N91-12418

19. ACQUISITION AND ANALYSIS OF ACCELEROMETER DATA

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ABSTRACT

Acceleration data reduction must be undertaken with a complete understanding of the physical process, the means by which the data are acquired, and finally, the calculations necessary to put the data into a meaningful format.

This paper will discuss the acceleration sensor requirements dictated by the measurements desired. Sensor noise, dynamic range, and linearity will be determined from the physical parameters of the experiment.

The digitizer requirements will be discussed next. Here the system from sensor to digital storage medium will be integrated, and "rules of thumb" for experiment duration, filter response, and number of bits will be explained.

Data reduction techniques after storage are next. Time domain operations including decimating, digital filtering, and averaging will be covered. Frequency domain methods, including windowing and the difference between power and amplitude spectra, will be discussed. Simple noise determination via coherence analysis shall be included.

Finally, an example experiment using the Teledyne Geotech Model 44000 Seismometer to measure accelerations from 10^{-4}g to 10^{-10}g over a bandwidth from 1 Hz to 10^{-6} Hz will be discussed. The sensor, data acquisition system, and example spectra will be presented.

INTRODUCTION

In order to acquire and analyze data in such a way as to characterize the acceleration environment, one must consider three primary areas:

1. The sensor(s) themselves. The accelerometers must be selected carefully in order to be certain that they are suited to measuring the anticipated signals.

2. The Data Acquisition System. For later analysis, the data must be converted to a digital form and stored. This process involves signal conditioning, analog to digital conversion, and storage. These steps must dovetail with the sensor such that they do not adversely affect the experimental goals.

3. The Analysis Techniques. The digital representation of the data must be operated upon in some way to provide meaningful, standard data. The methods used have inherent limitations and assumptions. One must have a good working knowledge of these factors in order to make quantitative judgments.

This paper will briefly discuss each of these three areas to explain techniques and terminology. Finally, an example experiment involving vertical accelerometer data will be presented. The data set is from a Teledyne Geotech Model 44000 Seismometer and was collected continuously over an interval of 72 hours. Signals due to teleseismic earthquakes, tidal fluctuations, and system noise were recorded. Acceleration levels as low as 10^{-12} g were discernible from system noise. The data was Fourier Transformed to yield spectral components with frequencies as low as 3×10^{-6} Hz.

SENSOR REQUIREMENTS

Dynamic Range

The dynamic range refers to the ratio between the largest and smallest signals measured. Obviously, the sensor selected must meet or exceed that dynamic range.

In the case of the Spacelab experiments, the anticipated signal is not well defined. Accelerations as low as 10^{-8} g may be expected, particularly at low frequencies. At higher frequencies, 10^{-3} g is likely during a typical experiment. The sensor chosen must therefore have an intrinsic dynamic range of at least 10^5 or 100 dB. A single figure can be misleading, however. Generally, we know what the peak acceleration permissible without distortion is. The minimum measurement will be limited by the system noise, which will in general have some variation with frequency. Bandwidth, too, is important in determining dynamic range requirements.

Consider a simple case where an instrument has 10^{-18} g^2/Hz noise, flat from dc to 100 Hz. If we were to integrate this power over a bandwidth from dc to 1 Hz, then the rms power would be 10^{-9} g. However, if we were interested in low frequencies and limited our bandwidth to 10^{-4} Hz, the rms power would be only 10^{-11} g. The important point here is that the average signal amplitude in time can translate to vastly different power spectral densities, dependent upon the bandwidth of interest.

Dynamic range is then actually a function of frequency. The sensor noise and full scale range are intertwined with the dynamic range.

Noise

After determining the dynamic range, this scale must be referred to an absolute value. The sensor should not have noise any higher than the smallest anticipated signal. In fact, a good rule of thumb is that the sensor noise at any given frequency should be at least 10 dB below the anticipated signal. Specific processing methods may require an even lower noise margin. This must be remembered as part of the sensor selection.

For the space station case already mentioned, we might want an accelerometer with internal noise no greater than 10^{-18} g^2/Hz, particularly at low frequencies. This also implies that the dynamic range requirements have been increased to 10^6 or 120 dB.

Linearity

Imagine that a full scale pure sine wave signal at a given frequency is applied to our sensor. If the response of our sensor is not purely linear, then the output signal will have components at all integer multiples of that frequency. If a pair of sine waves are applied, then both the difference and sum frequencies will appear at the sensor output. Hence, large signals at high frequencies can produce apparent signals at much lower frequencies due to response nonlinearity.

Since nonlinear behavior is not a one-to-one function, processing cannot extract the original input signal. We must rely on the instrument itself to provide output with high fidelity.

The expected signal range must not be such that harmonics or sum/difference signals will be created in other parts of the spectrum. If we define nonlinearity to be the size of the first harmonic of a full scale signal, then the nonlinearity must be less than 10^{-5} if the full dynamic range is to be utilized at the harmonic frequency.

All the above is really only an introductory discussion of noise and dynamic range requirements. A hypothetical case is shown in Figure 1. Here, the three performance specifications, dynamic range, noise, and linearity are drawn as a function of frequency from 1 to 10^{-6} Hz. Now consider a signal spectrum as shown in Figures 2a and 2b. The sensor has gain that is 20 dB at all frequencies. The input and output spectra are shown superimposed on our hypothetical sensor performance curves. Figure 2a will be represented properly at the sensor output, as it does not exceed the performance of the sensor. Figure 2b shows the effects of nonlinearity and excessive noise. The noise is apparent in the output spectrum, as is the harmonic of the input peak that exceeded the sensor linearity.

DATA ACQUISITION SYSTEM

With an appropriate sensor selected, the signal must be interfaced with some system in order to provide digital data for later pro-

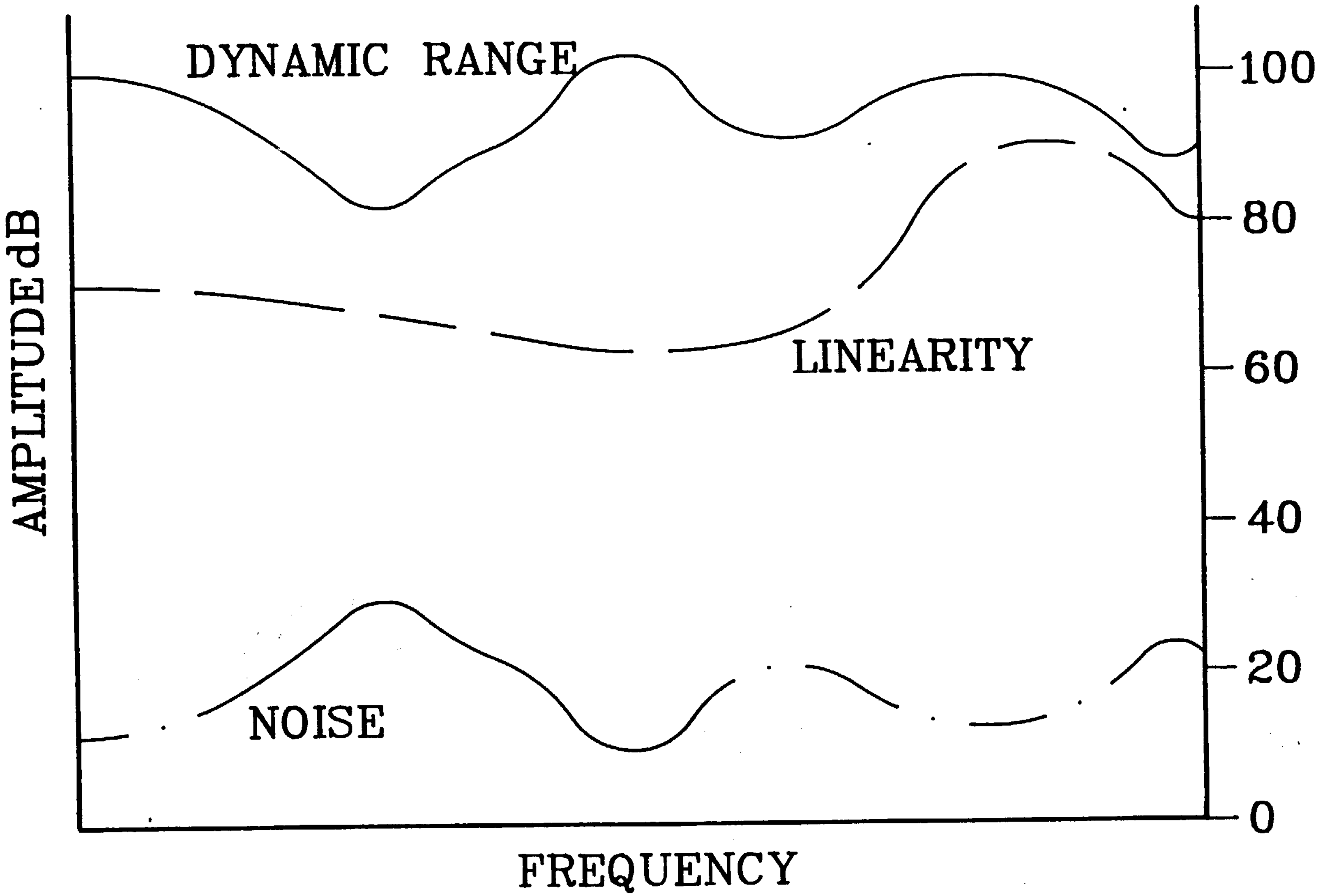


FIGURE 1. SENSOR CHARACTERISTICS

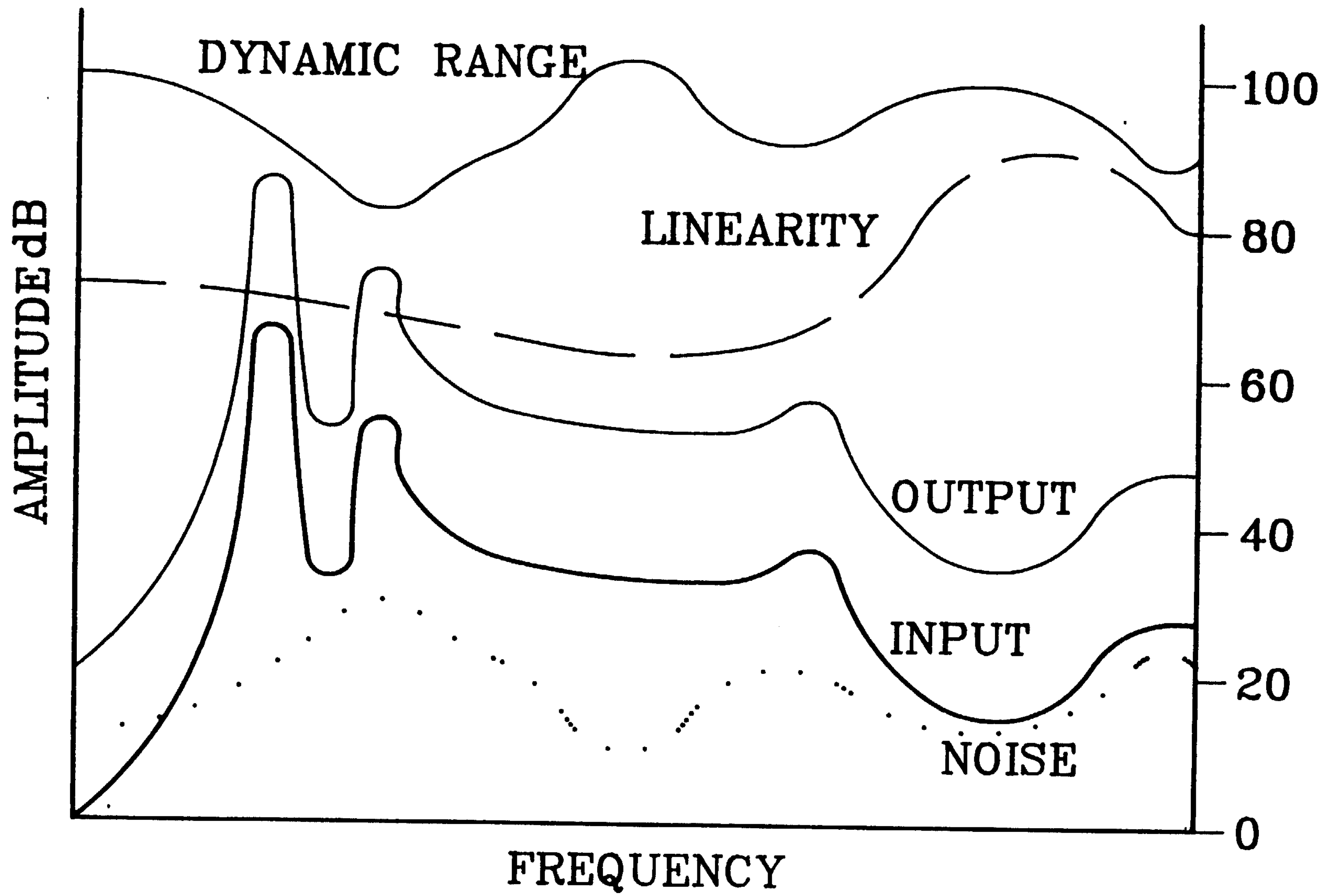


FIGURE 2a. SIGNAL THROUGHPUT (ADEQUATE SENSOR SELECTION)

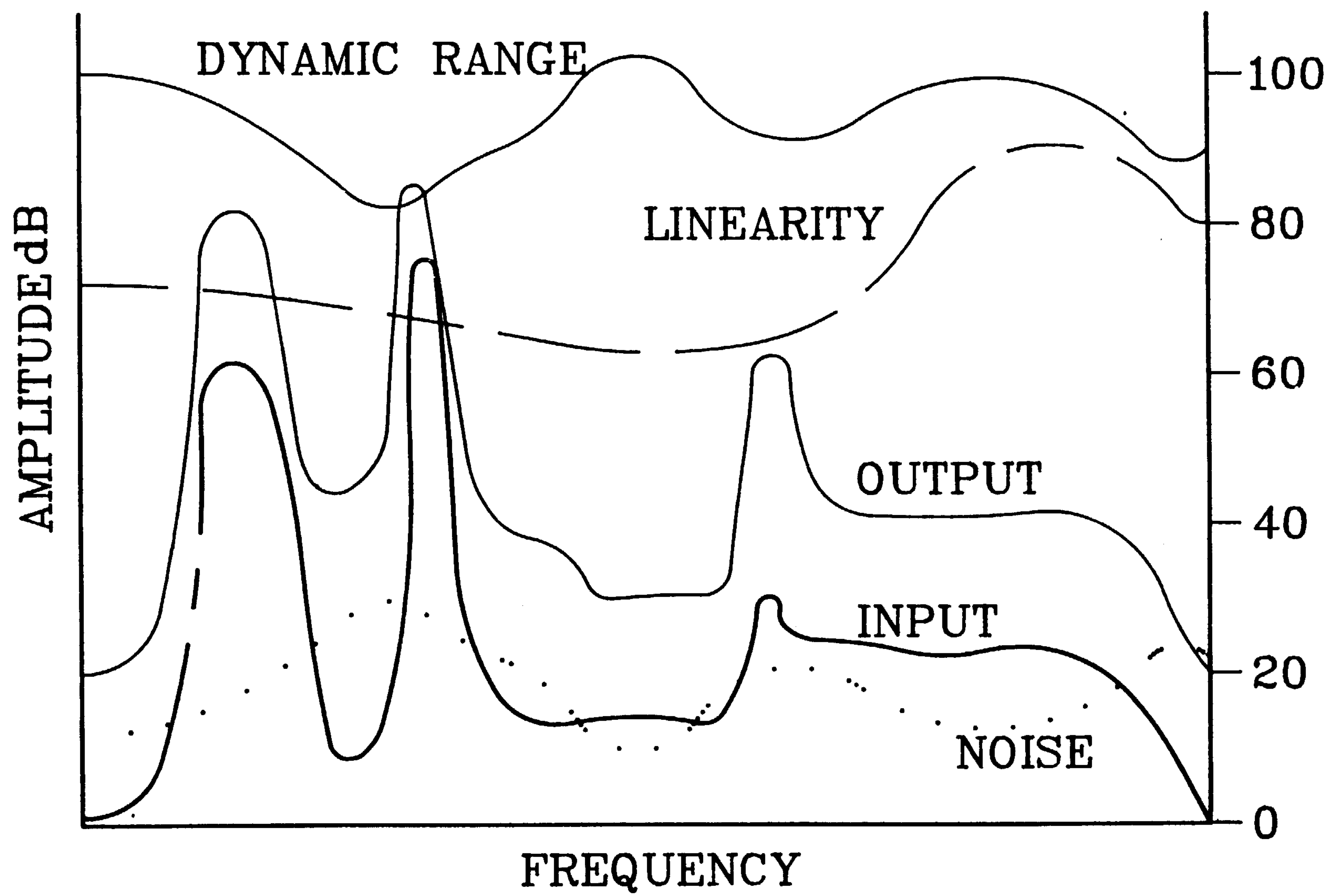


FIGURE 2b. SIGNAL THROUGHPUT (POOR SENSOR SELECTION)

cessing. The data must be stored on some medium suitable for the processing computer to read and work with. Discussion of some of the most important specifications follow.

Signal Conditioning

In the case where the sensor signal is an analog voltage, there will most likely be a requirement to condition that data prior to digitization. The most important areas include gain and filtering.

The gain serves to match the noise and dynamic range of the sensor to that of the digitizer.

Filtering prevents aliasing. Figure 3 shows the effect of sampling on a spectrum. The signal spectrum is folded about the axis at $w=0$ and repeated at every interval of the sampling rate. If at frequencies greater than the sampling rate divided by two (Nyquist Frequency) the input signal has non-zero frequency components, then they will be "folded" back into the spectrum. This can be avoided by filtering the input spectrum such that the signal at the Nyquist frequency is below the resolution of the digitizer. The sample rate and digitizer resolution then dictate the required filter response. Figure 4 shows the filter requirements as a function of sample rate, digitizer dynamic range, and desired signal bandwidth.

Sample Rate

The sample rate requirements have been briefly mentioned already. The sample rate is driven by two factors:

1. Signal Bandwidth. The desired signal bandwidth along with the anti-alias filter response help specify the minimum Nyquist frequency.

2. Oversampling requirements. If the quantization noise spectrum is too large for a given digitizer, the noise spectrum can be reduced by an increase in sample rate followed by digital filtering (as opposed to building/buying a higher resolution digitizer).

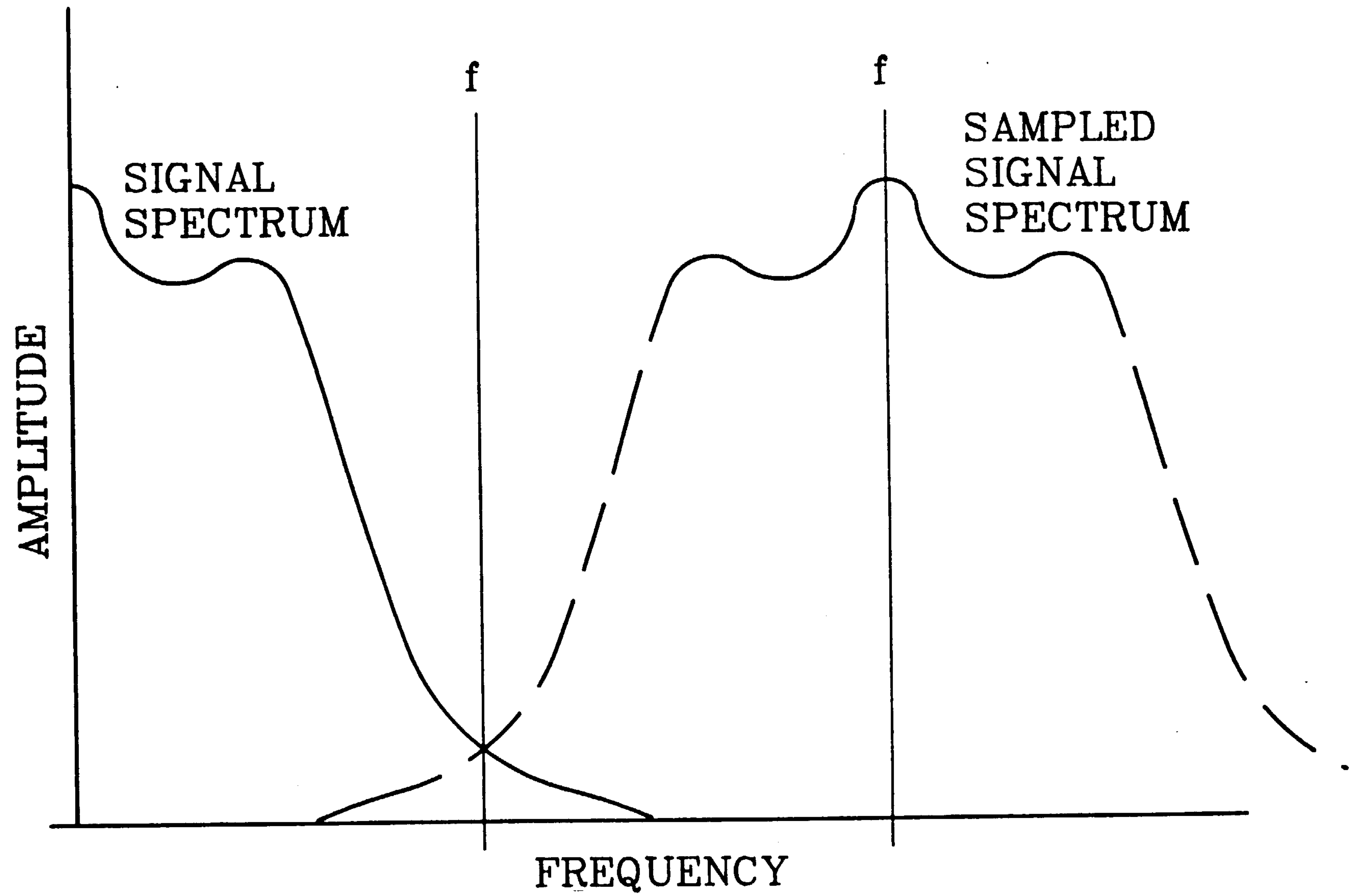


FIGURE 3. ALIASED SIGNAL

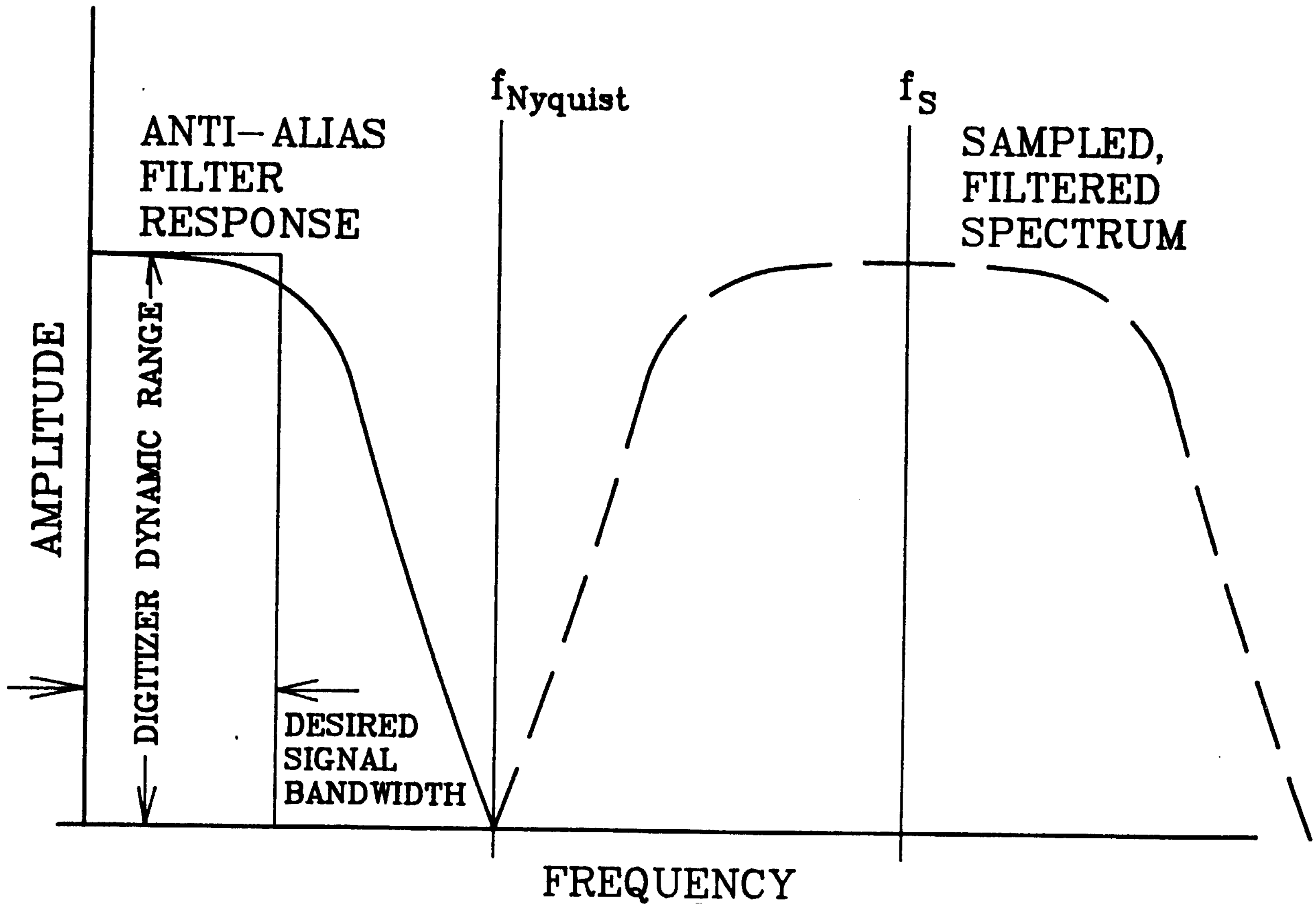


FIGURE 4. ANTI-ALIAS FILTER

Signal bandwidth refers here to the frequency interval over which we want to see the data. If the filter has a 3 dB corner at the edge of this band, it will roll off to a point below the digitizer resolution at some higher frequency f_{co}. The Nyquist frequency must be at least this great, hence the sample rate $f_s > 2f_{co}$. Refer to Figure 4, where $f_{co} = fN_y$. Of course the f_{co} can be decreased by a steeper roll-off in the filter response. The tradeoffs here are between the filter response and sample rate. A more complex filter is required to have the signal bandwidth a larger fraction of the Nyquist Frequency.

Oversampling is mentioned here as a means of decreasing the in band quantization noise. The enhancement as a function of sample rate will be discussed further in the following section.

Resolution or Number of Bits

A digitizer with sufficient dynamic range to complement the sensor must be selected. It would be foolish to use an 8-bit ADC for the purpose of digitizing a voltage with 100 dB dynamic range. Here the digitizer must be specified in much the same way as the sensor so that they will complement one another in terms of dynamic range and noise.

Scaling: The input amplifier must be set up to match the full scale range of the sensor to that of the digitizer. Figure 5 shows a simplified block diagram of a digital acceleration measurement system. If we have a maximum voltage for the ADC defined as V_{MAX}, then we do not want to exceed this value through the sensor and filter/amplifier. If the transfer functions for the sensor and filter/amplifier are F_s ($V/m/s^2$) and F_A (V/V) respectively, then we can express the desired V_{MAX} to a_{MAX} (both values are 0 to peak measurements) relationship:

$$V_{MAX} = F_s F_A \, a_{MAX}.$$

We simply adjust F_A to make the maximum anticipated acceleration a_{MAX} yield the V_{MAX} for the ADC. The anticipated acceleration should be estimated conservatively, as exceeding this limit will result in a hard clip of the data acquisition system.

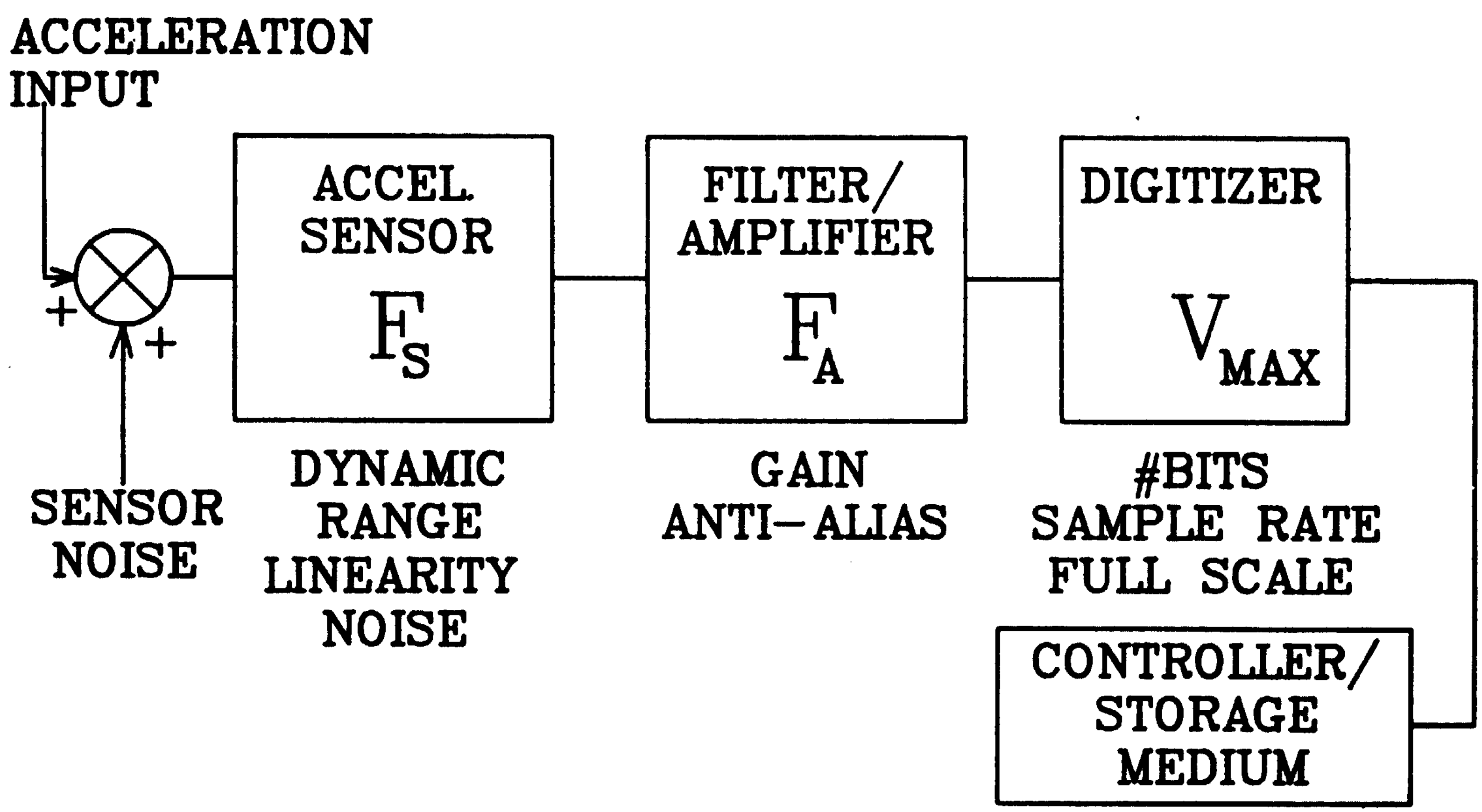


FIGURE 5. SYSTEM BLOCK DIAGRAM (SIMPLIFIED, SINGLE CHANNEL)

Dynamic Range: The required dynamic range for the ADC can be specified in number of bits N. This is a function of the sample rate and least significant bit Q. The root mean square noise for a digitizer is:

$$\bar{V}_{ND}^2 = Q^2/12$$

If the signal is much larger than Q, then this noise is white and its spectrum is distributed over the band from $-fN_y$ to fN_y. fN_y is the Nyquist frequency and is one half of the sampling frequency f_s. Figure 6 shows the spectra of two different sampling processes. The lower plot has twice the sample rate of the upper plot. The rms amplitude in each case is the same, but the energy is spread out over a greater bandwidth in the lower case. The spectrum of the quantization noise has amplitude:

$$\bar{V}_{ND}^2 = Q^2/12fs$$

Relating this to acceleration:

$$\bar{V}_{ND}^2 = F_s^2 F_A^2 a_{MAX}^2 / 3fs 2^{2n}$$

Here we have used the fact that for a digitizer of resolution N, Q is related to V_{MAX}:

$$Q = V_{MAX}/2^{N-1}$$

If we look at the quantization noise spectrum referred to acceleration, we can get an estimate of the dynamic range requirements:

$$\bar{A}_N^2 = a_{MAX}^2 / 3f_s 2^{2N}$$

For example, imagine an experiment where the maximum acceleration was to be 10^{-3} g. If we used a 16-bit ADC and 100 Hz sample rate, then the power spectral density of the quantization noise can be calculated:

$$\bar{A}_N^2 = 7.8 \times 10^{-19}\ g^2/Hz$$

or

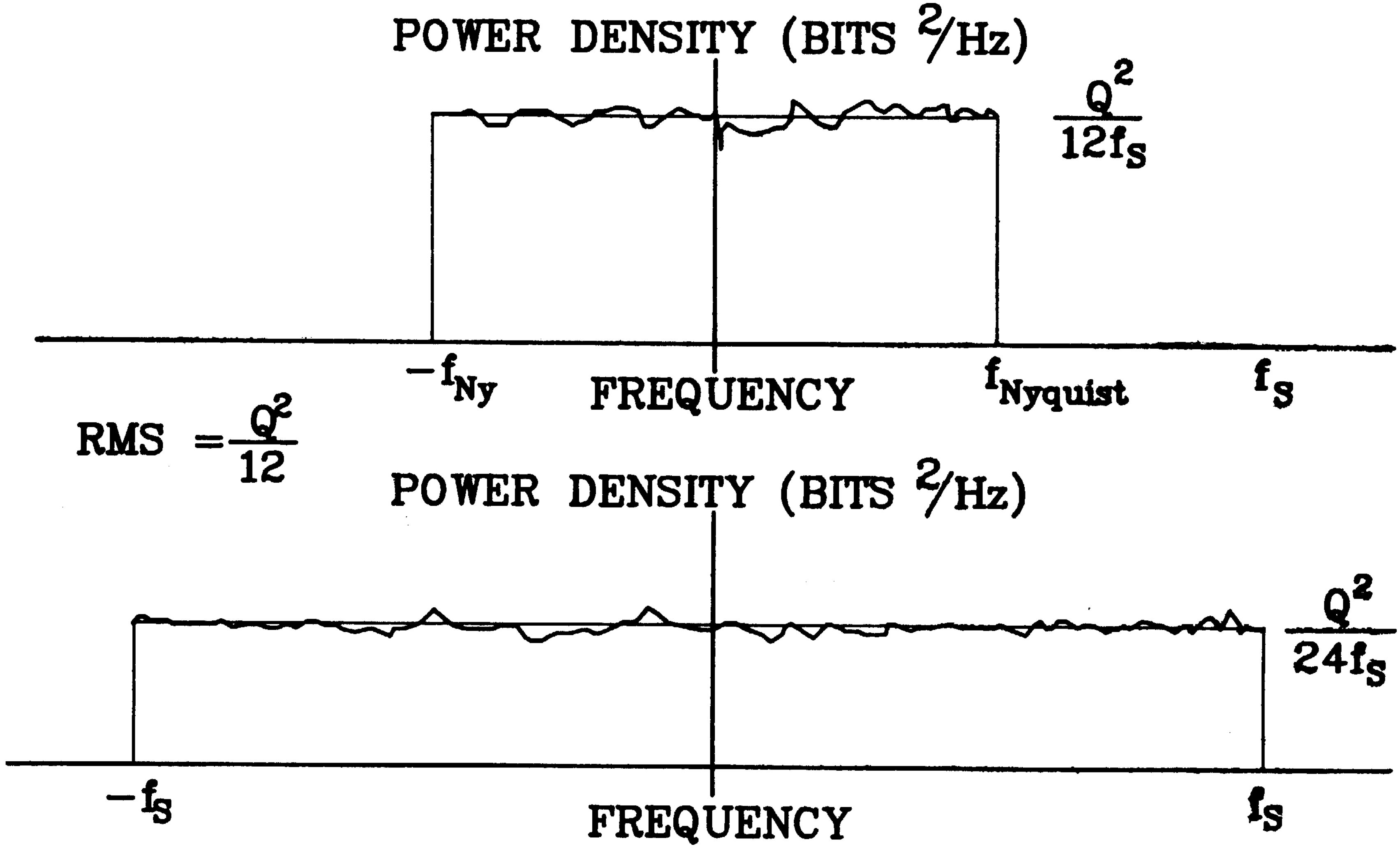


FIGURE 6. QUANTIZATION NOISE (FOR DIFFERENT SAMPLE RATES)

$$\bar{A}_N = 8.8 \times 10^{-10}\ g/\sqrt{Hz}$$

We see that we can reduce the quantization noise density simply by increasing the sample rate. If storage capacities permit, the sample rate can be increased as much as desired to capitalize on this phenomenon. The tradeoff here is with the maximum sample rate achievable, along with storage and computation limitations.

Importantly, no single figure for the dynamic range has been specified here. This is because the scaling was done using an estimate of the time signal. The spectral nature of this signal will change experimental signal-to-noise ratio. If, for example, the signal were a pure sine wave, then the spectral power would be confined to a small part of the spectrum and would be very large. If, on the other hand, the signal was Gaussian and distributed evenly over all frequencies, then the spectral power at each frequency would be far smaller. Clearly, the dynamic range calculations would be very different for these two cases.

ANALYSIS AND COMPUTING

After the experiment is over, we will have some data set consisting of time samples in a digital format. This data will be of interest in both the time and frequency domain.

Time Analysis

For a start, we may at least want to see the signal as a function of time, "strip chart" style. However, if we have data that was sampled every 10 milliseconds, the strip chart over a period of a day could be pretty long. Furthermore, the computation time to operate on such a large array can be prohibitive.

Size Reduction

The physical process of filtering and sampling the acceleration data during the experiment limits the bandwidth and resolution obtainable in the analysis. A typical broad-band experiment can produce many

Mbytes of data for any given channel. This data set cannot be operated on in any but the most rudimentary way as a whole. We will want to break the entire set down or compile it in such a way as to make these subsets more manageable.

If we simply take a subset of the data, this makes things more manageable, but places limitations. A Fourier transform of a set of data with duration T_{EXP} and sample spacing T_s will have spectral resolution and bandwidth:

Minimum frequency	$= 1/T_{EXP}$
Frequency resolution	$= W/T_{EXP}$
Maximum frequency (or bandwidth)	$= 1/2T_s$

Where W is the window factor, always greater than unity. Windowing is discussed further below.

The simple subset has as high a bandwidth as could be possible under the constraints of the experiment, but the frequency resolution and minimum frequency suffer from the truncation process.

Decimation

We would decimate the data set to a more manageable size. By picking every N^{th} sample the array size will be decreased by an equal factor. This process can induce aliasing, so the decimation process must be preceded by digital filtering.

Digital Filtering

The raw digital data set can have a bandwidth too great to permit practical calculation of Fourier transforms of low-frequency components with high resolution. We can simply filter the data digitally down to a smaller bandwidth and decimate in order to "zoom in" on the lower frequency components.

Two basic techniques for digital filtering are in common use. The filters are often designated as recursive or nonrecursive.

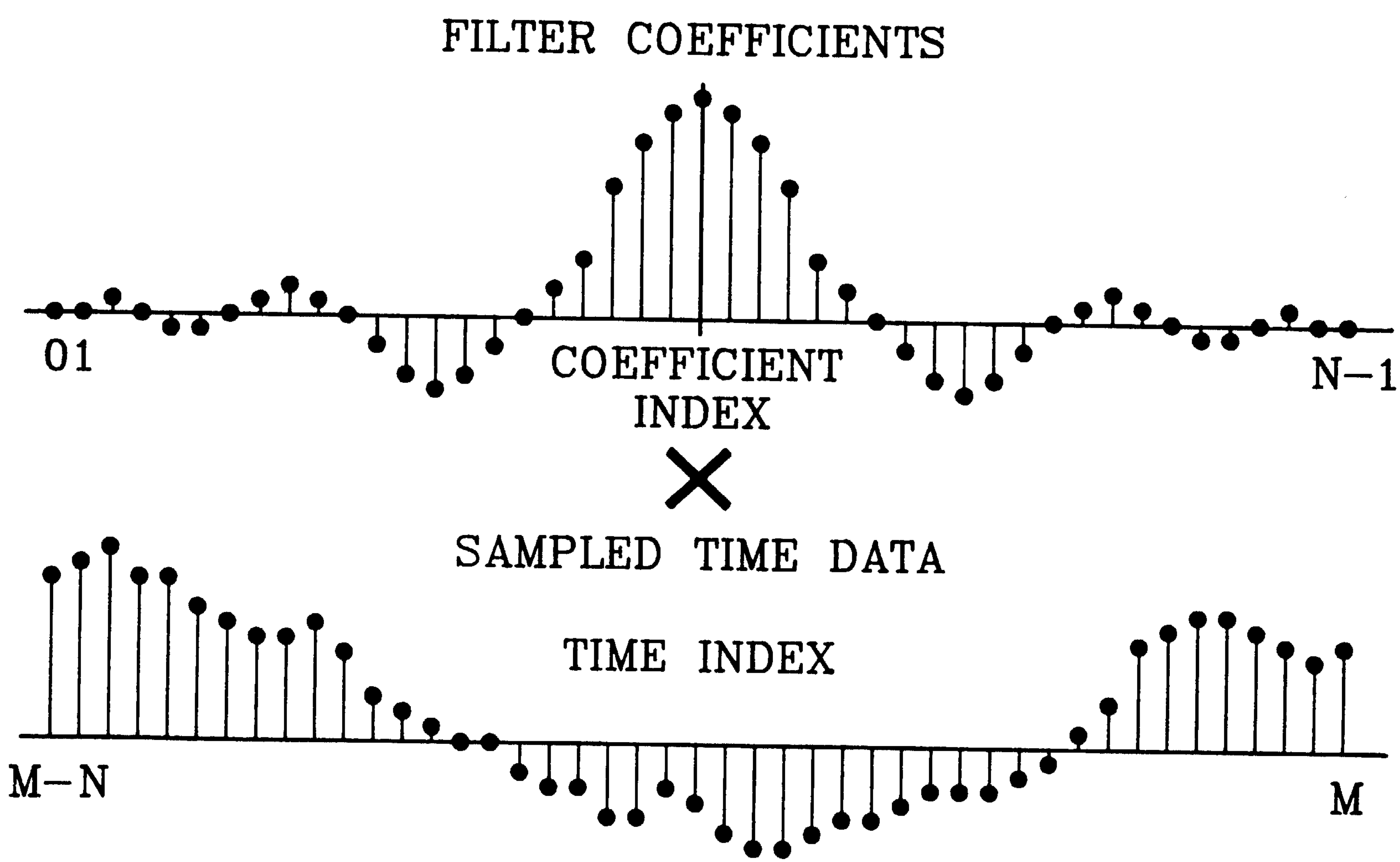
Recursive filters: These filters calculate a single time element from a linear combination of previous input and output time elements. "Recursive" refers to the fact that previously output samples are "fed back" to the calculation process. They require little computational time, since typically only a few previous elements are used in each calculation. However, a precise desired response can be hard to form from this technique.

A typical recursive filter design is based on analogy with analog filters. Desired poles and zeros are used to determine the Laplace transform of an analog filter. The Laplace transform can then be converted to a Z-transform. Finally, the Z-transform coefficients yield the time series domain function required for the filter. This process is generally approached from a pole-zero starting point. An arbitrary frequency response cannot in general be obtained from a recursive filter.

Other disadvantages of recursive filters arise from their close analogy with analog filters. The existence of poles in their response can cause calculations to "explode" during the filtering process. Furthermore, the phase response of a recursive filter is nonlinear function of frequency. Hence different frequency components of the time series will have different time delays through the filter. This can distort the time series of some discernible "event" significantly.

Nonrecursive or Finite Impulse Response (FIR) Filters: Another class of digital filter is the FIR filter. This filter is simply a linear array of coefficients that are convolved with an equal number of elements of the time series to yield a single filtered time series element. Figure 7 shows the operation of calculating a single filtered element from a time series. The filter coefficients (top) operate on the time series (below):

$$Y_m = \sum_{n=o}^{N} a_n X_{m-n}$$



$$Y_M = \sum_{N}^{n=0} a_n X_{M-n}$$

FIGURE 7. FIR FILTER OPERATION

Where N is the length of the filter, whose coefficients are the a_n. Y_m is the output and X_m is the input of the filter.

As you might expect, this can require a significant amount of computation for long filters. The beauty of this technique is the fact that the Fourier transform of the FIR filter coefficients is the response. Likewise, any desired frequency response can be simply inverted to yield a filter. The limitation here is that there must be a sufficient number of coefficients calculated in order to provide adequate accuracy in the frequency response.

This filter suffers from none of the weaknesses of the recursive filter. Phase response is linear with frequency. Furthermore the finiteness of the number of elements used in the calculation precludes the possibility of a numerical overflow.

Typically, unless the application is real time, FIR filters are the best choice for data reduction. They can be shortened at the expense of response accuracy and suffer from none of the other ailments of recursive filters.

Decimation versus Different Sample Rates

The process of digitally filtering a huge data set in order to band limit it sufficiently for decimation can be an excessively long effort. Furthermore, if we were to sample at 100 Hz continuously over a period of 10 days, we would generate 173 Mbytes of data! (assuming 2 bytes per sample). This can present something of a storage problem in addition to requiring large processing times. An alternative method exists, namely to filter several channels from each sensor and record them each at different sample rates. These separate data sets would be almost the same as postprocessed data already discussed.

The largest weakness to this scheme is the loss of bit enhancement for the low frequencies. Since they would be sampled differently, their quantization noise spectral densities would differ accordingly.

Obviously, additional hardware in the form of filters, multiplexers and/or digitizers would be required. This option is not considered in detail here, but the same techniques used can be applied to each of the channels at their different sample rates.

Frequency Domain Processing

The time data, even after filtering, is not terribly useful for quantitative evaluation. We can determine peak and rms amplitudes of given time series. Usually, only if a known process is occurring, is the time series useful for calculation. For example, an individual walking very near the accelerometer might leave a discernible "fingerprint" in the data. For environmental calculation, we will likely need an estimate of the frequency dependence of the measured acceleration.

Fourier Transforms

A function in time will have a Fourier transform defined by:

$$F(w) = \int_{-\infty}^{+X} f(t)e^{-iwt}dt$$

Likewise, the inverse transform is as follows:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+X} F(w)e^{-iwt}dw$$

A function that is sampled in time is in essence multiplied by the sampling function. Figure 8 shows the effect of sampling on a waveform. The sampling function is simply a series of infinitely narrow pulses of unity amplitude.

If this sampling function in time is assumed to be evenly spaced at time intervals T_s, with unit amplitude, it can be shown that the Fourier transform of the sampling function is a series of Dirac delta functions at all positive and negative integer multiples of $1/T_s$.

A sine wave can be shown to have a Fourier transform that consists of a pair of delta functions at $+/-(1/T)$, where T is the period of the sine wave.

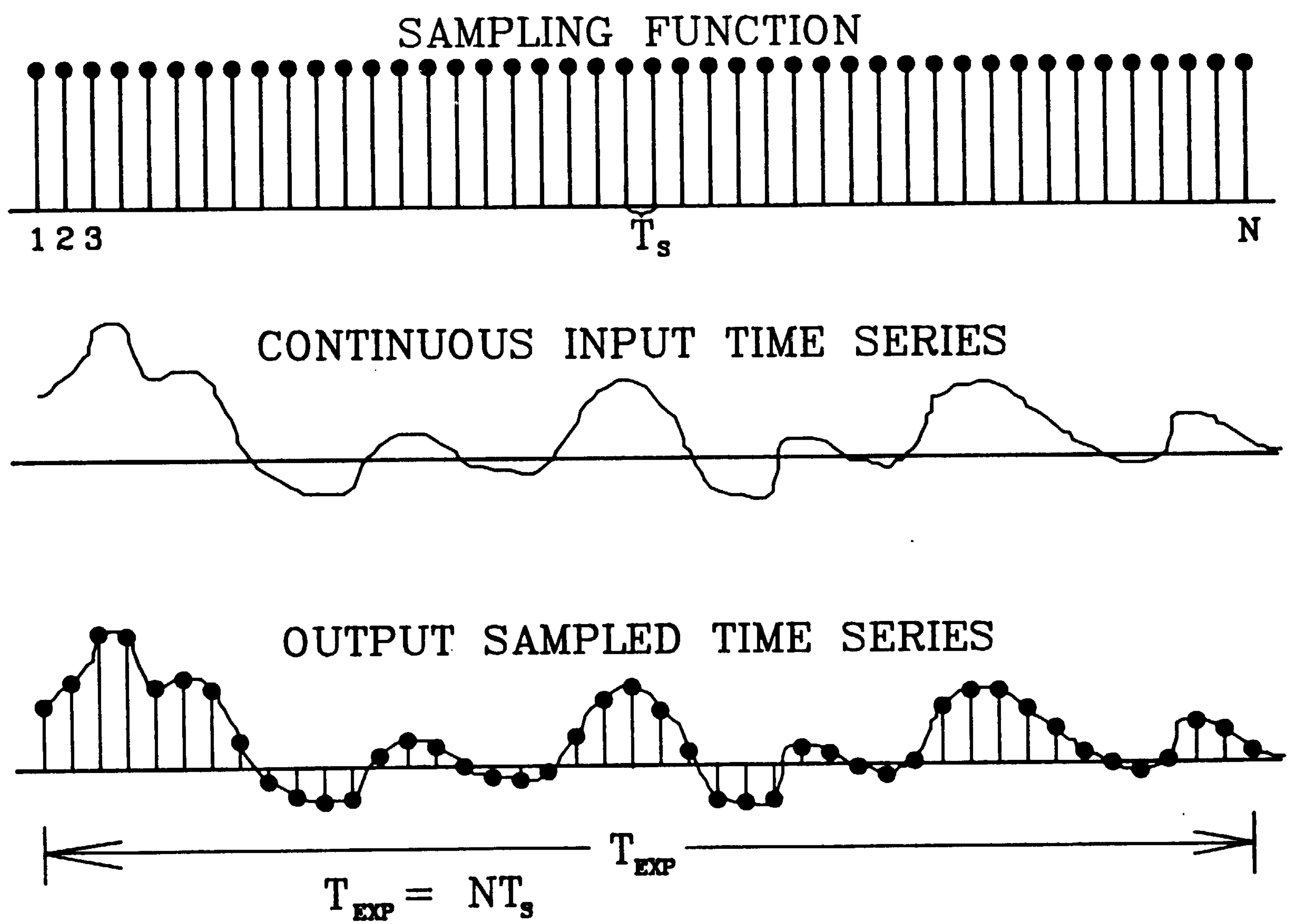


FIGURE 8. SAMPLED CONTINUOUS DATA

Of course, we cannot measure the signal for infinite time, so we must window the signal somehow. Figure 9 shows the Fourier transforms of a sine wave, the sampling function, and a rectangular window, each alone.

So now we are ready to calculate the Fourier transform of a finite set of data. We simply multiply (signal) X (sampling function) X (window) and transform. The convolution theorem states that the result is simply the convolution in the frequency domain of the three individual Fourier transforms.

For simplicity let the signal be the pure sine wave. Then the result is easy, since delta functions convolve quite nicely with anything else. Figure 10 shows the result of this process. We see immediately why we were so careful about aliasing, since the spectrum repeats itself every $1/T_s$. Also very important is the fact that the window effectively spreads the energy of the sine wave into other frequencies.

Windowing

The fact that we cannot look at a signal for infinite time necessitates the use of a window. As shown in Figure 10, the presence of a window will distort the spectrum in comparison with that in the case of a perfect transform. In the case of a rectangular window, the sidelobes are down in amplitude only 13 dB. The main lobe is only $1/T_w$ wide, however. Figure 11 shows the Fourier transforms of three popular windows, the rectangular, Hann, and Flat Top. The Flat Top window has the minimum amplitude sidelobes, but the main lobe is now five times as wide. If we want to distinguish two different frequencies, the minimum spacing between the two is now $5/T_w$.

This provides the criteria for selection of time span for the experiment and the subsets of the data. For example, if we want to distinguish frequencies as small as 10^{-6} Hz, then the window must be at least 5 x 10^6 sec long if we use the Flat Top window. This translates to an experiment duration of almost 58 days. If we used the

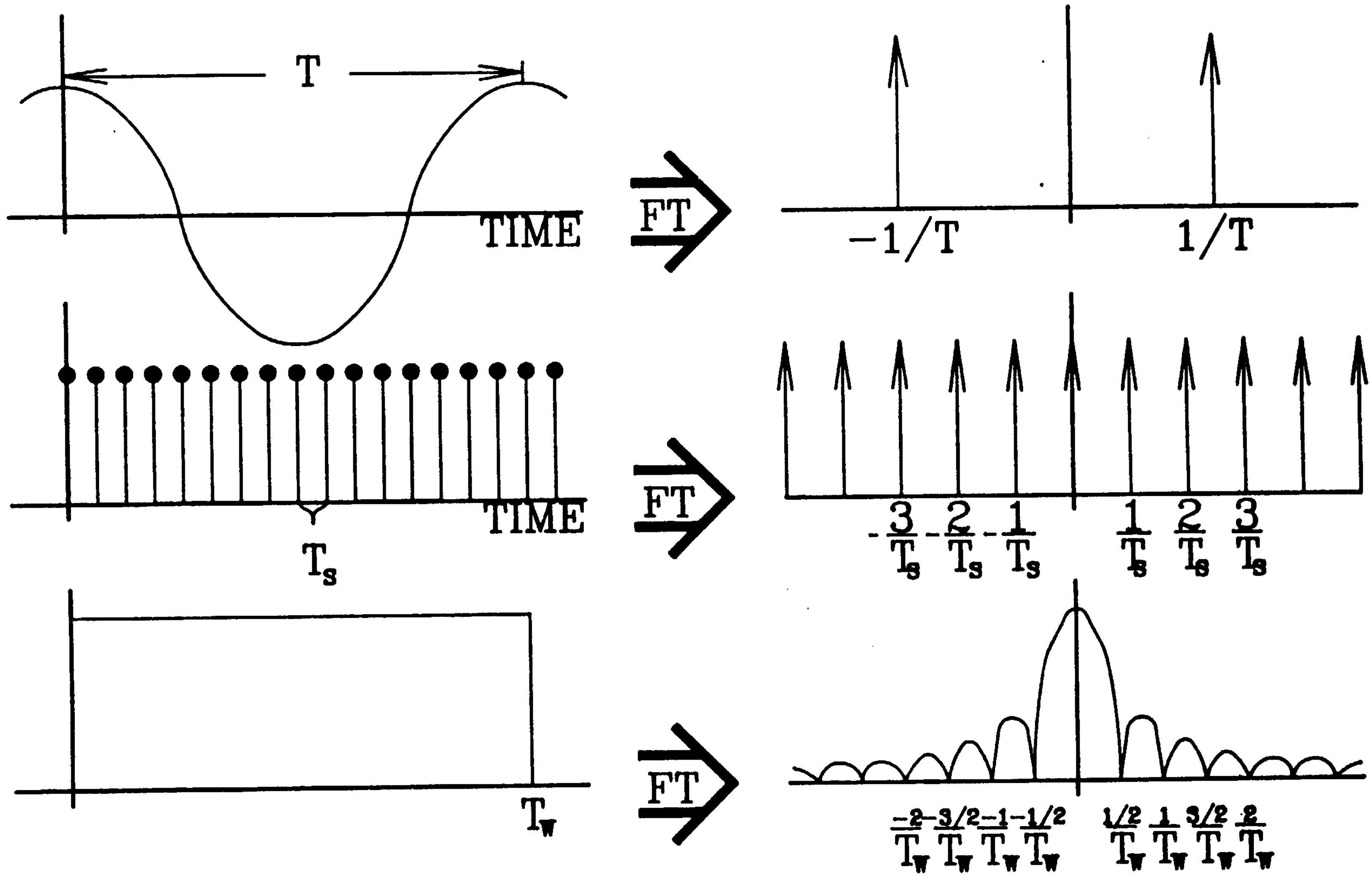


FIGURE 9. FOURIER PAIRS

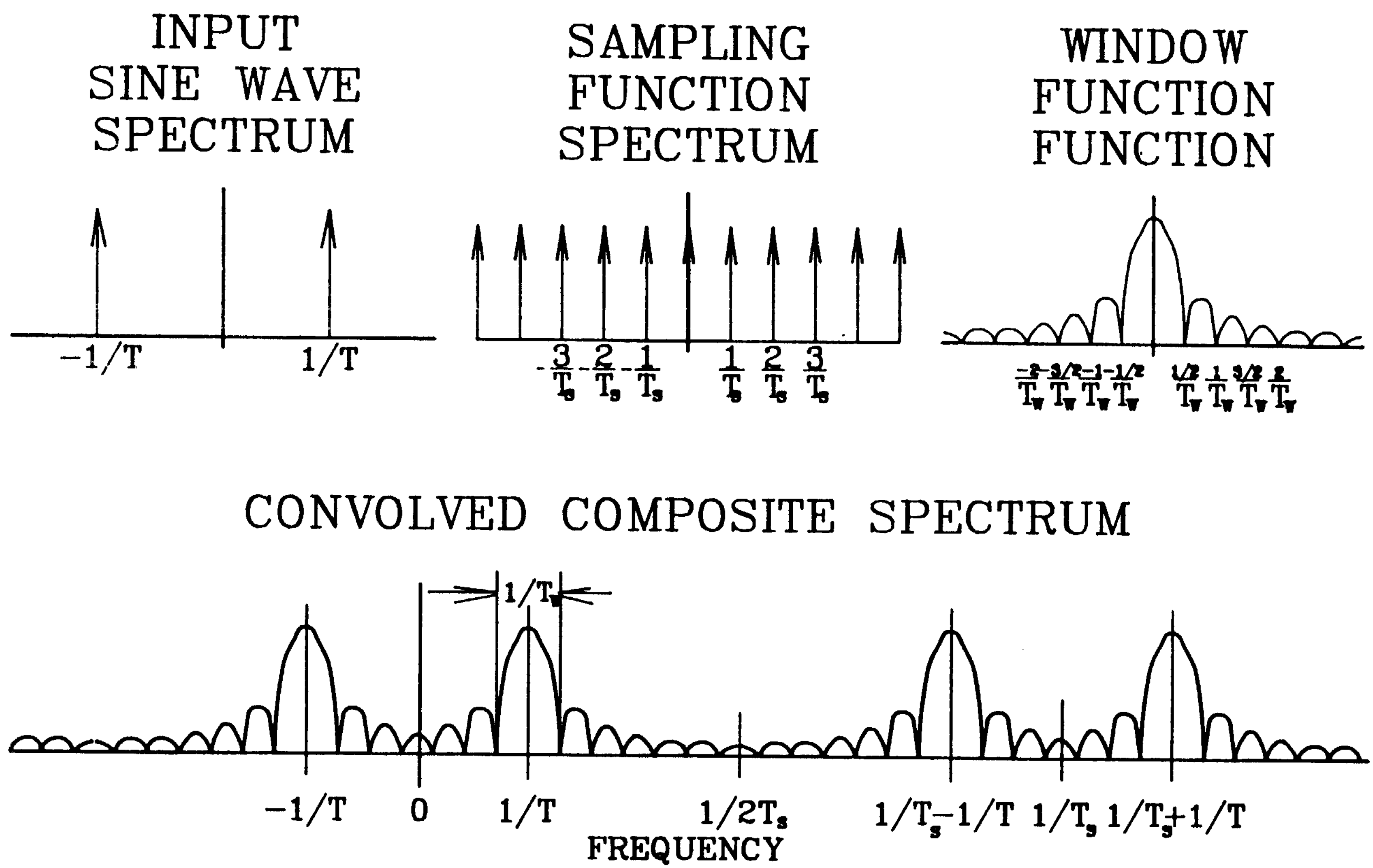


FIGURE 10. SINE WAVE SPECTRUM

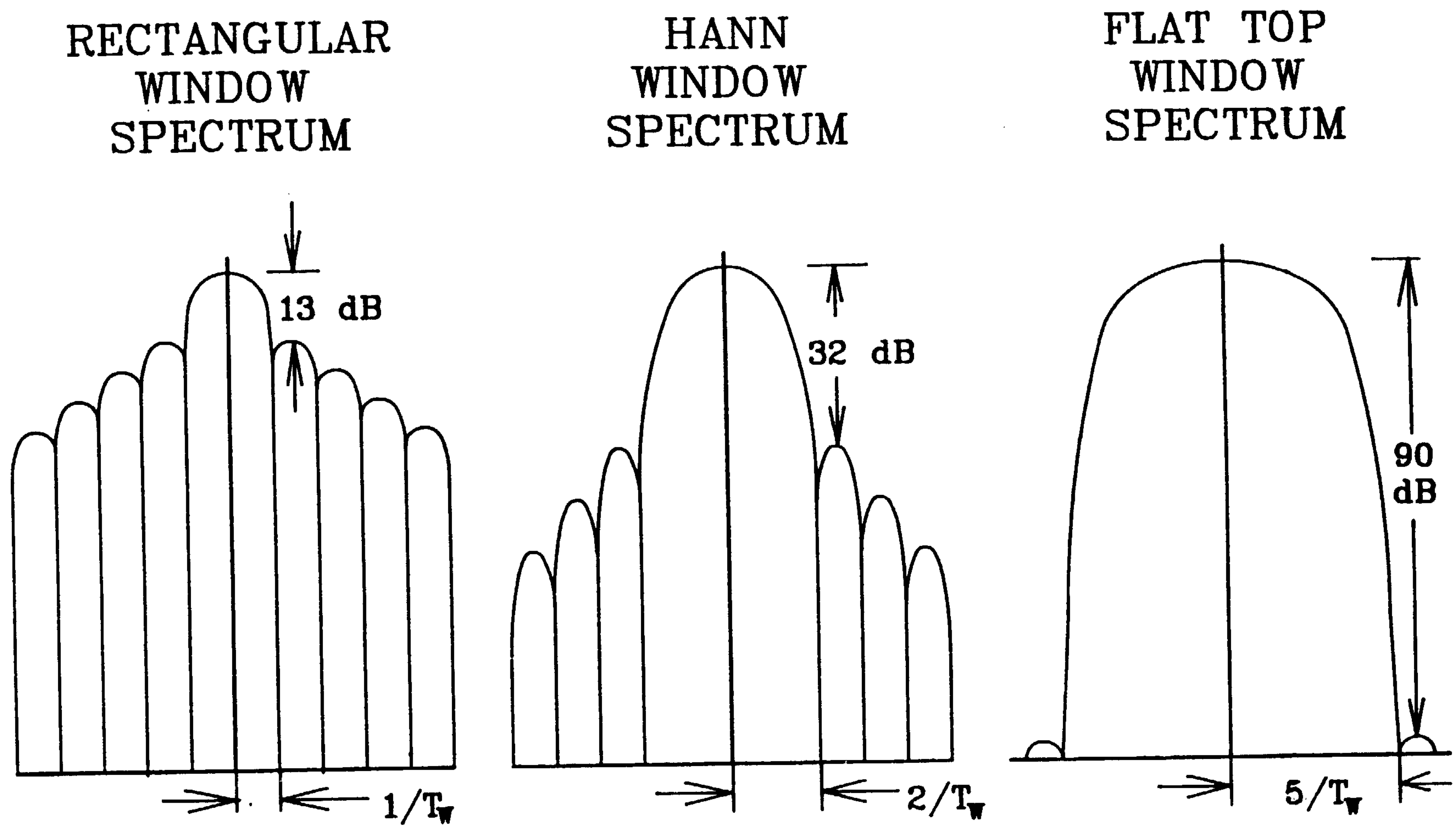


FIGURE 11. WINDOW FUNCTIONS

rectangular window, we would still need over 11 days. Also, if the spectral intensity of two adjacent (about $1/T_{EXP}$ apart) peaks differs by more than 13 dB, then the rectangular window will not be suitable for picking them out.

Averaging

If we have a data set that is sufficiently long that the required window length for our frequency resolution measurements is some fraction of the entire length, we have averaging options. The purpose of averaging is to improve the measurement of random processes.

Power Spectrum Averaging: If we average successive power spectra, the main improvement is in the standard deviation of the random signal measurements. For a Gaussian process whose amplitude is constant across the window bandwidth, the normalized standard deviation of the amplitude measurement, σ is as follows:

$$\sigma = 1/\sqrt{(B_w K T_{RECORD})}$$

Where B_w is the window bandwidth, K is the number of records averaged, and T_{RECORD} is the record time length.

This process will not enhance the signal-to-noise ratio. Any signal that is "buried" in the noise will not be enhanced, since the noise amplitude will not be altered. Only our confidence in the accuracy of the random process measurements will be improved.

Time Averaging: Subsets of the time data can be averaged before taking the spectra and the signal-to-noise ratio for some periodic signal in a noisy environment enhanced. We must have some reference for this process in order that the phase for the periodic process be the same in the successive time records. If not, destructive interference between averaged data sets can eliminate the signal-to-noise improvement.

Numerically, the signal will be increased by a factor K by adding K spectra. Random processes will only increase by a factor of $\sqrt{K}$.

Hence, time averaging K spectra will yield a signal to noise ratio improvement of $\sqrt{K}$.

This might be useful if we wanted to see the effect of some process like the repeated firing of a correction rocket, whose times could be accurately determined from some other source. These times could then be used to properly synchronize the time averaged data sets.

In general, the process most often used to estimate acceleration background on the earth is to average spectra. Here we assume that the processes are random in nature. A qualitative look at the time data will show whether there are any discernible nonrandom processes. These signals require special attention to identify the source. Otherwise, spectral averaging will provide the most accurate estimates of amplitude.

EXAMPLE EXPERIMENT

During July 1986, a Teledyne Geotech Model 44000 Borehole Seismometer was used to record acceleration data over a period of several days. The data set discussed here spanned 72 hours, from approximately 12 PM 15JUL86 to 12 PM 18JUL 86. The primary purpose of the experiment was to observe earth background from 1 Hz to the primary tidal frequencies, 12 and 24 hours.

Sensor Selection

The Model 44000 Seismometer is a very high resolution accelerometer. The package is tubular, approximately 6 feet in length and 3.75 inches in diameter. Three orthogonal axes of accelerometer reside within. The sensors use a capacitive position sensor and active feedback for response stabilization.

Anticipated tidal accelerations were approximately 100 nano-g peak. The instrument noise floor is lower than 10^{-20} g^2/Hz, with a bandwidth of 4 Hz in its present configuration. Optional bandwidths at higher frequencies are possible by changing the feedback parameters.

Small signal linearity is greater than 10^6, and dynamic range is greater than 120 dB. The response was set for 2 X 10^4 V/m/s^2 from dc to 4 Hz. The maximum anticipated signal output was 10^{-6} g. We anticipated no significant dynamic range or noise problems, as the predicted spectral intensity of the tidal fluctuations was expected to be greater than 10^{-13} g^2/Hz.

Amplifier and Filter

A special filter with dc to 0.1 Hz bandwidth was employed with a gain of 50 in the passband. The filter has response that is down by over 100 dB at 2.5 Hz, the Nyquist frequency for this experiment.

Digitizer

A 16-bit digitizer running at a 5-Hz sample rate was used to collect the filtered data. The quantization noise, referred to as acceleration, was 1.6 X 10^{-23} g^2/Hz. The digitizer LSB was 3 X 10^{-11} g. With 96 dB of dynamic range, the anticipated spectral intensity was within the limits of the digitizer. The data were recorded on standard 1/2 inch, 9-track, 1600 BPI magnetic tape.

Analysis

The raw data were decimated by 10, since the spectral content was not sufficient to induce aliasing in this process. Next, a linear trend was removed. The final time domain process was to digitally filter and decimate by another factor of 10 in order to yield a data set of 12,960 samples over a period of 259,200 sec with a sample rate of 0.05 samples/sec. Figure 12 shows the time record in comparison to pure calculated tidal data. The bar shows an amplitude of 50 μgal, which is equal to 50 nano-g. The glitches are teleseismic earthquakes. Quite good agreement with the theoretical data is shown.

The data set was transformed using a standard Fast Fourier Transform (FFT). The result is shown in Figure 13. The minimum

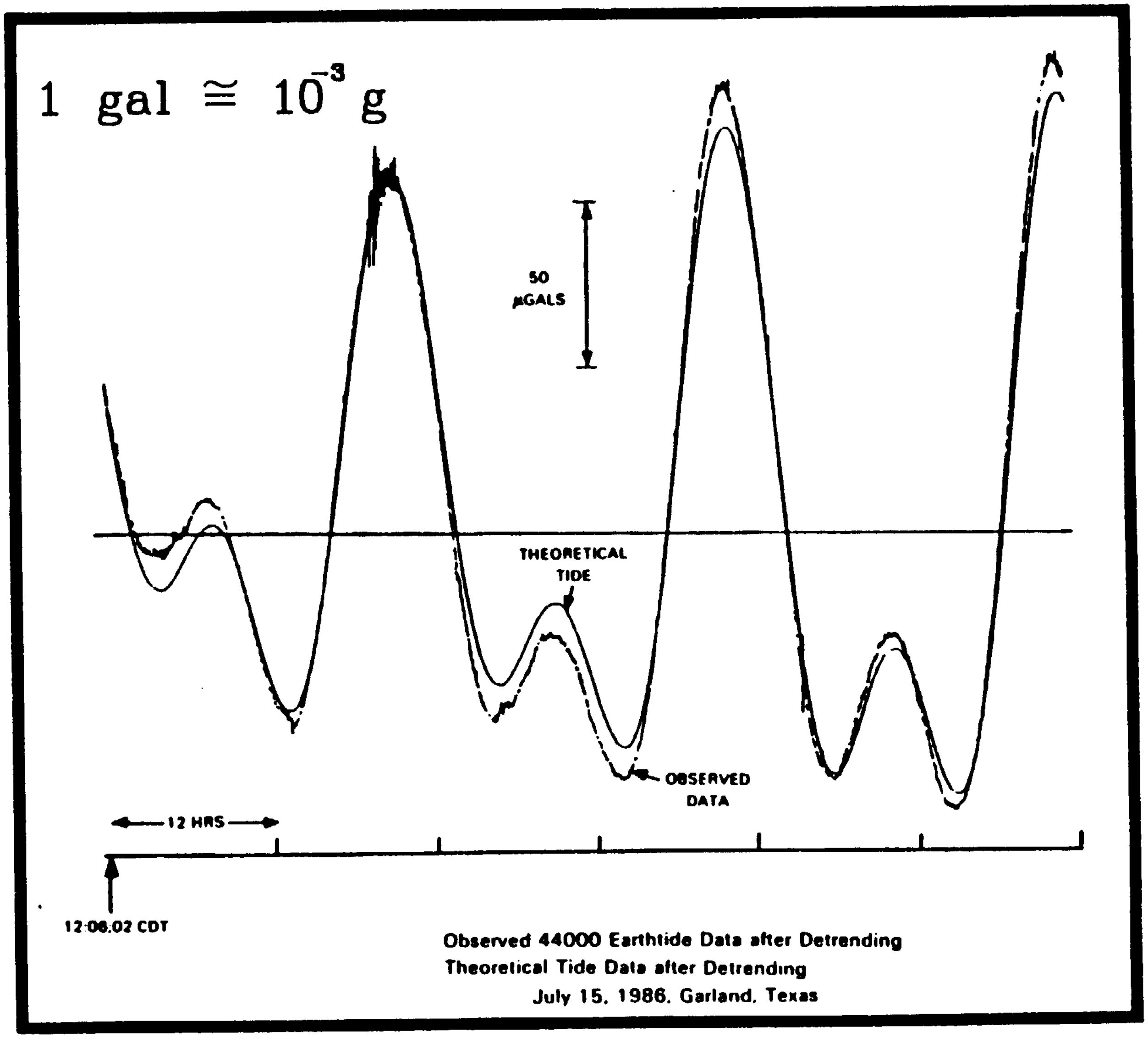


FIGURE 12. TIME SERIES DATA

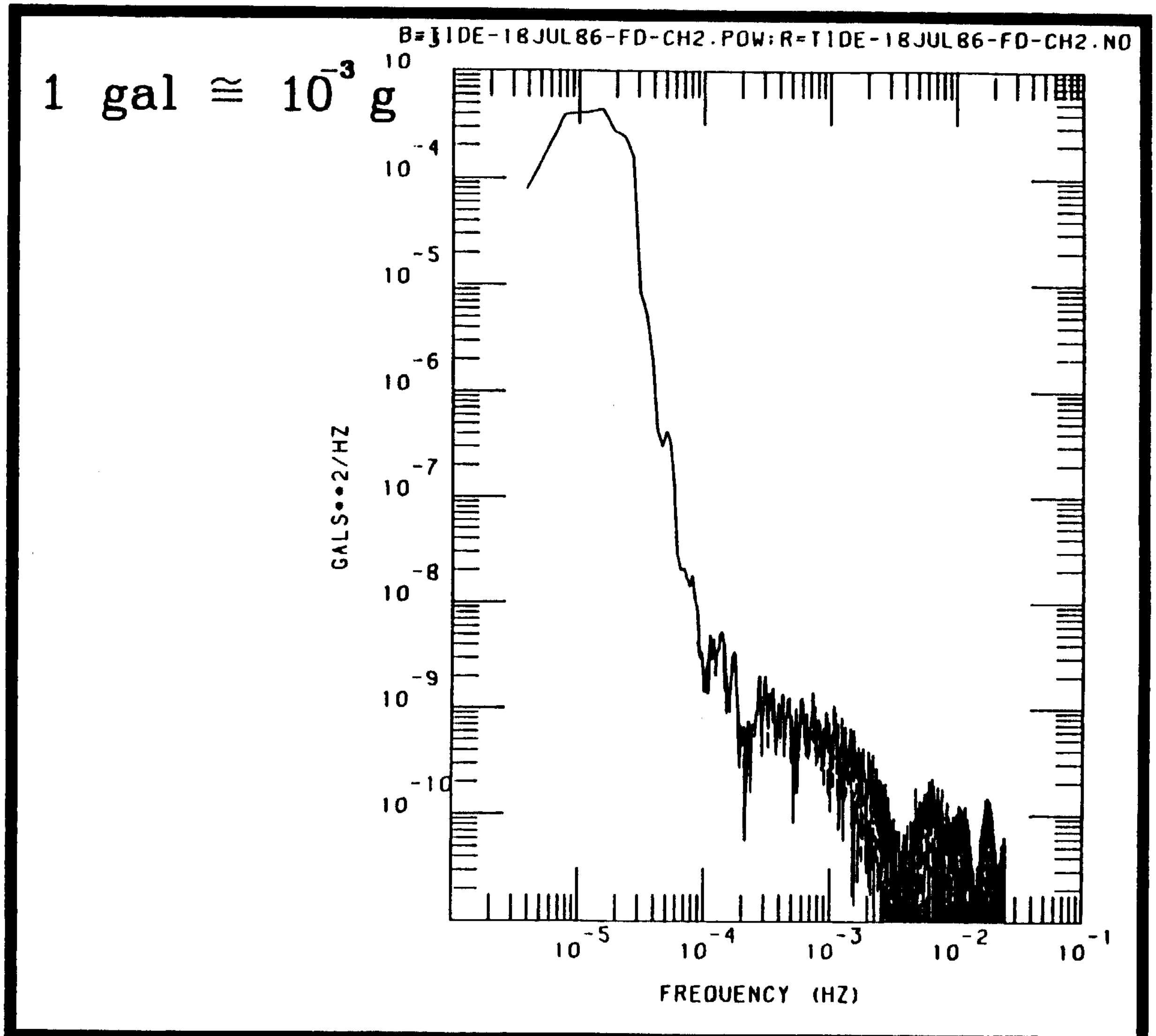


FIGURE 13. POWER SPECTRUM

frequency was 3.86 X 10^{-6} Hz, which is simply the reciprocal of the record length. Peaks at 1.2 X 10^{-5} Hz and 2.3 X 10^{-5} Hz overlap one another. In this case, the experiment was not long enough to discern both peaks spectrally. The time series clearly shows the presence of both, however.

This experiment is intended to show the potential resolution at long periods, and the problems associated with long period data analysis. We were able to obtain good data for frequencies as low as 10^{-5} Hz, even though our minimum frequency was lower. The experiment duration was 3 days. To resolve frequencies as low as 10^{-6} Hz, a continuous experiment of one month would be necessary!

CONCLUSION

The discussion above has been intended essentially as a primer on the topics of acquisition and analysis of acceleration data. Both areas require careful consideration if an experiment is to be successful and meaningful.

It is the author's belief that careful design of the experiment is the first step in data analysis. For long periods, the experiment duration must be adequate. Issues such as power stability, data storage capacity, and reliability become very important for ultra low frequency measurements.

The sensor must also be selected carefully, since stability and low noise at ultra long periods are essential to the acquisition of high quality data.

Finally, even simple analysis has caveats and assumptions that must be kept in mind throughout any experiment. The very nature of sampling and the finiteness of the experiment alter the Fourier transform of the measured signal. An understanding of the basics is important to proper interpretation of end results.

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