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Likelihood Function for Single Top Search with 162 pb^{-1}

Catalin Ciobanu[c], Thomas Junk[c], Thomas Müller[a], Pierre Savard[b], Bernd Stelzer[b], Wolfgang Wagner[a], Thorsten Walter[a]

[a] *Universität Karlsruhe*
[b] *University of Toronto*
[c] *University of Illinois, Urbana-Champaign*

Abstract

We describe the new likelihood function for our single top search with 162 pb^{-1}. The new function directly includes systematic uncertainties and thereby eliminates the need to convolute the marginalized likelihood function with Gaussian.

In previous single top searches at CDF systematic uncertainties were included into the calculation of the upper limit by convoluting the reduced likelihood function $L^*(\beta)$ with a Gaussian whose width was given by the quadratic sum of all systematic uncertainties [1, 2, 3]. We call this method the *convolution method*. It dates back to early $t\bar{t}$ searches in CDF [4]. In the meanwhile several conceptual problems concerning this technique were brought up:

1. In pure Bayesian theory all nuisance parameters, including the systematic errors, should occur in the likelihhood function. The convolution technique leads to a two-step process which assumes a factorization between background cross-sections and systematic uncertainties. The nuisance parameters are thereby somewhat artificially divided into two classes. The convolution technique can be seen as an approximation to a full Bayesian likelihood treatment. However, it is difficult to map out the limitation and the region of applicability of this approximation.

2. Correlations between the uncertainties of different physics processes are not taken into account in the convolution technique.

These points of criticism motivate the definition of a new likelihood function which will address these issues.

1 Separate t- and s-Channel Search

Definition of variables:

1. We consider 4 physics processes:

   $j =$ index over processes
   $j = 1:$ t-channel, $j = 2:$ s-channel, $j = 3:$ $t\bar{t}$, $j = 4:$ non-top.

2. The cross-section of process j is σ_j. In the likelihood function we use the parameter β_j, the cross-section normalized to its SM prediction: $\beta_j = \sigma_j/\sigma_{SM,j}$.

3. The effective acceptance of process j is ν_j. It is the event detection efficiency multiplied with the integrated luminosity.

4. We fit the $q \cdot \eta$ histogram in the 1-tag-bin and the number of events in the double-b-tag bin. The corresponding acceptances are: ν_{j1} and ν_{jd}. The bins of the $q\eta$ histogram are indexed with k.

5. The normalized content of bin k of the template histogram for process j is α_{jk}.

6. We consider 6 effects which cause systematic uncertainties in acceptance and in the shape of the template histograms: jet energy measurement, PDF, ISR, FSR, top mass, signal generator. The sources of systematic uncertainties are indexed with i. The relative acceptance uncertainties due to these sources are named: ϵ_{ji}. The relative uncertainties in the bin content of bin k of the template histograms are called κ_{jik}. Since some errors are asymmetric, we have to introduce positive and negative uncertainties: ϵ_{ji+}, ϵ_{ji-}, κ_{jik+} and κ_{jik-}.

7. Two other uncertainties influence only the acceptance: (1) the error on the event detection efficiency (b-tag scale factor, lepton identification efficiency etc.) and (2) luminosity. Those are added in quadrature and they enter as ϵ_7 in the likelihood.

8. So in total we take $S = 7$ systematic effects into account.

9. Gaussian functions of variable x with mean x_0 and standard deviation σ are denoted $G(x, x_0, \sigma)$.

10. H(x) denotes the Heaviside step function.

11. The variation in strength of a systematic effect i is measured with the variable δ_i.

12. The expected mean value of bin k for the $q \cdot \eta$ histogram is μ_k, for the double-b-tag-bin μ_d.

13. The data observed in bin k of the $q \cdot \eta$ histogram are denoted n_k, the data in the 2-tag-bin n_d.

14. The cross-section uncertainties are denoted as Δ_j. For the non-top background this is a rate uncertainty, since this is a sum of several processes.

15. s denotes the index of the signal process, which is $s = 1$ for the t-channel search and $s = 2$ for the s-channel search. If $s = 1$, we define $t = 2$; if $s = 2$, we define $t = 1$.

The likelihood: Using the above definitions we write down the likelihood function L:

$$
\begin{aligned}
L_s &= L_s(\beta_1, \dots, \beta_4; \delta_1, \dots, \delta_S) \\
&= \left\{ \prod_{k=1}^{B} \frac{e^{-\mu_k} \cdot \mu_k^{n_k}}{n_k!} \right\} \cdot \frac{e^{-\mu_d} \cdot \mu_d^{n_d}}{n_d!} \cdot \prod_{j=1, j\neq s}^{4} G(\beta_j, 1.0, \Delta_j) \cdot \prod_{i=1}^{S} G(\delta_i, 0, 1)
\end{aligned}
$$

The first factors are Poisson terms for the bins of the $Q \cdot \eta$ histogram (index k) and the double-tag-bin (index d). The second set of factors are Gaussian integration kernels which constrain the nuisance parameters to the expectation. The expected mean in bin k is:

$$
\begin{aligned}
\mu_k &= \sum_{j=1}^{4} \beta_j \cdot \nu_{j1} \cdot \left\{ \prod_{i=1}^{S} (1 + |\delta_i| \cdot (\epsilon_{ji+} H(\delta_i) + \epsilon_{ji-} H(-\delta_i))) \right\} \\
&\quad \cdot \alpha_{jk} \cdot \left\{ \prod_{l=1}^{S} (1 + |\delta_l| \cdot (\kappa_{jlk+} H(\delta_l) + \kappa_{jlk-} H(-\delta_l))) \right\} \\
\mu_d &= \sum_{j=1}^{4} \beta_j \cdot \nu_{jd} \cdot \left\{ \prod_{i=1}^{S} (1 + |\delta_i| \cdot (\epsilon_{ji+} H(\delta_i) + \epsilon_{ji-} H(-\delta_i))) \right\}
\end{aligned}
$$

The marginalized Likelihood is defined as:

$$
L_s^*(\beta_s) = \int_0^{+\infty} \int_0^{+\infty} \int_0^{+\infty} \int_{dV_\delta} L_s(\beta_1, \dots, \beta_4; \delta_1, \dots, \delta_S)\ d\delta^{S+1} d\beta_3 d\beta_4 d\beta_t
$$

The volume integral in δ space is defined as:

$$
\int_{dV\delta} d\delta^{S+1} = \int_{a_1}^{+\infty} \dots \int_{a_S}^{+\infty} \int_{a_N}^{+\infty} \ d\delta_1 \dots d\delta_S d\delta_N
$$

The integration will be implemented as a Monte Carlo integration. The lower cut offs a_j have to be chosen such, that no bin obtains unphysical, negative entries. Practically, in a Monte Carlo integration, this is done by rejecting negative factors of $1 + \epsilon_{ij}\delta_j$ and $1 + \kappa_{ijk}\delta_j$. If any one of these factors is negative for a particular δ_j we have to re-throw a new random number for δ_j. Effectively, this alters the Gaussian prior distributions, leading to truncated Gaussians as effective priors.

Comments:

1. To set 95% C.L. we have to marginalize L_s, i.e. integrate out the nuisance parameters: β_j ($j = 2, 3, 4$ for t-channel and $j = 1, 3, 4$ for the s-channel) and δ_i ($i \in \{1, ..., S\}$). In total 10 parameters.

2. The ϵ_{ji} are obtained from determining the acceptance for shifted Monte Carlo samples, for which a particular effect as been changed by $1\,\sigma$. The κ_{jlk} are obtained from shifted and normalized template histograms γ_{jlk}: $\kappa_{jlk} = (\gamma_{jik} - \alpha_{jk})/\alpha_{jk}$. We subtract the bin content of the template histogram (α_{jk}) from the shifted histogram (γ_{jlk}) and divide by the standard template value.

3. The uncertainties ϵ_{ji} and κ_{jlk} are constants. They do not depend on β_j. There is no need anymore to map out a β_j dependence of the errors.

2 Combined Search

The combined search, where t- and s-channel single top are taken as one signal, we use a binned likelihood for smoothed template histograms for the H_T variable. The likelihood function is analogously defined as for the separate search.

Definitions: Essentially, we use the same definitions as for the separate search. Except, we consider 3 physics processes:
$j = 1:$ t- + s-channel single top, $j = 2:$ $t\bar{t}$, $j = 3:$ non-top.

$$
\begin{aligned}
L_1 &= L_1(\beta_2, \beta_3; \delta_1, \ldots, \delta_S) \\
&= \left\{ \prod_{k=1}^{B} \frac{e^{-\mu_k} \cdot \mu_k^{n_k}}{n_k!} \right\} \cdot \frac{e^{-\mu_d} \cdot \mu_d^{n_d}}{n_d!} \cdot \prod_{j=1,\, j\neq s}^{4} G(\beta_j, 1.0, \Delta_j) \cdot \prod_{i=1}^{S} G(\delta_i, 0, 1)
\end{aligned}
$$

$$
\begin{aligned}
\mu_k &= \sum_{j=1}^{4} \beta_j \cdot \nu_{j1} \cdot \left\{ \prod_{i=1}^{S} \left(1 + |\delta_i| \cdot (\epsilon_{ji+} H(\delta_i) + \epsilon_{ji-} H(-\delta_i))\right) \right\} \\
&\quad \cdot \alpha_{jk} \cdot \left\{ \prod_{l=1}^{S} \left(1 + |\delta_l| \cdot (\kappa_{jlk+} H(\delta_l) + \kappa_{jlk-} H(-\delta_l))\right) \right\}
\end{aligned}
$$

3 Acceptance uncertainties

To evaluate our systematic acceptance uncertainties we use several Monte Carlo samples where systematic effects have been altered such to resemble 1 σ shifts. We determine the change in acceptance for these samples and call the relative change compared to the standard sample our systematic acceptance uncertainty. These are the ϵ_{ij}. Table 1 gives the numbers we obtain for the signal samples, Table 2 for the background samples.

4 Changes in the shape of the template histograms

For each systematic sample we also determine shifted template histograms and calculate the difference to the standard templates. Some examples for the $q \cdot \eta$ histogram are shown in Fig. 1. The $\alpha\kappa$ distributions for the combined search are shown in Figure 2.

		Separate Search		Combined Search
No.	Source	t-channel	s-channel	t- and s-channel
1	Jet energy scale	$^{+2.4}_{-6.7}\%$	$^{+0.4}_{-3.1}\%$	$^{+0.07}_{-4.3}\%$
2	ISR	$\pm1.0\%$	$\pm0.6\%$	$\pm1.0\%$
3	FSR	$\pm2.2\%$	$\pm5.3\%$	$\pm2.6\%$
4	PDF	+4.4%	+2.5%	+3.8%
5	Generator	-1.3%	+0.3%	-2.0%
6	Top quark mass	$^{+0.7}_{-6.9}\%$	-2.3%	-4.4%
7	ϵ_{trig}, ϵ_{id}, luminosity	$\pm9.8\%$	$\pm9.8\%$	$\pm9.8\%$

Table 1: Systematic acceptance uncertainties ϵ_{ij+} and ϵ_{ij-} for t- and s-channel single-top and the combined signal.

		Separate Search		Combined Search	
No.	Source	$t\bar{t}$	non-top	$t\bar{t}$	non-top
1	Jet energy scale	$^{+25}_{-20}\%$	$\pm15.1\%$	$^{+25}_{-20}\%$	$^{+10.1}_{-12.7}\%$
6	Top quark mass	$\pm4.4\%$	—	$\pm4.4\%$	—

Table 2: Systematic acceptance uncertainties ϵ_{ij+} and ϵ_{ij-} for $t\bar{t}$ and non-top background.

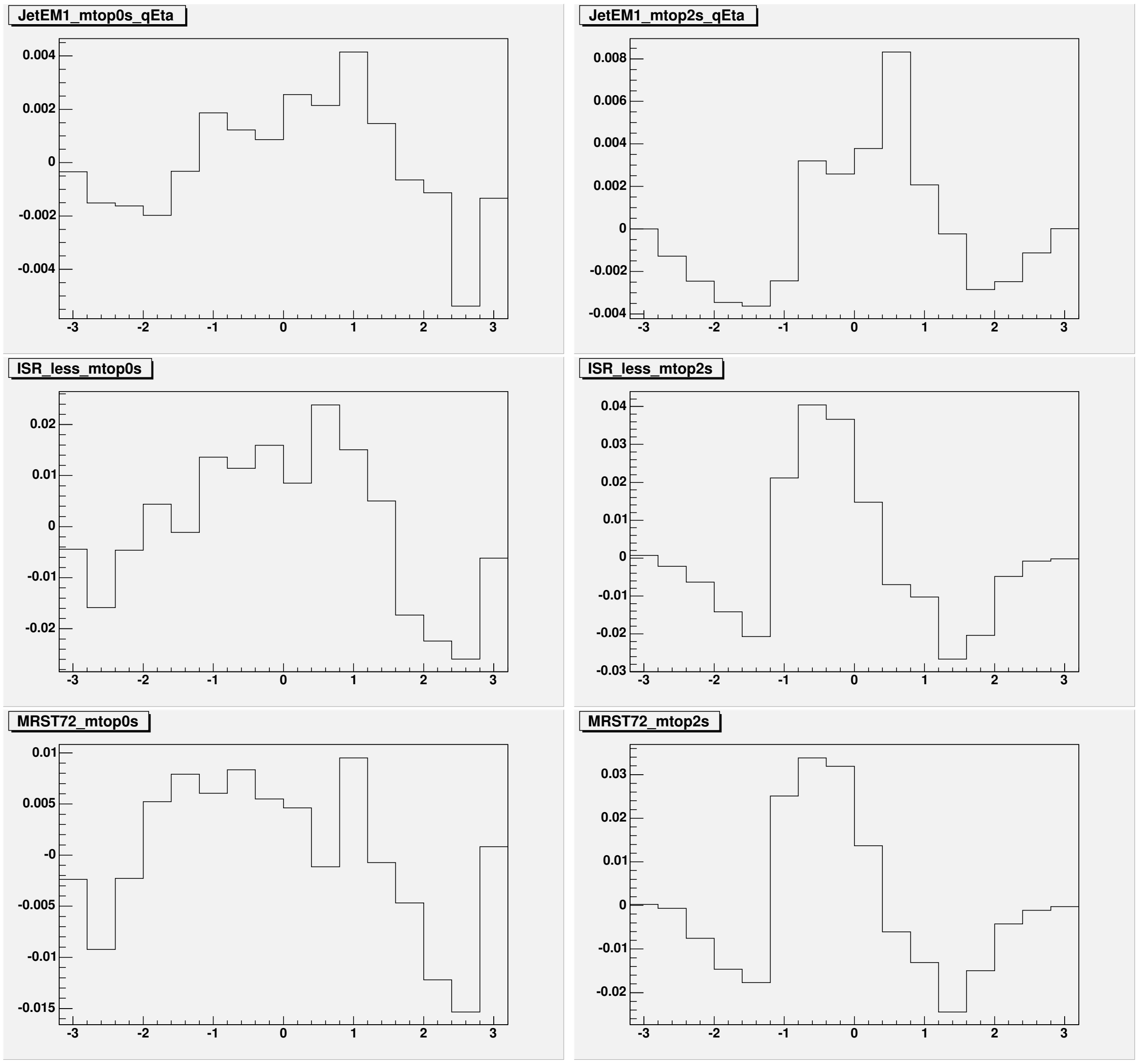

Figure 1: Changes of $Q \cdot \eta$ template histograms due to systematic uncertainties for t- and s-channel single-top.

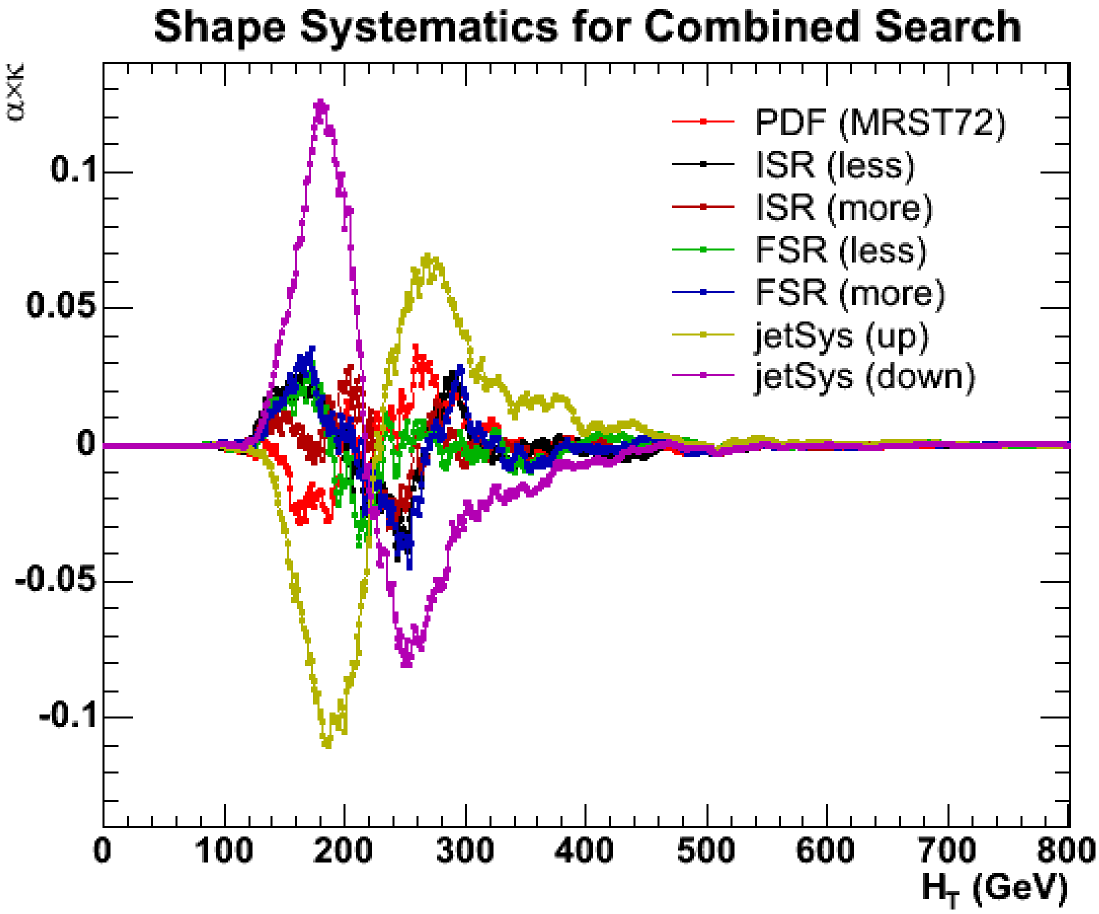

Figure 2: Changes of H_T template histograms due to systematic uncertainties for the cmobined t- and s-channel single-top.

References

[1] D. Acosta *et al.*, Phys. Rev. D **65**, 091102 (2002).

[2] C. Ciobanu et al., *Search for Single Top Production at* $\sqrt{s} = 1.96$ *TeV with CDF*, CDF note 6572, Version 1.4, January 2004 (preliminary analysis results presented Lepton-Photon 2003).

[3] C. Ciobanu et al., *Updated Search for Single Top Production using 162* pb^{-1}, CDF note 6795, Version 1.11, February 2004 (preliminary analysis results presented on Winter conferences 2004).

[4] F. Abe et al., Phys. Rev. D 43, 664–686 (1991).

Appendix A: Approximations

The formula for μ_k includes products of the terms $(1 + \epsilon_{ij}\delta_i)$ and $(1 + \kappa_{jik}\delta_i)$. If you multiply out all products and neglect higher order terms in ϵ and κ, the formula simplifies to:

$$\mu_k = \sum_{j=1}^{4} \beta_j \cdot \nu_{j1} \cdot \alpha_{jk} \left\{ 1 + \epsilon_N \delta_N + \sum_{i=1}^{S} \delta_i \cdot ((\epsilon_{ji+} + \kappa_{jik+}) H(\delta_i) + (\epsilon_{ji-} + \kappa_{jik-}) H(-\delta_i)) \right\}$$

In our analysis we implement the full likelihood without any approximation. We just want to point out here that going from the full likelihood over a set of approximations and assumptions one could possibly derive conditions and specific case in which the previous convolution technique is a valid approximation. For time constaints we do not do that here.