Cosmological relics from metric perturbations *

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❑ *RELICS PROBLEM*

- Cosmological relics: Massive, long-lived particles (gravitinos, moduli, ...)
- Gravitinos (spin 3/2) $\rightarrow$ gravitational couplings
- Long lifetime: $\tau_{3/2} \sim M_P^2/m_{3/2}^3 \rightarrow 10^5 s$ ($m_{3/2} \sim 1\,\mathrm{TeV}$)
- Decay during nucleosynthesis (10^2 s) ($^4He \rightarrow D + {}^3He$)
- Nucleosynthesis bounds

$$\boxed{\frac{n}{s} \lesssim 10^{-14} \frac{m_{3/2}}{1\,\mathrm{TeV}}}$$

- Primordial abundance: $n/s \simeq 10^{-3}$

(Inflation solves the problem)

❑ *PRODUCTION AFTER INFLATION*

- Thermal production

$$\boxed{\frac{n}{s} \simeq 10^{-14} \frac{T_R}{10^9 \mathrm{GeV}}}$$

- Non-thermal production

  – Gravitational production (M. Lemoine '99)

  – Production during preheating
  (A.L.M & A. Mazumdar '00, R. Kallosh et al.'00, A.L.M & J.R. Peláez '00, Nilles et al. '01)

  Conformally coupled particles in FRW (photons, massless fermions, ...) are <u>not created</u>

  – **Production from metric perturbations**

❑ *METRIC PERTURBATIONS*

- Scalar perturbation in the longitudinal gauge:

$$ds^2 = a^2(\eta)((1 + 2\Phi)d\eta^2 - (1 - 2\Phi)\gamma_{ij}dx^i dx^j)$$

- Power spectrum

$$\langle 0|\Phi_k^* \Phi_{k'}|0\rangle = \mathcal{P}_\Phi(k)\frac{2\pi^2}{k^3}\delta_{\vec{k}\vec{k}'}$$

$$\boxed{\mathcal{P}_\Phi(k) \simeq A_S^2 \left(\frac{k}{k_C}\right)^{n_s - 1}}$$

$$A_S \simeq 5 \times 10^{-5}, \qquad \lambda_C = \frac{2\pi}{k_C} \simeq 3000 \text{ Mpc}$$

- Breaking of conformal invariance $\Rightarrow$ particle production (Zeldovich, Starobinsky '72)

$$\Phi_k \sim C_k(\eta)e^{i\vec{k}\vec{x}} \to C^{\mu\nu}{}_{\rho\sigma} \sim \frac{C_k\, k^2}{a^2(\eta)}$$

$\boxed{\text{Strong production expected right after inflation}}$

❑ *GRAVITINO FIELD EQUATIONS*

- Rarita-Schwinger equation

$$\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho\psi_\sigma + \frac{m}{2}[\gamma^\mu, \gamma^\nu]\psi_\nu = 0.$$

  ψ_σ Majorana spinor $\psi_\sigma = C\bar{\psi}_\sigma^T$

$$D_\mu\psi_\sigma = (\partial_\mu + \Omega_\mu)\psi_\sigma - \Gamma^\lambda_{\mu\sigma}\psi_\lambda$$

- Two constraints $\rightarrow$ $\pm 3/2$, $\pm 1/2$ helicity states (flat space)

- Massless limit: only $\pm 3/2$ states.

- Background metric $\rightarrow$ FRW (asymptotically)

  $\Phi(\eta, \vec{x})$ vanishes asymptotically in the past & future (high k).

❑ GRAVITINO FIELD EQUATIONS

- Mode quantization

$$\psi_\mu(x) = \sum_\lambda \sum_{\vec{k}} \left(\psi_\mu^{\vec{k}\lambda} \mathbf{a}_{\vec{k}\lambda} + \psi_\mu^{\vec{k}\lambda C} \mathbf{a}^\dagger_{\vec{k}\lambda} \right)$$

$$\{a_{\vec{k}\lambda}, a^\dagger_{\vec{k}'\lambda'}\} = 2k\delta_{\lambda\lambda'}\delta_{\vec{k}\vec{k}'}$$

- In the asymptotic (FRW) regions
  RS $\rightarrow i \not{D}\, \psi_\mu^{\pm 3/2} = 0$, (A.L.M. & A. Mazumdar '00)

- Asymptotic solutions (positive frequency)

$$\psi_\mu^{(0)p,\pm}(x) = \frac{1}{\sqrt{2pVa(\eta)}} u(\vec{p},\pm)\epsilon_\mu(\vec{p},\pm)e^{i\vec{p}\vec{x}-ip\eta}$$

- Perturbative expansions in Φ

$$\begin{aligned} \psi_\mu &= \psi_\mu^{(0)p,\lambda} + \psi_\mu^{(1)} + \ldots \\ D_\mu &= D_\mu^{(0)} + D_\mu^{(1)} + \ldots \\ \gamma_\mu &= \gamma_\mu^{(0)} + \gamma_\mu^{(1)} + \ldots \end{aligned}$$

❑ *GRAVITINO FIELD EQUATIONS*

- Project RS equations on helicity $\pm 3/2$ states

$$P^{\mu}_{\pm 3/2}(\vec{k}) = \epsilon^{\mu *}(\vec{k}, \pm) P_{\pm 1/2}(\vec{k})$$

- Mode equation

$$\boxed{\left(\frac{\partial^2}{\partial \eta^2} + k^2\right) \tilde{\psi}^{(1)}_{\pm 3/2}(\vec{k}, \eta) + J_{\pm 3/2}(\vec{k}, \eta) = 0}$$

- Spinor current

$$J_{\pm 3/2}(\vec{k}, \eta) = \frac{1}{2}\Big[(p^2 + k^2 - 2pk_j \hat{\gamma}^0 \hat{\gamma}^j) \Phi(\vec{k} + \vec{p}, \eta)$$
$$- i\hat{\gamma}^0 (k_j \hat{\gamma}^j - p\hat{\gamma}^0) \Phi'(\vec{k} + \vec{p}, \eta)\Big] P^i_{\pm 3/2}(\vec{k}) \tilde{\psi}^{(0)\lambda}_i(\vec{p}, \eta)$$

❑ *GRAVITINO PRODUCTION*

In the *out* region the solution will become a superposition of positive and negative frequency modes with different helicities and different momenta.

$$\psi_\mu^{(in)\,p,\lambda}(x) \;\to\; \sum_{\lambda'=\pm}\sum_k \left(\alpha_{pk\lambda\lambda'}\frac{u(\vec{k},\lambda')\epsilon_\mu(\vec{k},\lambda')}{\sqrt{2kVa(\eta)}}e^{i(\vec{k}\vec{x}-k\eta)} + \beta_{pk\lambda\lambda'}\frac{u^C(\vec{k},\lambda')\epsilon^*_\mu(\vec{k},\lambda')}{\sqrt{2kVa(\eta)}}e^{-i(\vec{k}\vec{x}-k\eta)}\right)$$

From mode quantization, we find for the creation and annihilation operators $\mathbf{a}^\dagger_{\mathbf{k}\lambda}$, $\mathbf{a}_{\mathbf{k}\lambda}$:

$$\mathbf{a}^{\mathbf{out}}_{\mathbf{k}\lambda} = \sum_{\lambda'}\sum_q \left(\alpha_{qk\lambda'\lambda}\mathbf{a}^{\mathbf{in}}_{\mathbf{q}\lambda'} + \beta^*_{qk\lambda'\lambda}\mathbf{a}^{\mathbf{in}\,\dagger}_{\mathbf{q}\lambda'}\right)$$

Occupation number:

$$\langle 0, in|N^{out}_{k\lambda}|0, in\rangle = \sum_q\sum_{\lambda'}|\beta_{qk\lambda\lambda'}|^2 \neq 0$$

❑ *NUMBER AND ENTROPY DENSITIES*

- Bogolyubov coefficients

$$\beta_{pk\lambda\mp} = -\frac{i}{2k\sqrt{2kV}} \int_{\eta_0}^{\eta_1} u^{C\dagger}(\vec{k},\mp) J_{\pm 3/2}\, e^{-ik\eta}$$

- Total comoving number density $(\kappa \simeq a_I\, H_I)$

$$N = 2\int d^3k N_{k\lambda} \simeq 8\kappa^3 N_\kappa$$

$$\boxed{N_\kappa \simeq \frac{\sqrt{2\pi^3}}{3} \mathcal{P}_\Phi(\kappa)}$$

- Inflaton potential close to the minimum:

$$V \sim \phi^\alpha$$

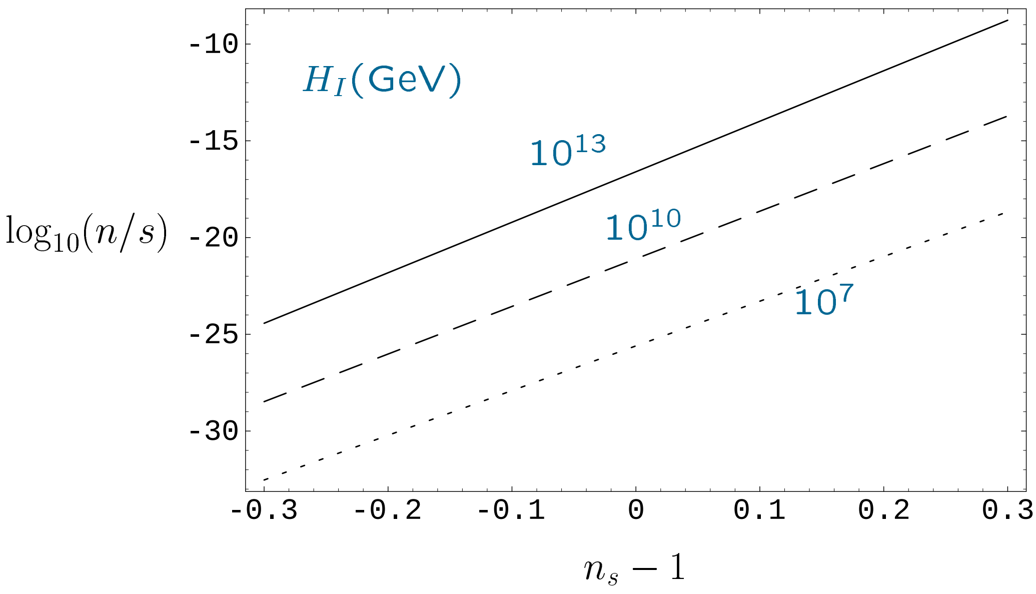
- Entropy density: $s \simeq \rho^{3/4} \sim a^{-9\alpha/(2(\alpha+2))}$

❑ n/s: *RESULTS*

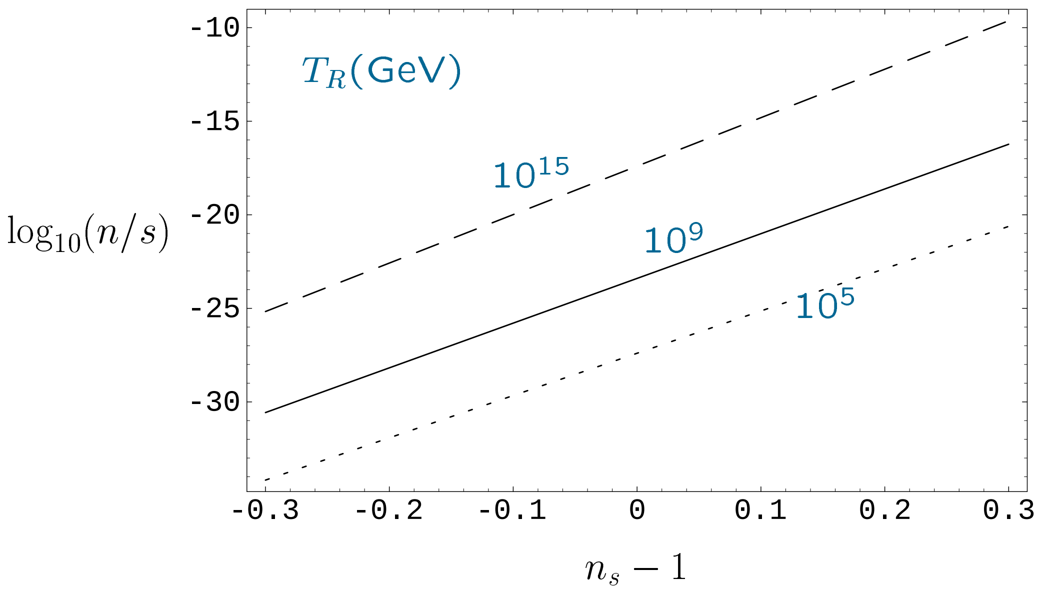
- Relative abundance

$$\boxed{\frac{n}{s} \simeq \frac{\sqrt{2}A_S^2}{3}\left(\frac{\pi\, H_I}{\sqrt{3}\; M_P}\right)^{3/2}\left(\frac{T_R^2}{\sqrt{3}\; H_I\; M_P}\right)^{\frac{4-\alpha}{2\alpha}} e^{(n_s-1)N(k_C)}}$$

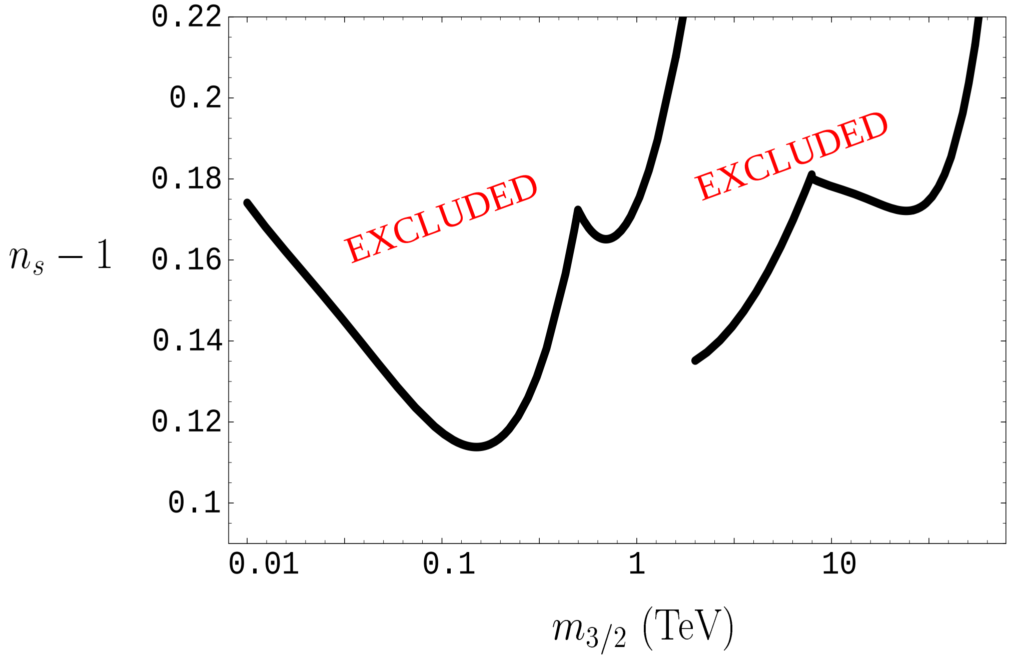
- n/s vs. spectral index ($V \sim \phi^4$)



- n/s vs. spectral index ($V \sim \phi^2$, $H_I \sim 10^{13}$ GeV)



- Spectral index vs. gravitino mass,
  ($V \sim \phi^4$, $H_I \sim 10^{13}$ GeV)



Left: D photofission, D and ^{3}He photoproduction

Right: D and ^{4}He hadroproduction

Black-hole production: $n_S < 1.28$ (Green, Liddle '97)

❑ *CONCLUSIONS*

- Primordial metric perturbations can produce cosmologically relevant amounts of relics after inflation, including conformally coupled particles.

- In models with blue spectra, i.e., $n_s > 1$ (hybrid inflation), the production could conflict with nucleosynthesis.

- This effect is sensitive to the small wavelength region of the primordial spectrum of metric perturbations.

- In some models, the bounds on the spectral index can be stronger than those coming from primordial black-hole production.