR schemes show only a small performance degradation relative to the ALR schemes as was the case for the comparable signal detection configurations in [6] and [8].

Finally, the performance of the "ping-pong" scheme of Figure 4 is superimposed on the curves of Figure 5. As is done for cross-spectrum bit synchronizers [12], the bandwidth of the lowpass filter preceding the delay-and-multiply operation should be optimized to provide the best performance. The optimum bandwidth for this filter in the "ping-pong" hop timing scheme is equal to 1.4 times the hop rate. This filter bandwidth results in the best compromise between signal x signal envelope distortion and reduction of the power in the signal x noise and noise x noise components that arise as a result of the square-law operations following the bandpass filters. We observe that the performance of the "ping-pong" scheme tracks that of the autocorrelation scheme (based on the average value of α) with about a 2.5 dB additional degradation in input SNR.

Conclusions

The optimum hop timing estimators (based on likelihood-ratio (LR) theory) for noncoherent slow and fast frequency-hopped M-FSK have been analytically derived and their performance in terms of rms timing jitter have been evaluated via computer simulation. When compared to previously documented suboptimum schemes such as the multiple-hop autocorrelation approach and the "ping-pong" approach, the average-likelihood ratio (ALR) and maximum-likelihood ratio (MLR) optimum estimators offer an improvement in performance on the order of 15 dB or better in input SNR. As a result of this large disparity in performance, it is reasonable to justify further investigation into finding suboptimum schemes with performances closer to the optimum ones and which offer the advantage of implementational practicality.

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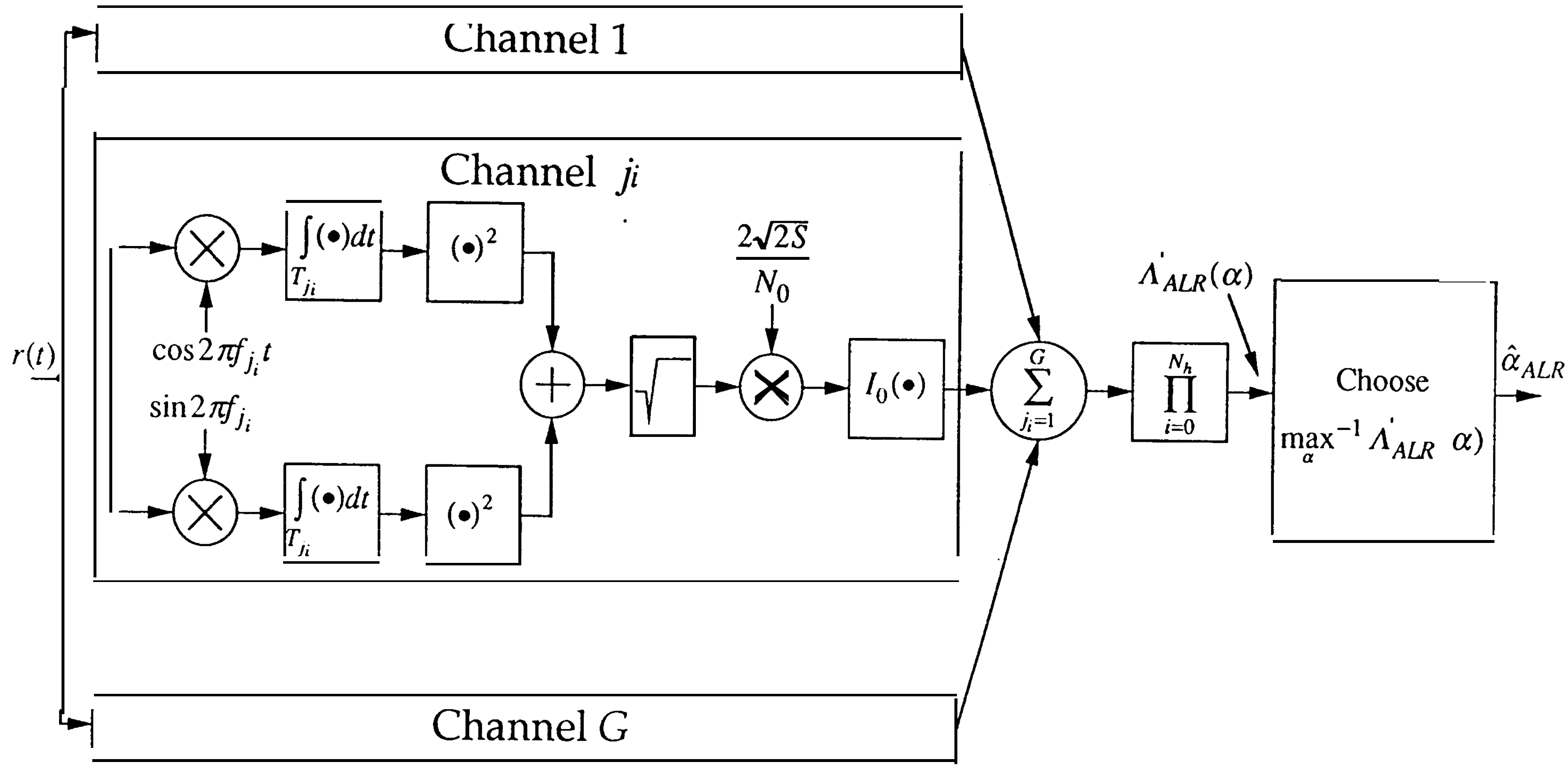
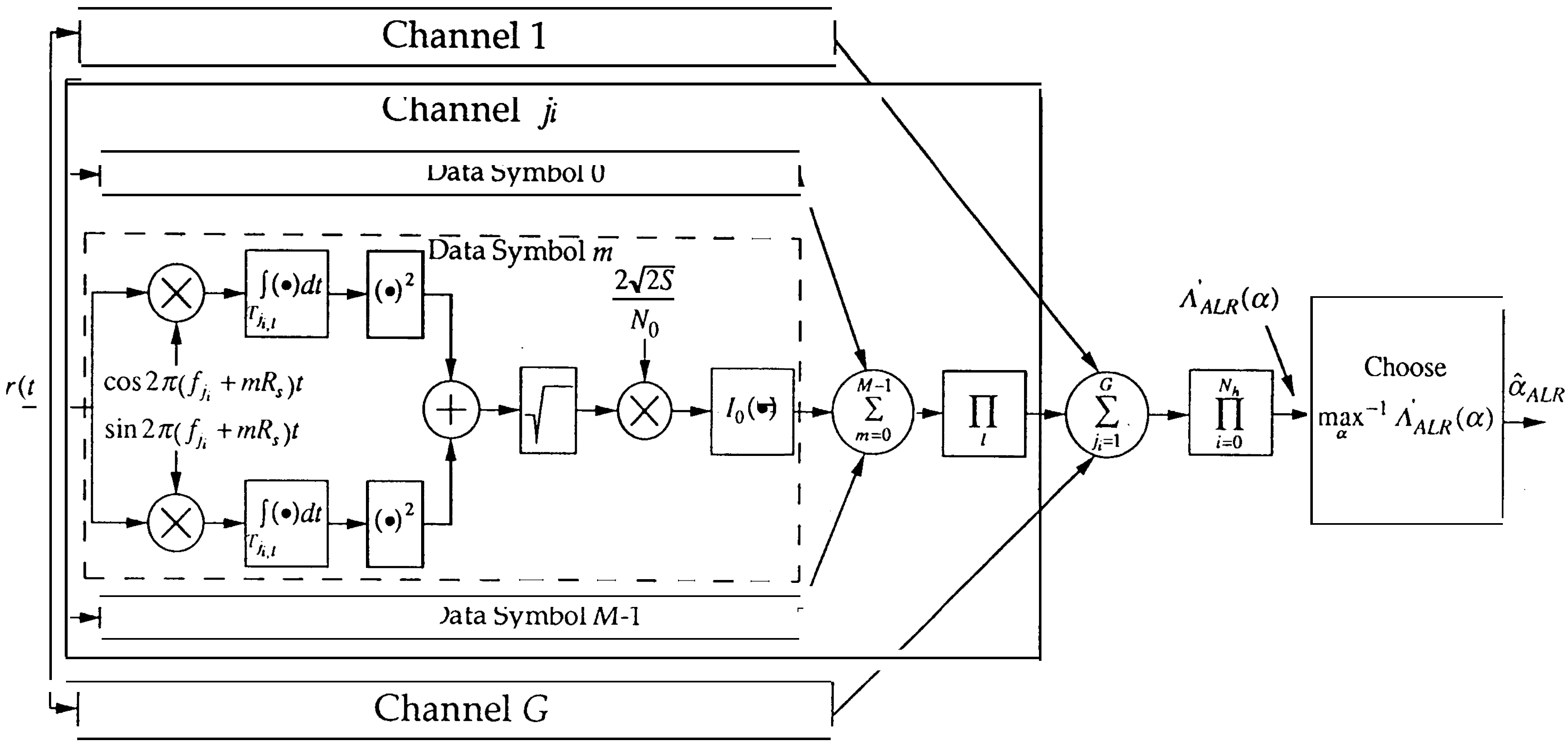


$$T_{j_i} \Rightarrow \begin{cases} 0 \le t \le \alpha T_h, & i = 0 \\ \alpha T_h \le t \le (1+\alpha)T_h, & i = 1,2,\ldots,N_h - \\ (N_h - 1 + \alpha)T_h \le t \le N_h T_h, & i = N_h \end{cases}$$

Figure 1. The Optimum ALR Hop Timing Estimator or FFH/*M*-FSK



$$T_{j_i,l} \Rightarrow \begin{cases} t_{\min} \le t \le (\alpha-1)T_h + (l+1)T_s, & i=0, l = N_s - N_a, N_s - N_a + 1, \ldots, N_s - 1 \\ (\alpha+i-1)T_h + lT_s \le t \le (\alpha+i-1)T_h + (l+1)T_s, & i = 1,2,\ldots,N_h - 1, l = 0,1,\ldots, N_s - 1 \\ (\alpha+N_h-1)T_h + lT_s \le t \le t_{\max}, & i = N_h, l = 0,1,2,\ldots,N_s - N_a \end{cases}$$

$$t_{\min} = \max(0, (\alpha-1)T_h + lT_s), \quad t_{\max} = \min((\alpha+N_h-1)T_h + (l+1)T, T)$$

$$N_\alpha \stackrel{\Delta}{=} \lceil \alpha T_h / T \rceil = \lceil \alpha N_s \rceil$$

Figure 2. The Optimum ALR Hop Timing Estimator for SFH/M-FSK

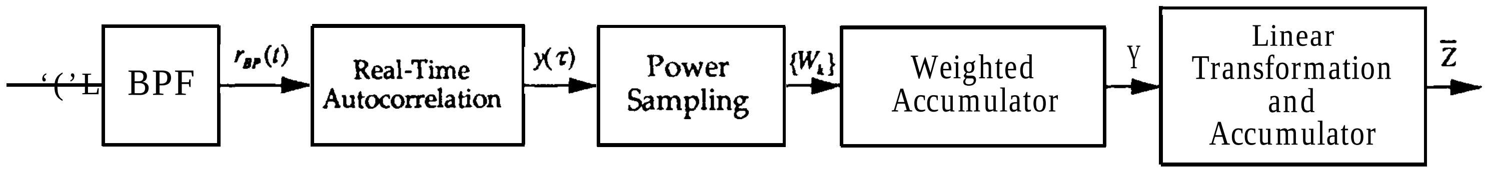


Figure 3. Autocorrelation Processing for Hop Time Estimation

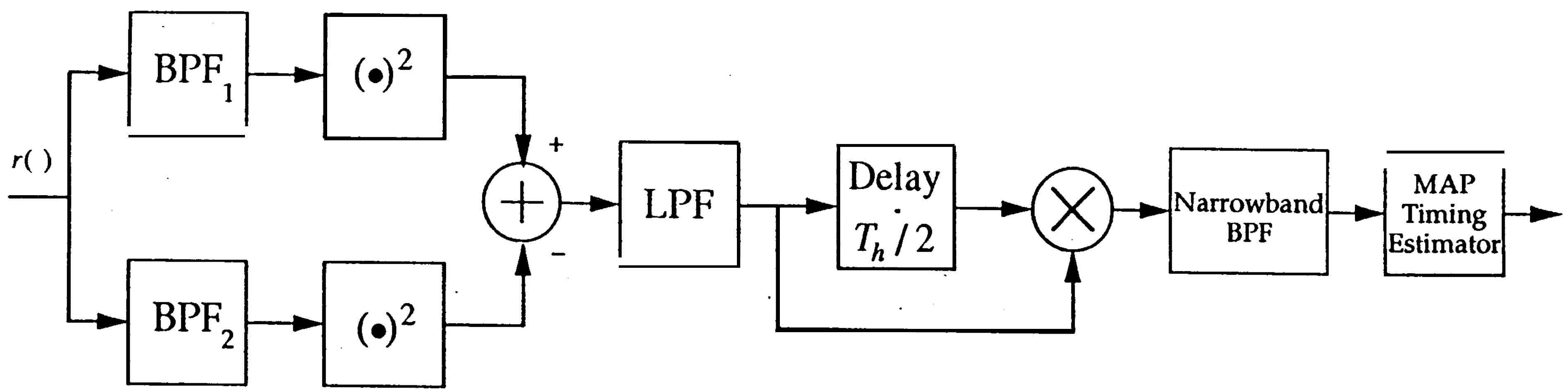
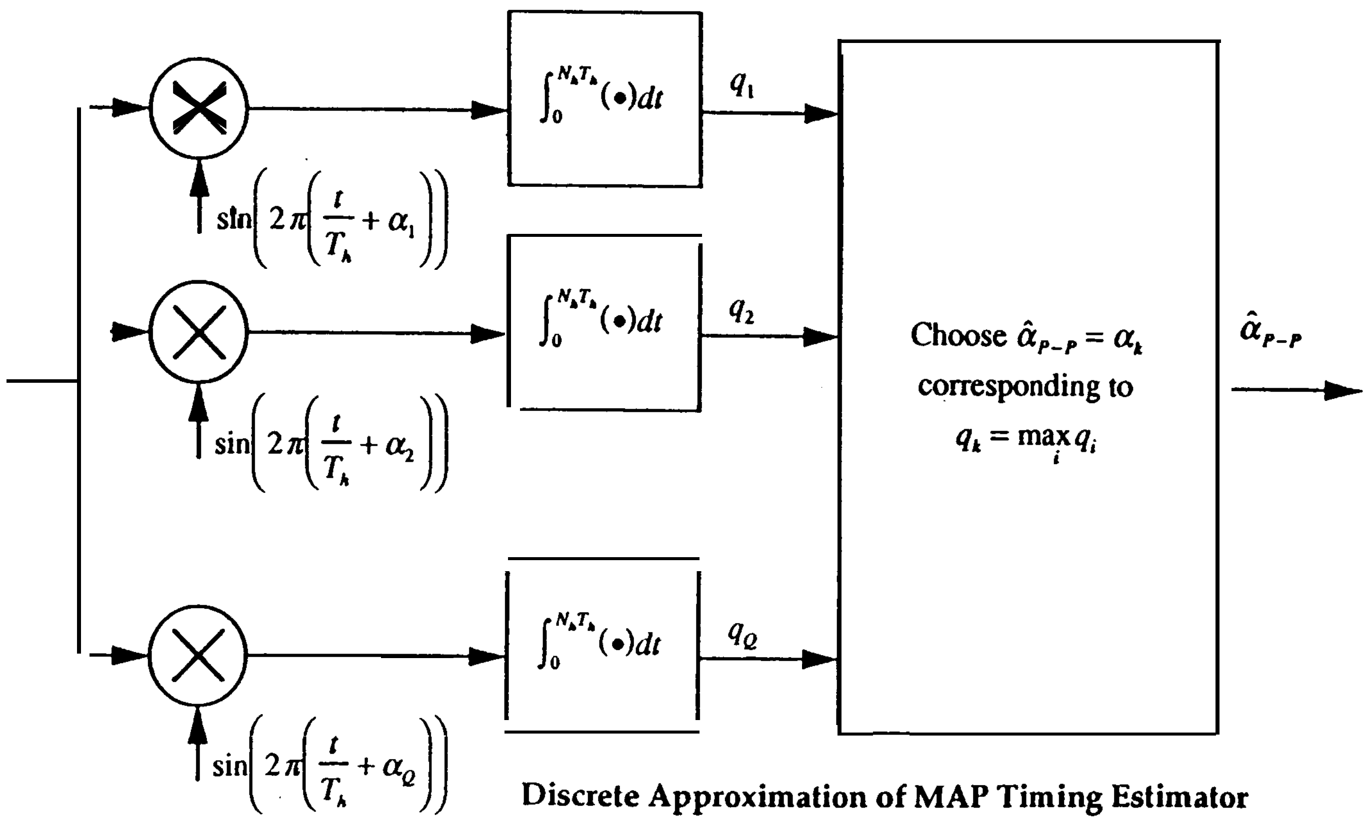


Figure 4. Ping-Pong Hop Timing Estimator (Variation #1)



Discrete Approximation of MAP Timing Estimator

Figure 5. A Comparison of the RMS Timing Error Performances of the ALR Estimator with the MHAC and Ping-Pong Estimators – FFH

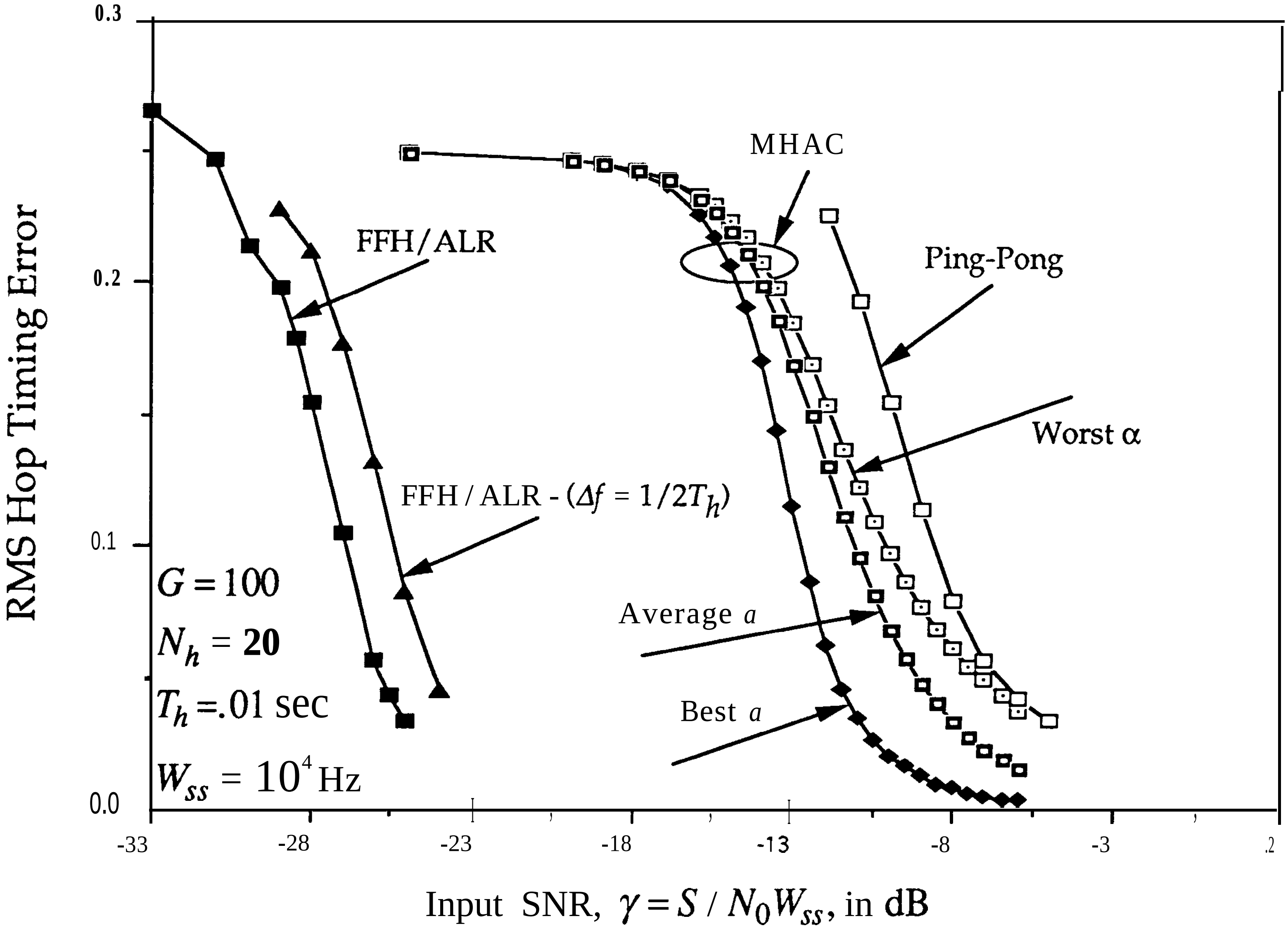


Figure 6. RMS Timing Error Performances of the ALR Estimator for SFH

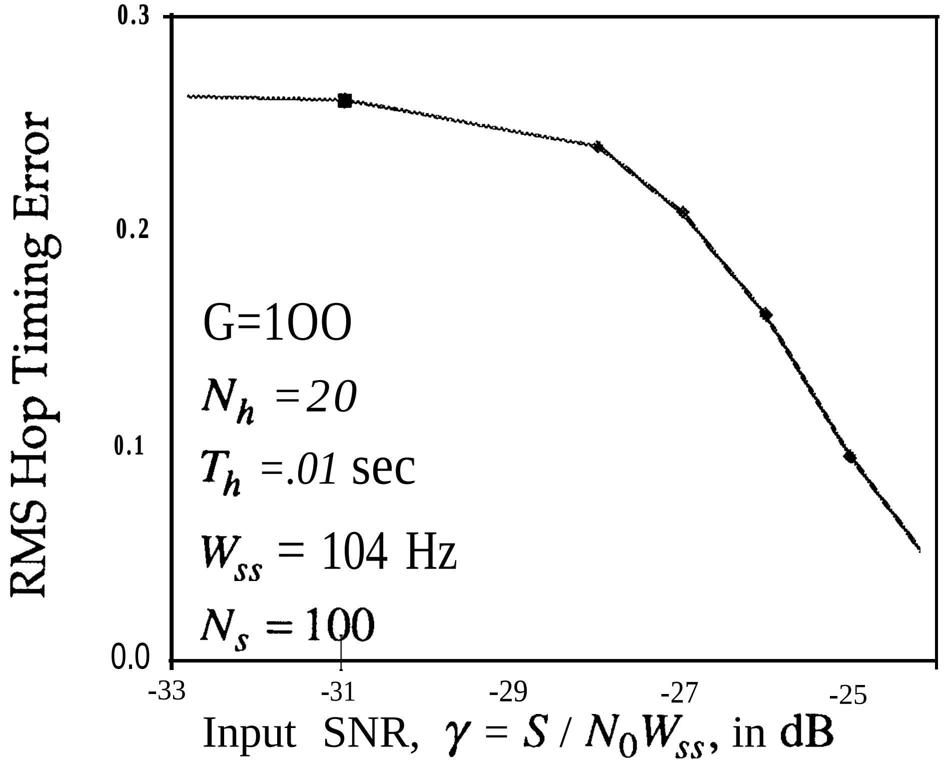


Figure 7. A Comparison of the RMS Timing Error Performances of the ALR and MLR Estimators for FFH

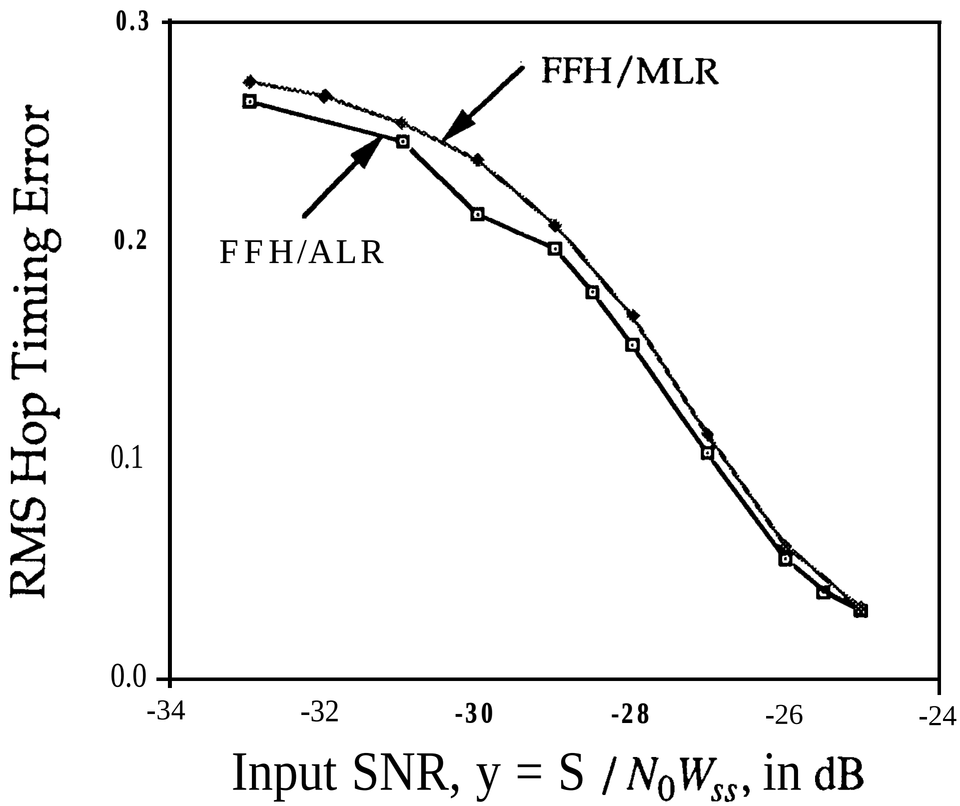


Figure 8. A Comparison of the RMS Timing Error Performances of the ALR and MLR Estimators for SFH