TESTING CPT AND LORENTZ INVARIANCE WITH THE ANOMALOUS SPIN PRECESSION OF THE MUON

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This article discusses tests of CPT and Lorentz invariance with data from the muon g-2 experiment at Brookhaven National Laboratory. According to an extension of the Standard Model by Kostelecký et al., CPT/Lorentz violating terms in the Lagrangian induce a shift of the anomaly frequency ω_a of muons in a magnetic field. This shift is predicted to be different for positive and negative muons and to oscillate with the Earth's sidereal frequency. We discuss the sensitivity of the g-2 experiment to different parameters of this Standard Model extension and propose an analysis method to search for sidereal variations of ω_a.

1 The Muon g-2 Experiment at BNL

The muon g-2 experiment determines the g value anomaly $a_\mu = (g-2)/2$ of the muon by measuring the difference ω_a between the spin precession angular frequency ω_s and the cyclotron angular frequency ω_c of highly polarized muons of momentum $3.09\,\mathrm{GeV}/c$ in a $14.2\,\mathrm{m}$ diameter storage ring with a homogeneous magnetic field of $1.45\,\mathrm{T}$ strength. The field is determined from the NMR frequency of protons in water, calibrated relative to the free proton NMR frequency ω_p. The anomaly a_μ is determined via

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p} \tag{1}$$

where the ratio μ_μ/μ_p is taken from the measurement by W. Liu et al. [1] For muons with $\gamma = 29.3$, ω_a is not affected by the electrostatic field from the quadrupoles used for vertical focussing. Experimental details and the result for a_{μ^+} based on the data of 1999 with an uncertainty of 1.3 ppm are described elsewhere. [2]

2 Predicted Effects of CPT/Lorentz Violation on ω_a

Kostelecký et al. [3] suggested an extension of the Standard Model which retains SU(3)×SU(2)×U(1) gauge invariance, renormalizability and energy conservation, but allows for violation of CPT and Lorentz invariance. The additional muon terms in the Lagrangian are given by [4]

$$\begin{aligned}\mathcal{L}' = & -a_\kappa \bar\psi\gamma^\kappa\psi - b_\kappa\,\bar\psi\gamma_5\gamma^\kappa\psi - \tfrac{1}{2}H_{\kappa\lambda}\,\bar\psi\sigma^{\kappa\lambda}\psi \\ & +\tfrac{1}{2}ic_{\kappa\lambda}\bar\psi\gamma^\kappa \overset{\leftrightarrow}{D^\lambda}\,\psi + \tfrac{1}{2}i\,d_{\kappa\lambda}\,\bar\psi\gamma_5\gamma^\kappa\,\overset{\leftrightarrow}{D^\lambda}\,\psi \quad .\end{aligned} \tag{2}$$

All terms violate Lorentz symmetry whereas CPT is only broken by the terms involving a_κ and b_κ. The additional terms are expected to be very small, based on established experimental bounds on CPT violation for other particles like $K^0 - \bar{K}^0$ [5] or the electron. [6] From Eq. (2) the correction to ω_a can be calculated. [4] The result is

$$\delta\omega_a^{\mu^\pm} = 2\check{b}_Z^{\mu^\pm}\cos\chi + 2(\check{b}_X^{\mu^\pm}\cos\Omega t + \check{b}_Y^{\mu^\pm}\sin\Omega t)\sin\chi \tag{3}$$

where

$$\check{b}_J^{\mu^\pm} \equiv \pm\frac{b_J}{\gamma} + m_\mu d_{J0} + \frac{1}{2}\varepsilon_{JKL}H_{KL} \quad . \tag{4}$$

Here, the model parameters are expressed in terms of a non-rotating celestial frame of reference whose Z-direction is oriented along the Earth's rotational axis. The angle χ is the geographic colatitude at the experiment, i.e. the angle between the vertical direction of the laboratory frame and the Earth's axis. Due to the rotation of the Earth, $\delta\omega_a^{\mu^\pm}$ has a component oscillating with the sidereal frequency $\Omega \equiv 2\pi/23\text{h}56\text{min}$. Eq. (3) predicts two experimental signatures of CPT/Lorentz violation, discussed below.

2.1 Sidereal time dependence of $\omega_a^{\mu^+}$ and $\omega_a^{\mu^-}$

A g-2 experiment measuring ω_a for μ^+ or μ^- over a period of several days is sensitive to the oscillating terms in Eq. (3). From the oscillation amplitude $\hat{\omega}_a^{\mu^+}$ or $\hat{\omega}_a^{\mu^-}$ a combination of model parameters can be extracted:

$$\sqrt{(\check{b}_X^{\mu^\pm})^2 + (\check{b}_Y^{\mu^\pm})^2} = \frac{\hat{\omega}_a^{\mu^\pm}}{2\,|\sin\chi|} \tag{5}$$

Since ω_a is proportional to the magnetic field B, the test has to be performed at constant B.

The parameter combination probed with this test is very similar to the one determined from muonium spectroscopy.[7] The only difference is that for muonium the quantities $\check{b}_J^{\mu^\pm}$, defined in Eq. (4), are replaced by their non-relativistic limits with $\gamma = 1$.[4] Therefore the sensitivity of g-2 tests to the basic parameter components b_X and b_Y is suppressed by the factor $\gamma = 29.3$ with respect to muonium, for a given sensitivity to $\check{b}_X^{\mu^\pm}$ and $\check{b}_Y^{\mu^\pm}$.

An analysis procedure for sidereal variations is currently being developed using the μ^+ data of 1999 whose uncertainty for the time average of ω_a is $\sigma(\langle\omega_a\rangle) = 1.3\,\text{ppm}$. The error on the sidereal oscillation amplitude $\hat{\omega}$ is determined analytically from the second derivatives of χ^2 for the fit function (11) discussed in Section 3. A simple calculation yields $\sigma(\hat{\omega}_a) \approx \sqrt{2}\sigma(\langle\omega_a\rangle) = 1.8\,\text{ppm}$ for the 1999 data. This leads to an achievable uncertainty for $\sqrt{(\check{b}_X^{\mu^\pm})^2 + (\check{b}_Y^{\mu^\pm})^2}$ of about 1.2×10^{-24} GeV. Taking into account the above-mentioned suppression by $\gamma = 29.3$, the parameters b_X and b_Y can be probed with a sensitivity of the order 3.5×10^{-23} GeV, whereas muonium tests achieve a limit of 2×10^{-23} GeV.

To quantify the level of CPT/Lorentz violating effects, the dimensionless figure of merit $r_{\hat{\omega}_a} \equiv \frac{\hat{\omega}_a}{m_\mu}$ was introduced[4], interpreting $\delta\omega_a$ as an energy shift of the muon and comparing its amplitude with the rest energy m_μ. With the 1999 data set $r_{\hat{\omega}_a}$ can be probed down to a level of $0.19\times10^{-22} \ll \frac{m_\mu}{m_{\text{Planck}}}$, implying sensitivity to physics beyond the Planck scale. With the total data set for μ^+ the sensitivity will be significantly better since in 2000 approximately four times more data were recorded. The final statistics for μ^- are expected to be similar.

2.2 *Comparison of ω_a for positive and negative muons*

From Eq. (3) follows that two experiments at the same colatitude χ and at the same magnetic field will observe the time-averaged ω_a-difference

$$\Delta\omega_a \equiv \langle\omega_a^{\mu^+}\rangle - \langle\omega_a^{\mu^-}\rangle = 4\frac{b_Z}{\gamma}\cos\chi \quad . \tag{6}$$

Thus the measurement of $\Delta\omega_a$ sets a bound on b_Z and is therefore complementary to the measurement of sidereal variations which involve the parameters b_X and b_Y. The CERN g-2 experiment [8] which measured both $\omega_a^{\mu^+}$ and $\omega_a^{\mu^-}$ with an uncertainty of 10 ppm, obtained $\Delta\omega_a = 2\pi(-5.5\pm3.3)$ Hz, corresponding to $b_Z = (-2.3 \pm 1.4) \times 10^{-22}$ GeV, or a figure of merit [4] $r_{\Delta\omega_a} \equiv \frac{\Delta\omega_a}{m_\mu} = (-2.2 \pm 1.3) \times 10^{-22}$. With the final statistics of the g-2 experiment at BNL the sensitivity is expected to improve by roughly a factor 15.

One can also compare $\omega_a^{\mu^+}$ from BNL with $\omega_a^{\mu^-}$ from CERN. However, one needs to consider that the two experiments are situated at different geographic colatitudes χ_1 and χ_2 and that the magnetic fields B_1 and B_2 were slightly different. Using Eq. (3), the measured frequencies can be written as

$$\langle\omega_a^{\mu^+}\rangle = \frac{e}{m}aB_1 + \langle\delta\omega_a^{\mu^+}\rangle \tag{7}$$

$$\langle\omega_a^{\mu^-}\rangle = \frac{e}{m}aB_2 + \langle\delta\omega_a^{\mu^-}\rangle \tag{8}$$

To cancel the B-field dependent terms, we scale the CERN result $\langle\omega_a^{\mu^-}\rangle$ by $\frac{B_1}{B_2}$ and subtract it from $\langle\omega_a^{\mu^+}\rangle$:

$$\begin{aligned}\Delta\omega_a &= \langle\omega_a^{\mu^+}\rangle - \frac{B_1}{B_2}\langle\omega_a^{\mu^-}\rangle \\ &= 2\frac{b_Z}{\gamma}\left(\cos\chi_1 + \frac{B_1}{B_2}\cos\chi_2\right) + 2\left(m_\mu d_{Z0} + H_{XY}\right)\left(\cos\chi_1 - \frac{B_1}{B_2}\cos\chi_2\right)\end{aligned} \tag{9}$$

The usefulness of such a test is its sensitivity to a parameter combination involving not only b_Z but also d_{Z0} and H_{XY}: From the 1999 result [2] for $\omega_a^{\mu^+}$ and the CERN value [8] for $\omega_a^{\mu^-}$ we obtain $b_Z + \gamma\frac{\cos\chi_1 - \frac{B_1}{B_2}\cos\chi_2}{\cos\chi_1 + \frac{B_1}{B_2}\cos\chi_2}(m\, d_{z0} + H_{xy}) = (-1.4 \pm 1.0) \times 10^{-22}$ GeV, where $\chi_1(\text{BNL}) = 49.1^\circ$, $\chi_2(\text{CERN}) = 43.8^\circ$ and $\frac{B_1}{B_2} = 0.984$. This corresponds to $r_{\Delta\omega_a} = (-1.3 \pm 0.9) \times 10^{-22}$.

3 Proposed Analysis Procedure for Sidereal Variations of $\omega_\mathbf{a}$

At present a search for sidereal variations of ω_a in the data set of 1999 for positive muons is underway. These data with a statistical uncertainty of 1.25 ppm in ω_a were collected in 806 runs distributed over 25 days. The typical length of a run was half an hour. While in the g-2 analysis [2] the decay positron time spectra of all runs were summed before the fit, the search for time variations in ω_a requires analyzing each run individually. After applying a correction for pulses arriving with a time separation smaller than the resolution of the pulse-finding algorithm, the g-2 analysis fits the time spectra with the function

$$N(t) = N_0 \, \mathrm{e}^{-\frac{t}{\tau}} [1 + A\cos(\omega_a t + \phi)] \cdot f_{\mathrm{CBO}}(t) \cdot f_{\mathrm{ml}}(t) \tag{10}$$

The data of single runs are only sensitive to the exponential decay modulated by the g-2 oscillation. The parameters pertaining to perturbations due to coherent betatron oscillations, $f_{\mathrm{CBO}}(t)$, and muon losses, $f_{\mathrm{ml}}(t)$, are fixed to the values found in the g-2 analysis. This is a valid approach as long as $f_{\mathrm{CBO}}(t)$ and $f_{\mathrm{ml}}(t)$ are not affected by sidereal variations from CPT/Lorentz violating effects.

The values $\omega_{a\,i}$ determined for each run at a time t_i are fitted with the function

$$\omega_a(t_i, \omega_{p\,i}) = K\,\omega_{p\,i} + \hat{\omega}_a \cos(\Omega t_i + \Phi) \tag{11}$$

taking into account the magnetic field monitored with NMR probes in terms of the free proton precession frequency ω_p. K, $\hat{\omega}$ and Φ are free parameters whereas the sidereal frequency Ω is fixed.

Including the magnetic field into the fit is necessary because its variations of up to 0.5 ppm cause variations in ω_a of equal relative size. Particularly dangerous are day-night field variations caused by temperature changes which could fake a sidereal oscillation.

Another systematic concern involves possible effects of CPT/Lorentz violation on the time and frequency references used in the experiment. They could exhibit their own sidereal variations and hence either mask the variation for muons or introduce false signatures. A sidereal change of ω_p – the basis of the NMR field measurement – could counteract with the muon-related change in ω_a. However, atomic clock comparisons [9] provide an upper bound on the shift of ω_p on the mHz level, well below a ppb of the measured frequency. Finally, the Loran-C frequency standard used for the timing of the positron pulse readout and for the NMR measurement is based on Cesium hyperfine transitions with $m_F = 0$, which are insensitive to any preferential direction in space and will therefore not induce any sidereal variations.

4 Conclusions

Tests based on muon g-2 experiments are sensitive to the muon sector of the CPT/Lorentz violating extension (2) of the Standard Model. The search for sidereal variations of ω_a and the comparison of ω_a for μ^+ and μ^- probe complementary parts of the parameter space of the extension. An analysis searching for sidereal variations of ω_a is currently being done.

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