

# Integrated Modeling Methodology Validation Using the Micro-Precision Interferometer Testbed

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## Abstract

This paper validates the integrated modeling methodology used for design and performance evaluation of complex opto-mechanical systems, particularly spaceborne interferometers. The methodology integrates structural modeling, optical modeling, and control system design into a common environment, the Integrated Modeling of Optical Systems (IMOS) software package. The validation utilized the Micro Precision Interferometer (MPI) testbed, a ground-based full-scale hardware model of a spaceborne interferometer. This paper presents a comparison of integrated model predictions with MPI laboratory measurements, indicating that the integrated modeling methodology has the accuracy required to evaluate interferometry mission designs with confidence.

## 1 Introduction

Spaceborne optical interferometers provide the only feasible method to significantly improve the angular resolution and astrometric accuracy of current astronomical telescopes such as the 11111111( Space Telescope (IST). This partial-aperture approach offers a number of important advantages over the traditional full-aperture approach including: control of systematic errors, a significantly lighter instrument yielding the same performance, and improved performance for a given amount of (III(IV, area. However, this approach requires positional stability of optical elements down to the nanometer level as well as laser metrology resolution to the picometer level [1, 2]. Achieving these control and sensing requirements is particularly difficult because the instrument consists of many optical elements distributed across a 10 m flexible truss structure. To further complicate the problem, the structure is excited by mechanical disturbances emanating predominantly from the attitude control system actuators.

The charter for the Jet Propulsion Laboratory Interferometer Technology Program is to mitigate risk for this optical interferometer mission class [3]. A number of ongoing complementary activities address these challenges including: integrated modeling methodology development and validation, hardware metrology testbeds, hardware vibration testbeds, and flight qualification of the interferometer components. Though all of these activities are necessary to buy down mission risk, it is integrated modeling that ultimately will be used in the mission and instrument design. Specifically, modeling will enable definition and flow down of spacecraft/instrument requirements, performance of spacecraft/instrument design trades, and prediction of instrument performance in the anticipated on orbit disturbance environment. This paper investigates whether the integrated modeling methodology has the necessary capability to meet these demanding analysis needs.

In anticipation of these needs, the Integrated Modeling of Optical Systems (IMOS) and the Controlled Optics Modeling Package (COM1) software packages were developed at JPL [4, 5]. The integrated modeling methodology combines structural modeling, optical modeling, and control system design within a common software environment, using these packages. To enable validation of the methodology, its development intentionally coincided with the phased delivery of the Micro Precision Interferometer testbed (MPI).

The MPI testbed is a ground-based, suspended hardware model of a future space-based interferometer (III (at JPL [6, 7, 8]. The primary objective of the testbed is integration of the vibration attenuation technologies required to demonstrate the end-to-end operation of a space-based interferometer. Figure 1 shows a bird's eye view of the testbed which contains all systems necessary to perform a space-based, optical interferometry measurement. These systems include a

7.3 m x 1.1 m x 6.5 m softly suspended truss structure with mounting plates for subsystem hardware, a six-axis vibration isolation system which supports a reaction wheel assembly to provide a flight-like input disturbance source, a complete Michelson interferometer with high-bandwidth optical control systems, internal and external metrology systems, and a star simulator that injects the "stellar" signal into the interferometer collecting apertures.

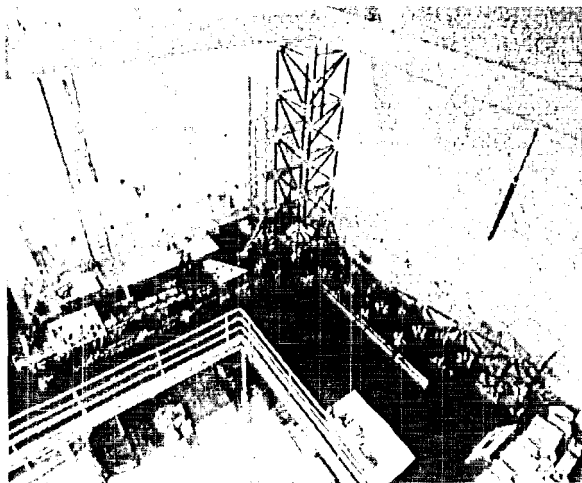


Figure 1: Bird's eye view of the MPI Testbed

A 1:1 integrated model of MPI was developed in parallel with the testbed. This modeling/hardware synergy resulted in a unique opportunity to validate the modeling methodology by comparing model predictions with testbed measurements. Of the three functions required from the modeling methodology (performance prediction, requirements definition, and design trades), performance prediction was initially deemed the most critical validation, due to the associated nanometer-level positional stability requirements. As such, this paper reports validation of the integrated modeling methodology by comparing predicted and measured performance metrics. Although it is necessary to validate the modeling methodology as it is applied to vibration isolation and high bandwidth optical control, validation must begin for the open-loop, hard-mounted disturbance case (i.e., no active optics nor vibration isolation). This paper presents the hard-mounted, open-loop MPI integrated model, the performance validation procedure, and the validation results.

## 2 Integrated Modeling

Integrated modeling is performed with two software packages: Integrated Modeling of Optical Systems (IMOS) [4] and the Controlled Optics Modeling Package (COMP) [5]. Both packages have been developed at the Jet Propulsion Laboratory. IMOS is a set of func-

tions that run within the MATLAB environment [9], whereas COMP is a stand-alone, FORTRAN-compiled program. A commercial version of COMP is available under the name MACOS (Modeling and Analysis of Controlled Optical Systems).

IMOS is a collection of MATLAB m-files that can be used to perform structural finite element modeling and analysis, optical ray tracing, and thermal analysis. IMOS also provides graphics functionality that enables viewing of structural geometries, structural deformations, optical ray traces, and optical element prescriptions. The core programs are easily coupled in MATLAB, and can be extended by the user by writing his own MATLAB functions. MATLAB toolboxes for control system design and analysis, signal processing, and optimization are available for enhancing the capability of IMOS. Detailed optical and thermal analysis are accommodated in IMOS by interfaces with COMP, and TRASYS and SINDA (for thermal analysis). The interface with COMP has been made effortless by creating a MATLAB mex-file version of MACOS. The result is an extremely flexible tool that enables the user to integrate models from different disciplines and conduct analysis, design, and optimization trades that would otherwise be exceedingly difficult [4].

COMP is an optical analysis and modeling program, providing geometric ray-trace, differential ray-trace, and diffraction modeling capability. COMP concentrates on providing detailed optical models for integrated design and analysis tasks. In particular, the differential ray trace capability of COMP can be used to generate linear perturbation models of optical systems [5].

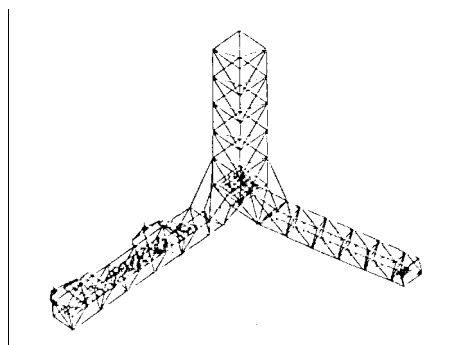
## 3 MPI Integrated Model

The MPI integrated model consists of a structural finite element model and a linear optical model that are integrated together. The structural model is generated with IMOS, whereas both IMOS and COMP are used to create the optical model. The integration and analysis are performed in MATLAB with the aid of IMOS functions.

### 3.1 Structural Model

The structural model is specified in IMOS as a finite element geometry, shown in Figure 2. This geometry consists of plate, beam, truss, and rigid body elements, modeling the base truss structure and the components. The base truss structure is made up of three main parts: the horizontal optics boom, the vertical tower, and the cantilevered metrology boom. The components consist of inboard and outboard optics plates, a disturbance mount plate, two siderostat mounts, an optics cant containing

an active delay line, the optics cart support structure, a hexapod isolation system, a passive delay line, and an external metrology beam launcher plate. The finite element model uses 2,577 degrees of freedom (DOFs) of which 1,837 DOFs are independent with respect to the multi-point constraints (MPCs) of the rigid body elements (RBEs) [4].



**Figure 2:** MPI Finite Element Geometry (compare with Figure 1)

Since the focus of this effort is to validate the integrated modeling methodology, it is important to have a structural model with accurate properties. If this input to the integrated modeling methodology is incorrect, then the validation results would be poor, regardless of the accuracy of the modeling methodology. For this reason, the plate and beam properties as well as the finite element geometry itself have been refined by incorporating MPI modal test data into the model. The structural model updating has been done in two phases, following the phased delivery of the MPI testbed. The first phase involved estimating the parameters of the beams comprising the base truss structure from modal testing performed on the bare truss [10, 11]. The second phase involved geometry modification and parameter estimation of the optics cart support structure, using *in situ* component modal test data [12, 13].

From the finite element geometry and its associated properties the system mass and stiffness matrices are built. The result is a second-order, state space description of the form:

$$M\ddot{d} + Kd = B_f f \quad (1)$$

where  $M$  and  $K$  are the system mass and stiffness matrices,  $d$  is the nodal state,  $f$  is a vector of force input, and  $B_f$  is the force influence matrix.

After the system mass and stiffness matrices are built, multi-point constraints are generated using RBE elements. These constraints take the form of [4]:

$$A \begin{bmatrix} \mathbf{d} \\ d_m \end{bmatrix} = \begin{bmatrix} I_n \\ G_m \end{bmatrix} d_n = G d_n \quad (2)$$

where  $d_n$  are the independent degrees of freedom and  $d_m$  are the constrained degrees of freedom. These constraints are then applied to Equation 1, reducing the state of the system to the independent degrees of freedom:

$$\begin{aligned} G^T M G \ddot{d}_n + G^T K G d_n &= G^T B_f f \\ M_{nn} \ddot{d}_n + K_{nn} d_n &= B_{nf} f \end{aligned} \quad (3)$$

The eigensolution of Equation 3 is found, yielding flexible-body modes and modeshapes. The resultant diagonalized system is:

$$\begin{aligned} \ddot{\eta} + 2Z\Omega\dot{\eta} + \Omega^2\eta &= \Phi_n^T B_{nf} f \\ d &= G\Phi_n\eta \end{aligned} \quad (4)$$

where  $\eta$  is the modal state vector,  $Z$  is a diagonal modal damping matrix,  $\Omega$  is the diagonal modal frequency matrix, and  $\Phi_n$  is the eigenvector matrix.  $Z$  is formed by assuming a modal damping of 0.3% for flexible body modes above 32 Hz and damping ranging from 0.15% to 0.45% for modes below 32 Hz. These damping values correspond to estimates obtained from the second phase of modal tests.

### 3.2 Optical Model

The optical model begins with a specification of the optical prescription. This prescription includes the shapes, positions, and orientations of the optical elements. A ray trace of the optical prescription is shown in Figure 3. This optical prescription is generated in IMOS based on the prescription of the actual optical elements of MPI (see Figure 4). The model generation uses the structural finite element geometry in order to simplify the prescription definition and to ease the succeeding structural-optical model integration. This allows the location of the actual optical elements to be measured with respect to reference points on the structure as opposed to with each other. Furthermore, structural nodes that correspond to optical element attachment points are easily identified or defined.

Once the optical prescriptions are specified, they are exported to COMSOL, where linear optical models are created. These linear models are calculated by performing an analytic differential ray trace [5]. The result is a model of the form:

$$y = C_{opt} d \quad (5)$$

where  $d$  is a vector of optical element position and orientation perturbations,  $y$  is a vector of optical output, and  $C_{opt}$  is the optical sensitivity matrix. The optical output can be path length, wavefront tilt, or spot motion.

### 3.3 Structural-Optical Model Integration

Once the structural modal model and the linear optical model have been created, they are integrated to

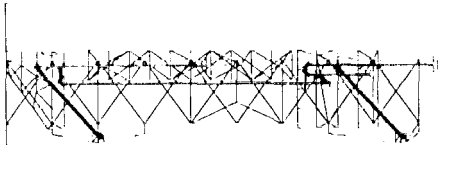


Figure 3: Ray trace of the MPI optical prescription on the finite element geometry of the optics boom.



Figure 4: Actual optical layout on the MPI optics boom.

form a structural-optical model. This integrated model is specified in first-order, state-space form, lending itself most easily to analysis with existing MATLAB functions. In particular, the state-space integrated model can be used for frequency-domain analysis, time domain simulation, and closed-loop synthesis.

First, the structural model is truncated to remove modes above the bandwidth of expected disturbances (*i.e.*, above 900 Hz) [14, 15]. The truncated modal model is then converted into first-order, state-space form by using the substitution [4]:

$$\dot{x} = \begin{bmatrix} \eta_k \\ \dot{\eta}_k \end{bmatrix} \quad (6)$$

Resulting in:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ d &= C_d x + Du \end{aligned} \quad (7)$$

with:

$$\begin{aligned} A &= \begin{bmatrix} 0 & I \\ -2Z_k\Omega_k & -\Omega_k^2 \end{bmatrix} & B &= \begin{bmatrix} 0 \\ \Phi_{nk}^T G^T B_f \end{bmatrix} \\ C_d &= \begin{bmatrix} G\Phi_{nk} & 0 \\ 0 & G\Phi_{nk} \end{bmatrix} & D &= 0 \end{aligned} \quad (8)$$

where the subscript  $k$  refers to the set of kept mode-shapes.

Finally, the linear optical model is incorporated into the first-order model. The optical output is obtained by premultiplying  $d$  by the optical sensitivity matrix,  $C_{opt}$ . In this case the matrix  $C$  of the measurement equation of Equation 7 becomes:

$$C = C_{opt} C_d \quad (9)$$

Note that the matrix  $D$  of Equation 7 is still zero but now has different dimension.

The MPI testbed is dedicated to the development and evaluation of vibration attenuation technologies. In part, this evaluation consists of observing the improvement in interferometer performance due to the application of the various technologies. Interferometer performance is primarily degraded by variation in optical pathlength difference (OPD), *i.e.*, the difference in the distances that the light travels from the stellar source, through each arm of the interferometer to the interference optical detector. This difference must be stabilized to the 10 nm (1 MS) level in the on-orbit mechanical disturbance environment [2].

In contrast to estimating modal characteristics as in [12, 13], disturbance input to stellar OPD output transfer functions were measured since they completely characterize (in a linear sense) the propagation of disturbances to OPD. Figure 5 shows the (disturbance input location relative to the 0.1") output location for the MPI testbed. This disturbance transfer function was measured for three force disturbance directions: (x, y, z). An 111" data analyzer was used to collect the data. A 10 N shaker, mounted at the base of the tower, applied the force input in each of the three directions. The force input was measured with a load cell mounted between the shaker and the structure. The analyzer calculated the transfer function from force input to 0.1" output.

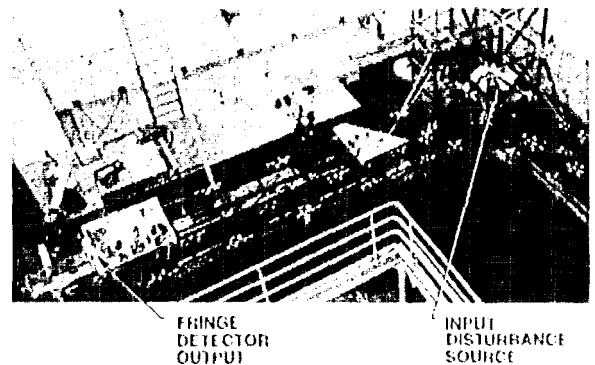


Figure 5: Locations of disturbance input and 0.1" output on the MPI testbed.

## 5 Comparison Metric and Results

In general on space-based interferometers, mechanical disturbances will be either broadband or narrowband with the energy varying over broad frequency ranges as a function of time [14, 15]. In either case, the powerspectral density of the disturbance is broadband. Therefore, the integrated model should be accurate in a broadband sense. More specifically, we desire  $\sigma_{opd}$  to

be accurate, where [16]:

$$\sigma_{opd}^2 = \frac{1}{\pi} \int_{f_{min}}^{f_{max}} |G(j\omega)|^2 \Phi_d(\omega) d\omega \quad (10)$$

for a broadband disturbance power spectral density,  $\Phi_d(\omega)$ , and a disturbance to OIPD transfer function,  $G(j\omega)$ .

Since Equation 10 yields the quantity that we wish to accurately predict, we can use this same equation as a metric to characterize the measured and predicted transfer functions. As opposed to picking a particular expected disturbance power spectral density, bandlimited white noise (over  $[f_{min}, f_{max}]$ ) is used:

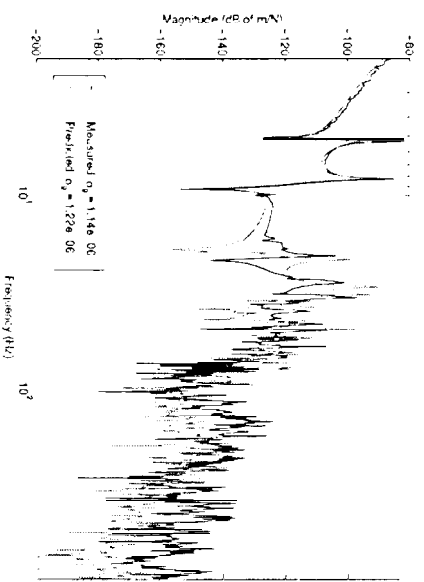
$$\sigma_g^2 = \frac{A_d}{\pi} \int_{f_{min}}^{f_{max}} |G(j\omega)|^2 d\omega \quad (11)$$

where  $A_d$  is the amplitude of the bandlimited white noise disturbance power spectral density with  $f_{min}$  and  $f_{max}$  defining the frequency range of interest.  $\sigma_g$  is used instead of  $\sigma_{opd}$  in order to stress that the result is a metric of the transfer function itself.

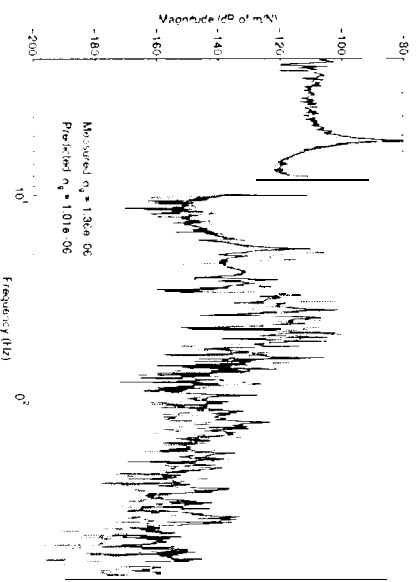
Using this metric, the accuracy of the model can be quantified by comparing  $\sigma_g$  for the predicted and measured transfer functions. As such, the particular value of the disturbance amplitude is immaterial. The amplitude is chosen so that the variance of the disturbance is one. This choice is arbitrary, and the value of  $\sigma_g$  has no significance by itself. It is the *comparison* of the metrics for corresponding measured and predicted transfer functions that is meaningful. Generally, it is desired that OIPD variation predictions be accurate to within a factor of 2. Since the metric  $\sigma_g$  is closely related to OIPD variations for broadband noise, the factor of 2 is applied as a requirement to the ratio of  $\sigma_g$  for the measured and predicted transfer functions.

The modulus of the measured transfer functions, along with the corresponding predicted transfer functions, are shown in Figures 6-8. The predicted transfer functions were calculated by applying standard MATLAB functions to the integrated model with disturbance force input and OIPD output. The value of the broadband metric, also calculated with MATLAB functions, is given in the legend for each transfer function.

The results of these comparisons are shown in Table 1. The bandwidth of interest is [4, 900] Hz. Below 4 Hz the force capability of the shaker is limited and the tested suspension modes pollute the measurement. Above 900 Hz the mechanical disturbances are expected to have no energy. This bandwidth is further broken roughly into decades and comparisons are shown for these "decades." Units are not given in the table so as to discourage the reader from attaching significance to the separate values.



**Figure 6:** Predicted and measured MIP1 disturbance to OIPD transfer function: x-axis force input.



**Figure 7:** Predicted and measured MIP1 disturbance to OIPD transfer function: y-axis force input.

The comparisons for the three force disturbance transfer functions show that the broadband metrics ([4, 900] Hz) for the predicted transfer functions are accurate with respect to the measured transfer functions to well within the desired factor of two. Furthermore, the comparisons for each "decade" show accuracies of better than a factor of three. This indicates that OIPD variation estimates for colored broadband input should also be accurate.

## 6 Conclusion/Future Work

This paper validates the performance prediction capabilities of the integrated modeling methodology which incorporates the IMOS and COMP analysis tools. A metric is proposed that characterizes the disturbance transfer functions over a broad frequency range. This metric is simply the expected OIPD variation assuming a bandlimited white noise disturbance input. Comparisons between predicted and measured transfer function metrics show that the model is accurate to within a factor of 1.5 over the frequency range of interest,

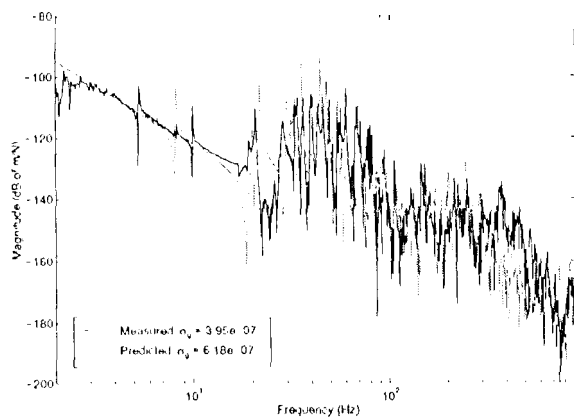


Figure 8: Predicted and measured MTF disturbance to OPD transfer function: z-axis force input.

Disturbance Input		$\sigma$			
		4 - 10 Hz	10 - 100 Hz	100 - 900 Hz	4 - 900 Hz
x-axis Force	measured	997	541	70	<b>1,137</b>
	predicted	666	1,025	21	<b>1,223</b>
	factor	0.67	1.89	0.30	<b>1.08</b>
y-axis force	measured	1,313	360	69	<b>1,363</b>
	predicted	864	521	23	<b>1,009</b>
	factor	0.66	1.45	0.34	<b>0.74</b>
z-axis Force	measured	<b>185</b>	346	50	<b>395</b>
	predicted	<b>177</b>	591	44	<b>618</b>
	factor	0.95	1.71	0.88	<b>1.56</b>

Table 1: Broadband transfer function metric comparison between the predicted and measured transfer functions of the MPJ Testbed.

[4, 900] Hz. This is comfortably better than the desired factor of two. Furthermore, comparisons of the metric applied over each decade in the frequency range show an accuracy to within a factor of three. This indicates that performance predictions for colored broadband input should also be accurate.

However, this study only addressed the hard-mounted-disturbance, open-loop-control configuration. Ongoing activities include validation of the methodology for various closed-loop configurations, inclusion of disturbance torque transfer functions in the validation, and assessment of the sensitivity of these results to the accuracy of the structural model.

## References

[1] M. Shao, "Orbiting Stellar Interferometer," Proceedings of the SPIE Symposium on OE/Aerospace, Science and Sensing, Conference on Spaceborne Interferometry, vol. 1947, pp. 89, Orlando, FL, April 1993.

[2] M. Shao and D. M. Wolf, "Orbiting Stellar Interferometer," Proceedings of the SPIE Symposium 011 OE/Aerospace, Science and Sensing, Conference on Spaceborne Interferometry, vol. 2447, pp. 228-239, Orlando, FL, April 1995.

[3] R. A. Laskin, "Technology for Space Optical Interferometry," Proceedings from the 33<sup>rd</sup> Aerospace Sciences Meeting and Exhibit, AIAA vol. 950825, Reno, NV, January 1995.

[4] M. Milman, et al., "Integrated Modeling of Optical Systems User's Manual, Release 2.0," JPL D-13040, November 15, 1995.

[5] D. Redding, "Controlled Optics Modelling Package User Manual Release 1.0" JPL D-9816, June 1, 1992.

[6] G. W. Neal, J. F. Sword, B. E. Hines, and R. J. Calvet, "Micro-Precision Interferometer Testbed: End-to-End System Integration of Control Structure Interaction Technologies," Proceedings of the SPIE Symposium 011 OE/Aerospace, Science and Sensing, Conference on Spaceborne Interferometry, vol. 1947, pp. 91-103, Orlando, FL, April 1993.

[7] B. E. Hines, "Optical Design Issues for the Micro-Precision Interferometer Testbed for Space-Based Interferometry," Proceedings of the SPIE Symposium on OE/Aerospace, Science and Sensing, Conference on Spaceborne Interferometry, vol. 1947, pp. 114-125, Orlando, FL, April 1995.

[8] G. W. Neal, J. F. O'Brien, N. M. Nerheim, R. J. Calvet, H. Singh, and S. Shaklan, "Micro-Precision Interferometer Testbed: First Stabilized Stellar Fringes," Proceedings of the SPIE International Symposium on AeroSense, Conference on Spaceborne Interferometry II, vol. 2477, pp. 104-115, Orlando, FL, April 1995.

[9] "Matlab User's Guide," The Math Works Inc., August 1992.

[10] P. G. Carne, R. L. Mayes, M. B. Levine-West, "A Modal Test of a Space Truss for Structural Parameter Identification," Proceedings of the 11<sup>th</sup> IMAC, Kissimmee, FL, February 1993.

[11] J. R. Red-Horse, E. L. Marek, M. B. Levine-West, "System Identification of the JPL Micro-Precision Interferometer Truss: Test-Analysis Reconciliation," Proceedings of the 34<sup>th</sup> SDM, La Jolla, CA, April 1993.

[12] M. B. Levine West, J. W. Melody, "NIocIcI Updating of Evolutionary Structures," Proceedings of the 15<sup>th</sup> ASME Biennial Conference on Mechanical Vibration and Noise, Boston, MA, September 1995.

[13] J. W. Melody, M. B. Levine-West, "High Fidelity Modeling of Evolutionary Structures in IMOS," Proceedings of the First World Conference on Structural Control, pp. WP1-98 WP1-107, Los Angeles, CA, August 1994.

[14] J. W. Melody and H. C. Briggs, "Analysis of Structural and Optical Interactions of the Precision Optical Interferometer in Space (POINT S)," Proceedings of the SPIE Symposium on OE/Aerospace, Science and Sensing, Conference on Spaceborne Interferometry, vol. 1947, pp. 44-57, Orlando, FL, April 1993.

[15] S. Shaklan, J. Yu, H. C. Briggs, "Integrated Structural and Optical Modeling of the Orbiting Stellar Interferometer," SPIE: Space Astronomical Telescope and Instrument II Conference, Orlando, FL, April 1993.

[16] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3<sup>rd</sup> Edition, McGraw-Hill, New York, 1991.