Sally Dawson, BNL
email: dawson@bnl.gov
http://quark.phy.bnl.gov/~dawson/

EWSB3

- Particles of a SUSY Model
  - Scalar Interactions
    - 2-component notation
  - Anomaly cancellation
  - Superpotential of MSSM
    - R parity
    - Some phenomenological implications
  - Gauge fields and gauginos
- How to break SUSY?
- Higgs potential
  - Higgs mass bounds
  - Higgs search strategies

Many excellent references on the MSSM
(many are TASI summer schools)

- N. Polonsky, hep-ph/0108236
- M. Drees, hep-ph/9611409
- J. Bagger, hep-ph/9604232
- M. Dine, hep-ph/9612389
- H. Haber, hep-ph/9306207
- S. Martin, hep-ph/9709356
- S. Dawson, hep-ph/9712464

See also:
Wess and Bagger, “Supersymmetry and Supergravity”
Gates, Grisaru, Rocek, Siegel, “Superspace”

- Supersymmetric generators, Q, must have property

$$Q \mid \text{Boson}\rangle = \mid \text{Fermion}\rangle$$
$$Q \mid \text{Fermion}\rangle = \mid \text{Boson}\rangle$$

- Q must be fermionic, anti-commuting, spin- $\frac{1}{2}$ operator
- Required properties:

$$\begin{aligned} \{Q_\alpha, \overline{Q}_{\dot\beta}\} =& 2\sigma^\mu_{\alpha\dot\beta} P_\mu \\ \{Q_\alpha, Q_\beta\} =& \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \\ [P_\mu, P_\nu] =& 0 \\ [Q_\alpha, P_\mu] =& [\overline{Q}_{\dot\alpha}, P_\mu] = 0 \end{aligned}$$

$\alpha, \beta = 1, 2$
$Q, \overline{Q}$ are $2-$ component Weyl spinors
P_μ is momentum generator
$N = 1$ SUSY (only 1 set of Q's)

- Particles classified into representations of SUSY algebra called supermultiplets

- $(mass)^2 = -P^2$ commutes with Q, $\overline{Q}$

$\longrightarrow$ particles in same supermultiplet have equal masses
$\longrightarrow$ also equal gauge quantum numbers

Supermultiplets have equal fermionic/bosonic degrees of freedom:

Define operator: $(-1)^{2s}$, $s \equiv$ spin angular momentum

$$(-1)^{2s} \mid \text{Boson}\rangle = + \mid \text{Boson}\rangle$$
$$(-1)^{2s} \mid \text{Fermion}\rangle = - \mid \text{Fermion}\rangle$$

- Fermionic operators, $\{(-1)^{2s}, Q\} = \{(-1)^{2s}, \overline{Q}\} = 0$
- All states, $\mid i\rangle$ in supermultiplet have same eigenvalue of P_μ
- Completeness relation, $\Sigma_i \mid i\rangle\langle i \mid = 1$

$$\begin{aligned}\Sigma_i \langle i \mid (-1)^{2s}\, P^\mu \mid i\rangle =& \Sigma_i \langle i \mid (-1)^{2s} \left(Q\overline{Q} + \overline{Q}Q\right) \mid i\rangle \\ =& 0\end{aligned}$$

Final result:

$$\Sigma_i \langle i \mid (-1)^{2s}\, P^\mu \mid i\rangle = p^\mu Tr\left[(-1)^{2s}\right] \sim n_B - n_F$$

Simplest possibility: Single Weyl Fermion, complex scalar

[cf. Wess and Bagger]

Aside on 2-component Notation

4-component fermion, Ψ, can be written in terms of 2-component spinors, ζ and η

$$\Psi = \begin{pmatrix} \zeta_\alpha \\ \eta^{\dagger\dot{\alpha}} \end{pmatrix} \qquad \overline{\Psi} = \left(\eta^\alpha, \zeta^\dagger_{\dot{\alpha}}\right)$$

Lagrangian for 4-component Dirac fermion

$$\mathcal{L} = \overline{\Psi}\left(i\partial_\mu \gamma^\mu - m\right)\Psi$$

Define basis:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}$$

$\overline{\sigma} = (1, -\vec{\sigma})$

Spinor indices raised/ lowered with ϵ:

$$\zeta_\alpha = \epsilon_{\alpha\beta}\zeta^\beta \qquad \zeta^\alpha = \epsilon^{\alpha\beta}\zeta_\beta$$

$$\begin{aligned} \zeta\chi =& \zeta^\alpha \chi_\alpha = \zeta^\alpha \epsilon_{\alpha\beta}\chi^\beta \\ =& -\chi^\beta \epsilon_{\alpha\beta}\zeta^\alpha = \chi^\beta \epsilon_{\beta\alpha}\zeta^\alpha \\ =& \chi^\beta \zeta_\beta = \chi\zeta \end{aligned}$$

$$\begin{aligned} \zeta^\dagger_{\dot{\alpha}} \left(\overline{\sigma}^\mu\right)^{\dot{\alpha}\alpha} \eta_\alpha =& -\eta\sigma^\mu\zeta^\dagger \\ =& \left(\eta^\dagger \overline{\sigma}^\mu \zeta\right)^* = -\left(\zeta\sigma^\mu\eta^\dagger\right)^* \end{aligned}$$

[see Haber]

Dirac equation in 2-component notation:

$$\mathcal{L} = -i\zeta^\dagger \overline{\sigma}^\mu \partial_\mu \zeta - i\eta^\dagger \overline{\sigma}^\mu \partial_\mu \eta - m\left(\eta\zeta + \eta^\dagger \zeta^\dagger\right)$$

4-component fermions of definite helicity:

$\Psi_{R,L} = P_\pm \Psi$ with $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$:

$$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{aligned} \Psi_L &= P_- \Psi = \eta^\dagger \\ \Psi_R &= P_+ \Psi = \zeta \end{aligned}$$

Can write fermion bi-linears in 2- or 4-component notation

$$\Psi_i = \begin{pmatrix} \zeta_i \\ \eta_i^\dagger \end{pmatrix}$$

Show:

$$\begin{aligned} \overline{\Psi}_1 P_L \Psi_2 &= \eta_1 \zeta_2 \\ \overline{\Psi}_1 \gamma^\mu P_L \Psi_2 &= \zeta_1^\dagger \overline{\sigma}^\mu \zeta_2 \end{aligned}$$

Particles of a Supersymmetric Model

- Supersymmetric theories constructed from supermultiplets
- Chiral superfield, Φ_i, has:
  - $\cdot$ Complex scalar field, ϕ_i
  - $\cdot$ 2-component Weyl fermion field, ψ_i
  - $\cdot$ Auxiliary Field, F_i (no kinetic energy term)

  [Cancellation of quadratic divergences: 2 complex scalars, Dirac fermion]
- Interactions described in terms of chiral superfields

$$\Phi_i(x) \equiv \phi_i(x) + \sqrt{2}\theta\psi_i(x) + \theta\theta F_i(x)$$

[Taylor series stops at θ^2 since
θ is anti-commuting Grassman variable, $\theta^3 = \frac{\theta}{2}\{\theta, \theta\}$]

- Components of Φ have identical quantum numbers (except spin) and masses
- Construct model with supermultiplets corresponding to known particles
- More than doubles spectrum

Chiral Superfields of MSSM

Superfield	SU(3)	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{Q}$	3	2	$\frac{1}{6}$	$(u_L, d_L),\ (\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	$\overline{3}$	1	$-\frac{2}{3}$	$\overline{u}_R,\ \tilde{u}^*_R$
$\hat{D}^c$	$\overline{3}$	1	$\frac{1}{3}$	$\overline{d}_R,\ \tilde{d}^*_R$
$\hat{L}$	1	2	$-\frac{1}{2}$	$(\nu_L, e_L),\ (\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^c$	1	1	1	$\overline{e}_R,\ \tilde{e}^*_R$
$\hat{H}_1$	1	2	$-\frac{1}{2}$	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	$\frac{1}{2}$	$(H_2, \tilde{h}_2)$

- Scalar partners with degenerate masses to fermions don't exist $\longrightarrow$ supersymmetry must be a broken symmetry

- Note: this is 2-Higgs doublet model where Higgs doublets have opposite Y

- Fermions of SM have just the right quantum numbers to cancel anomalies

- Fermion partner of single Higgs would give uncancelled contribution to anomaly:

SM has couplings of form

$$\mathcal{L} \sim g\overline{\psi} T^{\alpha} \gamma_{\mu} \gamma_{5} \psi A^{\alpha\mu} + \hat{g} \overline{\psi} T^{\alpha} \gamma_{\mu} \psi V^{\alpha\mu}$$

- Triangle diverges; grows with energy
- Anomaly independent of mass $\sim Tr[\eta_i t^a \{t^b, t^c\}]$
  $\eta_i = \pm 1$ for $\psi_{L,R}$

[Peskin and Schroeder, Quantum Field Theory]

$$Tr\left(Y^3\right) = 0 = \frac{1}{6}^3 (6) + \left(-\frac{2}{3}\right)^3 (3) + \left(\frac{1}{3}\right)^3 (3) + \left(-\frac{1}{2}\right)^3 (2) + 1 + \left(-\frac{1}{2}\right)^3 (2) + \left(\frac{1}{2}\right)^3 (2)$$

$$Tr\left(T_3^2 Y\right) = 0 = \left(\frac{1}{2}\right)^2 \left(\frac{1}{6}\right) (3) + \left(\frac{1}{2}\right)^2 \left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 \left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)$$

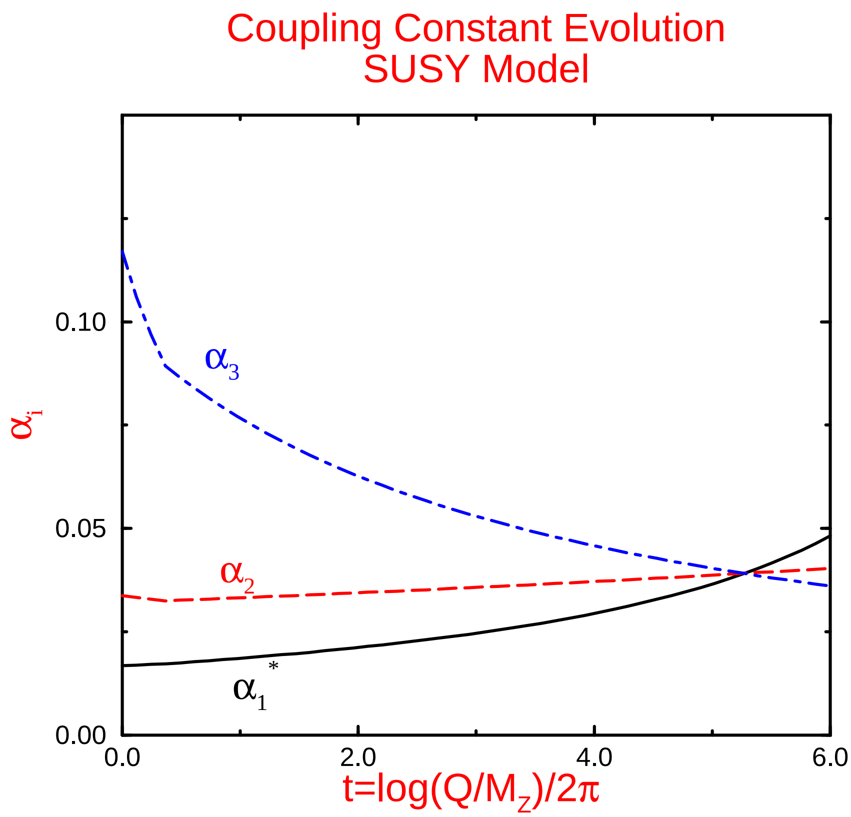
ALL SUSY Models are multi-Higgs Models

Interesting phenomenology in Higgs sector

What about unification now?

- Gauge couplings evolve differently in SUSY models
- Running is slower than SM

$$
\begin{aligned}
b_1 &= 2N_g + \frac{3N_h}{10} \\
b_2 &= -6 + 2N_g + \frac{N_h}{2} \\
b_3 &= -9 + 2N_g
\end{aligned}
$$



The cynic:

- Assume supersymmetric at some scale, Λ_{SUSY} (say $1\ TeV$)
- Input 2 couplings, say g_1, g_2, at M_Z
- Evolve couplings until they meet at scale M_{GUT}
- Assume unification at M_{GUT}, $g_i(M_{GUT}) = g_{GUT}$
- Evolve $3rd$ coupling, say g_3, down to M_Z
- If $g_3(M_Z)$ disagrees with measured number, change Λ_{SUSY}
- Keep going until it works!

Amazing fact: consistency occurs for
$\Lambda_{SUSY} \sim 1\ TeV$

(Actually, $\alpha_s(M_z)$ typically a little large)

Simple Example

Complex Scalar, ϕ, and 2-component Weyl Fermion, ψ:

$$
\begin{aligned}
S =& \int d^4x \, (\mathcal{L}_S + \mathcal{L}_F) \\
\mathcal{L}_S =& - \partial^\mu \phi^* \partial_\mu \phi \\
\mathcal{L}_F =& - i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi
\end{aligned}
$$

Want SUSY transformation to take $\phi \to (...)\psi$

Ansatz:

$$
\begin{aligned}
\delta\phi =& \theta\psi \\
\delta\phi^* =& \theta^\dagger \psi^\dagger
\end{aligned}
$$

Under this transformation,

$$
\delta\mathcal{L}_S = -\theta\partial^\mu\psi\partial_\mu\phi^* - \theta^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi
$$

Need to cancel with $\delta\mathcal{L}_F$. Guess:

$$
\begin{aligned}
\delta\psi_\alpha =& i \left(\sigma^\mu\theta^\dagger\right)_\alpha \partial_\mu\phi \\
\delta\psi^\dagger_{\dot{\alpha}} =& - i \left(\theta\overline{\sigma}^\mu\right)_{\dot{\alpha}} \partial_\mu\phi^*
\end{aligned}
$$

This gives:

$$\mathcal{L}_F = -\theta\sigma^\mu\overline{\sigma}^\nu\partial_\nu\psi\partial_\mu\phi^* + \psi^\dagger\overline{\sigma}^\nu\sigma^\mu\theta^\dagger\partial_\mu\partial_\nu\phi$$

Use Pauli matrix identity:

$$[\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu]_\alpha^\beta = -2g^{\mu\nu}\delta_\alpha^\beta$$

$$\begin{aligned}\delta\mathcal{L}_F =& \theta\partial^\mu\psi\partial_\mu\phi^* + \theta^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi \\ &- \partial_\mu\left(\theta\sigma^\nu\overline{\sigma}^\mu\psi\partial_\nu\phi^* + \theta\psi\partial^\mu\phi^* + \theta^\dagger\psi^\dagger\partial^\mu\phi\right)\end{aligned}$$

First line cancels $\delta\mathcal{L}_S$; 2nd line is total derivative

$$\delta S = \int d^4x\,(\delta\mathcal{L}_S + \delta\mathcal{L}_F) = 0$$

Are we done?

- Must show that SUSY algebra closes

$$\left[\delta_{\theta_1}, \delta_{\theta_2}\right] \phi = i \left(\theta_1 \sigma^\mu \theta_2^\dagger - \theta_2 \sigma^\mu \theta_1^\dagger\right) \partial_\mu \phi$$

Good.... Now try it for ψ

$$\left[\delta_{\theta_1}, \delta_{\theta_2}\right] \psi_\alpha = i \left(\sigma^\mu \theta_1^\dagger\right)_\alpha \theta_2 \partial_\mu \psi - i \left(\sigma^\mu \theta_2^\dagger\right)_\alpha \theta_1 \partial_\mu \psi$$

Use same Fiertz transform as before

$$\begin{aligned}\left[\delta_{\theta_1}, \delta_{\theta_2}\right] \psi_\alpha =& i \left(\theta_1 \sigma^\mu \theta_2^\dagger - \theta_2 \sigma^\mu \theta_1^\dagger\right) \partial_\mu \psi_\alpha \\ & - i \theta_{1\alpha} \theta_2^\dagger \overline{\sigma}^\mu \partial_\mu \psi + i \theta_{2\alpha} \theta_1^\dagger \overline{\sigma}^\mu \partial_\mu \psi\end{aligned}$$

2nd line is left over....

Introduce complex scalar field, F, with no kinetic energy

$$\mathcal{L}_{aux} = F^* F$$

As before, ansatz for δF:

$$\begin{aligned}\delta F =& i \theta^\dagger \overline{\sigma}^\mu \partial_\mu \psi \\ \delta F^* =& - i \partial_\mu \psi^\dagger \overline{\sigma}^\mu \theta\end{aligned}$$

$$\delta\mathcal{L}_{aux} = i\theta^\dagger\overline{\sigma}^\mu\partial_\mu\psi F^* - i\partial_\mu\psi^\dagger\overline{\sigma}^\mu\theta F$$

Cancel this by adding extra piece to $\delta\psi$:

$$\delta\psi_\alpha = i\left(\sigma^\mu\theta^\dagger\right)_\alpha\partial_\mu\phi + \theta_\alpha F$$

NOW....

$$[\delta\theta_1, \delta\theta_2]\, B = i\left(\theta_1\sigma^\mu\theta_2^\dagger - \theta_2\sigma^\mu\theta_1^\dagger\right)\partial_\mu B$$

$$\text{for } B = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$$

This symmetry is valid symmetry of Lagrangian

Now construct interactions

$$\mathcal{L} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i$$

Invariant under supersymmetry transformations:

$$\begin{aligned} \delta \phi_i =& \theta \psi \\ \delta \left(\psi_i\right)_\alpha =& i \left(\sigma^\mu \theta^\dagger\right)_\alpha \partial_\mu \phi_i + \theta_\alpha F_i \\ \delta F_i =& i\theta^\dagger \overline{\sigma}^\mu \partial_\mu \psi_i \end{aligned}$$

Most general set of renormalizable interactions:

$$\mathcal{L}_{int} = -\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + c.c.$$

Renormalizability implies W^{ij}, W^i functions of ϕ_i, ϕ_i^* only

What are conditions on W^{ij}, W^i such that $\delta \mathcal{L}_{int} = 0$?

$$\delta \mathcal{L}_{int}^{3\psi} = -\frac{1}{2} \frac{\delta W^{ij}}{\delta \phi_k} \left(\theta \psi_k\right) \left(\psi_i \psi_j\right) - \frac{1}{2} \frac{\delta W^{ij}}{\delta \phi_k^*} \left(\theta^\dagger \psi^{\dagger k}\right) \left(\psi_i \psi_j\right)$$

Each term has to cancel separately

Use Fiertz identity:

$$\chi_\alpha \left(\zeta \eta\right) = -\zeta_\alpha \left(\eta \chi\right) - \eta_\alpha \left(\chi \zeta\right)$$

[proof in Haber]

$$\left(\theta \psi_i\right) \left(\psi_j \psi_k\right) + \left(\theta \psi_j\right) \left(\psi_k \psi_i\right) + \left(\theta \psi_k\right) \left(\psi_i \psi_j\right) = 0$$

First term vanishes if $\frac{\delta W^{ij}}{\delta\phi_k}$ symmetric under interchange of i, j, k

No way to cancel 2nd term $\longrightarrow$ $W^{ij} = W^{ij}(\phi_k)$

$W^{ij} = M^{ij} + g^{ijk}\phi_k \equiv \frac{\delta^2 W}{\delta\phi_i\delta\phi_j}$

Define superpotential:

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{g^{ijk}}{6}\phi_i\phi_j\phi_k$$

Continue checking cancellation....

Terms involving partial derivative:

$$\delta\mathcal{L}^{\partial}_{int} = -iW^{ij}\partial_\mu\phi_j\psi_i\sigma^\mu\theta^\dagger - iW^i\partial_\mu\psi_i\sigma^\mu\theta^\dagger + c.c.$$

$$W^{ij}\partial_\mu\phi_j = \partial_\mu\left(\frac{\delta W}{\delta\phi_i}\right)$$

$\delta\mathcal{L}^{\partial}_{int}$ will be a total derivative if:

$$W^i = \frac{\delta W}{\delta\phi_i} = M^{ij}\phi_j + \frac{g^{ijk}}{2}\phi_j\phi_k$$

[Remainder of terms in $\delta\mathcal{L}_{int}$ cancel easily]

What about auxilliary fields, F_i?

$$\mathcal{L}_F \sim F_i F^{*i} + W^i F_i + W_i^* F^{*i}$$

Use equations of motion:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial F_i} =& F^{*i} + W^i = 0 \\ \frac{\partial \mathcal{L}}{\partial F^{*i}} =& F_i + W_i^* = 0\end{aligned}$$

This give scalar potential:

$$\begin{aligned}V(\phi, \phi^*) =& W^i W_i^* > 0 \\ =& M_{ij}^* M^{kj} \phi^{*i} \phi_j + \frac{M^{in}}{2} g_{jkn}^* \phi_i \phi^{*j} \phi^{*k} \\ &+ \frac{M_{in}^*}{2} g^{jkn} \phi_i^* \phi_j \phi_k + \frac{1}{4} g^{ijk} g_{kln}^* \phi_i \phi_j \phi^{*k} \phi^{*l}\end{aligned}$$

Problems breaking EWS....

Put it all together,

$$
\begin{aligned}
\mathcal{L} = & - \partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi \\
& - \frac{1}{2} M^{ij} \psi_i \psi_j - \frac{1}{2} M^*_{ij} \psi^{i\dagger} \psi^{\dagger j} - V\left(\phi, \phi^*\right) \\
& - \frac{g^{ijk}}{2} \phi_i \psi_j \psi_k - \frac{g^{ijk*}}{2} \phi^{*i} \psi^{*j} \psi^{*k}
\end{aligned}
$$

- Superpotential has Yukawa interactions of fermions with scalars and quartic interactions of potential:

$$\mathcal{L}_W = -\sum_i \mid \frac{\partial W}{\partial \phi_i} \mid^2 - \frac{1}{2} \sum_{ij} \left[\frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j + \text{c.c.} \right]$$

- Most general $SU(3) \times SU(2)_L \times U(1)_Y$ invariant superpotential:

(Hats denote scalar component of supermultiplet)

$$W = -\epsilon_{ij}\mu \hat{H}_1^i \hat{H}_2^j + \epsilon_{ij}\left[\lambda_L \hat{H}_1^i \hat{L}^{cj}\hat{E}^c + \lambda_D \hat{H}_1^i \hat{Q}^j \hat{D}^c + \lambda_U \hat{H}_2^j \hat{Q}^i \hat{U}^c\right]$$
$$+ \epsilon_{ij}\left[\lambda_1 \hat{L}^i \hat{L}^j \hat{E}^c + \lambda_2 \hat{L}^i \hat{Q}^j \hat{D}^c\right] + \lambda_3 \hat{U}^c \hat{D}^c \hat{D}^c$$

i, j $SU(2)$ indices

- Could have $\epsilon_{ij}\hat{L}^i \hat{H}_2^j$
- Rotate $\hat{L}$ to make this vanish
- λ_i 3×3 matrices
- $\mu \hat{H}_1 \hat{H}_2$ term gives mass terms for the Higgs bosons
- Physics sensitive to the sign of μ
- Need 2 Higgs doublets to give mass to u and d quarks
- No mass terms for SUSY partners yet

R Parity

Allowed terms in superpotential:

$$W_{RP} = \lambda_1^{ijk} \hat{L}^i \hat{L}^j \hat{E}^c + \lambda_2^{ijk} \hat{L}^i \hat{Q}^j \hat{D}^c + \lambda_3^{ijk} \hat{U}^c \hat{D}^c \hat{D}^c$$

- W_{RP} gives lepton/baryon number violating interactions

$$\mathcal{L} \sim \lambda_2 u_L e_L \tilde{d}_R^* + \lambda_3 \overline{u}_R \overline{d}_R \tilde{d}_R^*$$

- Could just make coefficients small
- Limits on proton decay require:

$$\mid \lambda_2^{11i} \lambda_3^{11i} \mid < 10^{-27} \left(\frac{M_{\tilde{d}_i}}{100\ GeV} \right)^2$$

Impose *symmetry* which forbids W_{RP}

- R parity is multiplicative quantum number: Discrete Z_2 symmetry
- Imposed by hand

$$R \equiv (-1)^{3(B-L)+2s}$$

- SM particles have $R = 1$, SUSY partners have $R = -1$
- $\theta \to -\theta$ does same thing

$$\Phi_i(x) \equiv \phi_i(x) + \sqrt{2}\theta\psi_i(x) + \theta\theta F_i(x)$$

- SM dimension- 4 baryon/lepton number violating interactions forbidden by the gauge symmetries

Consequences of R parity:

- SUSY partners are pair produced
- Lightest SUSY particle (LSP) is stable
- LSP neutral (cosmological bounds on stable light charged or colored particles)
- The LSP interacts very weakly with ordinary matter
- Generic signal for R parity conserving SUSY theories is missing transverse energy from the non-observed LSP.

Signal for R parity violation:

- HERA experiments saw excess of events at large Q^2, ($Q^2 > 1.5 \times 10^4 GeV^2$), in e^+p deep inelastic scattering events
- Explained by resonant squark production

$$e^+ d \to \tilde{u}_L, \tilde{c}_L, \tilde{t}_L$$

Include Gauge fields in the Theory

Add:

- Massless gauge boson, A_μ^a
- 2-component Weyl fermion gaugino, λ^a (adjoint representation of group)
- Auxilliary Field, D^a (adjoint representation of group; $[mass]^2$)

Gauge invariant interactions are:

$$\mathcal{L}_{gauge} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} - i\lambda^{\dagger a}\overline{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2}D^a D^a$$

As usual:

$$\begin{aligned} F_{\mu\nu}^a =& \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c \\ D_\mu \lambda^a =& \partial_\mu \lambda^a - g f^{abc} A_\mu^b \lambda^c \end{aligned}$$

As before:

Guess form of SUSY transformations, verify algebra closes

Transformations which work:

$$\begin{aligned} \delta A_\mu^a =& -\frac{1}{\sqrt{2}}\left(\theta^\dagger \overline{\sigma}_\mu \lambda^a + \lambda^{\dagger a}\overline{\sigma}_\mu \theta\right) \\ \delta \lambda_\alpha^a =& -\frac{i}{2\sqrt{2}}\left(\sigma^\mu \overline{\sigma}^\nu \theta\right)_\alpha F_{\mu\nu}^a + \frac{\theta_\alpha}{\sqrt{2}} D^a \\ \delta D^a =& \frac{i}{\sqrt{2}}\left(\theta^\dagger \overline{\sigma}^\mu D_\mu \lambda^a - D_\mu \lambda^{\dagger a} \overline{\sigma}^\mu \theta\right) \end{aligned}$$

Next: couple scalar supermultiplets to gauge fields

Turn ordinary derivatives into covariant derivatives:
(couples vector bosons to scalar supermultiplet)

$$
\begin{aligned}
\partial_\mu \phi_i \rightarrow & D_\mu \phi_i = \partial_\mu \phi_i + i g A_\mu^a \left(T^a \phi\right)_i \\
\partial_\mu \phi^{*i} \rightarrow & D_\mu \phi^{*i} = \partial_\mu \phi^{*i} - i g A_\mu^a \left(\phi^* T^a\right)^i \\
\partial_\mu \psi_i \rightarrow & D_\mu \psi_i = \partial_\mu \psi_i + i g A_\mu^a \left(T^a \psi\right)_i
\end{aligned}
$$

- Are there other gauge invariant terms (involving λ, D) which we need to make $\mathcal{L}$ supersymmetric?

- Possible renormalizable terms:

$$
\begin{gathered}
\left(\phi^* T^a \psi\right) \lambda^a \\
\lambda^{\dagger a} \left(\psi^\dagger T^a \phi\right) \\
\left(\phi^* T^a \phi\right) D^a
\end{gathered}
$$

- Add these terms, with arbitrary couplings to $\mathcal{L}$
  $\longrightarrow$ demand $\mathcal{L}$ invariant under SUSY transformations (up to total derivative)
- Need to modify transformation laws:

$$
\begin{aligned}
\delta \phi_i = & \theta \psi_i \\
\delta \left(\psi\right)_\alpha = & i \left(\sigma^\mu \theta^\dagger\right)_\alpha D_\mu \phi_i + \theta_\alpha F_i \\
\delta F_i = & i \theta^\dagger \overline{\sigma}^\mu D_\mu \psi_i + \sqrt{2} g \left(T^a \phi\right)_i \theta^\dagger \lambda^{\dagger a}
\end{aligned}
$$

Final result:

$$
\begin{aligned}
\mathcal{L} =& \mathcal{L}_{gauge} + \mathcal{L}_{chiral} \\
&- \sqrt{2} g \left[\left(\phi^* T^a \psi \right) \lambda^a + \lambda^{\dagger a} \left(\psi^\dagger T^a \phi \right) \right] \\
&+ g \left(\phi^* T^a \phi \right) D^a
\end{aligned}
$$

What about terms involving D^a?

$$\mathcal{L} \sim \frac{1}{2} D^a D^a + g \left(\phi^* T^a \phi \right) D^a$$

Use equation of motion:

$$D^a = -g \left(\phi^* T^a \phi \right)$$

Complete scalar potential:

$$V \left(\phi, \phi^* \right) = F^{*i} F_i + \frac{1}{2} D^a D^a = W_i^* W^i + \frac{1}{2} \Sigma_a g_a^2 \left(\phi^* T^a \phi \right)^2$$

Remember 2-Higgs doublet model:

Scalar potential had 6 couplings/phase

SUSY theories:

Scalar potential determined by gauge, Yukawa couplings

Gauge Multiplets of MSSM:

Superfield	SU(3)	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{G}^a$	8	1	0	$g,\ \tilde{g}$
$\hat{W}^i$	1	3	0	$W_i,\ \tilde{\omega}_i$
$\hat{B}$	1	1	0	$B,\ \tilde{b}$

Gauge fields will get masses from Higgs mechanism

Interactions of the form $q\tilde{q}\tilde{g}$ with strength fixed by gauge coupling

Scalar Potential

$$W \sim -\epsilon_{ij}\mu \hat{H}_1^i \hat{H}_2^j + \ldots$$

"F-Term":

$$V_F = \sum_i \mid \frac{\partial W}{\partial \phi_i} \mid^2$$
$$= \mid \mu \mid^2 \left(\mid H_1 \mid^2 + \mid H_2 \mid^2 \right)$$

"D Terms"

$$V_D = \frac{1}{2} D^a D^a$$
$$D^a = -\, g_a \phi_i^* T^a \phi_i$$

H_1 has $Y = -\frac{1}{2}$
H_2 has $Y = +\frac{1}{2}$

$$U(1): \quad D' = \frac{g}{2}\left(\mid H_2 \mid^2 - \mid H_1 \mid^2 \right)$$
$$SU(2): \quad D^a = \frac{g}{2}\left(H_1^{i*}\sigma_{ij}^a H_1^j + H_2^{i*}\sigma_{ij}^a H_2^j \right)$$

(Normalization $T^a = \frac{\sigma^a}{2}$)

$SU(2)$ identity: $\sigma_{ij}^a \sigma_{kl}^a = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}$

$$V_D = \frac{g^2}{8}\left(4 \mid H_1^* \cdot H_2 \mid^2 - 2\left(H_1^* \cdot H_1\right)\left(H_2^* \cdot H_2\right) + \left(\mid H_1 \mid^2 \right)^2 + \left(\mid H_2 \mid^2 \right)^2 \right)$$
$$+ \frac{g'^2}{8}\left(\mid H_2 \mid^2 - \mid H_1 \mid^2 \right)^2$$

Scalar potential:

$$V = \mid \mu \mid^2 \left(\mid H_1 \mid^2 + \mid H_2 \mid^2 \right) + \frac{g^2 + g'^2}{8} \left(\mid H_2 \mid^2 - \mid H_1 \mid^2 \right)^2$$
$$+ \frac{g^2}{2} \mid H_1^* \cdot H_2 \mid^2$$

Minimum at $\langle V \rangle = \langle H_1 \rangle = \langle H_2 \rangle = 0$

No EWSB, No SUSY breaking....

Good exercises related to todays lecture:

- Show that Majorana color octet fermion with gauge invariant coupling to gluon can only have vector coupling
- Derive $2 \leftrightarrow 4$ component bi-linear identities
- Fill in the blanks...
- Calculate some cross sections using 2-component notation.

$$u\overline{d} \rightarrow \tilde{u}\tilde{d}^*$$
$$u\overline{u} \rightarrow \tilde{g}\tilde{g}$$

[Dawson, Eichten,Quigg, PRD31, 1581 (1985)]