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Conservation Laws in the Quantum Mechanics of Closed Systems

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Abstract

We investigate conservation laws in the quantum mechanics of closed systems and begin by reviewing an argument that exact decoherence implies the exact conservation of quantities that commute with the Hamiltonian. However, we also show that decoherence limits the alternatives which can be included in sets of histories which assess the conservation of these quantities. In the case of charge and energy, these limitations would be severe were these quantities not coupled to a gauge field. However, for the realistic cases of electric charge coupled to the electromagnetic field and mass coupled to spacetime curvature, we show that when alternative values of charge and mass decohere they always decohere exactly and are exactly conserved. Further, while decohering histories that describe possible changes in time of the total charge and mass are also subject to the limitations mentioned above, we show that these do not, in fact, restrict *physical* alternatives and are therefore not really limitations at all.

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I Introduction.

Energy is conserved during the unitary evolution

$$|\psi(t)\rangle = e^{-iht/\hbar}|\psi(0)\rangle \tag{1.1}$$

of a quantum state of an isolated subsystem of the universe because the Hamiltonian, h, of that subsystem commutes with the unitary time evolution operator. However, energy is not generally conserved by the "second law" of quantum evolution that describes how the state of a subsystem evolves when an "ideal" measurement of it is carried out. If $|\psi\rangle$ is the state before an ideal measurement, the state afterwards is "reduced" to

$$|\psi\rangle \to \frac{s_\alpha|\psi\rangle}{\|s_\alpha|\psi\rangle\|} \tag{1.2}$$

where s_α is a Schrödinger-picture projection operator corresponding to the measurement outcome – one of a set of such operators $\{s_\alpha\}, \alpha = 1, 2, \ldots$ describing different possible outcomes. Even if $|\psi\rangle$ is an energy eigenstate, the reduced state vector will generally not be an energy eigenstate except in the special case that s_α commutes with the Hamiltonian of the subsystem h.

More generally, if a sequence of measurements is carried out on the subsystem at times $t_1, ..., t_n$, with outcomes represented by Heisenberg picture projection operators $\{s^k_{\alpha_k}(t_k)\}, k = 1, ..., n$, the joint probability of a particular sequence of outcomes $\alpha \equiv (\alpha_1, ..., \alpha_n)$ is

$$p(\alpha_1, \ldots, \alpha_n) = \|s^n_{\alpha_n}(t_n) \ldots s^1_{\alpha_1}(t_1)|\psi\rangle\|^2. \tag{1.3}$$

Energy is conserved if the joint probability vanishes whenever measurements of the energy at two times disagree. However, if measurements of quantities that do not commute with h intervene between the two determinations of the energy, then that joint probability will not be zero. More precisely, if $\{s^h_{\alpha_l}(t_l)\}$ and $\{s^h_{\alpha_m}(t_m)\}$ project to the same set of ranges of

h-eigenvalues, the joint probability has nonzero entries for $\alpha_l \neq \alpha_m$ when the intervening projections do not commute with h. Energy is thus not necessarily conserved in a sequence of measurements.

The conservation of energy by the unitary evolution and the general non-conservation by the reduction of the state vector are not surprising. The unitary law (1.1) describes the evolution of a subsystem in isolation. The reduction law (1.2) describes the evolution of a subsystem interacting with another system which is measuring it. Energy may be exchanged between the measuring apparatus and the measured subsystem.

The familiar "Copenhagen" quantum mechanics of measured subsystems sketched above is an approximation to a more general quantum mechanics of closed systems [1,2,3], most generally the universe as a whole. It is an approximation that is appropriate when certain approximate features of realistic measurement situations may be idealized as exact*. The most general predictions of quantum mechanics are the probabilities for individual members of sets of alternative histories of a closed system. One kind of alternative history set may be specified by giving exhaustive sets of exclusive alternatives at a sequence of times $t_1, \ldots, t_n$. The alternatives at one time are represented by a set of Heisenberg-picture projection operators $\{P^k_{\alpha_k}(t_k)\}$ satisfying

$$P^k_{\alpha_k}(t_k) P^k_{\alpha'_k}(t_k) = \delta_{\alpha_k \alpha'_k} P^k_{\alpha_k}(t_k), \quad \sum_{\alpha_k} P^k_{\alpha_k}(t_k) = I \tag{1.4}$$

showing that they represent a mutually exclusive, exhaustive set of alternatives. An individual history corresponds to a particular sequence of alternatives $\alpha \equiv (\alpha_1, \ldots, \alpha_n)$ and is represented by the corresponding chain of projections. When the theory assigns probabilities to a set of such alternative histories, the probabilities of the individual histories are given by

$$p(\alpha_1, \ldots, \alpha_n) = \| P^n_{\alpha_n}(t_n) \ldots P^1_{\alpha_1}(t_1) |\Psi\rangle \|^2 \tag{1.5}$$

* For example, as in [4], Section II.10.

assuming (for simplicity) that the initial condition of the closed system $|\Psi\rangle$ is pure.

Eq.(1.5), giving the probability of a history of a closed system, has the same form as eq.(1.2) giving the probability of a history of measurements of a subsystem. The only difference between the expressions is that in (1.2) states operators, *etc*, all act on the Hilbert space of the measured subsystem, while in (1.5) they act on the Hilbert space of a closed system including both the measured subsystem and any measurement apparatus. Thus, if $\{P^H_{\alpha_l}(t_l)\}$ and $\{P^H_{\alpha_m}(t_m)\}$ are projectors onto the same sets of ranges of the total energy H at two different times there is no reason to believe that the probability of histories with $\alpha_l \neq \alpha_m$ will vanish if the intervening projections do not commute with the Hamiltonian. Eq.(1.5) no more conserves energy than does (1.2). However, in the quantum mechanics of a closed system there is nothing "external" to cause a change in the total energy. Does this mean that the quantum mechanics of closed systems predicts non-zero probabilities for violations of energy conservation? Further, not only conservation of energy is at stake. Similar remarks hold for any other quantity that commutes with the Hamiltonian such as the total electric charge*. In the following, we shall show that no such violations are in fact predicted.

In posing the question of possible violations of fundamental conservation laws in the quantum mechanics of closed systems we should stress that we do not mean violations that might be revealed by successive measurements of a *subsystem*. The probabilities of the outcomes of ideal measurements on a subsystem are described by (1.3) to an accuracy far beyond the precision available in any experimental check of a conservation law. A sequence of two measurements that determines whether the value of a quantity a that commutes

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* Following the usual terminology, when no confusion should result, we shall often refer to quantities that commute with the Hamiltonian as "conserved quantities" even though it is their conservation that is being investigated!

with h is in one of a set of ranges $\{\Delta_\alpha\}$ is represented by the string of projections

$$s^a_{\alpha_2}(t_2)s^a_{\alpha_1}(t_1). \tag{1.6}$$

The Heisenberg equations of motion

$$s^a_\alpha(t) = \mathrm{e}^{iht/\hbar} s^a_\alpha(0)\mathrm{e}^{-iht/\hbar} \tag{1.7}$$

together with the analog of (1.4) show that, when a commutes with the Hamiltonian h, the operator string (1.6) is proportional to $\delta_{\alpha_1\alpha_2}$. The probabilities (1.5) of a measured change in the value of a quantity commuting with h, including the energy itself, are therefore zero**.

However, the quantum mechanics of closed systems does not only predict probabilities for the outcomes of measurements of a subsystem. We may consider, if we wish, the probabilities of histories which describe alternative values of the *total* value of a quantity commuting with the *total* Hamiltonian H for the whole closed system at various moments of time. Such total quantities are unlikely to be accessible to experiment, but their conservation, or lack of it, is still of theoretical interest, and it is this question which is the subject of this paper.

The expression (1.5) for the probabilities of the histories of a closed system would seem to allow non-zero probabilities for changes in a quantity commuting with the Hamiltonian if projections that appear between two projections associated with this conserved quantity do not commute with it. However, in the quantum mechanics of closed systems, probabilities are not predicted for an *arbitrary* set of alternative histories. They are predicted only for those sets for which there is negligible quantum mechanical interference between the individual histories in the set [1, 2, 3]. Such sets of histories are said to decohere. It would

** This is true for any two times t_1 and t_2 despite common misconceptions concerning the energy-time uncertainty principle be inconsistent to assign probabilities to sets of histories that did not decohere because the correct probability sum rules would not be obeyed. Decoherence of histories implies the validity of the probability sum rules so that decoherent sets of histories are consistent.

Conservation laws are obeyed by consistent sets of histories. In Section II we review an argument of Griffiths* [5] that exact decoherence implies exact conservation of quantities that commute with the Hamiltonian. However, as we also show in Section II, there are limits on the exactly decohering sets of histories through which the probabilities of changes in such a quantity, A, could even be defined. For a set of histories that test the conservation of A in some fixed way to some given accuracy, there is an associated 'smeared' version $\tilde{A}$ of the conserved operator A. The only alternatives permitted in such histories are of the values of quantities that effectively commute with $\tilde{A}$; i.e., commute when acting on the initial condition of the closed system. That limitation would prohibit, for instance, the exact decoherence of a set of histories that, in addition to describing the outcomes of our everyday observations, also specifies alternative values of the total energy to arbitrary precision. That is because the total Hamiltonian does not commute with the crudely specified positions, momenta, etc necessary to describe typical experimental situations. (Recall that as observers we are part of this closed system.)

However, an important feature of the two most important absolutely conserved quantities – electric charge and mass – is that they are coupled to gauge fields. This fact has two consequences: (1) It allows decoherent histories that describe possible changes in charge or mass together with other realistic, everyday alternatives, as the above condition is satisfied for *all* quasilocal operators. (2) It ensures the exact decoherence of the alternative values of these changes, and that the probability is zero for any non-vanishing value of such a change. That is, total charge and total energy are exactly conserved. The simplest case

* The argument appears well known to a number of people. We learned it from R. Griffiths.

of electric charge is discussed in Section III. Section IV discusses the conservation of total energy.

II Exact decoherence and Exact Conservation.

In this section we review Griffiths' demonstration that the probabilities for changes in the values of quantities that exactly commute with the Hamiltonian are exactly zero for exactly decohering sets of alternative histories of a closed system. We also show that, given a quantity A which commutes with the Hamiltonian, the only alternatives which can occur in an exactly decohering set of histories describing possible changes of A are values of operators which effectively commute with A when acting on the initial condition of the system.

Let A be any quantity satisfying

$$[A, H] = 0 \tag{2.1}$$

including the Hamiltonian itself. Let $\{\Delta_j\}, j = 1, 2, \ldots$ be an exhaustive set of non-overlapping ranges of the eigenvalues of A, and let $\{P_j^A(t)\}$ be the set of Heisenberg picture projections onto them. The same set of intervals will be used at every time so that the $\{P_j^A(t)\}$ obey (1.4).

Consider a set of histories (consisting of alternatives at a sequence of times) in which sets of projections onto ranges of A occur at two different times t_l and t_m. We will use such histories to test the conservation of A to a degree determined by the choice of intervals $\{\Delta_j\}$. Note that the argument below may easily be generalized to test conservation between more than two moments of time.

The individual histories in such a set are represented by chains of projections operators

$$C_\alpha = C^c_{\alpha_c} P^A_{j_m}(t_m) C^b_{\alpha_b} P^A_{j_l}(t_l) C^a_{\alpha_a} \tag{2.2}$$

where the $\{C^a_{\alpha_a}\},\{C^b_{\alpha_b}\},\{C^c_{\alpha_c}\}$ are chains of projections representing alternatives before t_l, between t_l and t_m, and after t_m respectively. More generally the $C^a_{\alpha_a}, C^b_{\alpha_b}, C^c_{\alpha_c}$ could be sums of chains of projections corresponding to alternative histories defined by partitions of the chains into exclusive classes, and they could be branch dependent in the sense of [4] without affecting the subsequent simple argument.

The decoherence functional whose off diagonal elements measure quantum interference between parts of histories is

$$D(\alpha', \alpha) = Tr(C_{\alpha'}\rho C^{\dagger}_{\alpha}) \tag{2.3}$$

where ρ is the density matrix representing the initial condition of the closed system. When $Re(D)$ vanishes for $\alpha' \neq \alpha$ the set of histories exactly (weakly) decoheres and the probabilities are given by the diagonal elements, as summarized in the equation

$$ReD(\alpha', \alpha) = \delta_{\alpha'\alpha} p(\alpha)\ . \tag{2.4}$$

We can now proceed with Griffiths' argument.

Consider the probabilities $p(\alpha_c, j_m, \alpha_b, j_l, \alpha_a)$ of the set of histories represented by (2.2). Exact weak decoherence implies that these probabilities are consistent. That is, they must obey the probability sum rules and in particular

$$\sum_{\alpha_a, \alpha_b, \alpha_c} p(\alpha_c, j_m, \alpha_b, j_l, \alpha_a) = p(j_m, j_l)\ . \tag{2.5}$$

The $p(j_m, j_l)$ are the probabilities for the set of histories represented by the chain

$$P^A_{j_m}(t_m) P^A_{j_l}(t_l)\ . \tag{2.6}$$

But the individual operators in the chain are in fact *independent* of t because A is conserved. Specifically, the Heisenberg equations of motion show that

$$P^A_j(t) = \mathrm{e}^{iHt/\hbar} P^A_j(0) \mathrm{e}^{-iHt/\hbar} = P^A_j(0) \tag{2.7}$$

because A commutes with H. Thus

$$P^A_{j_m}(t_m)P^A_{j_l}(t_l) = \delta_{j_l j_m} P^A_{j_l}(t_l) \tag{2.8}$$

and the probabilities $p(j_m, j_l)$ which follow from (2.4) vanish if $j_m \neq j_l$. Since the left hand side of (2.5) is the sum of positive numbers, they must vanish individually. We have

$$p(\alpha_c, j_m, \alpha_b, j_l, \alpha_a) = 0 \quad , \quad j_m \neq j_l \tag{2.9}$$

and the probability is zero for any non-vanishing change in the value of a quantity that commutes with the Hamiltonian. Energy in particular is conserved*.

While exact decoherence ensures exact conservation of the quantities commuting with the Hamiltonian, the following argument shows that it also limits the alternatives which can be included in a set of histories defining that conservation. Suppose the set of histories represented by (2.2) exactly decoheres. Then every coarse graining of it must also exactly decohere and in particular the set represented by

$$C_{j_m \alpha_b j_l} = P^A_{j_m}(t_m) C^b_{\alpha_b} P^A_{j_l}(t_l) \ . \tag{2.10}$$

is exactly decoherent. According to the result of Griffiths derived above, the probability of a change in the value of A is zero:

$$p(j_m, \alpha_b, j_l) \equiv Tr(C_{j_m \alpha_b j_l} \rho C^\dagger_{j_m \alpha_b j_l}) = 0 \quad , \quad j_l \neq j_m \ . \tag{2.11}$$

Write the density matrix ρ in the basis in which it is diagonal as

$$\rho = \sum_\lambda |\lambda\rangle \pi_\lambda \langle\lambda| \tag{2.12}$$

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* The argument for conservation depends only on the consistency of the set of histories. Although we introduced it by discussing weak decoherence which implies consistency, the argument could proceed directly from (2.5).

for positive probabilities π_λ. In that basis, (2.11) reads

$$\sum_{\lambda'\lambda} \pi_\lambda |\langle\lambda|C_{j_m\alpha_b j_l}|\lambda'\rangle|^2 = 0 \quad , \quad j_l \neq j_m \tag{2.13}$$

so that

$$C_{j_m\alpha_b j_l}|\lambda\rangle = 0, \quad \text{if} \quad j_m \neq j_l \quad \text{and} \; \pi_\lambda \neq 0. \tag{2.15}$$

Thus, $C_{j_m\alpha_b j_l}$ for $j_m \neq j_l$ must vanish on the subspace $\mathcal{S}_\rho$ of initial states with non-vanishing probabilities in the initial density matrix. In particular

$$C_{j_m\alpha_b j_l}\rho = 0 \quad , \quad j_m \neq j_l \; . \tag{2.16}$$

The result (2.15) can be used to show that $C^b_{\alpha_b}$ must commute (at least on the subspace $\mathcal{S}_\rho$) with a an operator $\tilde{A}$ formed from A by smearing over the ranges $\{\Delta_j\}$. To do so, choose from each range Δ_j some value a_j. From (2.15) and (2.10) we can write

$$(a_{j_m} - a_{j_l})P^A_{j_m}(t_m)C^b_{\alpha_b}P^A_{j_l}(t_l)|\lambda\rangle = 0, \quad \text{when} \; \pi_\lambda \neq 0 \tag{2.17}$$

now holding for all values of j_m and j_l. Consider the 'smeared' operator

$$\tilde{A}(t_m; \{\Delta_j, a_j\}) \equiv \sum_{j_m} a_{j_m} P^A_{j_m}, \tag{2.18}$$

where the notation indicates the the smearing depends on the choice of intervals $\{\Delta_j\}$. This operator is actually time independent and, for simplicity, we drop all labels on $\tilde{A}$ below.

By summing (2.17) over j_m and j_l we have

$$[\tilde{A}, C^b_{\alpha_b}]|\lambda\rangle = 0, \quad \text{when} \quad \pi_\lambda \neq 0 \tag{2.19}$$

and in particular

$$[\tilde{A}, C^b_{\alpha_b}]\rho = 0 \; . \tag{2.20}$$

The result (2.20) shows that, given $[A, H] = 0$, the only operators permitted in an exactly decohering set of histories which test the conservation of A in the manner specified by the choice of intervals $\{\Delta_j\}$ must commute with an operator $\tilde{A}(t; \{\Delta_j, a_j\})$ formed by smearing A over these intervals. There are thus a variety of constraints, depending on how these ranges are chosen.

When A has a point spectrum with no accumulation points, the intervals $\{\Delta_j\}$ are naturally taken to contain only one eigenvalue. Then, $\tilde{A} = A$. When the spectrum of A has accumulation points, we may investigate histories in which its value is defined with arbitrary precision by using small intervals $\{\Delta_j\}$. In the limit of infinitesimal intervals, we may again replace $\tilde{A}$ in (2.20) with A itself. In this limit, the only permissible alternatives $C^b_{\alpha_b}$ in a set of exactly decohering histories of the form (2.2) that test the conservation of A are alternatives of quantities that effectively commute with A in the initial condition ρ.

For some conserved quantities, the limitations of the condition (2.20) are of little consequence. For example, if the conserved quantity is the total number of particles in a non-relativistic system, then many interesting operators commute with it and (2.20) is easily satisfied. However, in other cases, the restriction is more severe. Consider in particular the case of the total energy.

The Hamiltonian corresponding to the total energy of course commutes with itself. In the limit of infinitesimal spectral intervals, the result (2.20) is sufficient to show that histories of the form (2.2) that test conservation of energy can only exhibit trivial dynamics. Consider the case when $\{C^b_{\alpha_b}\}$ is a set of histories of alternative ranges of values of a quantity B at two times t_1, t_2 such that $t_l < t_1, t_2 < t_m$. Then,

$$C^b_{\alpha_b} = P^B_{\alpha_2}(t_2) P^B_{\alpha_1}(t_1) \ . \tag{2.21}$$

From (2.20) we conclude

$$[H, P^B_{\alpha_1}(t_1)]\rho = 0 \quad \text{and} \quad [H, P^B_{\alpha_2}(t_2) P^B_{\alpha_1}(t_1)]\rho = 0. \tag{2.22}$$

Eq.(2.22) and the Heisenberg equation of motion

$$P_\alpha^B(t) = \mathrm{e}^{iHt/\hbar} P_\alpha^B(0) \mathrm{e}^{-iHt/\hbar} \tag{2.23}$$

are enough to show that for $t_l < t_1 < t_2 < t_m$

$$P_{\alpha_2}^B(t_2) P_{\alpha_1}^B(t_1) \rho = 0 \quad , \quad \alpha_1 \neq \alpha_2 \tag{2.24}$$

and this implies that there is *zero* probability of any change in the value of B for the histories in which changes in the energy can be defined.

If only finite sized energy intervals are considered, some change of B with time will be allowed. A description of the operators satisfying (2.20) would then depend on the details of the dynamics of the system and the choice of the intervals $\Delta_{j_m}^m$. However, the allowed change must vanish in the limit in which either the intervals become small or we require (2.20) to hold independently of the choice of intervals $\Delta_{j_m}^m$.

To appreciate the strength of this result, imagine a model universe consisting of a large box of non-relativistic particles interacting by potentials. Sets of histories describing just changes in the total energy of the model universe, say,

$$C_\alpha = P_{j_2}^H(t_2) P_{j_1}^H(t_1) \tag{2.25}$$

always exactly decohere. That is because the $P_j^H(t)$'s are constant in time [*cf* (2.7)] so that the $\{C_\alpha\}$ are a set of orthogonal *projections.* The cyclic property of the trace in (2.3) shows that histories consisting of orthogonal projections always decohere exactly. Further, as a consequence of the orthogonality of the projections in (2.25) for different ranges of the total energy, the C_α representing non-vanishing changes in the total energy vanish identically. The probability of a change in the total energy is exactly zero. However, the result (2.24) shows that it is not possible to fine grain the histories (2.25) to include alternatives of any quantity other than the total energy without the values of that quantity being constant in time with probability one. Thus, were we part of such a closed system, we could not consider a set of histories that describe the changes in our everyday lives and at the same time describe the changes in time of the precise values of the total energy.

The above discussion considered tests of conservation laws in exactly decohering sets of histories. It is true that it might be more realistic to consider approximately decohering sets of histories and the approximate violations of conservation laws that could be expected to occur. This is an interesting problem. However, more important consequences can be derived for quantities like electric charge and mass by including their couplings to the gauge fields that we have so far neglected. This coupling provides a natural mechanism to guarantee that *every* interesting operator will satisfy (2.20). We show this in the next two sections.

III Electric Charge.

The most firmly established examples of absolutely conserved quantities are electric charge and total energy. Both are coupled to gauge fields and the conservation of each is connected with a fundamental symmetry principle. These fundamental symmetry principles limit the sets of decoherent histories which can describe changes in quantities that commute with the Hamiltonian and the values of the probabilities of these changes. The simplest case is electric charge which we discuss in this section.

The quantum theory of the electromagnetic field can be conveniently studied in temporal, $A_0(x)=0$, gauge where $A_\mu(x)$ are the components of the potential. The states may be represented by vectors in the fermion Fock space with components that are functionals of the vector potential $\vec{A}(\mathbf{x})$. Thus formulated on a space which contains electromagnetic degrees of freedom beyond the true physical ones, the theory has a constraint represented by the operator

$$\mathcal{C}(\Lambda)=\int d^3x\Lambda(\mathbf{x})[\nabla\cdot\vec{E}(\mathbf{x})-\rho(\mathbf{x})] \tag{3.1}$$

where $\rho(\mathbf{x})$ is the charge density and $\mathbf{x}$ denotes the three spatial coordinate of some particular Lorentz frame. The function $\Lambda(\mathbf{x})$ is the parameter of the gauge transformation which is generated by $\mathcal{C}(\Lambda)$ via

$$\delta A_i(\mathbf{x}) = -\frac{i}{\hbar}[\mathcal{C}(\Lambda), A_i(\mathbf{x})] = \partial_i\Lambda(\mathbf{x}) \tag{3.2}$$

More generally, $\mathcal{C}(\Lambda)$ generates gauge transformations for an arbitrary quasilocal (see, e.g. [6]) operator $\mathcal{O}$ that vanishes sufficiently fast outside some bounded region of space (which we will call the effective support) according to

$$\delta\mathcal{O} = -\frac{i}{\hbar}[\mathcal{C}(\Lambda), \mathcal{O}] \ . \tag{3.3}$$

Physical, gauge invariant, quasilocal operators must commute with the constraint

$$[\mathcal{C}(\Lambda), \mathcal{O}_{phys}] = 0 \tag{3.4}$$

and physical, gauge invariant states are annihilated by the constraint.

As a particular case of (3.4) we may take $\Lambda = const$. Then Gauss' law may be applied to reduce the $\nabla \cdot \vec{E}$ term in (3.1) to a surface term with spacelike separation from the effective support of $\mathcal{O}_{phys}$ yielding the result*

$$[Q, \mathcal{O}_{phys}] = 0 \tag{3.5}$$

where

$$Q = \int d^3x\rho(x) \tag{3.6}$$

is the total charge operator. Quasilocal physical quantities therefore commute with the total charge*.

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* This derivation is naive form a rigorous point of view. For a better one see [7].

* In representing physical quantities by operators in this way we are considering, as usual, quantities defined at one moment of time. For more general spacetime alternatives see [8], Chap.VI.

An exhaustive set $\{\Delta_\alpha\}, \alpha = 1, 2, \ldots$ of ranges of a gauge invariant quantity $\mathcal{O}$ define a set of alternatives for a closed system at a moment of time. These are represented by a set of Heisenberg picture projection operators $\{P^{\mathcal{O}}_\alpha(t)\}$. Sets of histories for the closed system may be defined by giving a series of such sets at a sequence of times $t_1, \ldots, t_n$. The individual histories correspond to particular sequences of alternatives $(\alpha_1, \ldots, \alpha_n)$ and are represented by the corresponding chains of projectors as in (1.5). More general examples of histories can be obtained by partitioning such sequences into classes $\{c_\alpha\}$ represented by a set of class operators $\{C_\alpha\}$ that are sums of the chains in the class. Thus, a gauge invariant set of histories is generally represented by a set of class operators of the form

$$C_\alpha = \sum_{(\alpha_1 \ldots \alpha_n) \in \alpha} P^{\mathcal{O}_n}_{\alpha_n}(t_n) \ldots P^{\mathcal{O}_1}_{\alpha_1}(t_1) \ . \tag{3.7}$$

When electromagnetism is formulated in this way, with redundant as well as true physical degrees of freedom, the decoherence functional is not given by a formula like (2.2) in which the Hilbert space is a space of wave functionals of the vector potential $\vec{A}(\mathbf{x})$. Rather, it is given by that formula utilizing the Hilbert space of functionals of the true physical degrees of freedom; that is, just the transverse part $A^T(\mathbf{x})$ of the vector potential. The class operators (3.7) must first be reduced to class operators $\{C^T_\alpha\}$ on the Hilbert space of physical degrees of freedom by integrating their matrix elements over the redundant (longitudinal) degrees of freedom with appropriate gauge fixing conditions. The details of this process are not important for us**; it suffices to note that the decoherence functional may be written

$$D(\alpha', \alpha) = Tr_T(C^T_{\alpha'} \rho C^{T\dagger}_\alpha) \tag{3.8}$$

where ρ is a density matrix in the Hilbert space of the matter degrees of freedom and the true physical degrees of freedom of the electromagnetic field.

** The construction is standard, but for more details in the present notation see [8].

We next consider fine-graining a set of histories (3.7) by including alternative values of the total electric charge Q at a sequence of times $t'_k, k = 1, \dots, m$. We consider, for simplicity the same set of ranges $\{\Delta_\beta\}, \beta = 1, 2, 3, \dots$ of Q at each of these times and let $\{P^Q_{\beta_k}(t'_k)\}$ be the projections of the total charge operator onto them. The class operators for such a finer grained set are

$$C_{\alpha\beta} = \sum_{(\alpha \dots \alpha_n)\in\alpha} P^{\mathcal{O}_n}_{\alpha_n}(t_n) \dots P^Q_{\beta_m}(t'_m) \dots \dots P^Q_{\beta_1}(t'_1) \dots P^{\mathcal{O}_1}_{\alpha_1}(t_1) \tag{3.9}$$

where the $P^Q_{\beta_k}(t'_k)$ have been inserted at the positions dictated by time ordering.

The projections $\{P^Q_\beta(t')\}$ have two important properties: First, they commute with all gauge invariant quantities as a consequence of (3.5), and therefore, in particular

$$[P^{\mathcal{O}_k}_{\alpha_k}(t_k), P^Q_{\beta_l}(t'_l)] = 0 \ . \tag{3.10}$$

Second, they are conserved

$$[H, P^Q_{\beta_l}(t'_l)] = 0 \tag{3.11}$$

and therefore are independent of the times t'_l. Eq.(3.10) means all P^Q's may be commuted to the right or left in (3.9) and (3.11) means that the class operator is zero unless all the β_l are the same

$$\begin{aligned} C_{\alpha\beta} &= \delta_{\beta_n\beta_1} \dots \delta_{\beta_2\beta_1} P^Q_{\beta_1} C_\alpha \\ &= \delta_{\beta_n\beta_1} \dots \delta_{\beta_2\beta_1} C_\alpha P^Q_{\beta_1} \end{aligned} \tag{3.12}$$

The first of the relationships (3.12) shows that, for *any* set of histories, the alternative values of the charge always decohere *exactly*. That is because, as a consequence of the cyclic property of the trace, the decoherence functional $D(\alpha', \beta'; \alpha, \beta)$ is always proportional to $\delta_{\beta'_1\beta_1}$. The δ-functions in (3.12) thus ensure that histories in which the total charge fluctuates have probability zero. Total charge decoheres exactly and is exactly conserved.

Allowing approximate decoherence does not permit non-zero probabilities for changes in Q.

The restrictions derived in Section II on histories that include alternative values of the total charge are still valid. The alternatives in a decohering set of histories must commute with the total charge. However, *all* quasilocal physical alternatives satisfy this condition as a consequence of gauge invariance. It is therefore no restriction at all.

In general, if a set of histories $\{C_\alpha\}$ decoheres, then the finer-grained set (3.9) that includes alternatives values of the charge does not necessarily decohere. However, it does in one interesting and natural case. That is when the initial condition has a definite, fixed total charge q.

$$Q\rho = \rho Q = q\rho \ . \tag{3.13}$$

Then when $C_{\alpha\beta}$, in the form of the second of (3.12), acts on ρ there will be a non zero result only for that interval Δ_β, which contains q. $D(\alpha'\beta';\alpha\beta)$ is thus non-zero only when both β'_1 and β_1 have this value and is therefore diagonal. The finer-grained set $\{C_{\alpha\beta}\}$ decoheres if the set $\{C_\alpha\}$ does. It follows that when the universe has a definite value of the total charge*, we may always fine-grain any set of decoherent histories to ask about the total charge without disturbing decoherence and receive from the quantum mechanics of closed systems the reassuring answer that it is conserved with probability one.

Finally, note that all of our results follow directly from (3.5). Any operator with the property satisfied by Q in this equation is said to be superselected (see, e.g. [6]). In specific restricted models, this occurs for quantities like baryon number, lepton number, and non-Abelian charges as well as electric charge. Thus, alternative values of superselected charges always decohere exactly and are exactly conserved. Furthermore, when the total state has

* As it does for instance in the "no-boundary" [9] initial condition where the total charge is zero because the universe is spatially closed.

a definite value of such a charge, projections onto its eigenvalues may be added to any set of histories without affecting decoherence.

IV Total Mass.

Energy universally couples to spacetime curvature which itself can carry energy in the form of gravitational waves. As a consequence, a realistic classical discussion of the total energy of a closed system, which in relativity is the same thing as its total mass, must be carried out in the context of general relativity and a discussion of possible quantum fluctuations in the total energy in the context of quantum gravity.

There is no local definition of mass-energy in general relativity because a general spacetime does not exhibit a time-translation symmetry. Neither is it possible to define the total mass of a spatially closed cosmology except by assigning it the value zero in which case its conservation is trivial. Conservation of energy becomes an interesting issue in asymptotically flat spacetimes possessing *asymptotic* time translation symmetries enabling the *total* mass of the system to be defined.

For asymptotically flat spacetimes, the mass on a spacelike surface can be determined from the asymptotic behavior of the spatial metric on that surface. Using coordinates which asymptotically become rectangular Minkowski coordinates at spatial infinity the deviations from flat space must, at the very least, fall off as

$$g_{\alpha\beta} = \eta_{\alpha\beta} + \frac{M_{\alpha\beta}(t,\theta,\phi)}{r} + O\left(\frac{1}{r^2}\right) . \tag{4.1}$$

Here, $\eta_{\alpha\beta}$ is the flat metric in rectangular Minkowski coordinates, $\eta_{\alpha\beta} = \mathrm{diag}(-1,1,1,1,)$, $t = x^0$ and the polar coordinates (r,θ,ϕ) are connected to the rectangular coördinates (x^1,x^2,x^3) by the usual relations, *e.g.* $x^1 = r\sin\theta\cos\phi$.

Consider a spacelike surface that asymptotically is a surface of constant t. The ADM total mass [10] is defined by evaluating the following integral on a two-surface at large r

$$M(t) = \frac{1}{16\pi} \lim_{r\to\infty} \int_t dS_j (\partial_k g_{kj} - \partial_j g_{kk}) \ . \tag{4.2}$$

Here ∂_i is the flat-space gradient and we have followed the usual convention of indicating a summation in asymptotic expressions by repeated lower indices. The asymptotic behavior of the metric (4.1) ensures that $M(t)$ is finite.

Whether total mass-energy is conserved in a quantum theory of asymptotically flat spacetimes depends on the probabilities of decoherent histories that describe differing values of $M(t)$ on different spacelike surfaces. There are, of course, a variety of approaches to a quantum theory of spacetime. We shall analyze the question in the sum-over-histories generalized quantum theory of spacetime geometry. A generalized quantum theory is specified by three elements: (1) The fine-grained histories, which here are a class of four-dimensional metrics and matter field configurations. The metrics $g_{\alpha\beta}(x)$ are asymptotically flat at least in the sense of (4.1) but with possibly more restrictive conditions to be discussed below and dwell on a manifold with two spacelike boundaries σ' and σ'' representing the "endpoints" of the history. To keep the notation manageable we shall indicate only a single matter field $\phi(x)$. (2) The allowed coarse-grainings, which here are *diffeomorphism invariant* partitions of the fine-grained histories into exclusive classes $\{c_\alpha\}, \alpha = 1, 2, \cdots$ called coarse-grained histories. (3) A decoherence functional defining the measure of interference between pairs of coarse-grained histories. The precise details of the construction of this decoherence functional will not be important for us. Its form is similar to (2.3) but with notions of ρ, Tr, etc. appropriate to gravity. It is the form of the class operators corresponding to coarse-grained histories that is important for the present discussion of the conservation of the total mass. These class operators act on the space of wave-functionals defined on the space of three-metrics $h_{ij}(\mathbf{x})$ and spatial matter field configurations $\chi(\mathbf{x})$ on a spacelike surface. The matrix elements of the class operator corresponding to a diffeomorphism invariant class c_α of asymptotically flat four geometries and field configurations are defined by the sum-over-fine-grained-histories:

$$\langle h''_{ij}, \chi'' || C_\alpha || h'_{ij}, \chi' \rangle = \int_{[(h',\chi'),c_\alpha,(h'',\chi'')]} \delta g \delta\phi \, \exp\{iS[g(x),\phi(x)]/\hbar\} \ . \tag{4.3}$$

Here, $h'_{ij}(\mathbf{x})$ and $\chi'(\mathbf{x})$ are the induced metrics and matter field configurations on the boundary σ'. There are similar definitions on σ''. $S[g,\phi]$ is the action for geometry coupled to matter fields. The sum is over four-metrics $g_{\alpha\beta}(x)$ and four dimensional field configurations $\phi(x)$ which are in the diffeomorphism invariant class c_α and match the prescribed conditions on σ' and σ''. Of course, the expression (4.3) is only formal and must be augmented by gauge fixing machinery, regularization procedures, *etc* to make sense, but its form will be sufficient for the level of argument we are able to give. Further details can be found in, for example [8].

With these preliminaries in hand we may return to the issues of the conservation of total ADM mass at spatial infinity and whether histories that define possible changes in the total mass are limited to trivial dynamics as they were in the simple model of Section II which neglected gravitation. To calculate the probability of a fluctuation in the mass, we must consider partitions of the set of fine-grained histories into classes by ranges of the value of the total mass $M(\sigma)$ on at least two different spacelike surfaces σ_1 and σ_2, in addition to whatever other alternatives define the classes under consideration. Such histories are the analog of those represented by (2.2) when A is the total energy, H, in the non-gravitational case. When such sets decohere, the issue of conservation of ADM mass is then the question of whether the probability is zero for those with $M(\sigma_1) \neq M(\sigma_2)$. For this case, the arguments of Section III are not satisfactory as quasilocal diffeomorphism invariant operators are difficult to construct – in fact, because there are no *local* diffeomorphism invariant operators for gravity, strict use of the definition in [6] shows that there are *no* quasilocal invariant operators at all. To find a more satisfying argument we must look more closely at what is meant by "asymptotically flat".

Penrose's notion of conformal completion [11,12] gives a standard definition of a spacetime which is asymptotically flat*. A consequence of this definition is that asymptotically flat metrics have a more restricted asymptotic behavior than that given by (4.1). In particular the Riemann tensor must decay at large r as

$$R_{\alpha\beta\gamma\delta} = O\big(1/r^3\big). \tag{4.4}$$

It is not difficult to show that, in any $3+1$ decomposition of spacetime into space and time, this implies

$$\dot{M}_{ij} = 0 \tag{4.5}$$

where a dot denotes a time derivative and Roman indices range over spatial directions. This means that the ADM mass, as defined by (4.2) is constant in time. The conservation of ADM mass in this context does not follow from the equation of motion, but from the definition of an asymptotically flat spacetime. Of course, the asymptotically flat context would be uninteresting except that solutions of this form *do* exist. The finite propagation velocity of gravitational radiation ensures that any solution with suitably localized initial data will be asymptotically flat.

The Penrose diagram for the conformally completed asymptotically flat spacetime makes the reason for this "conservation" intuitively clear. Spacelike infinity is a single two-sphere where all spacelike surfaces terminate. A common value of the ADM mass is therefore shared by all.

Were we to use conformal completion to define the asymptotically flat metrics which enter into the sum-over-histories (4.1) the question of conservation of total mass would be

* For a lucid review see Ashtekar[13]

trivial. Only geometries with constant total mass contribute to the sum, therefore partitions into classes with different masses on different spacelike surfaces would be vacuous.

However, while the conservation of total mass at spatial infinity is trivial, the dynamics permitted in histories that define this conservation is not. In the model without gauge fields discussed in Section II, only alternatives of quantities that effectively commuted with the total energy were permitted in exactly decohering sets of histories which also described changes in the total energy. However, in the presence of the gravitational field, the analog of (2.17) which led to that result is trivially satisfied for *any* diffeomorphism invariant partition of the fine-grained histories, regardless of whether it is associated with projections onto eigenvalues of quasi-local operators. That is because there are no fine-grained histories at all with changes in the total mass. We therefore expect that in generalized quantum theory we are permitted arbitrary sets of physical histories that also describe changes in the total mass. If any set of alternatives decoheres, we may always consider the finer graining which in addition describes changes in the total mass. If that finer graining continues to decohere, the alternatives referring to the total mass decohere exactly. Total mass, or total energy which is the same thing, is conserved with probability one.

The above discussion was carried out using the conformal completion definition of asymptotic flatness. However, from the perspective of quantum gravity it appears more natural to define asymptotic flatness from a property of the *action* rather than from a notion of conformal completion. We now show that if the sum-over-histories in (4.3) is restricted to a class of metrics with the fall-off (4.1) that (1) have finite action and (2) are invariant under diffeomorphisms, then the ADM mass is conserved. To understand this it is sufficient to look at the action for pure gravity.

The action for gravity on a domain of spacetime $\mathcal{M}$ is

$$(16\pi)S_E[g] = \int_{\mathcal{M}} d^4x\sqrt{-g}R + 2\int_{\partial\mathcal{M}} d^3x\sqrt{h}K \tag{4.6}$$

where R is the scalar curvature and K is the extrinsic curvature scalar of the boundary of $\mathcal{M}$. In order to discuss the properties of metrics at spatial infinity, it is useful to consider a standard $3+1$ decomposition of the metric

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \tag{4.7}$$

which need only hold near spatial infinity for our purposes. Consider the action for a region of spacetime lying between two spacelike surfaces of constant t and bounded by a timelike surface $\partial\mathcal{M}_s$ near infinity. This may be written as

$$\begin{aligned}(16\pi)S_E[g] = &\int dt \int d^3x N h^{1/2}[K_{ij}K^{ij} - K^2 +^3 R] \\ &- \int_{\partial\mathcal{M}_s} d\Sigma_i(-2KN^i + 2D^i N) + 2\int_{\partial\mathcal{M}_t} d^3x\sqrt{\tilde{h}}(\tilde{K} - \tilde{K}_0)\end{aligned} \tag{4.8}$$

where K_{ij} is the extrinsic curvature of surfaces of constant t,

$$K_{ij} = \frac{1}{2N}\Big[-\dot{h}_{ij} + D_{(i}N_{j)}\Big] \ , \tag{4.9}$$

D_i being the derivative in the surface, $\tilde{K}_{ij}$ is the extrinsic curvature of $\partial\mathcal{M}_t$, and the tilde indicates that a quantity is to be evaluated for a timelike surface.

The general form of the metric (4.1) is not sufficient to ensure the convergence of the action (4.8). Eq.(4.1) implies that the asymptotic behavior of ${}^3R$ is $O(1/r^3)$ but the asymptotic behavior of K_{ij} is

$$K_{ij} = -\frac{1}{2r}\dot{M}_{ij} + O(\frac{1}{r^2}) \ . \tag{4.10}$$

If we evaluate the volume term in the action (4.8) out to a large radius r_l, the coefficient of the leading term as $r_l \to \infty$ is

$$r_l^2 \int dt \int d\Omega[(\dot{h}_{ij})^2 - (\dot{h}_{kk})^2] = \int dt \int d\Omega[(\dot{M}_{ij})^2 - (\dot{M}_{kk})^2] \tag{4.11}$$

where $d\Omega$ is an element of solid angle at infinity. Thus metrics in the class (4.1) must be further restricted so that the right hand side of (4.11) vanishes in order to ensure finite action. If we assume that this should hold for any choice of the time interval between the boundary surfaces we must have

$$r_l^2 \int d\Omega[(\dot{h}_{ij})^2 - (\dot{h}_{kk})^2] = \int d\Omega[(\dot{M}_{ij})^2 - (\dot{M}_{kk})^2] = 0 \ . \tag{4.12}$$

This is not enough to show that the ADM mass is constant in time, but when coupled with the requirements of diffeomorphism invariance it will be.

The asymptotic behavior of (4.1) refers to a particular decomposition of the spacetime into space and time, and the condition (4.12) ensures that there is no linear divergence of the action when evaluated between two constant time surfaces in that decomposition. However, since the class operators (4.3) are to be defined by integrals over diffeomorphism invariant partitions, the notion of asymptotic flatness and of finiteness of the action must be independent of the $3+1$ decomposition. In particular it must be invariant under diffeomorphisms which Lorentz transform the asymptotic slices. This leads to stronger conditions than (4.12) as we shall show.

Consider infinitesimal diffeomorphisms (gauge-transformations)

$$g_{\alpha\beta}(x) \rightarrow g_{\alpha\beta} + \nabla_{(\alpha}\xi_{\beta)}(x) \ , \tag{4.13}$$

and in particular those which asymptotically correspond to Lorentz boosts

$$\begin{aligned} \xi^i &\approx v^i t + d^i(t,\theta,\phi) + O(1/r) \ , \\ \xi^0 &\approx v_i x^i + O(1) \ . \end{aligned} \tag{4.14}$$

Lorentz boosts preserve the asymptotic behavior (4.1). The supertranslations d^i, however, must be independent of time to preserve (4.1), as substitution into (4.7) will show. The covariant components of $\xi_i(x)$ relevant for the transformation of the $h_{ij}(x)$ are thus

$$\xi_i \approx v_i t + d_i(\theta,\phi) + N_i(x)(v_j x^j) + O(1/r) \tag{4.15}$$

displaying explicitly the $O(r)$ and $O(1)$ terms. Since $N_i(x) \approx s_i(t,\theta,\phi)/r$ the third term is of $O(1)$. The spatial part of the metric important for the asymptotic form of the metric thus transforms as

$$h_{ij} \to \delta_{ij} + \frac{M_{ij}}{r} + \partial_{(i}\xi_{j)} + O(\frac{1}{r^2}) \tag{4.16}$$

with $\xi_j(x)$ of the form (4.15).

Diffeomorphism invariance requires that the condition (4.11) be enforced for $h_{ij}(x)$ of the form (4.16) with $\xi_i(x)$ given by (4.15). The first term in (4.15) does not change the spatial metric. In determining the effect of the rest of (4.15) on $h_{ij}(x)$ the boost parameter v_i is arbitrary. But it is also important to note that the $s_i(t,\theta,\phi)$ determining $N_i(x)$ is arbitrary; s_i merely defines how the spacetime is sliced internally, consistent with a given asymptotic slicing. We must therefore enforce the condition (4.12) for $h_{ij}(x)$ of the form (4.16) with arbitrary $\xi_i(x)$ of $O(1)$. One further invariance should be enforced. In (4.11) we evaluated the action inside spheres of constant r_l and considered the limit $r_l \to \infty$. The same results should hold for arbitrary shaped surfaces $r_l = r_l(\theta,\phi) = Rf(\theta,\phi)$ as $R \to \infty$. To first order in $\xi_i(x)$, the gauge transformed condition (4.12) becomes the condition that the linear divergence in R of

$$\int^{r_l(\theta,\phi)} r^2 dr \int d\Omega [u_{ij}(x)\partial_i \xi_j(x)] \tag{4.17}$$

vanish. Here we have abbreviated

$$u_{ij}(x) = \dot{h}_{ij}(x) - \delta_{ij}\dot{h}_{kk}(x) \tag{4.18}$$

and assumed, as discussed above, that the $O(1)$ part of $\xi_i(x)$ is an arbitrary function of (t,θ,ϕ). Integrating (4.17) by parts and retaining only the leading terms in large R in the resulting condition following from the arbitrary form of $\xi_i(x)$ we have

$$R[-Rf(\theta,\phi)\partial_i u_{ij}(x) + n_i(\theta,\phi)u_{ij}(x)] = 0 \tag{4.19}$$

for large R where n_i is the normal to the bounding surface proportional to $\partial_i f$. Since f is arbitrary (4.19) can be satisfied only if both terms vanish and, in particular, if $Ru_{ij}(x) = 0$. That implies

$$\dot{M}_{ij} = \delta_{ij}\dot{M}_{kk} \ . \tag{4.20}$$

This condition together with (4.12) is enough to guarantee

$$\dot{M}_{ij}(t,\theta,\phi) = 0 \ . \tag{4.21}$$

The result (4.21) is enough to guarantee that the Riemann tensor falls off as $O(1/r^3)$ as in (4.4) and ensure the conservation of the ADM mass. To see that, simply note that from (4.2), the ADM mass is determined by the coefficients M_{ij} all of whose time derivatives vanish.

Thus restricting attention to metrics with the minimal asymptotic behavior (4.1) for which the action is finite and diffeomorphism invariant means restricting to metrics for which the Riemann tensor falls off as $O(1/r^3)$ and for which the ADM mass is constant. It is an interesting question whether the above rather clumsy argument could be pushed further and whether there is full equivalence between asymptotic flatness defined by finiteness and diffeomorphism invariance of the gravitational action and asymptotic flatness defined by conformal completion. It would be of special interest to investigate and compare the conditions necessary to define total angular momentum at spatial infinity, a quantity that we have not touched upon.

Thus, whether asymptotic flatness is defined by conformal completion or by the behavior (4.1), finiteness of the action, and covariance, the result is the same for the ADM mass. It is conserved in each history. The set of fine-grained histories includes histories with differing values of the ADM mass but within each history it does not vary. Arbitrary diffeomorphism invariant alternatives may therefore be considered in addition to those necessary to describe the conservation of total mass of a closed system and, if all alternatives decohere, total mass, which is the same as the total energy, is conserved.

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