QUASILINEAR AND NONLINEAR EVOLUTION OF OPTIMAL INITIAL PERTURBATIONS IN BAROTROPIC FLOW

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1. INTRODUCTION.

Farrell (1988) and others have investigated the linear optimal initial value problem for idealized atmospheric flows. Many of the resulting structures are strongly localized, leading us to wonder whether strong local interactions can limit the global growth of the initial wave to values significantly less than linear theory would predict. We choose some rapidly growing initial conditions from Farrell (1988) and follow their evolution in quasilinear and nonlinear models for a variety of initial amplitudes. Departures from linear evolution are due to local depletion of mean-flow energy and to changes in the wave's propagation which inhibit global energy growth. These effects are greatly enhanced by increased localization of the initial condition vorticity, by the presence of boundaries, and by wave-wave interaction. Though the focus of this study is on the growth of these perturbations, it is worth noting that in none of the cases we considered did the wave undergo equilibration of the type commonly seen in nonlinear treatments of unstable normal modes.

2. MODEL AND INITIAL CONDITIONS.

A barotropic beta-plane channel model is used in linear, quasilinear, and nonlinear versions to follow the development of three types of initial conditions derived in Farrell (1988) for a range of initial amplitudes. These initial conditions, shown in figure 1, consist of a single wave in the x (zonal) direction and the following structures in the y (meridional) direction:

- E3: The third energy-norm adjoint mode (the optimal projection onto the normal mode with the third highest phase speed). This mode shows the greatest transient energy growth of the three adjoint modes that exist for this problem.
- EG8: The perturbation with greatest energy growth over 8 nondimensional time units.
- EG2: The same as EG8, except optimization is over 2 time units.

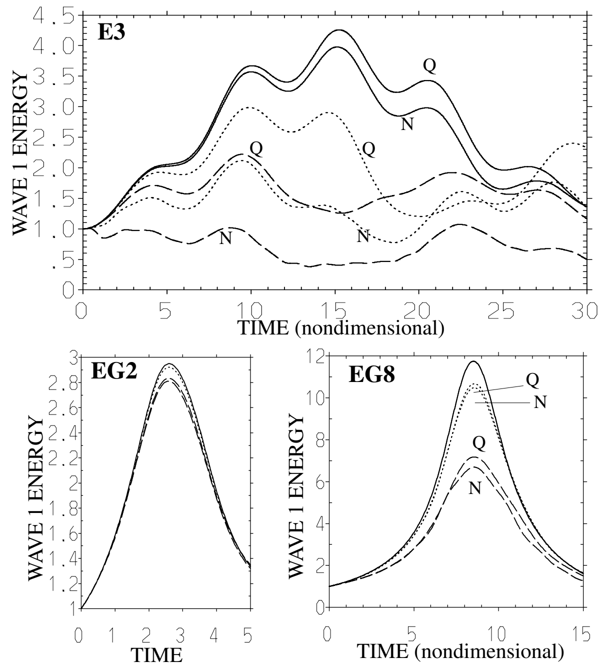

Figure 2. Normalized wave 1 energy growth defined as E´(t)/E´(0). Quasilinear (Q) and nonlinear (N) runs are shown for initial wave amplitudes of .01 (solid), .05 (dotted), and .1 (dashed), corresponding to initial wave energies of .0001, .0025 and .01 respectively. In the EG2 case several of the curves overlap, and only the highest amplitude nonlinear curve is shown. By comparison, the mean flow energy is initially equal to .3333.

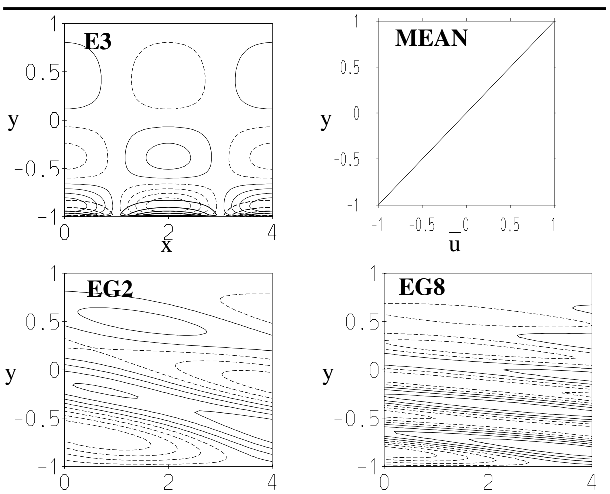

Figure 1. The initial condition perturbation streamfunctions and the mean flow profile as described in the text. The E3 plot also includes contours of vorticity (which is strongly concentrated near the southern boundary).

The wave streamfunction is set to zero at the north and south boundaries and is periodic in x. The basic state is linear shear (Couette) flow. The linear normal modes of this problem are all neutrally stable.

The model geometry and nondimensionalization are as in Farrell (1988) and correspond to planetary scale dynamics. Time is nondimensionalized by the mean-flow shear. The model is semi-spectral with a grid spacing of .01 in y. In the quasilinear runs the zonal mean flow is allowed to change due to the self-interaction of wave 1. In the nonlinear model, 16 waves are kept in the x-direction. A numerical smoother in the form of biharmonic diffusion is applied to points in the interior of the flow. The damping time scale at the smallest spatial scales resolved by the model in each direction is .25 time units except in the high amplitude case EG8 case, where the diffusion was increased by a factor of 10. This was necessary to maintain numerical stability in the decay phase (t > 8); results prior to this time are generally similar to those for the standard diffusion.

3. LINEAR GROWTH

A wave tilted against the mean flow shear can gain energy from the mean flow according to:

$$\frac{d}{dt}E' = -\int_A \overline{u'v'}\,\bar{u}_y \qquad \text{(EQ 1)}$$

Primes denote wave quantities, and an overbar denotes the zonal mean. The shear causes the wave phase tilt to rotate until it eventually tilts with the shear, after which the wave loses energy to the mean flow. For the shear in Figure 1, a negatively tilted Rossby wave packet will propagate northward as it gains energy, be tilted upright at its turning point (for the EG cases this occurs near the northern boundary), and then propagate southward to a stagnation level as it loses energy (Tung, 1983).

The E3 initial condition can be thought of as a superposition of northward and southward propagating waves with equal and opposite phase tilts, for which a similar process occurs (Boyd, 1983). The presence of boundaries is essential for the existence of normal modes. If a northward propagating, growing wave reaches the northern boundary before it reaches its turning point, it will be reflected as a southward propagating, decaying wave. Thus the wave energy growth is not monotonic in this case as there are short periods in which the southward-propagating wave will lose energy at rates comparable to the energy gain of the northward-propagating waves. Conversely, a southward wave hitting the southern boundary before its stagnation level will reflect as a northward, growing wave.

4. GROWTH-LIMITING MECHANISMS

Because the optimization is done in terms of global eddy energy growth we use that quantity as the primary basis for comparing the runs; results are displayed in figure 2. The quasilinear runs showed decreased eddy energy growth compared to the linear solution, particularly in the case of the adjoint mode initial condition. The nonlinear runs closely parallel the quasilinear runs with a few important exceptions to be noted later.

As the fluid is stirred, the mean-flow velocity gradient is reduced and in some cases even reversed. This locally limits the mechanism by which waves extract energy from the mean flow, as written in equation 1. This limitation can be significant if the mean flow shear is reduced in a region of high wave amplitude–right where the wave could otherwise best take advantage of the shear. This local effect is demonstrated in the E3 case depicted in figure 3, particularly between t = 5 and t = 10. To take advantage of the mean flow energy in the entire domain, however, the localized wave enstrophy must be able to propagate northward.

Wave energy growth can also be interpreted as a conversion of wave enstrophy to wave energy, since

$$\int_A \frac{dE'}{dt} + \int_A \frac{\bar{u}}{\bar{q}_y}\frac{dQ'}{dt} = 0. \qquad \text{(EQ 2)}$$

,where Q´ is the eddy enstrophy and $\bar{q}_y$ the zonal mean

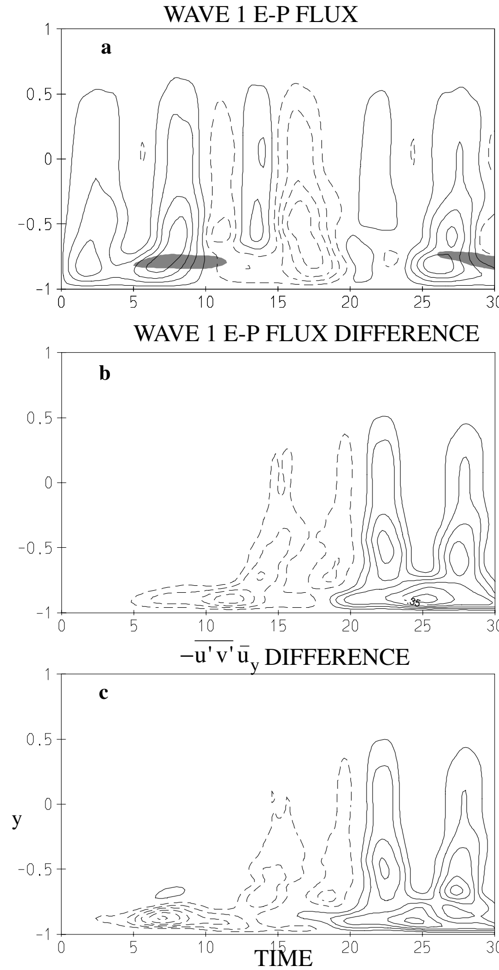

Figure 3. Latitude-time plots for case E3 with amplitude .05. (a) Quasilinear wave 1 Eliassen-Palm (EP) flux defined as $-\overline{u'v'}$. Shading depicts regions where the zonal mean absolute vorticity gradient is less than half the planetary vorticity gradient. (b) Quasilinear EP flux – linear EP flux. (c) Quasilinear $(-\overline{u'v'}\bar{u}_y)$ – linear $(-\overline{u'v'}\bar{u}_y)$. The contour interval is approximately .0007 in all the plots.

absolute vorticity gradient. Wave energy will increase if wave activity is redistributed from regions of low $\bar{u}$ to regions of higher $\bar{u}$. Since the group velocity of a Rossby wave packet is proportional to the mean vorticity gradient, $\bar{q}_y$, a reduction in $\bar{q}_y$ can reduce the speed at which wave activity moves northward. Taken to the extreme, the vorticity gradient can change sign leading to a region which blocks wave propagation and leads to a “trapping” of the wave. This appears to happen in the quasilinear E3 case for amplitude .1 (not shown).The resulting feature is suggestive of a reflecting critical line. This self-limiting mechanism seems to be a consequence of the strength and localization of the initial perturbation enstrophy.

The local reduction in shear mentioned above can also affect the global redistribution of enstrophy by slowing the northward propagation of strongly tilted waves away from their initial location, in effect reducing the localized wave “source”.

The presence of boundaries strongly affects the development of the E3 cases. This is only natural because the set-up of a normal mode involves multiple reflection off both boundaries. In this sense, this optimization problem is the least realistic. However, adjoint modes on a sphere for realistic flows can also show regions of concentrated enstrophy as in Branstator (1983). The wave-mean-flow interaction interferes with the balance required to set up the normal mode. The plot of the difference between the quasilinear and linear runs for amplitude .05 shows how the northward wave propagation is diminished, first near the southern boundary beginning at t = 5 and then extending northward into the fluid after t = 12. The similarity of figures 3b and 3c indicates that the reduction in energy growth in the quasilinear case is primarily due to this change in wave propagation. In fact, in the amplitude .1 run the southward EP flux eventually extends throughout the domain at t = 10 and reaches the southern boundary. This produces such strong wave decay that the peak wave energy is reached much earlier than in the linear run.

Although it has local velocity gradient changes which are as great as those in the E3 case, the EG8 case (figure 4) experiences less diminishment of energy growth as its initial amplitude is increased. As the wave extracts energy from the mean flow, it creates "stripes" of stronger and weaker shear. These stripes in turn are regions of differential rotation of the wave's phase tilt, which affects the wave's northward propagation and energy generation. In the growth phase, weak shear regions roughly coincide with relatively larger EP flux, and strong shear regions with relatively weaker EP flux, so that less eddy energy is gained. EG2 is less affected by increased initial amplitude than EG8, because it is initially less concentrated, and has a much shorter time to change the mean flow and/or interact with higher wavenumbers.

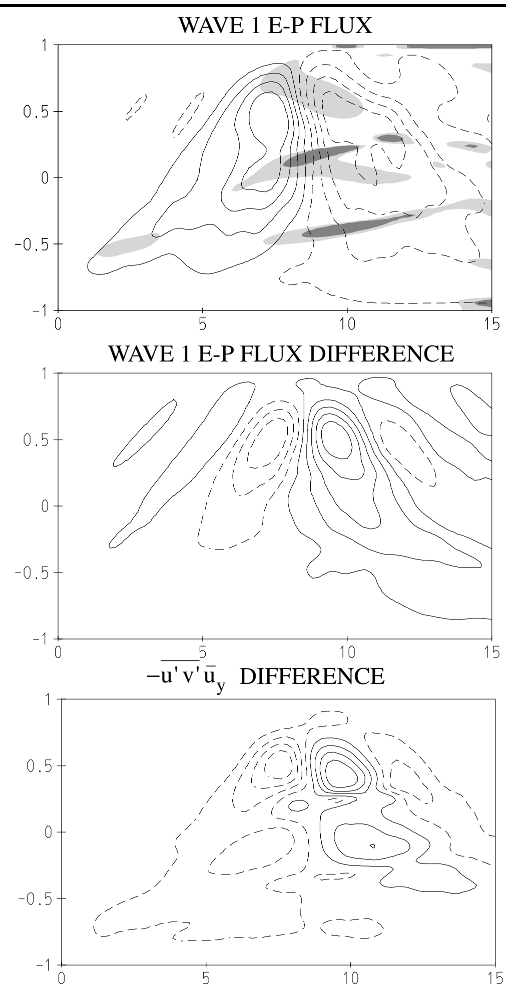

Figure 4. Same as Figure 3, except for the quasilinear EG8 case with amplitude .1. In (a), regions of $\bar{q}_y$ less than 75% (light shading) and 50% (dark shading) of the planetary vorticity gradient are noted.

The primary effect of wave-wave interaction in the nonlinear runs is to produce a rapid decrease in wave 1 enstrophy in regions where it is concentrated and at high amplitudes. This is mostly transferred to wave 2, which propagates and grows. Because the mechanism for wave 1 energy growth is still primarily the conversion of wave 1 activity into energy (that is, equation 2 is still approximately true for the initial stages of wave 1 development), the nonlinear loss of enstrophy results in less wave 1 energy growth. In fact, we could obtain similar results for wave 1 if we ran the quasilinear model with linear damping of the appropriate time scale. Despite the energy growth in higher wavenumbers, the total wave energy in the nonlinear case is still smaller than in either the linear or quasilinear cases.

5. SUMMARY

The adjoint mode initial condition has a concentration of enstrophy located near the southern boundary which, when it propagates away from the boundary, causes large local changes in the mean flow. The time evolution of adjoint mode cases involves reflections off the north and south boundaries. Changes in the mean flow can alter the balance between northward and southward wave propagation and reduce the projection onto the normal mode. In the quasilinear and nonlinear runs, regions of low vorticity gradient help confine the enstrophy to a region of low shear, inhibiting global growth. In the nonlinear runs transfer of enstrophy to higher zonal harmonics can drastically reduce the growth of the primary wave. In addition, the growth of the higher zonal harmonics can somewhat compensate for the loss of the primary wave growth.

For a given total initial eddy energy and enstrophy the finite-time growth initial conditions tend to be less sensitive to quasilinear and nonlinear effects than the adjoint modes, and behave much like Tung's(1983) unbounded-shear wave packets.

6. ACKNOWLEDGEMENTS.

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