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Passage Through a Transverse Magnetic Slab

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Passage through a transverse magnetic slab

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The purpose of this short note is to derive the simple but remarkable result that the tranverse impulse given to a particle passing through a "slab" of homogeneous, stationary, transverse magnetic field depends only on the properties of the slab and the particle's charge, and not at all on its initial state or its mass. Let a constant magnetic field, $\vec{B}$, exist in a region bounded by two parallel planes separated by a distance, l. If the triple, $\langle \hat{e}_1, \hat{e}_2, \hat{e}_3 \rangle$, is an orthonormal frame of vectors with $\hat{e}_3$ orthogonal to the planes, then the condition that $\vec{B}$ is "transverse" can be written as either $\vec{B} \cdot \hat{e}_3 = 0$ or $\vec{B} = B_1 \hat{e}_1 + B_2 \hat{e}_2$. We want to show that the tranverse impulse, $(\hat{e}_1 \hat{e}_1 + \hat{e}_2 \hat{e}_2) \cdot \Delta\vec{p}$, given to a particle passing through the region is independent of its dynamical coordinates at the point of entry. (Since the field is homogeneous, this is obviously true with regard to its position.)

Begin with the Lorentz equation.

$$\dot{\vec{p}} = e \, \vec{v} \times \vec{B} \ ,$$

where e is the particle's electric charge. The impulse given to the particle is obtained by integrating this through the field region.

$$\begin{aligned} \Delta\vec{p} &= e \int dt \, (\vec{v} \times \vec{B} \,) \\ &= e \int d\vec{s} \times \vec{B} \\ &= e \left(\int d\vec{s} \right) \times \vec{B} \\ &= e \, \vec{L} \times \vec{B} \ , \end{aligned} \tag{1}$$

where $\vec{L}$ is a displacement vector connecting the entry point to the exit point. Because $\vec{B} \cdot \hat{e}_3 = 0$, we can introduce the auxiliary field $\vec{U}$, defined by

$$\begin{aligned} \vec{B} &= \hat{e}_3 \times \vec{U}, \ \text{or, equivalently} \\ \vec{U} &= \vec{B} \times \hat{e}_3 \ . \end{aligned}$$

Note in passing that

$$\text{if } \vec{B} = B_1 \hat{e}_1 + B_2 \hat{e}_2, \ \text{then } \vec{U} = B_2 \hat{e}_1 - B_1 \hat{e}_2 \ .$$

That is, $U_1 = B_2$ and $U_2 = -B_1$. We rewrite Eq.(1) in terms of $\vec{U}$.

$$\begin{aligned} \vec{L} \times \vec{B} &= \vec{L} \times (\hat{e}_3 \times \vec{U}) \\ &= (\vec{L} \cdot \vec{U})\hat{e}_3 - (\vec{L} \cdot \hat{e}_3)\vec{U} \end{aligned} \tag{2}$$

It will be convenient to make the angular dependence explicit and express everything in terms of l, the length of the field region. We write $\vec{L} = L\hat{n}$ and use $l = \vec{L} \cdot \hat{e}_3 = L\hat{n} \cdot \hat{e}_3$. Substituting these and Eq.(2) back into Eq.(1), we obtain.

$$\Delta\vec{p} = (\, el\,) \left[\left(\hat{n} \cdot \vec{U} / \hat{n} \cdot \hat{e}_3 \right) \hat{e}_3 - \vec{U} \right]$$

Only the longitudinal component, Δp_3, depends on orientation.

$$\begin{aligned} \Delta p_3 &= e\, l\, \frac{\hat{n} \cdot \vec{U}}{\hat{n} \cdot \hat{e}_3} \\ &= e\, l\, \frac{\hat{n} \cdot (\, \vec{B} \times \hat{e}_3\,)}{\hat{n} \cdot \hat{e}_3} \end{aligned}$$

However, in practice, we never compute this component directly but merely infer it from the transverse components and the fact that magnetic fields cannot alter the energy of a particle. The transverse components are free from any angular dependence.

$$\begin{aligned} \Delta p_1 &= -e\, l\, U_1 \\ &= -e\, B_2 l \\ \Delta p_2 &= -e\, l\, U_2 \\ &= e\, B_1 l \end{aligned}$$

These equations express the desired result, as their right hand sides depend only on properties of the field region and the particle's charge. Usually, we divide by a reference momentum,

$$p_{\rm ref} = e|B\rho|_{\rm ref}$$

and rewrite them as follows.

$$\begin{aligned} \Delta p_1 / p_{\rm ref} &= -B_2 l / |B\rho|_{\rm ref} \\ \Delta p_2 / p_{\rm ref} &= B_1 l / |B\rho|_{\rm ref} \end{aligned}$$

This is an exact result, but it is extended via the "thin element" approximation, in which we indulge ourselves with the polite fiction that, if l is sufficiently small, the particle's transverse coordinates remain "essentially constant" in passing through the field. The approximation enables us to allow transverse variations in B_2 and B_1,

$$\begin{aligned} \Delta p_1 / p_{\rm ref} &\approx -B_2(x,y) l / |B\rho|_{\rm ref}\ , \\ \Delta p_2 / p_{\rm ref} &\approx B_1(x,y) l / |B\rho|_{\rm ref}\ . \end{aligned}$$

These equations then form the basis of a number of "kick codes" for tracking particles.