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Synchro-Betatron Resonances in the 8 GeV Proton Driver

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I. Background and motivation

Two reports on the design of future proton drivers have been issued at Fermilab:

[1] The Proton Driver Design Study, FERMILAB-TM-2136, December 2000.

[2] Proton Driver Study II, Part 1, FERMILAB-TM-2169, May 2002.**

The major difference of these two versions is the size (circumference) and the maximum energy. In the first study, the circumference is chosen to be 711.3m, which is 1.5 times the present Booster, with the maximum energy of 16 GeV. In the second version, it is mandated to be the same as Booster together with the same maximum energy of 8 GeV. One of the major impacts of the reduced size of the ring is the inevitable reduction in the total length of available space for injection/collimation/extraction systems and for rf cavities, 14 slots of 7.43m each in the smaller ring compared with 24 slots of 6.15m each in the larger ring. Since each cavity occupies a slot of 2.35m and 22 cavities are desirable, seven or eight slots out of 14 in the smaller ring must be reserved for rf, only six or seven remaining for all other systems. The constraint in space is particularly troublesome for the extraction system since the beam loss at extraction (at the highest beam energy) is the major concern of any high intensity proton machines. This concern is clearly stated in [2], **7.2 Extraction**, p. 7-11.

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** Part 2 of this report, which is the design study of an 8 GeV superconducting linac as the proton driver, has not been made available so far.

It is possible to make more slots available for the extraction system (as well as for other systems) *if one abandons the requirement of placing rf cavities at dispersion-free locations.* In two arcs, there are altogether ten slots of 3.37m each, which should be adequate for one cavity plus two correction magnets, most likely a chromaticity correcting sextupole and a steering dipole. Cavities are not there because of the non-zero dispersion, D = 2.1m and D' = -0.22. In Chapter 3 of [2], p. 3-2, one sees a statement

> "Although it has not been observed in proton machines, it is prudent to avoid any possibility of synchro-betatron coupling resonances, especially in view of a relatively large value of synchrotron frequency, ($\nu_s \approx 0.06$), at low energies."

An almost identical statement is in [1], p. 3-2 but the impact of this "prudence" is quite different. While one can afford to be prudent in the larger ring with many available free slots with zero dispersion, it is not the case in the smaller ring in which this luxury of prudence is possible only with the sacrifice in the effectiveness of extraction system. By installing up to ten cavities in the arcs, one can substantially relieve the tightness of space allocation in the dispersion-free long straight sections, thereby increasing the overall robustness of the machine.

Anton Piwinski has been associated with the question of synchro-betatron resonances more than anyone else [3]. In particular, excitation of resonaces by non-zero dispersions in rf cavities is treated in his DESY report with Albin Wrulich [4].

When vertical dispersions are zero at rf cavities, the resonance condition can be expressed in the form

$$\nu_x \pm m\nu_s = n \quad \text{(m, n = positive integers)}, \tag{1}$$

the dominant one being m = 1, the linear coupling resonance. Since ν_s is less than 0.1 at all energies in the 8GeV ring, one expects the dangerous range of ν_x to be between integer $\pm$ 0.1 or possibly $\pm$0.2. According to Piwinski [3], a true instability (i.e., the amplitudes growing exponentially) occurs for the condition (1) with plus sign below transition, which is the case here, while with minus sign it is simply an exchange of energy between synchrotron and betatron motions. This behavior is similar to the more familiar horizontal-vertical linear coupling. Above transition, which is the case for all electron machines, the situation is reversed. The base lattice of the 8GeV ring has ν_x = 11.747 but this could go up to 11.880 ("a possible tuned lattice") so that it is on the borderline of resonance condition, (1). There are, however, several factors one must take into account for a better understanding of what would happen in rapid-cycling proton machines.

1. Many machine parameters such as rf voltage and synchronous phase angle are changing during the machine cycle and this results in non-stationary synchrotron oscillation number ν_s.

2. Since the longitudinal energy is much larger than the transverse energy, there will be no practical difference between sum and difference resonances; both are potentially dangerous as the source of an intolerable increase in the horizontal beam emittance.

3. Any increase in the beam emittance will be compensated by the usual damping as the beam is accelerated. This is important since the resonance is likely to be at low energies near injection.

4. With multiple cavities in the ring, betatron phase advance between cavities is expected to play a role. This has been pointed out by Piwinski [4] and also by Grahame Rees [5]. If cavities are placed in locations with the same dispersion parameters D_x and D_x', the relevant quantity should be, in the lowest order,

$$|\Sigma \exp(i\psi_x)| \quad \text{(summation for all cavities)}, \tag{2}$$

where ψ_x is the horizontal betatron phase at each cavity location. Although it is possible to make an analytical estimate of the synchro-betatron resonance effect for an idealized case [3], Piwinski himself has suggested that one should try simulations for a particular lattice in question [6] to obtain a reliable estimate of the possible emittance growth. The work presented here is the result of such a simulation.

II. Model

The model used for the simulation is a faithful replica of the synchrotron given in [2] with a few inevitable simplifications. In the lattice, rf cavities are introduced as thin elements (that is, the length much shorter than the betatron wave length). Chromaticity correcting sextupoles are not included but the tune is assumed to be independent of the beam momentum. Tunes are varied with the strength of all quadrupoles, in arcs as well as in long straights, multiplied by a certain common factor. This will change the horizontal phase advance in each arc from the design value of 8π and create nonzero dispersions in long straights. Since tunes will be varied in the real operation without introducing nonzero dispersions in long straights, dispersions are artificially put to zero in the simulation but without restoring the phase advance in each arc to 8π. This is simply to avoid time-consuming procedures for matching. In order to see the effect of nonzero (but small) dispersions in long straights where majority of cavities will be located, dispersions are kept as they are in some cases when the horizontal tune is changed from the nominal value of 11.747.

Since no explicit formulas are given in [2] for various acceleration parameters during the acceleration cycle, they are "read" from **Figure 5.1.2.** Particles are tracked from the injection kinetic energy of 600 MeV to 800 MeV, which corresponds to **Time ms.** of 3.7ms in the figure and 1880 turns altogether. Note that the injection is before **Time ms.** = 0 because of the second harmonic component added to the acceleration ramp. Painting is not included in the simulation so that the phase motion during the painting is ignored. One consequence of this is that particles with less than $\pm 180^o$ in the initial phase are captured in the rf bucket.

Ideally speaking, particles distributed in the four-dimensional phase space, (x, x', rf phase, $\Delta p/p$), should be tracked. Instead, particles with initial $\Delta p/p = 0$ and initial phase between 5^o and 155^o (with 5^o interval) are tracked in the longitudinal phase space. In the horizontal phase space (x, x'), twenty particles are distributed initially on the border of 40π.mm.mr (*normalized*) matched ellipse. The total number of particles is therefore

20x31=620. The corresponding *unnormalized* emittance at injection is 31π.mm.mr. Since there are no nonlinear elements in the ring, the normalized emittance of 40π.mm.mr remains the same when all rf cavities are at zero dispersion locations.

III. Simulation code

(x, x') horizontal coordinates of a particle measured from the central orbit

(x_c, x_c') closed orbit of a particle

(x_β, x_β') betatron oscillation of a particle around the closed orbit

$x_\beta \equiv x - x_c$, $x_\beta' \equiv x' - x_c'$

(ϕ, E) rf phase and the total energy of a particle

(ϕ_s, E_s) rf phase and the total energy of the synchronous particle

$(q, \Delta E) \equiv (\phi - \phi_s, E - E_s)$

$(\Delta p/p)$ fractional momentum deviation of a particle from the synchronous value

h (= 84) rf harmonic number

V_k rf voltage of the k-th cavity

R ring circumference/(2π) = 75.47m

$$\mathbf{M} \equiv \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & 1 \end{pmatrix}$$: transfer matrix for each element

$m_{13} = m_{23} = 0$ for drift space and quadrupole

$$\delta(x_c) = (m_{11}-1)(x_c)_{in} + m_{12}(x_c')_{in} + m_{13}(\Delta p/p),$$

$$\delta(x_c') = m_{21}(x_c)_{in} + (m_{22}-1)(x_c')_{in} + m_{23}(\Delta p/p),$$

$$\delta(x_\beta) = (m_{11}-1)(x_\beta)_{in} + m_{12}(x_\beta')_{in},$$

$$\delta(x_\beta') = m_{21}(x_\beta)_{in} + (m_{22}-1)(x_\beta')_{in}$$

δ (“X”) = change in quantity “X” through an element

$\delta(q) = (hL/R)[\Delta L/L - (\Delta p/p)/\gamma_s^2]$, L = length of element, $\gamma_s = E_s/m_pc^2$

With θ = bend angle and ρ = bend radius of a bending magnet,

$$\Delta L = \sin\theta\,(x)_{in} + \rho(1-\cos\theta)(x')_{in} + \rho(\theta - \sin\theta)(\Delta p/p),$$

$\Delta L = 0$ in drift space and quadrupole

At k-th rf cavities,

$\delta(\Delta E) = eV_k[\sin(\phi_s+q) - \sin(\phi_s)]$,

$\delta(x_c) = D_k\,\delta(\Delta p/p)$, $\delta(x_c') = D_k'\,\delta(\Delta p/p)$;

$\delta(x_ß) = -D_k\,\delta(\Delta p/p)$, $\delta(x_ß') = -D_k'\,\delta(\Delta p/p) - [\delta(p_s)/p_s](x_ß')$

IV. Simulation results

Altogether 620 particles are tracked from injection to 800MeV corresponding to 1880 turns. The launching point is the entrance to the first quadrupole slot of an arc. At the end of n-th turn, "emittance" $\varepsilon_k(n)$ for each particle is computed,

$$\varepsilon_k(n) = \pi[\gamma(x_{ß,k})^2 + 2\alpha(x_{ß,k})(x_{ß,k}') + \beta(x_{ß,k}')^2]_n,\ \ k = 1\sim620,\ \ n=1\sim1880. \qquad (3)$$

If there is no effect from synchro-betatron resonances, this will decrease from the 31π.mm.mr at injection to 26π.mm.mr at n = 1880 and the decrease will be inversely proportional to the particle momentum. In order to see the increase in $\varepsilon_k(n)$, two quantities are recorded for a given horizontal tune:

$$E_1 = \frac{1}{620}\sum_{k=1}^{620} \text{ (largest value of } \varepsilon_k(n) \text{ over } n = 1 \text{ to } 1880), \qquad (4)$$

$$E_2 = \text{largest of } \varepsilon_k(n) \text{ for any k over } n = 1 \text{ to } 1880. \qquad (5)$$

Obviously, E_2 is always larger than E_1 and both of them will be 31π.mm.mr if there is no increase in emittance. Another way of seeing the increase may be to evaluate the "rms" emittance

$$\varepsilon_{rms} = (1/620)\{\Sigma\,(x_{ß,k})^2\ \Sigma(x_{ß,k}')^2 - [\Sigma\,(x_{ß,k})(x_{ß,k}')]^2\}^{1/2}\ . \qquad (6)$$

This, however, is not an appropriate quantity to use when one is concerned about any particle loss due to synchro-betatron resonances.

Table 1. Ten cavities in two arcs and twelve cavities in two long straights, total 22. Dispersion is zero at cavities in long straights.

ν_x	($E_1/31\pi$.mm.mr)	($E_2/31\pi$.mm.mr)
11.25	1.0030	1.0084
.30	1.0031	1.0079
.35	1.0029	1.0074
.40	1.0027	1.0071
.45	1.0026	1.0066
.50*	1.0016	1.0061
.55	1.0021	1.0055
.60	1.0018	1.0048
.65	1.0014	1.0047
.70	1.0015	1.0104
.75	1.0034	1.0307
.80	1.0180	1.263
.85	1.0423	1.621
.90	1.108	3.70
.95	2.29	> 20
12.05	1.32	3.40
.10	1.0102	1.122
.15	1.0057	1.0204
.20	1.0068	1.0185
.25	1.0030	1.0084

One sees that, with this cavity arrangement, synchro-betatron resonance effect on the horizontal emittance is negligible outside of $11.75 < \nu_x < 12.10$. It is also clear that it is safer to be above an integer than below as predicted by the theory. [See pp. 2-3.] Since the emittance of only one out of 620 particles can grow to E_2 during the entire acceleration cycle, it should be regarded as an overestimate of the emittance increase for the beam.

* Since periodicity is two and no quadrupole errors are included, ν_x= 11.50 is stable.

When the horizontal tune is changed from its design value of 11.747 by simply multiplying a common factor to all quadrupoles, this just for the sake of convenience, all linear optical parameters within the arcs (β,α,ψ, D, D') change so that the results listed in **Table 1** may not faithfully represent the true dependence on the tune. In order to clarify this point, a quality figure "**QF**" is defined,

$$\mathbf{QF} \equiv |\sum_k \; [(\gamma D^2+2\alpha\Delta\Delta \; +\beta D'^2)_k]^{1/2}\exp(\iota\psi_k)|, \; (k=1\text{-}22) \qquad (6)$$

which should be more suitable than the one defined by (2) when optical parameters are different at different cavity locations.

Table 2. Quality figure "QF" defined by (6).

ν_x	**QF**	ν_x	**QF**	ν_x	**QF**
11.25	0.484	11.60	0.356	11.95	1.127
.30	0.527	.65	0.256		
.35	0.550	.70	0.136	12.05	0.138
.40	0.553	.75	0.025	.10	0.635
.45	0.535	.80	0.178	.15	0.907
.50	0.496	.85	0.377	.20	1.115
.55	0.437	.90	0.641	.25	1.290

QF takes the minimum (the best) value at the design tune of 11.75, indicating a near perfect cancellation (at least in the lowest order) among ten cavities in two arcs. It is more than an order of magnitude larger at tune values away from 11.75.

A question arises as to whether one can improve the cancellation or make it worse by different arrangements of cavities in the arcs. An exhaustive study on this has not been done but the following examples may reveal the importance of cavity placement.

1. $\nu_x = 11.90$. Remove the middle cavity in one arc to a long straight.
   QF changes from 0.641 to 0.138, which should improve the cancellation.
   $E_1/31\pi$.mm.mr = 1.036 (compared with 1.108 in **Table 1**.),

$E_2/31\pi$.mm.mr = 2.48 (compared with 3.70 in **Table 1**.)

2. ν_x = 11.90. Remove the first cavity in one arc to a long straight.
   QF changes from0.641 to 1.26, which should reduce the cancellation.
   $E_1/31\pi$.mm.mr = 1.181, $E_2/31.\pi$mm.mr = 6.83.

3. ν_x = 11.75. Remove the first cavity in each arc to long straights.
   QF changes from 0.025 to 1.30, which should increase E_1 and E_2.
   $E_1/31\pi$.mm.mr = 1.017 (compared with 1.003 in **Table 1.**)
   $E_2/31\pi$.mm.mr = 1.155 (compared with 1.031 in **Table 1.**)

Note that, because of a near perfect cancellation, ten cavities in arcs result in a smaller increase of emittance compared with eight cavities in arcs.

4. ν_x = 11.45. Remove the second and the third cavities in one arc, the third and the fourth in another arc to long straights.
   QF changes from 0.535 to 2.50, which should reduce the cancellation.
   $E_1/31\pi$.mm.mr = 1.011 (compared with 1.002 in **Table 1.**)
   $E_2/31\pi$.mm.mr = 1.027 (compared with 1.007 in **Table 1.**)

From these examples (and more that are not listed here), one can conclude that

1. It is important to pay attention to QF when the horizontal tune is larger than 11.75 or so.
2. When the tune is below 11.75, the impact of QF is much less serious so that one can use almost any arrangement of cavities in arcs without affecting the emittance increase.

So far, the dispersion in two long straights was artificially set to zero in spite of the fact that the phase advance in each arc is not kept at 8π. What would be the increase in emittance if the dispersion in long straights is small but not exactly zero? This situation may arise in the real operation of the machine when a tedious process of using long straights as a “phase trombone” is bypassed for the sake of convenience, as was done in the simulation. **Table 3** is identical to **Table 1**, with ten cavities in two arcs and twelve in long straights, but retaining the nonzero dispersions in long straights so that all 22 cavities contribute to the synchro-betatron resonances

Table 3. Ten cavities in two arcs, twelve cavities in two long straights.
Dispersions are **not** zero in long straights.

ν_x	(E_1/31π.mm.mr)	(E_2/ 31π.mm.mr)	max. |D| at LS cavities
11.30	1.0026	1.0068	0.112m
.45	1.0021	1.0055	0.058m
.65	1.0013	1.0047	0.011m
.80	1.0187	1.271	0.007m
.85	1.0477	1.706	0.030m
.90	1.151	5.0	0.091m
.95	4.3	46.	0.31m
12.05	3.8	24.	0.71m
.10	1.23	4.8	0.47m
.15	1.068	1.7	0.40m
.20	1.033	1.29	0.38m
.25	1.008	1.053	0.39m

Comparing this with **Table 1**, one sees that

1. From 11.25 to 11.80, there is no difference in E_1 or E_2.
2. *For this lattice*, dispersions at long straight cavities are large so that E_1 and E_2 increase significantly. It is advisable to stay below 11.8 if the horizontal tune is changed without maintaining the phase advance of 8π in each arc.

V. Concluding remarks

The purpose of this note is definitely **not** to advocate installing rf cavities at locations where dispersions are nonzero. It is still prudent to avoid dispersions at rf cavities although $|D| < 10$cm would be harmless as far as the synchro-betatron resonances in rapid-cycling machines are concerned. This note **does** advocate, however, that rf cavities at nonzero dispersions should be an option if other considerations such as the robustness of extraction system are at issue. Furthermore, one must keep in mind that, once the machine is built, the most precious item in the ring for the future upgrade is "empty space" (which should perhaps be qualified as "warm" empty space for superconducting rings). When locations with nonzero dispersion are to be considered for cavities, simulation studies are essential for finding the optimum arrangement, especially if the fractional part of the horizontal tune is above 0.8 or so. In general, it is advisable to design a lattice with the horizontal tune between 0.2 and 0.8 .

References

[1], [2] See p. 1.

[3] *Handbook of Accelerator Physics and Engineering (edited by A.W.Chao and M. Tigner)*, pp. 72-75.

[4] A. Piwinski and A. Wrulich, DESY 76/07, February 1976.

[5] Grahame Rees, private communication.

[6] Anton Piwinski, private communication.