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Branching ratio for $\pi^0 \to \nu\bar{\nu}$ in the case of massive neutrinos

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The decay $\pi^0 \to \nu\bar{\nu}$ is forbidden by angular momentum conservation if neutrinos are purely massless left-handed particles, but a finite neutrino mass permits the decay to occur. If neutrinos, with mass m_ν less than half of the π^0 mass, couple to the Z^0 with standard weak-interaction strength, the theoretical branching ratio for the $\pi^0 \to \nu\bar{\nu}$ decay is given, for a single neutrino type, as

$$Br(\pi^0 \to \nu\overline{\nu}) = \quad \mathbf{C}_{\pi^0\to\nu\bar{\nu}} \quad \times \quad (m_\nu/m_{\pi^0})^2 \sqrt{1-4\,(m_\nu/m_{\pi^0})^2}$$

This technote explains why

$$\mathbf{C}_{\pi^0\to\nu\bar{\nu}} = \mathbf{3 \times 10^{-8}}$$

for a single Dirac-neutrino type in the E949 publication [1].

1 $C_{\pi^0\to\nu\bar{\nu}}$ for a single Dirac-neutrino type

- $\underline{C_{\pi^0\to\nu\bar{\nu}} = 2.9 \times 10^{-8}}$
  T. Kalogeropoulos, J. Schechter and J. Valle, A test for neutrino masses, Phys. Lett. B86, 72 (1979) [2].
- $\underline{C_{\pi^0\to\nu\bar{\nu}} = 3 \times 10^{-8}}$
  P. Herczeg and C.M. Hoffman, On the decays $\pi^0 \to \nu\nu'$, Phys. Lett. B100, 347 (1981).
- $\underline{C_{\pi^0\to\nu\bar{\nu}} = 3 \times 10^{-8}}$
  L. Arnellos, W.J. Marciano and Z. Parsa, The decay $\pi^0 \to \nu\bar{\nu}\gamma$, Nucl. Phys. B196, 365 (1982).

2 $C_{\pi^0\to\nu\bar{\nu}}$ for a single Majorana-neutrino type

- $\underline{C_{\pi^0\to\nu\bar{\nu}} = 5.5 \times 10^{-8}}$
  W.J. Marciano and Z. Parsa, Rare kaon decays with "missing energy", Phys. Rev. D53, R1 (1996) [3].

[1] Upper limit on the branching ratio for the decay $\pi^0 \to \nu\bar{\nu}$, Phys. Rev. D72, 091102(R) (2005).

[2] $Br(\pi^0 \to \nu\overline{\nu})$ has a maximum Br^{max} at $m_\nu = m_{\pi^0}/\sqrt{6}$=55MeV/c^2, in which $(m_\nu/m_{\pi^0})^2\sqrt{1-4\,(m_\nu/m_{\pi^0})^2}$ is 0.0962 and thereby $C_{\pi^0\to\nu\bar{\nu}} \simeq 10.4 \times Br^{max}$. In this paper, Br^{max} is given as 0.28×10^{-8}.

[3] They wrote, "That formula (the branching ratio for $\pi^0 \to \nu_i\nu_i$ in the case of massive Majorana neutrinos) is a factor of 2 larger than those in the literature which were derived for Dirac neutrinos."

- $\underline{C_{\pi^0\to\nu\bar{\nu}} = 6.3\times10^{-8}}$
  R. Shrock, private communication [4].

2.1 $K\to\pi\nu\bar{\nu}$ in the case of Majorana neutrinos

If the branching ratio for $K\to\pi\nu\bar{\nu}$ in the case of massive Majorana neutrinos were a factor 2 larger than for Dirac neutrinos, the E949 experiment would immediately become the center of attention in the physics community because the central value of the $K^+\to\pi^+\nu\bar{\nu}$ branching ratio is about a factor 2 larger than the Standard Model prediction. Nature was, unfortunately, not kind to us. In the limit of vanishing mass it is impossible to distinguish between the Dirac and Majorana character of neutrinos from $K\to\pi\nu\bar{\nu}$. I explain the reason by following the paper [5]:

- J.F. Nieves and P.B. Pal, Majorana neutrinos and photinos from kaon decay, Phys. Rev. D32, 1849 (1985).

Firstly, a Majorana neutrino is identical with its antiparticle. So, the final state of $K\to\pi\nu\bar{\nu}$ contains two identical particles in the Majorana case. This gives a symmetry factor 1/2! in the calculation of the decay rate. Secondly, in the Dirac case the right-handed components of the neutrinos, ν_R, are singlets of $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ and hence do not interact with W and Z bosons. In the Majorana case, on the other hand, a right-handed neutrino forms an $\mathrm{SU}(2)_L$ doublet with the right-handed component of a charged anti-lepton. So they have gauge interactions as well. If we take them into account, the effective leptonic current, with is simply $\bar{\nu}\gamma^\mu(1-\gamma_5)\nu$ for the Dirac case, becomes $\bar{\nu}\gamma^\mu(1-\gamma_5)\nu\ -\ \bar{\nu}\gamma^\mu(1+\gamma_5)\nu = -2\bar{\nu}\gamma_5\nu$. The dependence of the decay width on the nature of the neutrino appears through the expression $(a^2+b^2)/n!$, where a and b are the coefficients of $(a+b\gamma_5)$ ($a=-b=1$ for Dirac neutrinos and $a=0$, $b=-2$ and $n=2$ for Majorana neutrinos, as explained above) and n is the number of identical particles in the final state ($n=1$ for Dirac neutrinos and $n=2$ for Majorana neutinos). $(a^2+b^2)/n!$ is the same for Dirac neutrinos ($(1^2+(-1)^2)/1!\ =\ 2$) and Majorana neutrinos ($(0^2+2^2)/2!\ =\ 2$).

The decay width of $Z\to\nu\bar{\nu}$ is also the same for Dirac and Majorana neutrinos (rather than the difference of factor 2 for $\pi^0\to\nu\bar{\nu}$) [6].

3 $C_{\pi^0\to\nu\bar{\nu}}$ beyond the Standard Model

- $\underline{C_{\pi^0\to\nu\bar{\nu}} = 12\times10^{-8}}$
  M.A.B. Bég, W.J. Marciano and M. Ruderman, Properties of neutrinos in a class of gauge

[4] We asked him to calculate $Br(\pi^0\to\nu\overline{\nu})$ for our publication; he gave 6.3×10^{-8} to us and we wrote $C_{\pi^0\to\nu\bar{\nu}}$ $= 6.3\times10^{-8}$ in the first submission (hep-ex/0506028, v1). After we noticed the Marciano-Parsa paper, we raised a question to him and he replied he had (naturally) assumed Majorana neutrinos in his calculation.

[5] See also N.G. Deshpande and G. Eilam, Neutrino-mass limits from $K^+\to\pi^+\nu\bar{\nu}$ decay, Phys. Rev. Lett. 53, 2289 (1984).

[6] W.J. Marciano, private communication.

theories, Phys. Rev. D17, 1395 (1978),
in a class of gauge theories characterized by manifest Left-Right symmetry and spontaneous genesis of parity nonconservation [7].

- $\underline{C_{\pi^0 \to \nu\bar{\nu}} = 7 \times 10^{-8}}$
  P.D. Meyers, Measurement of the π^0 inefficiency and an upper limit on the process $\pi^0 \to \nu\bar{\nu}$, E787 Technical Note No. 165 (1989). [8]

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[7] This paper was cited as [1] in the E787 paper: Upper limit on the branching ratio for the decay $\pi^0 \to \nu\bar{\nu}$, Phys. Rev. Lett. 66, 2189 (1991). I think that was a mistake (and really misled us in E949 !).

[8] He wrote, "Taking the first possibility (the neutrinos have finite mass) as an example, the decay looks very much like a neutral version of $\pi\mu2$, mediated by a Z instead of W." I do not expect he assumed Majorana neutrinos; I have no idea how he reached 7×10^{-8} in his calculation.